The Chisholm Paradox and the Situation Calculus

Robert Demolombe[1] and Pilar Pozos Parra[2]

[1] ONERA-Toulouse, 2 Avenue E. Belin BP 4025, 31055 Toulouse, France
Robert.Demolombe@cert.fr
[2] Department of Computing, Macquarie University, NSW 2109, Australia
pilar@ics.mq.edu.au

Abstract. Deontic logic is appropriate to model a wide variety of legal arguments, however this logic suffers form certain paradoxes of which the so-called Chisholm is one of the most notorious. We propose a formalisation of the Chisholm set in the framework of the situation calculus. We utilise this alternative to modal logic for formalising the obligations of the agent and avoiding the Chisholm paradox. This new approach makes use of the notion of obligation fluents together with their associated successor state axioms. Furthermore, some results about automated reasoning in the situation calculus can be applied in order to consider a tractable implementation.

1 Introduction

Deontic logic, the logic of obligations and permissions, is appropriate to model a wide variety of legal arguments, however this logic suffers form certain paradoxes of which the so-called Chisholm is one of the most notorious. The Chisholm paradox [1] comes from the formalisation of secondary obligations (they are also called "contrary-to-duty") that are obligations that come into force when primary obligations have been violated. Chisholm has proposed a set of sentences defining secondary obligations in natural language that are used to check whether a proposed formal logic can be applied to represent these sentences whithout leading to unexpected properties such as: inconsistency, redundancy, pragmatic oddity or others. Many logics have been proposed in the literature to solve the Chisholm paradox. A weakness of the approaches is their practical drawbacks. The purpose of this paper is to propose a simple representation of Chisholm sentences in the framework of situation calculus which avoids unexpected properties and can be used for practical applications.

The situation calculus offers formalisms for reasoning about actions and their effects on the world [3], on agent's mental states [4], and on the obligations [5]. These approaches propose a solution to the corresponding "frame problem", namely the problem of specifying what properties of the world, beliefs, goals, intentions or obligations remain unchanged after the execution of an action. In [3] the properties of the world that undergo change are represented by *fluents*, to solve the frame problem the *successor state axioms* were introduced. The formalisms proposed in [4, 5] consider the introduction of suitable new fluents such as *belief, goal* and *intention fluents* (or *obligation fluents*) that represent the mental states (or ideal worlds). Their successor state axioms are introduced in order to solve the corresponding frame problem in the mental states (or in ideal worlds). Although the scope of beliefs, goals, intentions and obligations in the proposals is limited to accept only literals, the method utilized for automated reasoning about ordinary fluent change [6] can easily be extended to consider cognitive and social change. We employ this proposal to represent the Chisholm sentences. By avoiding the use of modal logic, the proposal avoids the Chisholm paradox.

Section 2 briefly describes the situation calculus and its use in the representation of issues involving the evolution of the world and mental states. Section 3 presents the formalisation of obligations. In Section 4, we propose a representation of the Chisholm set that avoids the paradox. Section 5 considers an alternative representation in order to avoid the pragmatic oddity problem. We conclude with a discussion of some issues and future work.

2 Situation Calculus

2.1 Dynamic Worlds

The situation calculus involves three basic components: actions, situations and fluents. It is assumed that every change is caused by an action. Situations are sequences of actions which represent possible histories of the world. Fluents are world's properties (in general, relations) that may change. If s represents an arbitrary situation, and a an action, the result of performing a in s is a situation which is denoted by the term $do(a, s)$. The fluents are represented by predicates whose last argument is of type *situation*. For any fluent p and situation s, the expression $p(s)$ denotes the truth value of p in s. The evolution of a fluent p is represented by the successor state axiom of the form:[3]

$$(\mathbf{S_p}) \quad p(do(a, s)) \leftrightarrow \Upsilon_p^+(a, s) \vee (p(s) \wedge \neg\Upsilon_p^-(a, s))$$

where $\Upsilon_p^+(a, s)$ represents the exact conditions under which p turns from false to true when a is performed in s, and similarly $\Upsilon_p^-(a, s)$ represents the precise conditions under which p turns from true to false when a is performed in s. It is assumed that no action can turn p to be both true and false in a situation, i.e. $\neg\exists s\exists a(\Upsilon_p^+(a, s) \wedge \Upsilon_p^-(a, s))$.

