Wavelet Leader multifractal analysis for texture classification

Conference Paper *in* Proceedings / ICIP ... International Conference on Image Processing · November 2009

DOI: 10.1109/ICIP.2009.5414273 · Source: DBLP

Herwig Wendt
University of Toulouse

Patrice Abry
Ecole normale supérieure de Lyon

Stéphane Jaffard
Université Paris-Est Créteil Val de Marne - Université Paris 12

Hui Ji
National University of Singapore

WAVELET LEADER MULTIFRACTAL ANALYSIS FOR TEXTURE CLASSIFICATION

Herwig Wendt, Patrice Abry, Stéphane Jaffard†*

Purdue Univ. - CNRS, ENS Lyon - Univ. Paris 12
Math. Dept. - Physics Dept. - Math. Dept.
Lafayette, USA - Lyon, France - Créteil, France

Hui Ji, Zuowei Shen

National University (NUS)
Math. Dept.
Singapore

ABSTRACT

Image classification often relies on texture characterization. Yet texture characterization has so far rarely been based on a true 2D multifractal analysis. Recently, a 2D wavelet Leader based multifractal formalism has been proposed. It allows to perform an accurate, complete and low computational and memory costs multifractal characterization of textures in images. This contribution describes the first application of such a formalism to a real large size (publicly available) image database, consisting of 25 classes of non traditional textures, with 40 high resolution images in each class. Multifractal attributes are estimated from each image and used as classification features within a standard k nearest neighbor classification procedure. The results reported here show that this Leader based multifractal analysis enables the effective discrimination of different textures, as performances in both classification scores and computational costs compare favorably against those of procedures previously proposed in the literature on the same database.

***Index Terms*—** Image Multifractal Analysis, Wavelet Leader, Texture Characterization, Image Classification.

1. INTRODUCTION

In Image processing, classification constitutes a standard task that can be based on image texture analysis. In an important number of research articles, e.g., [1, 2, 3, 4, 5], it is argued that texture characterization can be obtained by measuring the fluctuations of image amplitude regularity in space, and hence achieved by means of a multifractal analysis. In [4], Xu et al. proposed a texture descriptor, termed the multifractal spectrum vector (MFS) that aims at providing a viewpoint and illumination invariant characterization of image textures. However, so far multifractal analysis of images has rarely been conducted in a satisfactory manner: Either it remains incomplete (failing to explore entirely the multifractal spectrum, cf. Section 2), or images are analyzed as a set of independent 1D slices (or signals). This is mostly because until a recent past, there existed only one multifractal formalism for images (i.e., a practical procedure enabling to actually measure the multifractal properties of 2D data), based on the skeleton of a 2D Continuous Wavelet Transform (CWT): the Modulus Maxima Wavelet Transform (MMWT) [6]. This approach suffers from high computational costs, severe implementation difficulties, and still lacks theoretical foundations. Recently, a new multifractal formalism based on *wavelet Leaders* (WLMF) has been proposed [7, 8]: It is constructed from the coefficients of a 2D Discrete Wavelet Transform (DWT) and hence benefits from low computational costs and a simple implementation; It is backed up by a strong mathematical framework which shows that it enables accurate measurements of the multifractal properties of 2D fields, hence of images. This has been detailed in [7, 8]. The goal of this contribution is to illustrate the potential of this WLMF by showing it, for the first time, at work in a classification task conducted over a large size database of high resolution images. Estimates of multifractal attributes are used as features for a standard nearest neighbor classification procedure. The results reported here indicate that the WLMF enables the effective discrimination of intra-class textures, with robust invariance to inter-class textures, and compares favorably against those of previous attempts described in the literature, with much lower computational and memory costs.

*work conducted while in position at ENS Lyon.

†P. Abry and S. Jaffard acknowledge the support of IMS/NUS.

2. MULTIFRACTAL ANALYSIS

The analysis of the texture of the image $X(k_1, k_2)$ is conducted using the following multifractal formalism.

