Transitivity and Propagation of Trust in Information Sources. An Analysis in Modal Logic

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Abstract. The paper is about trust in information sources in the context of Multi Agents Systems and it is focused on information and trust propagation. Trust definition is inspired from Cognitive Science and it is seen as a truster's belief in some trustee's properties which are called: sincerity, competence, vigilance, cooperativity, validity and completeness. These definitions are formalized in Modal Logic and it is shown that even if trust, in that sense, is not transitive, we can find interesting sufficient conditions based on trust that guarantee that the truth of an information is propagated along a chain of information sources.

1 Introduction

In the context of Multi Agent Systems trust plays a quite significant role with respect to interactions between agents. That is because in many applications agents only have a partial knowledge of the agents they have to interact with. For instance, in the context of electronic commerce the buyer has to trust the seller in the properties of the goods he wants to buy and the seller has to trust the buyer in the fact that he will be paid for the goods he wants to sell.

In the context of information retrieval the agents have to trust the information sources in the fact that the transmitted information is true or in the fact that information sources are competent or sincere or both competent and sincere. Moreover, in many cases the information transmitted by the information sources is not supported by direct observations but by information reported by other information sources. Then, information may be propagated along a chain of information sources and it is definitely not easy to decide wether we can trust such or such information source. That is the main issue which is investigated in this paper.

The analysis of this problem is not easy because the notion of trust itself is not simple. Even if some authors do not analyse this notion in detail and define trust by a relationship of the kind: agent i trusts agent j, we consider here that this is an over simplification and that trust is a complex mental attitude as people in the field of Cognitive Science [4, 5] or in Philosophy [1, 13, 12] have pointed out.

In this approach concepts definitions and reasoning rules related to trust have to be carefully defined. That is why a formalism based on mathematical logic is adopted in this work in order to investigate trust transitivity and propagation. Thanks to this formalism it will be shown that even if trust is not transitive, other properties related to information propagation and to trust propagation hold.

The paper is structured as follows. In section 2 are informally compared several trust definitions and the definition adopted in the rest of the paper is presented. The logical framework which is used for formal definitions and reasoning rules is defined in section 3. This formalism is used in section 4 to define different kinds of trust. Since properties that guarantee the truth of transmitted information or of trust propagation are quite complex a case study is presented in section 5 to give to the reader an intuitive understanding of these properties. The main results of the paper are presented in section 6 in terms of theorems. In section 7 our work is compared with related works and the last section presents our conclusions.

2 Informal trust definitions

As mentioned above there are many different trust definitions. In [4, 5] Castelfranchi and Falcone mention that they have found around 60 different definitions. However, their own definition is one of the most popular in this field. It is informally presented below.

The main feature is that trust is a truster's belief about some trustee's properties. This belief is motivated by truster's goal and the properties he ascribes to the trustee are such that by doing some action α the trustee will reach the truster's goal. More precisely trust is defined by the fact that the truster has the following beliefs:

- truster's goal is to reach a situation where the proposition ϕ holds
- the action α has the effect that ϕ holds
- the trustee has the ability and opportunity to do the action α
- the trustee has the intention to do α

This approach has been expressed by Lorini and Demolombe in Modal Logic in [14, 15] and it has been shown that a logical consequence of truster's beliefs is that the truster believes that his goal will be reached after α performance by the trustee.

In this paper the trust definition we have adopted has some similarities with the above definition. However, there are significant differences. The first one is that truster's goal is not involved in trust definition itself even if this goal may be a motivation for the truster to adopt some beliefs about the trustee. The second one is that the properties ascribed to the trustee are all in a conditional form. That is, the truster believes that some fact entails another fact, and at least one of these facts is about a trustee's property.

In the specific context of trust in information sources these facts may be that (1) the trustee has informed the truster about some proposition or that (2) the trustee believes that some proposition is true or the fact that (3) it is the case that some proposition is true. The combination of these three kinds of facts in terms of the entailment relationship leads to six different kinds of trustee's properties and six kinds of trust in the trustee's properties [7–9]. These properties are presented below.

Trust in sincerity: the truster believes that if he is informed by the trustee about some proposition, then the trustee believes that this proposition is true.

Trust in competence: the truster believes that if the trustee believes that some proposition is true, then this proposition is true.

Trust in vigilance: the truster believes that if some proposition is true, then the trustee believes that this proposition is true.

Trust in cooperativity: the truster believes that if the trustee believes that some proposition is true, then he is informed by the trustee about this proposition.