2.2 Dynamic Mental States

Mental states are represented by cognitive (belief, goal or intention) fluents which are a syntactic combination of modal operators and standard fluents or its negations. We say that the cognitive fluent $\mathcal{M}_i p$ holds in situation s iff the attitude of i about p is positive in situation s and represent it as $\mathcal{M}_i p(s)$. Similarly $\mathcal{M}_i \neg p(s)$

[3] In what follows, it is assumed that all the free variables are universally quantified.

represents the fact that the fluent $\mathcal{M}_i \neg p$ holds in situation s: the attitude of i about $\neg p$ is positive in the situation s.[4]

In this case, the evolution of the mental state needs to be represented by two axioms, each allowing the representation of two attitudes out of four i's attitudes concerning the fluent p, namely $\mathcal{M}_i p(s)$, $\neg\mathcal{M}_i p(s)$, $\mathcal{M}_i \neg p(s)$ and $\neg\mathcal{M}_i \neg p(s)$. The corresponding successor state axioms for an agent i and a fluent p are of the form:

- $\mathcal{M}_i p(do(a,s)) \leftrightarrow \Upsilon^+_{\mathcal{M}_i p}(a,s) \vee (\mathcal{M}_i p(s) \wedge \neg\Upsilon^-_{\mathcal{M}_i p}(a,s))$
- $\mathcal{M}_i \neg p(do(a,s)) \leftrightarrow \Upsilon^+_{\mathcal{M}_i \neg p}(a,s) \vee (\mathcal{M}_i \neg p(s) \wedge \neg\Upsilon^-_{\mathcal{M}_i \neg p}(a,s))$

where $\Upsilon^+_{\mathcal{M}_i p}(a,s)$ are the precise conditions under which the attitude of i (with regard to the fact that p holds) changes from one of negative to positive (for example in the case of beliefs, it turns from one of disbelief to belief) when a is performed in s, and similarly $\Upsilon^-_{\mathcal{M}_i p}(a,s)$ are the precise conditions under which the state of i changes from one of positive to negative. The conditions $\Upsilon^+_{\mathcal{M}_i \neg p}(a,s)$ and $\Upsilon^-_{\mathcal{M}_i \neg p}(a,s)$ have a similar interpretation (with regard to the fact that p does not hold). Note that the axioms proposed to solve the frame problem in this context are similar to ($\mathbf{S_p}$). Moreover, to suppose knowing all the conditions representing the change in the mental states is an assumption more realistic than to suppose knowing all the conditions representing the change in the real world. As in the successor state axioms, some constraints must be imposed to prevent the derivation of inconsistent mental states [4].

3 Deontic Modalities

In the last section we outlined the approach that allows designing of cognitive agents (an example of application about planning can be found in [4]). However, in the development of multiagent systems not only the cognitive aspect is crucial, but also social issues such as norms play an important role. For example, in a planning application, the agent must include in her plan only the "permitted" actions. Since deontic modalities, which deal with obligations, permissions and prohibitions, consider ideal behaviour, the integration of such modalities into a cognitive model makes it possible to draw not only the distinction between real and mental entities, but also between real and ideal behaviours.

3.1 Dynamic Norms

As in the case of mental states, a deontic (obligation or prohibition) fluent is a syntactic combination of a modal operator and a standard fluent. We say that the *obligation fluent* Op holds in situation s iff the "law"[5] says that p must hold in

[4] We abuse of notation $\mathcal{M}_i p$ and $\mathcal{M}_i \neg p$ in order to have an easy identification of the agent and proposition. An "adequate" notation should be $\mathcal{M}ip$ and $\mathcal{M}inotp$.