Wavelet Leaders are constructed from the wavelet coefficients, $D_X^{(m)}(j, k_1, k_2)$, $m = 1, 2, 3$, of a 2D orthonormal and separable DWT with finite response decomposition filters. The chosen mother wavelet possesses N_ψ vanishing moments. Readers are referred to, e.g., [9] for definitions and details. Wavelet coefficients are renormalized to a L^1-norm: $d_X^{(m)}(j, k_1, k_2) = 2^{-jd/2} D_X^{(m)}(j, k_1, k_2)$. *Wavelet Leaders* are defined as [7, 8]: $L_X(j, k_1, k_2) = \sup_{m=1,2,3,\, \lambda' \subset 3^2\lambda_{j,k_1,k_2}} |d_X^{(m)}(\lambda')|$, where $\lambda_{j,k_1,k_2} = \left\{[k_1 2^j, (k_1+1)2^j), [k_2 2^j, (k_2+1)2^j)\right\}$, and $3^2\lambda_{j,k_1,k_2} = \bigcup_{m,n=\{-1,0,1\}} \lambda_{j,k_1+m,k_2+n}$. They hence consist of the supremum of wavelet coefficients taken within a certain spatial neighborhood, and over all finer scales.

The wavelet Leader Multifractal Formalism is based on the *structure functions* $S(j,q) = \frac{1}{n_j}\sum_{k_1,k_2} L_X(j,k_1,k_2)^q$. From these $S(j,q)$, the *scaling function* is defined as $\zeta(q) = \liminf_{2^j\to 0} \ln S(2^j,q)/\ln 2^j$. Then, the Legendre transform of $\zeta(q)$ is taken, which defines the multifractal spectrum $\mathcal{L}(h) = \inf_{q\in\mathbb{R}}(d + qh - \zeta(q))$. This $\mathcal{L}(h)$ is deeply related to the *multifractal spectrum* $\mathcal{D}(h)$ of X, which characterizes image texture in terms of local regularity fluctuations and Hölder exponents h (cf. [7, 8] for details).

Log-cumulants consist of the coefficients c_p of the polynomial expansion $\zeta(q) = \sum_{p=1}^{\infty} c_p \frac{q^p}{p!}$. The first order $c_1, c_2, \ldots$ constitute valuable approximate summaries for both $\zeta(q)$ and $\mathcal{L}(h)$, satisfactory for most practical purposes.

Positive minimum regularity of X is a sufficient condition for the WLMF to yield a correct measure of $\mathcal{L}(h)$ from a given image. Minimum regularity can be measured as: $h_{min} = \liminf_{2^j\to 0} \ln \sup_{m,k_1,k_2} |d_X^{(m)}(j,k_1,k_2)|/\ln 2^j$. If $h_{min} < 0$, X needs to be fractionally integrated (of an order $\eta > -h_{min}$) prior to applying the WLMF. This equivalently amounts to replacing its $d_X^{(m)}(j,k_1,k_2)$ with $d_X^{(m),\eta}(j,k_1,k_2) = 2^{\eta j} d_X^{(m)}(j,k_1,k_2)$ (cf. [8]).

Estimations of $\zeta(q)$, $\mathcal{L}(h)$ and c_p can be performed by linear regressions in log-log plots, since the definition of $\zeta(q)$ above essentially implies $S(2^j,q) \sim \lambda_q 2^{j\zeta(q)}$, $2^j \to 0$. The computation of the Legendre transform yielding $\mathcal{L}(h)$ is often conducted in a parametric form $\mathcal{L}(q)$ and $h(q)$, with estimates also based on linear regressions in log-log plots. This is detailed in [8] and not recalled here for space reason.

Practical multifractal analysis aims at obtaining estimates of the multifractal attributes $\zeta(q)$, $\mathcal{L}(q)$, $h(q)$ and c_p from the image X: First, the power law behaviors of the structure functions $S(j,q)$ w.r.t. scales, hence straight lines in log-log plots, are validated. Then, the estimates are obtained by linear regressions in log-log coordinates. A key issue lies in the selection of the range of scales, $2^j \in [2^{j_1}, 2^{j_2}]$, over which to perform the regressions. To obtain estimates of the complete function $\mathcal{L}(h)$, structure functions have to be calculated for both positive and *negative* orders q. This is one of the major reasons why the WLMF must be used in place of previous formalisms based directly on wavelet coefficients, for which structure functions would be numerically unstable for negative q (cf. [8] for details).

3. IMAGE DATABASE AND MULTIFRACTAL

The image database analyzed here is publicly available at www.cfar.umd.edu/users/fer/website-texture/texture.htm (and referred to as the *UMD* dataset). It consists of 1000 digital 1280×960 pixel gray level images split into 25 different non-traditional natural texture classes, such as fruits, plants, floor textures or fabric (cf. Fig. 1 for samples). Each class contains 40 un-calibrated images, taken from different viewpoints and distances, and under varying illuminations.