Trust in validity: the truster believes that if he is informed by the trustee about some proposition, then this proposition is true.

Trust in completeness: the truster believes that if some proposition is true, then he is informed by the trustee about this proposition.

We can informally see how these trust definitions and the definition proposed by Castelfranchi and Falcone are related. Let us assume, for example, that the truster's goal is to be informed about the truth of some proposition ϕ. Then, if the truster trusts the trustee in his completeness about ϕ and about $\neg\phi$, the truster believes that if ϕ is true he will be informed by the trustee about the fact ϕ is true and if ϕ is false, he will be informed by the trustee about the fact that ϕ is false. If, in addition, the truster trusts the trustee in his validity about ϕ and $\neg\phi$, from the fact that he has been informed that ϕ is true, he will believe that ϕ is true, and, in the same way, if he has been informed that ϕ is false, he will believe that ϕ is false. Therefore, whatever ϕ is true or false, the truster goal will be achieved.

To express these definitions and derivations in formal terms we present in the next section an appropriate logical framework.

3 Logical framework

The logical framework is defined by its formal language L and its axiomatics.

We have adopted the following notations:

$ATOM$: set of atomic propositions denoted by p, q, r...

$AGENT$: set of agents denoted by i, j, k, l,...

The language L is the set of formulas defined by the following BNF:

$$\phi ::= p \mid \neg\phi \mid \phi \vee \phi \mid Bel_i\phi \mid Inf_{j,i}\phi$$

where p ranges over $ATOM$ and i and j range over $AGENT$.

The intuitive meaning of the modal operators is:

- $Bel_i\phi$: the agent i believes that ϕ is the case.
- $Inf_{j,i}\phi$: the agent j has informed the agent i about ϕ.

The axiomatics of the logic is the axiomatics of a Propositional multi Modal Logic (see Chellas in [6]).

In addition to the axiomatics of Classical Propositional Calculus we have the following axiom schemas an inference rules.

(K) $Bel_i(\phi \rightarrow \psi) \rightarrow (Bel_i\phi \rightarrow Bel_i\psi)$

(D) $\neg(Bel_i\phi \wedge Bel_i\neg\phi)$

(Nec) If $\vdash \phi$, then $\vdash Bel_i\phi$

For the modal operator $Inf_{j,i}$ we have the inference rule and axiom schemas:

(EQV) If $\vdash \phi \leftrightarrow \psi$, then $\vdash Inf_{j,i}\phi \leftrightarrow Inf_{j,i}\psi$

(CONJ) $Inf_{j,i}\phi \wedge Inf_{j,i}\psi \rightarrow Inf_{j,i}(\phi \wedge \psi)$

(OBS) $Inf_{j,i}\phi \rightarrow Bel_iInf_{j,i}\phi$

(OBS') $\neg Inf_{j,i}\phi \rightarrow Bel_i\neg Inf_{j,i}\phi$

According to Chellas's terminology, modalities of the kind Bel_i obey a normal system KD and modalities of the kind $Inf_{j,i}$ obey a particular kind of classical system which is not monotonic. Axioms (OBS) and (OBS') show how these two kinds of modalities interact.

The justification of (OBS) and (OBS') is that it is assumed that if an agent j informs or does not inform an agent i about ϕ, then i is aware of this fact. Roughly speaking, it is assumed that we have perfect communication channels.

The adoption of axiom schema (CONJ) is questionable. It can be adopted or rejected depending on the fact that performance of both actions $Inf_{j,i}\phi$ and $Inf_{j,i}\psi$, on one hand, and performance of the action $Inf_{j,i}(\phi \wedge \psi)$, on the other hand, have "equivalent" effects with respect to the issues we want to analyze. If an effect of the action $Inf_{j,i}\phi$ is denoted by $Effect(\phi)$, this "equivalence" holds as long as we have: (CLOS) $Effect(\phi) \wedge Effect(\psi) \rightarrow Effect(\phi \wedge \psi)$. In the context of our trust definitions the effects we are interested in, which are mentioned in the following section, are either $Effect(\phi) \stackrel{\text{def}}{=} \phi$ or $Effect(\phi) \stackrel{\text{def}}{=} Bel_j\phi$. For both of them the property of closure under conjonction (CLOS) holds. Therefore, the axiom schema (CONJ) does not lead to any counter intuitive consequence in our context. Nevertheless, it is worth noting that it is not needed to prove any theorem in this paper and, then, it could be rejected as well. Last comment: if in an other context (for example, if we are interested in resources that have been used for communication) we have to count the number of communication actions that have been performed, then the (CLOS) property does not hold any more and (CONJ) should be rejected.