[5] "Law" is the term utilised for denoting the normative entity.

the situation s and we represent it as $Op(s)$. Similarly $Fp(s)$ represents the fact that the *prohibition fluent* Fp holds in the situation s: the law says that p must not hold in the situation s. $O\neg p(s)$ (be obliged not) may be used to represent prohibition notion $Fp(s)$ (be forbidden). Furthermore, the relationship among obligation, permission and prohibition is considered in the *successor deontic state axioms*, specifically in the constraints that the Υ's must satisfy.

In this case, the four interpretations are: $Op(s)$, the law says that p must hold in situation s (p is obligatory); $\neg Op(s)$, the law says that it is not the case that p must hold in s (p is not obligatory); $Fp(s)$, the law says that p has not to hold in s (p is forbidden); and $\neg Fp(s)$, the law says that p can hold in s (p is permitted). Deontic fluents, unlike cognitive fluents, do not require the agent's identification since they are considered universals, i.e. every agent must obey the law. However, the parameterisation of agents allows the introduction of rules applied only to a subset of the community. For example: the people who are older that 18 years have right to vote, $\forall s, x, y(agent\_age(x, y, s) \wedge y >= 18 \rightarrow \neg Fvote(x, s))$.

The successor deontic state axioms for a fluent p are of the form:

$(\mathbf{S_{Op}})$ $Op(do(a, s)) \leftrightarrow \Upsilon^{+}_{Op}(a, s) \vee (Op(s) \wedge \neg\Upsilon^{-}_{Op}(a, s))$

$(\mathbf{S_{Fp}})$ $Fp(do(a, s)) \leftrightarrow \Upsilon^{+}_{O\neg p}(a, s) \vee (Fp(s) \wedge \neg\Upsilon^{-}_{O\neg p}(a, s))$

where $\Upsilon^{+}_{Op}(a, s)$ are the precise conditions under which p turns from not obligatory to obligatory (to oblige) when a is performed in s. Similarly $\Upsilon^{-}_{Op}(a, s)$ are the precise conditions under which p turns from obligatory to not obligatory (to waive). $\Upsilon^{+}_{O\neg p}(a, s)$ are the precise conditions under which p turns from permitted to forbidden (to forbid) when a is performed in s. Finally $\Upsilon^{-}_{O\neg p}(a, s)$ are the precise conditions under which p turns from forbidden to permitted (to permit) when a is performed in s. Note the resemblance between the successor deontic state axioms and $(\mathbf{S_p})$, hence for practical applications the method used to automated reasoning about real world can be easily adapted to deontic context.

A violated obligation is represented by $\neg p(s_1) \wedge Op(s_1)$: in s_1, p does not hold but in the same situation s_1, the law says that p must be. A violated prohibition is represented by $p(s_1) \wedge Fp(s_1)$: in s_1, p holds but in the same situation, the law says that p must not be.

3.2 Consistency Properties

As in successor state axioms, some constraints must be imposed to prevent the derivation of inconsistent norms.

To enforce consistent obligations, we need the following axiom:

- $\forall a \forall s \neg(\Upsilon^{+}_{Op} \wedge \Upsilon^{-}_{Op})$

To enforce consistent permissions (p cannot be permitted and forbidden at the same time), the following axiom is necessary:

- $\forall a \forall s \neg(\Upsilon^{+}_{O\neg p} \wedge \Upsilon^{-}_{O\neg p})$

To assure that p is not forbidden and obliged, simultaneously, we need an axiom of the form:

– $\forall a \forall s \neg(\Upsilon^{+}_{Op} \wedge \Upsilon^{+}_{O\neg p})$

Note that we can, for instance, have $\neg Op(S_1) \wedge \neg Fp(S_1)$ that means: p is not obligatory nor forbidden in S_1. A similar formula in the belief context represents the ignorance of the agent about p: $\neg Bp(S_1) \wedge \neg B\neg p(S_1)$. So such a formula represents the "passivity" of the law about p.