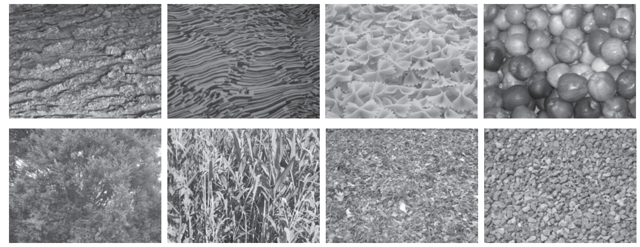

Fig. 1. **High resolution texture image database.** One example image out of the 40 samples per class for 8 out of the 25 classes of the UMD dataset: Cork, fabric, farfalle, apples, shrubbery, grass, fallen leaves, gravel.

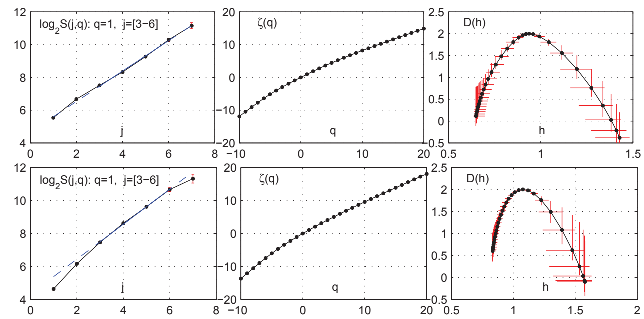

Fig. 2. **Multifractal estimation.** Wavelet Leader based estimates of structure functions $S(j,q)$ (left), scaling functions $\zeta(q)$ (center) and spectra $\mathcal{L}(h)$ (right) for example images in Fig. 1: "shrubbery" (top), "cork" (bottom).

Multifractal analysis is illustrated in Fig. 2 for two example images from the database. It shows wavelet Leader based structure functions and estimates of $\zeta(q)$, $\mathcal{L}(q)$ and $h(q)$ (obtained using a Daubechies mother wavelet with $N_\psi = 3$ vanishing moments, $[j_1, j_2] = [3, 6]$, and $\eta = 1$ as all images have $\hat{h}_{min} > -1$). Confidences intervals, existing on all plots but mostly visible on $D(h)$, are obtained for each image using a time-scale block bootstrap procedure applied to the wavelet Leaders (cf. [8] for details). The (log-log plots of the) structure functions (Fig. 2, left column) indicate that image textures exhibit power law behavior with scale 2^j over a large range of scales and hence do satisfy the multifractal paradigm. Also, the estimates of $\zeta(q)$, $\mathcal{L}(q)$ (Fig. 2, center and right column, respectively) possess concave shapes, which are classical for multifractal processes: The functions $\zeta(q)$ are clearly non-linear in q, and the estimated spectrum has support on a large range of values of h. These results clearly indicate that a multifractal description is relevant and fruitful for the characterization of texture images in this database. This motivates the use of WLMF attributes for texture image classification.

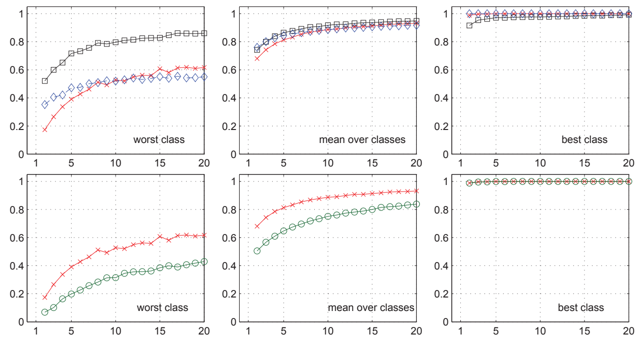

Fig. 3. **Classification results.** Mean estimated probabilities of correct classification for worst class (left) all classes (center) and best class (right) as a function of the number of training images T per class. Top row: Wavelet Leaders ('×'), MFS ('□'), HSLR ('◇'). Bottom row: Wavelet Leaders, 25 class ('×'), and 50 class ('○') database.