We have **not** accepted the following inference rule:

(CLOS) If $\vdash \phi \rightarrow \psi$, then $\vdash Inf_{j,i}\phi \rightarrow Inf_{j,i}\psi$

The reason is that it could lead to consequences that are counter intuitive. Let's consider, for example, a situation where, if j informs i about $p \vee q$, then $p \vee q$ is true, while the fact that j informs i about p does not entail that p is true. That could be the case, for instance, if p means that it is snowing and q means that it is raining, because there are much more situations where it is raining than situations where it is snowing.

If we accept (CONS), in a situation where j has informed i about p, from (CONS) we could infer that it is also true that j has informed i about $p \vee q$ and that $p \vee q$ is true.

4 Formal trust definitions

The different kinds of trust which have been informally presented in section 2 are formally represented in this framework as follows.

Sincerity.
$Sinc(j, i, \phi)$: if agent j has informed agent i about ϕ, then j believes ϕ.
$Sinc(j, i, \phi) \stackrel{\text{def}}{=} Inf_{j,i}\phi \rightarrow Bel_j\phi$
$TrustSinc(i, j, \phi)$: agent i trusts agent j in his sincerity about ϕ.
$TrustSinc(i, j, \phi) \stackrel{\text{def}}{=} Bel_i(Inf_{j,i}\phi \rightarrow Bel_j\phi)$
Competence.
$Comp(j, \phi)$: if agent j believes ϕ, then ϕ is true.
$Comp(j, \phi) \stackrel{\text{def}}{=} Bel_j\phi \rightarrow \phi$
$TrustComp(i, j, \phi)$: agent i trusts agent j in his competence about ϕ.
$TrustComp(i, j, \phi) \stackrel{\text{def}}{=} Bel_i(Bel_j\phi \rightarrow \phi)$
The "dual" of these properties are formally represented below.
Cooperativity.
$Coop(j, i, \phi)$: if agent j believes ϕ, then j informs agent i about ϕ.
$Coop(j, i, \phi) \stackrel{\text{def}}{=} Bel_j\phi \rightarrow Inf_{j,i}\phi$
$TrustCoop(i, j, \phi)$: agent i trusts agent j in his cooperativity about ϕ.
$TrustCoop(i, j, \phi) \stackrel{\text{def}}{=} Bel_i(Bel_j\phi \rightarrow Inf_{j,i}\phi)$
Vigilance.
$Vigi(j, \phi)$: if ϕ is true, then agent j believes ϕ.
$Vigi(j, \phi) \stackrel{\text{def}}{=} \phi \rightarrow Bel_j\phi$
$TrustVigi(i, j, \phi)$: agent i trusts agent j in his vigilance about ϕ.
$TrustVigi(i, j, \phi) \stackrel{\text{def}}{=} Bel_i(\phi \rightarrow Bel_j\phi)$
We also have the following less specific kinds of trust.
Validity.
$Val(j, i, \phi)$: if agent j has informed agent i about ϕ, then ϕ is true.
$Val(j, i, \phi) \stackrel{\text{def}}{=} Inf_{j,i}\phi \rightarrow \phi$
$TrustVal(i, j, \phi)$: agent i trusts agent j in his validity about ϕ.
$TrustVal(i, j, \phi) \stackrel{\text{def}}{=} Bel_i(Inf_{j,i}\phi \rightarrow \phi)$
The "dual" of validity is completeness.
Completeness.
$Cmp(j, \phi)$: if ϕ is true, then agent j informs i about ϕ.
$Cmp(j, \phi) \stackrel{\text{def}}{=} \phi \rightarrow Inf_{j,i}\phi$
$TrustCmp(i, j, \phi)$: agent i trusts agent j in his completeness about ϕ.
$TrustCmp(i, j, \phi) \stackrel{\text{def}}{=} Bel_i(\phi \rightarrow Inf_{j,i}\phi)$

Some of these properties are not independent. We can easily show that we have:

(V) $\vdash TrustSinc(i,j,\phi) \wedge TrustComp(i,j,\phi) \rightarrow TrustVal(i,j,\phi)$

(C) $\vdash TrustVigi(i,j,\phi) \wedge TrustCoop(i,j,\phi) \rightarrow TrustCmp(i,j,\phi)$

These definitions can be used to characterize the effects of the fact that an agent has informed or does not have informed another agent, depending on different kinds of assumption about the trust relationship between these two agents. We have the following properties.