The model considers the obligation of type "ought to be", examples of actions changing deontic fluents are of type "to create" or "to abrogate" a law. In the next section we consider special conditions (violations of norms) that allow an obligation comes into force. Moreover, the formalisation of primary and secondary (contrary-to-duty or CTD) obligations is introduced.

4 Chisholm Paradox

Let's consider the Chisholm set that consists of four sentences of the following form:

1. p ought to be,
2. if p is, then q ought not to be,
3. if p is not, then q ought to be,
4. p is not.

The first sentence denotes a primary obligation: p should ideally be. The third one denotes a CTD obligation, i.e. an obligation about q comes into force when the obligation about p (the primary obligation) is violated. If the obligation about p is obeyed, the secondary obligation (or sanction) must not be effective (second sentence). The last sentence denotes the violation of the primary obligation. Let's consider the following example: let p be "to be a law-abiding or non-delinquent" and q "to be in prison". Intuitively we have: (1) the agent ought to be a non-delinquent, (2) if the agent is not a delinquent, the agent ought not to be in prison, (3) if the agent is a delinquent, the agent ought to be in prison, and (4) the agent is a delinquent. The set of statements is intuitively consistent and none of four sentences seems to be redundant. However, if we try to formalise the Chisholm set in the framework of the Standard Deontic Logic (SDL), the reconciliation between consistence and redundancy does not take place. A usual way to represent the sentences in SDL is the following:

1′. $O(p)$,
2′. $O(p \rightarrow \neg q)$,
3′. $\neg p \rightarrow O(q)$,
4′. $\neg p$.

Using the properties of the system KD, which are satisfied by SDL, we can infer the paradox, i.e. we can conclude that there are simultaneously an obligation to be in prison and an obligation not to be in prison. Furthermore, it is questionable that $2'$ and $3'$ are represented in a different form. However, if we replace for instance $3'$ by

$3''$. $O(\neg p \rightarrow q)$,

we can deduce $3''$ from $1'$ and the axiom $O(A) \rightarrow O(A \vee B)$. Similarly, if we replace $2'$ by

$2''$. $p \rightarrow O(\neg q)$,

the sentence $2''$ can be deduced from $4'$. Both substitutions lead to redundancy in the set of sentences. In the SDL representation, contrary to intuition, everything is formalized in a static perspective. So the difference between the rules (first three sentences) that must be obeyed "all the time" and the facts (last sentence) that hold "sometimes" cannot be represented. Using deontic fluents, the sentences are represented as follows:[6]

C_1. $\forall s\ Op(s)$,
C_2. $\forall s\ (p(s) \rightarrow Fq(s))$,
C_3. $\forall s\ (\neg p(s) \rightarrow Oq(s))$,
C_4. $\neg p(S_1)$,

where S_1 denotes a situation in which p does not hold. In this context it denotes a situation in which the agent is a delinquent. The definition of the evolution of deontic fluents and their initial settings must satisfy the constraints $C_1 - C_3$. Without loss of generality, we consider only one agent. To create the successor deontic state axioms, let's simplify the problem and consider solely static ideal worlds, i.e. the primary and CTD obligations are considered fixes in the sense that there is not an abrogation of the rules. However, the real world may conserve its dynamism, and certainly the CTD obligations change whenever p changes. The representation of deontic fluents evolution is of the form:

A_1. $Op(do(a, s)) \leftrightarrow \top$
A_2. $Fq(do(a, s)) \leftrightarrow \Upsilon_p^+(a, s) \vee (Fq(s) \wedge \neg\Upsilon_p^-(a, s))$
A_3. $Oq(do(a, s)) \leftrightarrow \Upsilon_p^-(a, s) \vee (Oq(s) \wedge \neg\Upsilon_p^+(a, s))$

where the successor state axiom of p is of the form ($\mathbf{S_p}$) (see Section 2.1). To define the initial situation we consider two cases: (i) when $S_1 = S_0$, i.e. initially the primary obligation is been violated, the initial settings must be of the form: $\neg p(S_0)$, $Op(S_0)$, $\neg Fq(S_0)$ and $Oq(S_0)$; (ii) when initially the primary obligation is obeyed, the initial settings must be of the form: $p(S_0)$, $Op(S_0)$, $Fq(S_0)$ and $\neg Oq(S_0)$.