4. ANALYSES

The multifractal attribute feature vectors chosen here consist of the estimates of c_1, c_2 and $\zeta(q)/q$, $D(q)$, $h(q)$, for $q \in \{-4, -3, -2, -1, 1, 2, 3, 4\}$, plus the intercepts of the regression lines, obtained with Daubechies mother wavelet with $N_\psi = 3$. $\eta = 1$. As no method is available for the automatic selection of the scale regression range, various ranges of scales are used: $[j_1, j_2] \in \{[1, 3], [2, 4], [3, 5], [4, 6]\}$. This yields feature vectors of dimension 208.
A nearest neighbor classification procedure (k-NN) is used [10]. It is chosen for simplicity reasons as the focus of the present contribution is on the wavelet leader multifractal formalism potentials rather than on the derivation of original classification schemes. Following [4] against which we want to compare results, each class is split into a subset of T randomly chosen training images. All other images, called test images, remain to be classified.

5. RESULTS

Classification performances are evaluated after averaging results obtained from $R = 50$ randomly selected training samples (so as to avoid bias induced by the choice of a specific training sample). Various values of k have been experimented. Results are reported for $k = 1$ as it yields the best performances. Hence, the estimated class is that containing the training image whose feature vector has smallest distance to the feature vector of the test image. The mean estimated probabilities of correct classification are reported in Fig. 3 (top), as a function of T. It displays the mean estimated probabilities of correct classification for the best class, i.e., the class with largest mean estimated probability of correct classification amongst all classes (left), for mean estimated probability of correct classification for all classes (center) and for the worst class (right).
Performance comparisons can be obtained using the results reported in [4], on the same database: A box-agregation based multifractal formalism (MFS), combined with a support vector machine (SVM) based feature selection (*MFS+SVM*) is proposed and compared against two other state-of-art texture analysis procedures: *HLSR* (histogram orientation based on Harris and Laplacian operators, [11]) and the *VZ* (texton histogram based on pixel local joint distributions, [12]). The same k-NN classifier is used to evaluate the classification performances of the three texture features. The *VZ* approach is ruled out because of both its weak performances and high computational costs, [4, 11]. Comparing the results of the present contribution to those in [4] (cf. Fig. 3) shows that the *WLMF*, *MFS+SVM* and *HLSR* approaches show similar performances: They almost superimpose for the mean classification property (center); They are significantly in favor of *MFS+SVM* for the worst class; For the best class, though all approaches are doing well, interestingly, the *WLMF* score remains remarkably high and constant at small T. Also, it is essential to mention that these comparable performances are obtained with the *WLMF* having computational and memory costs of the order of those of a DWT ($O(n \log n)$, n being the number of pixels of the image), hence orders of magnitude lower than the two other approaches: The calculation of the high-dimensional multifractal attribute feature vectors takes roughly 10 seconds only per image. The processing (including classification) of the 1000 high-resolution images takes approximately a couple of hours on a standard PC (against a couple of days *M*FS+SVM and *H*LSR). These results constitute clear indications in favor of the use of wavelet Leader based multifractal analysis for texture classifications.
The feature vector for the WLMF is high-dimensional, and results are obtained without any specific feature selection or fine tuning, such as principal components analysis or SVM-based learning techniques. Hence, it potentially contains highly redundant attributes. Decreasing the intrinsic redundancy of the feature vector will further reduce the computational cost for classification. Also, some features may be numerically dominated by others and hence practically ineffective, though potentially theoretically discriminative. For example, the log-cumulants c_p of order $p \geq 2$ usually take on values that are relatively close to zero, as compared to other attributes, such as $D(q \approx 0) \simeq 2$. Exploring such normalization issues demands for further investigations and represents a large potential for future improvements of texture classifications based on multifractal analysis. This is beyond the scope of the present contribution, which concentrates on proposing the first quantifications of the WLMF texture classification performances obtained from a large size real world image database. Finally, the use of estimates obtained from other mother-wavelets (different N_ψ) does not significantly modify the classification results. This is a very satisfactory empirical conclusion, since it is theoretically proven that the

Leader based multifractal analysis does not depend on the choice of the mother wavelet, as soon as N_ψ is large enough (larger than the largest Hölder exponent that exists in the image) [7, 8]. Along the same line, combining multifractal attributes computed from different mother wavelets does not improve classification performance.

An extended dataset, with 50 classes of 40 images each, augmented from the first *UMD* dataset has also been easily processed. The results, reported in Fig. 3, second row, show that classification performances remain satisfactory. No comparisons against other methods is so far available.