(E1) $\vdash TrustSinc(i,j,\phi) \rightarrow (Inf_{j,i}\phi \rightarrow Bel_i Bel_j \phi)$

(E2) $\vdash TrustVal(i,j,\phi) \rightarrow (Inf_{j,i}\phi \rightarrow Bel_i \phi)$

(E3) $\vdash TrustCoop(i,j,\phi) \rightarrow (\neg Inf_{j,i}\phi \rightarrow Bel_i \neg Bel_j \phi)$

(E4) $\vdash TrustCmp(i,j,\phi) \rightarrow (\neg Inf_{j,i}\phi \rightarrow Bel_i \neg \phi)$

Property (E2) (resp. (E4)) shows sufficient conditions about trust that guarantee that performing (resp. not performing) the action $Inf_{j,i}\phi$ has the effect that i believes that ϕ is true (resp. false).

5 From case studies to generalization

Before to show in formal terms in which situations information and/or trust can be propagated along a chain of information sources a case study is presented in order to give an intuitive understanding of the general results which are presented in the next section.

John wants to invest money in stocks and bonds. Peter who is a journalist in the field of finance has told to John that Franck, who is a trader, told him that the stock value of the company AXB is going to increase. John trusts Peter and Peter trusts Franck. The question is: *can we infer that John trusts Franck?*.

It is tempting to answer: "*yes*". However, to answer this question we have to make more precise the kind of trust we have in mind.

Let us assume that we are thinking to trust in validity. Then, if the sentence: "the stock value of the company AXB is going to increase" is denoted by p, the sentence "*John trusts Peter*" means: "*John believes that if Peter tells to John that p, then p is true*", the sentence "*Peter trusts the Franck*" means: "*Peter believes that if Franck tells to Peter that p, then p is true*", and the sentence "*John trusts Franck*" means: "*John believes that if Franck tells to John that p, then p is true*".

According to this definition of trust, from the fact that John trusts Peter (i.e. $TrustVal(John, Peter, p)$) and Peter trusts Franck (i.e. $TrustVal(Peter, Franck, p)$)) we cannot infer that John trusts Franck (i.e. $TrustVal(John, Franck, p)$). The first reason is that John is not necessarily aware of what believes Peter, and in particular he may not be aware of the fact that Peter trusts Franck. In formal terms we may have: $\neg Bel_{John}(TrustVal(Peter, Franck, p))$.

Even if it is assumed that Peter has told to John that he trusts Franck (i.e. $Inf_{Peter,John}(TrustVal(Peter, Franck, p))$) it is not necessarily the case that John believes that what Peter told him is true. In other words, it may be that

John does not trust Peter in his validity about the fact that Peter trusts Franck in his validity about p (i.e. $\neg TrustVal(John, Peter, Val(Peter, Franck, p))$).

Indeed, it may be that John trusts Peter as a reliable information source about the value of the stocks but that he does not trust Peter as an evaluator of other information sources.

The reason why John does not trust Peter as an evaluator may be, for example, that John believes that Peter has the capacity to evaluate the trader's competence about p but Peter does not have the capacity to evaluate the trader's sincerity about p and it could happen that the trader Franck tells to Peter that the stock value of the company AXB is going to increase while Franck believes that it is going to decrease.

This simple example shows that trust in informations sources' validity, as it has been defined, is not transitive. In formal terms, it can easily be checked that the set of sentences: $TrustVal(John, Peter, p)$, $TrustVal(John, Franck, p)$ and $\neg TrustVal(Peter, Franck, p)$ is consistent.

Nevertheless, we can try to find if there are other kinds of trust relative to information transmission that "guarantees" the truth of the information which is transmitted.

Let us use q to denote the sentence: "Franck has told p to Peter and, if Franck has told p to Peter, then p is true".

Now, it is assumed that John trusts Peter in his validity about q.

If John is aware of the fact that the trader Franck has told p to Peter and of the fact that Peter has told q to him, we can infer that John believes that p is true.

To check the validity of this derivation and to understand how it can be generalized to an unlimited number of information sources we introduce the following notations: i: John, i_1: Peter, i_2: Franck and:

$q \stackrel{\text{def}}{=} Inf_{i_2,i_1}p \wedge (Inf_{i_2,i_1}p \rightarrow p)$

In that example we have: $Inf_{i_2,i_1}p$, $Inf_{i_1,i}q$ and $Bel_i(Inf_{i_1,i}q \rightarrow q)$.