[6] Note the similarity of the first sentences with the *state constraints*. They also describe global properties that must hold in all situations. The difference is that the obligations take effect over the idealised world while the state constraints take effect over the real world.

Following the example, suppose that the agent becomes a delinquent if she executes some of the following actions: *steal*, *kidnap*, *kill* or *rape*; and she becomes a non-delinquent after the end of her conviction (action *free*). So the successor state axiom of p is of the form:

$$p(do(a,s)) \leftrightarrow a = free \vee p(s) \wedge \neg(a = steal \vee a = kidnap \vee a = kill \vee a = rape)$$

Suppose that the agent is in prison if the police imprisons her (action *imprison*) and she is not in prison after she is freed (action *free*). So the successor state axiom of q is of the form:

$$q(do(a,s)) \leftrightarrow a = imprison \vee q(s) \wedge \neg(a = free)$$

We obtain the following successor deontic state axioms:

$$Op(do(a,s)) \leftrightarrow \top$$
$$Fq(do(a,s)) \leftrightarrow a = free \vee Fq(s) \wedge \neg(a = steal \vee a = kidnap \vee a = kill \vee a = rape)$$
$$Oq(do(a,s)) \leftrightarrow a = steal \vee a = kidnap \vee a = kill \vee a = rape \vee Oq(s) \wedge \neg(a = free)$$

In the following, the compact notation $do([a_1, a_2, \ldots, a_n], s)$ represents $do(a_n, \ldots, do(a_2, do(a_1, s)) \ldots)$ when $n > 0$ and s when $n = 0$.

Now, suppose the following facts: initially the agent was not a delinquent, $p(S_0)$. She bought a house, but it was so expensive that she had a loan from the bank. The rates were so high that she began to despair and stole from the bank. So the agent becomes finally a delinquent, $\neg p(do([buy, borrow, despair, steal], S_0))$. In S_0, the obligation is not been violated, so the initial settings are: the agent ought to be a non-delinquent and not to be in prison $Op(S_0)$, $\neg Oq(S_0)$; and it is forbidden to be in prison $Fq(S_0)$. From the successor deontic state axioms we can, for example, deduce $Op(do(buy, S_0))$, $\neg Oq(do[buy, borrow], S_0))$, $Fq(do([buy, borrow, despair], S_0))$, which mean: the agent ought to be a non-delinquent after buying her house, it is not obligatory that she be in prison after she has a loan from the bank, and it is not permitted to be in prison after feeling despair, respectively. What happens after she steals from the bank? In $S_1 = do([buy, borrow, despair, steal], S_0)$, we have: $\neg p(S_1)$, $Op(S_1)$, $Oq(S_1)$ and $\neg Fq(S_1)$, i.e. the agent becomes a delinquent, she ought to be a non-delinquent, she ought to be in prison and it is not forbidden to be in prison, respectively.

5 Pragmatic Oddity

Note that under the proposed representation $A_1 - A_3, C_4$, we can deduce $Op(S_1)$ and $Oq(S_1)$. In S_1, the agent must be a non-delinquent and must be in prison, i.e. in the ideal world, in S_1, the agent is non-delinquent and is in prison! This kind of problem has been called "pragmatic oddity". To solve the problem, we will distinguish the primary and CTD obligations, just as Carmo and Jones proposed in [7]. Thus, the timeless obligations, such as the primary obligations, are called *ideal obligations* (the set of all ideal obligations defines the ideal world) and the circumstance dependent obligations, such as CTD obligations, are called *situation dependent obligations* (the set of all situation dependent obligations about