6. CONCLUSIONS AND PERSPECTIVES

The results reported above lead to conclude that multifractal attribute estimates, as obtained by the WLMF, are highly relevant for the characterization of texture images. Used as features for classification, they give rise to effective image classification schemes whose performance compare favorably against those of schemes previously proposed in the literature.

Future analyses are required to lower feature vector dimensions and to select the most discriminant and relevant multifractal attributes. Moreover, the inclusion of the bootstrap based confidence intervals into the classification scheme is envisaged. Also, modern classification procedures such as support vector machines or other non linear machine learning concepts can be refined to further select optimal subsets of attributes or propose more advanced classification schemes. WLMF attributes can also be associated to more classical features. Such issues are under current investigations.

The WLMF (together with bootstrap confidences intervals) have been implemented by ourselves in MATLAB routines and documented. They are available upon request. Their low memory and time costs together with their satisfactory estimation performances pave the way toward a systematic application of multifractal analysis to possibly large size images of large databases, and therefore toward its use for image retrieval, computer vision or robotic purposes.

At a more conceptual level, it is worth noting that the multifractal attributes (scaling exponents, log-cumulants or Legendre spectrum) can be fruitfully used as relevant quantities for texture characterization, with no explicit interpretation in terms of Hölder exponents or multifractal spectrum. Yet, the theoretical and mathematical the multifractal framework remains the founding layer of the associated classification procedure.

At a higher semantic level, commonly referred to as scene recognition (i.e., discrimination of images of e.g., houses, landscapes, etc.), it could be interesting to validate whether multifractal analysis enables to perform scene classification. This is of importance in a large number of applications, such as automatized image retrieval and computer vision. Such a procedure may be considered by combining the feature vectors proposed here with the image function space and uniform regularity estimates calculated from wavelet coefficients [8].

7. REFERENCES

[1] A. Conci and L.H. Monteiro, "Multifractal characterization of texture-based segmentation," in *Proc. IEEE Int. Conf. Image Processing (ICIP)*, Vancouver, BC, Canada, 2000, vol. 1, pp. 792–795.

[2] L. M. Kaplan, "Extended fractal analysis for texture classification and segmentation," *IEEE Trans. Patt. Anal. Mach. Intell.*, vol. 8, no. 11, pp. 1572–1585, 1999.

[3] S. Peleg, J. Naor, R. Hartley, and D. Avnir, "Multiple resolution texture analysis and classification," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 6, no. 4, pp. 518–523, 1984.

[4] Y. Xu, J. Hui, and C. Fermüller, "Viewpoint invariant texture description using fractal analysis," *Int. J. Computer Vision*, 2009, To Appear.

[5] P. Chainais, "Multi-dimensional infinitely divisible cascades to model the statistics of natural images," in *Proc. IEEE Int. Conf. Image Processing (ICIP)*, 2005.

[6] A. Arneodo, N. Decoster, and S.G. Roux, "A wavelet-based method for multifractal image analysis. I. Methodology and test applications on isotropic and anisotropic random rough surfaces," *Eur. Phys. J. B*, vol. 15, no. 3, pp. 567–600, 2000.

[7] S. Jaffard, "Wavelet techniques in multifractal analysis," in *Fractal Geometry and Applications: A Jubilee of Benoît Mandelbrot, M. Lapidus and M. van Frankenhuijsen Eds., Proceedings of Symposia in Pure Mathematics*, M. Lapidus and M. van Frankenhuijsen, Eds. 2004, vol. 72(2), pp. 91–152, AMS.

[8] H. Wendt, S.G. Roux, P. Abry, and S. Jaffard., "Wavelet leaders and bootstrap for multifractal analysis of images," *Signal Process.*, vol. 89, pp. 1100–1114, 2009.

[9] S. Mallat, *A Wavelet Tour of Signal Processing*, Academic Press, San Diego, CA, 1998.

[10] E. Fix and J. Hodges, "Discriminatory analysis, nonparametric discrimination," Tech. Rep. 4, USAF School of Aviation Medicine, Randolph Field, February 1951.

[11] S. Lazebnik, C. Schmid, and J. Ponce, "A sparse texture representation using affine-invariant regions," *IEEE Trans. on Patt. Anal. Mach. Intell.*, vol. 8, no. 27, pp. 1265–1278, 2005.

[12] M. Varma and A. Zissermann, "Classifying images of materials: Achieving viewpoint and illumination independence," *ECCV*, vol. 3, pp. 255–271, 2002.