If it is assumed that i (John) is aware of what i_1 (Peter) told him q, we also have: $Bel_i Inf_{i_1,i}q$. Then, we can infer $Bel_i q$ and, by definition of q, we have: $Bel_i(Inf_{i_2,i_1}p \wedge (Inf_{i_2,i_1}p \rightarrow p))$, which entails $Bel_i p$.

Let us consider now a variant of the previous examples where some agents play the role of information sources and others play the role of evaluators of the information sources. Let us assume, for example, that Franck has told to Peter that if another agent, who is called Carlo, tells to John that p, then p is true. The reason why Franck ascribes this property to Carlo may be, for example, that he knows that Carlo is a manager of the company AXB. Let us assume in addition that Jack, who is a consultant in stocks and bonds, has told to John that Franck is a reliable evaluator of Carlo, and John believes that Jack is a reliable evaluator of Franck. Can we infer in that case that John trusts Carlo in p and that John believes p?

To find a formal answer to this question we adopt the following notations: e_2: Jack, j: Carlo and:

$p' \stackrel{\text{def}}{=} Inf_{j,i}p \rightarrow p$

$r \stackrel{\text{def}}{=} Inf_{i_2,i_1}p' \to p'$

$s \stackrel{\text{def}}{=} (Inf_{i_2,i_1}p') \wedge (Inf_{e_2,i_1}r) \wedge (Inf_{e_2,i_1}r \to r)$

It is assumed that John trusts Peter in his validity about s and John is aware of the fact that Peter has told him s and Carlo has told him p. Then, we have:

$Bel_i Inf_{j,i}p$, $Bel_i Inf_{i_1,i}s$ and $Bel_i(Inf_{i_1,i}s \to s)$.

From these assumptions we can infer: $Bel_i(s)$, which, by definition of s means: $Bel_i((Inf_{i_2,i_1}p') \wedge (Inf_{e_2,i_1}r) \wedge (Inf_{e_2,i_1}r \to r))$, which entails: $Bel_i((Inf_{i_2,i_1}p') \wedge r)$, which, by definition of r, means: $Bel_i((Inf_{i_2,i_1}p') \wedge (Inf_{i_2,i_1}p' \to p'))$, which entails: $Bel_i p'$, which, by definition of p', means: $Bel_i(Inf_{j,i}p \to p)$, and from $Bel_i Inf_{j,i}p$ we can infer: $Bel_i p$. That gives a positive answer to the above question.

Before to present a general analysis of trust and information propagation we introduce the following general notations.

In the case of the second example the information transmitted by an agent i_k to another agent i_{k-1} is denoted by $\Phi_{k,k-1}$. Then, we have: $\Phi_{2,1} = p$ and $\Phi_{1,0} = Inf_{i_2,i_1}\Phi_{2,1} \wedge Val(i_2, i_1, \Phi_{2,1})$. Roughly speaking we can say that the information transmitted by i_1 to i_0 represents the fact that i_1 has received the piece of information represented by $\Phi_{2,1}$ from i_2 (i.e. $Inf_{i_2,i_1}\Phi_{2,1}$) and the fact that i_2 is a valid information source for i_1 with respect to $\Phi_{2,1}$ (i.e. $Val(i_2, i_1, \Phi_{2,1})$).

If we have three information sources: i_3, i_2 and i_1, we have (see figure 1):

$\Phi_{3,2} = p$, $\Phi_{2,1} = Inf_{i_3,i_2}\Phi_{3,2} \wedge Val(i_3, i_2, \Phi_{3,2})$ and $\Phi_{1,0} = Inf_{i_2,i_1}\Phi_{2,1} \wedge Val(i_2, i_1, \Phi_{2,1})$

The fact that i trusts i_1 in his validity about $\Phi_{1,0}$ is represented by:
$Bel_i(Inf_{i_1,i}\Phi_{1,0} \to \Phi_{1,0})$.

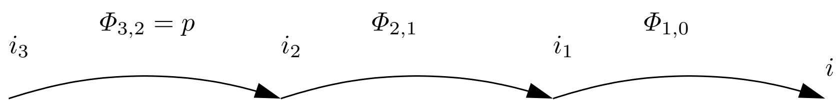

Fig. 1. Chain of information sources.