s defines the obligations in s).[7] So if $p(s)$ is a fluent, the ideal obligation about p is represented by the deontic predicate symbols Op which means that, in the ideal world, p must hold forever. The situation dependent obligation about p is represented by the deontic fluent $Op(s)$, meaning p must hold in the situation s. Clearly, the simultaneous introduction of an ideal and situation dependent obligation about p, is not allowed. Note that only the situation dependent obligations need to be revised. Therefore, the proposed representation of the Chisholm set suffers a small modification in order to avoid the pragmatic oddity problem. The successor deontic state axiom A_1, as well as the initial setting about p, are replaced by the situation independent predicate Op. The representation of first sentence is of the form:

A_1'. Op

So the set A_1', A_2, A_3 and C_4 can be utilised for representing the Chisholm set, which reconcile consistency and non-redundancy. Moreover, it avoids the pragmatic oddity problem since $Op \wedge Oq(S_1)$ is interpreted as: the agent must ideally be a non-delinquent but in S_1 she is obliged to be in prison.

The representation satisfies all requirements, proposed by Carmo and Jones in [2], that an adequate formalisation of the Chisholm set should met, i.e.

a. consistency,
b. logical independence of the members,
c. applicability to timeless and actionless CTD-examples,
d. analogous logical structures for the two conditional sentences (2) and (3),
e. capacity to derive actual obligations (in our case situation dependent obligations),
f. capacity to derive ideal obligations,
g. capacity to represent the fact that a violation of an obligation has occurred,
f. capacity to avoid pragmatic oddity.

The approach proposed in [7] also seems to consider these requirements. However, the pragmatic oddity problem reappears if we add a second level of CTD obligations, such as:

5. if p is not and q is not, then r ought to be,
6. q is not,

which is represented by the following constraints:

C_5. $\forall s\ (\neg p(s) \wedge \neg q(s) \rightarrow Or(s))$,
C_6. $\neg q(S_1)$.

We propose the following representation of the evolution of Or:

A_5. $Or(do(a,s)) \leftrightarrow (\Upsilon_p^-(a,s) \wedge \Upsilon_q^-(a,s)) \vee (\Upsilon_p^-(a,s) \wedge \neg q(s) \wedge \neg \Upsilon_q^+(a,s)) \vee$

[7] In order to avoid the belief that the obligations are only concerned with the actual situation, the term *actual obligations* is not used to indicate the second type of obligations, as Carmo and Jones proposed.

$(\Upsilon_q^-(a,s) \wedge \neg p(s) \wedge \neg \Upsilon_p^+(a,s)) \vee (Or(s) \wedge \neg(\Upsilon_p^+(a,s) \vee \Upsilon_q^+(a,s)))$

Following the example, we have (5) if the agent is a delinquent and she is not in prison, then there must be a warning sign: "loose delinquent", and (6) the agent is not in prison. The successor deontic state axiom of Or, after reducing some terms using the set of unique name axioms for actions [6], is of the form:

$Or(do(a,s)) \leftrightarrow (a = steal \vee a = kidnap \vee a = kill \vee a = rape) \wedge \neg q(s) \vee Or(s) \wedge \neg(a = free \vee a = imprison)$

Note that we can deduce $Oq(S_1) \wedge Or(S_1)$, which means that in S_1, it is obligatory that the agent is in prison and that a warning sign is advertised.