For the third example $\Psi_{k,k-1}$ is used to denote the information transmitted by an agent i_k to another agent i_{k-1}. Then, we have: $\Psi_{2,1} = Val(j, i, p)$ and $\Psi_{1,0} = (Inf_{i_2,i_1}\Psi_{2,1}) \wedge (Inf_{e_2,i_1}Val(i_2, i_1, \Psi_{2,1})) \wedge Val(e_2, i_1, Val(i_2, i_1, \Psi_{2,1}))$. In that case we can say that the information transmitted by i_1 to i represents the fact that i_1 has received the piece of information $\Psi_{2,1}$ from i_2 (i.e. $Inf_{i_2,i_1}\Psi_{2,1}$) and i_1 has received from the evaluator e_2 a piece of information which means that i_2 is a valid information source for i_1 with respect to $\Psi_{2,1}$ (i.e. $Inf_{e_2,i_1}Val(i_2, i_1, \Psi_{2,1})$) and the fact that the evaluator e_2 is a valid information source for i_1 with respect to the fact that i_2 is a valid information source for i_1 with respect to $\Psi_{2,1}$ (i.e. $Val(e_2, i_1, Val(i_2, i_1, \Psi_{2,1}))$).

In the case of a generalization of the third example to three information sources, the information transmitted by an information source to another one is represented by the following sentences (see figure 2).
$\Psi_{3,2} = Val(j,i,p), \Psi_{2,1} = (Inf_{i_3,i_2}\Psi_{3,2}) \wedge (Inf_{e_3,i_2}Val(i_3,i_2,\Psi_{3,2})) \wedge Val(e_3,i_2,\Psi_{3,2})$ and $\Psi_{1,0} = (Inf_{i_2,i_1}\Psi_{2,1}) \wedge (Inf_{e_2,i_1}Val(i_2,i_1,\Psi_{2,1})) \wedge Val(e_2,i_1,\Psi_{2,1})$

The fact that i trusts i_1 in his validity about $\Psi_{1,0}$ is represented by: $Bel_i(Inf_{i_1,i}\Psi_{1,0} \rightarrow \Psi_{1,0})$.

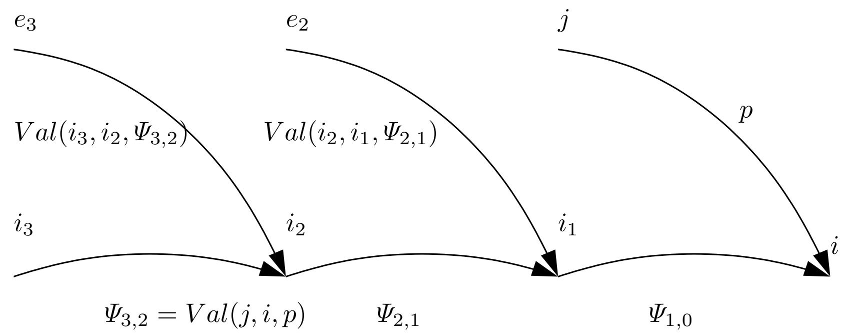

Fig. 2. Chain of information sources and of of their evaluators.

6 Trust in information sources propagation

In this section general results about information and trust propagation are presented in the form of theorems. We first prove the following Lemma 1.

Lemma 1 *If ϕ is in L and i and j are in $AGENT$, we have:*
$\vdash Inf_{j,i}\phi \wedge Bel_iVal(j,i,\phi) \rightarrow Bel_i\phi$
$\vdash \neg Inf_{j,i}\phi \wedge Bel_iCmp(j,i,\phi) \rightarrow Bel_i\neg\phi$

Proof. *This Lemma is a direct consequence of (OBS), (OBS') and of validity and completeness definitions.* **QED.**

The Theorem 1 shows a property which allows us to infer from an assumption about the fact that i trusts j in his competence about k's competence and i believes that j trusts k in his competence a conclusion about the fact i trusts k in his competence. However, this is not a property of transitivity because the assumptions do not have exactly the same form as the conclusion.

Theorem 1 *If ϕ is in L and i, j and k are in $AGENT$, we have:*
$\vdash TrustComp(i,j,Comp(k,\phi)) \wedge Bel_iTrustComp(j,k,\phi)$
$\rightarrow TrustComp(i,k,\phi)$

Proof. *From* $TrustComp(i, j, Comp(k, \phi))$ *definition we have:*
(1) $Bel_i(Bel_j Comp(k, \phi) \to Comp(k, \phi))$
From $Bel_i TrustComp(j, k, \phi)$ *we have: (2)* $Bel_i Bel_j Comp(k, \phi)$.
Then, from (1) and (2) we have: (3) $Bel_i Comp(k, \phi)$.
By definition of $TrustComp$, *(3) is:* $TrustComp(i, k, \phi)$. **QED.**

The Theorem 2 shows that if agent i_1 has informed agent i about the fact that agent i_2 has informed agent i_1 about the fact that...agent i_n has informed agent i_{n-1} about ϕ, and agent i believes that these agents are valid for what they have told each other, then agent i believes ϕ.

This theorem guarantees some kind of propagation of the proposition ϕ through a chain of information sources from i_n to i, provided agent i believes that they are all valid information sources for what they have told to the next information source in that chain.

Theorem 2 *If* ϕ *is in* L *and* i, i_1, i_2, ... ,i_n *are in* $AGENT$ *and we adopt the following notations:*

$\Phi_{n,n-1} \stackrel{def}{=} \phi$ *and for* k *in* $[1, n-1]$: $\Phi_{k,k-1} \stackrel{def}{=} Inf_{i_{k+1},i_k} \Phi_{k+1,k}$
we have:
$\vdash (Inf_{i_1,i} \Phi_{1,0}) \wedge Bel_i(Val(i_1, i, \Phi_{1,0}) \wedge Val(i_2, i_1, \Phi_{2,1}) \wedge \ldots \wedge Val(i_n, i_{n-1}, \Phi_{n,n-1}))$
$\to Bel_i \phi$

Proof. *From* Val *definition definition* $Val(i_1, i, \Phi_{1,0}) \wedge Val(i_2, i_1, \Phi_{2,1})$ *is:* $(Inf_{i_1,i} \Phi_{1,0} \to \Phi_{1,0}) \wedge (Inf_{i_2,i_1} \Phi_{2,1} \to \Phi_{2,1})$, *and from* $\Phi_{k,k-1}$ *definition* $\Phi_{1,0}$ *is* $Inf_{i_2,i_1} \Phi_{2,1}$. *Then,* $Val(i_1, i, \Phi_{1,0}) \wedge Val(i_2, i_1, \Phi_{2,1})$ *is logically equivalent to:* $Inf_{i_1,i} \Phi_{1,0} \to \Phi_{2,1}$, *and we can easily prove by induction that* $Val(i_1, i, \Phi_{1,0}) \wedge Val(i_2, i_1, \Phi_{2,1}) \wedge \ldots \wedge Val(i_n, i_{n-1}, \Phi_{n,n-1})$ *is logically equivalent to* $Inf_{i_1,i} \Phi_{1,0} \to \Phi_{n,n-1}$.

Then, from $Bel_i(Val(i_1, i, \Phi_{1,0}) \wedge Val(i_2, i_1, \Phi_{2,1}) \wedge \ldots \wedge Val(i_n, i_{n-1}, \Phi_{n,n-1}))$ *we infer that: (1)* $Bel_i(Inf_{i_1,i} \Phi_{1,0}) \to \Phi_{n,n-1})$, *and from (1),* $Inf_{i_1,i} \Phi_{1,0}$ *and Lemma 1 we infer:* $Bel_i(\Phi_{n,n-1})$, *that is* $Bel_i \phi$. **QED.**

The Theorem 3 shows that if it not the case that agent i_1 has informed agent i about the fact that agent i_2 has informed agent i_1 about the fact that...agent i_n has informed agent i_{n-1} about ϕ, and agent i believes that these agents are complete for what they might have told each other, then agent i believes $\neg\phi$.

This theorem can be seen as the dual of Theorem 2 where completeness plays a similar role as validity.

Theorem 3 *If* ϕ *is in* L *and* i, i_1, i_2, ... ,i_n *are in* $AGENT$ *and we adopt the following notations:*

$\Phi_{n,n-1} \stackrel{def}{=} \phi$ *and for* k *in* $[1, n-1]$: $\Phi_{k,k-1} \stackrel{def}{=} Inf_{i_{k+1},i_k} \Phi_{k+1,k}$
we have:
$\vdash (\neg Inf_{i_1,i} \Phi_{1,0}) \wedge Bel_i(Cmp(i_1, i, \Phi_{1,0}) \wedge Cmp(i_2, i_1, \Phi_{2,1}) \wedge \ldots$
$\wedge Cmp(i_n, i_{n-1}, \Phi_{n,n-1}) \to Bel_i \neg\phi$

Proof. *proof is very similar as the proof of Theorem 2.*

The difference between the assumptions in Theorem 4 and those in the Theorem 2 is that each agent i_k informs i_{k-1} about the validity of agent i_{k+1} for the information $\Phi_{k+1,k}$ transmitted by i_{k+1} to him and agent i only trusts agent i_1 in his validity. Roughly speaking, here it is not required that i knows the agents i_2, i_3, ... ,i_n.