In order to represent the CTD obligations, we made some assumptions. For example, the axiom A_3 assumes that p is the only cause that modifies the obligation about q, so the representation not only has an implication but rather an equivalence to satisfy, i.e. the constraint C_3'. $\forall s\ (\neg p(s) \leftrightarrow Oq(s))$ must be considered. In particular $\neg p(do(a,s)) \leftrightarrow Oq(do(a,s))$ must be satisfied for all a and s. Substituting $p(do(a,s))$ for the right hand side of its corresponding successor state axiom, we have $\neg(\Upsilon_p^+(a,s) \vee (p(s) \wedge \neg \Upsilon_p^-(a,s))) \leftrightarrow Oq(do(a,s))$. However, $\neg(\Upsilon_p^+(a,s) \vee (p(s) \wedge \neg \Upsilon_p^-(a,s))) \leftrightarrow \Upsilon_p^-(a,s) \vee (\neg p(s) \wedge \neg \Upsilon_p^+(a,s))$. Thus we have $(\Upsilon_p^-(a,s) \vee (\neg p(s) \wedge \neg \Upsilon_p^+(a,s))) \leftrightarrow Oq(do(a,s))$. Finally, replacing $\neg p(s)$ by its equivalent formula $Oq(s)$, we obtain the axiom A_3. An analogous method was used to find the representation of the evolution of Fq.

When the conditions that modify the obligation not only depend on a fluent, as in C_5, the constraint representing the CTD obligation is also considered as an equivalence. Implicitly we make the casual completeness assumption [6]. So we consider the case that p and q are all of the causes that can modify Or, i.e. the constraint C_5'. $\forall s(\neg p(s) \wedge \neg q(s) \leftrightarrow Or(s))$ must be satisfied. This assumption seems too strong, however, its intuitive interpretation corresponds with desired facts. For example, the warning sign "loose delinquent" must exist if and only if there is a delinquent and she is not in prison. Concerning Or, in particular we have $\neg p(do(a,s)) \wedge \neg q(do(a,s)) \leftrightarrow Or(do(a,s))$ for all a and s. Substituting $p(do(a,s))$ and $q(do(a,s))$ for the right hand side of their corresponding successor state axioms, we have $\neg(\Upsilon_p^+(a,s) \vee (p(s) \wedge \neg \Upsilon_p^-(a,s))) \wedge \neg(\Upsilon_q^+(a,s) \vee (q(s) \wedge \neg \Upsilon_q^-(a,s))) \leftrightarrow Or(do(a,s))$ which can be reduced to A_5.

Intuitively, the CTD obligations take effect when a primary obligation is violated. Thus Oq comes into force when Op is violated: $Op \wedge \neg p(S_1)$. Also Or comes into force when Oq is violated: $Oq(S_1) \wedge \neg q(S_1)$, recall that $Oq(s) \leftrightarrow \neg p(s)$. In general, the first violation must be done on an ideal obligation. So when the agent abandons the ideal world, she is obliged to "pay" the consequences of her behaviour, the "price" (sanction) is allocated by a situation dependent obligation, if she does not pay, a second situation dependent obligation can increase the "debt", and so on.

6 Conclusion

We have proposed a representation of the Chisholm set, in the framework of the situation calculus, that avoids the paradox and other intrinsic problems such as pragmatic oddity. In order to close the gap between the theoretical and realistic implementation of rational agent design, we have avoided the use of modal logic for formalising deontic notions and we have explored this alternative. An important advantage is that the regression mechanism that deals with fluents [6] can be extended in order to get a tractable automated deontic reasoning implementation.

A disadvantage, which accrues from the representations such as in Prakken and Sergot's approach [8], is the context-dependence of logical form. It is unclear how multiple levels of CTD can be handled by the approach. Unlike Prakken and Sergot's approach the proposed approach supplies a uniform representation of deontic conditionals.

We suppose that the conditions changing the truth value of p, $\Upsilon_p^+(a,s)$ and $\Upsilon_p^-(a,s)$ do not depend (directly or indirectly) on the deontic fluent Oq. In general, we suppose that the fluents are not affected by the deontic fluents. In other words, the real world is not affected by the ideals defined by the law. However, it is clear that the mental states are influenced by the ideals. Thus, one possible extension is the inclusion of obligations in a cognitive model based on the situation calculus such as in [4] which may assist in the determination of the agent behaviour, in particular, in the determination of her intentions or in the adoption of goals. Also an analysis of the philosophical issues will be carried out in our future investigations.

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