Theorem 4 *If ϕ is in L and i, i_1, i_2, ... ,i_n are in AGENT and we adopt the following notations:*

$\Phi_{n,n-1} \stackrel{def}{=} \phi$ *and*

for k in $[1, n-1]$: $\Phi_{k,k-1} \stackrel{def}{=} (Inf_{i_{k+1},i_k}\Phi_{k+1,k}) \wedge Val(i_{k+1}, i_k, \Phi_{k+1,k})$

we have:

$\vdash (Inf_{i_1,i}\Phi_{1,0}) \wedge TrustVal(i, i_1, \Phi_{1,0}) \rightarrow Bel_i\phi$

Proof. *Let us call (H) the formula: $(Inf_{i_1,i}\Phi_{1,0}) \wedge TrustVal(i, i_1, \Phi_{1,0})$.*
We prove by induction that for every l in $[1, n]$: (H) entails $Bel_i\Phi_{l,l-1}$.
For $l = 1$, from (H) definition and Lemma 1 we have: $Bel_i\Phi_{1,0}$.
Induction hypothesis: (H) entails $Bel_i\Phi_{l,l-1}$.
From $\Phi_{l,l-1}$ definition and the induction hypothesis (H) entails:
(1) $Bel_iInf_{i_{l+1},i_l}\Phi_{l+1,l}$ and (2) $Bel_iVal(i_{l+1}, i_l, \Phi_{l+1,l})$
From (1) and (2) we have: $Bel_i\Phi_{l+1,l}$. Therefore, (H) entails $Bel_i\Phi_{l+1,l}$.
Then for every l in $[1, n]$ we have: $Bel_i\Phi_{l,l-1}$, and in particular we have: $Bel_i\Phi_{n,n-1}$ which is, by definition of $\Phi_{n,n-1}$, $Bel_i\phi$. **QED**.

The following Theorem 5 is the dual of Theorem 4. The main difference is that here the meaning of the proposition $\Phi_{k,k-1}$ is that if i_{k+1} is a complete information source, then i_{k+1} informs i_k about $\Phi_{k+1,k}$.

Theorem 5 *If ϕ is in L and i, i_1, i_2, ... ,i_n are in AGENT and we adopt the following notations:*

$\Phi_{n,n-1} \stackrel{def}{=} \phi$ *and*

for k in $[1, n-1]$: $\Phi_{k,k-1} \stackrel{def}{=} Cmp(i_{k+1}, i_k, \Phi_{k+1,k}) \rightarrow Inf_{i_{k+1},i_k}\Phi_{k+1,k}$

we have:

$\vdash (\neg Inf_{i_1,i}\Phi_{1,0}) \wedge TrustCmp(i, i_1, i, \Phi_{1,0}) \rightarrow Bel_i\neg\phi$

Proof. *The proof is very similar as the proof of Theorem 4.*
If we call (H) the formula: $(\neg Inf_{i_1,i}\Phi_{1,0}) \wedge TrustCmp(i, i_1, \Phi_{1,0})$ we prove by induction that (H) entails $Bel_i\neg\Phi_{l,l-1}$.
We just have to notice that $\neg\Phi_{l,l-1}$ is equivalent to: $Cmp(i_{l+1}, i_l, \Phi_{l+1,l}) \wedge \neg Inf_{i_{l+1},i_l}\Phi_{l+1,l}$, which entails $\neg\Phi_{l+1,l}$. **QED**.

The following Theorem 6 has a similar meaning as Theorem 4.

The difference is that for each information source i_k there is another information source e_k who plays the role of an evaluator of i_k's validity and e_k himself is considered by i_{k-1} as a valid information source for this evaluation.

Theorem 6 *If ϕ is in L and j, i, i_1, i_2, ... ,i_n are in AGENT and we adopt the following notations:*

$\Psi_{n,n-1} \stackrel{def}{=} \phi$ *and*

for k *in* $[1, n-1]$: $\Psi_{k,k-1} \stackrel{def}{=} (Inf_{i_{k+1},i_k}\Psi_{k+1,k}) \wedge (Inf_{e_{k+1},i_k}Val(i_{k+1}, i_k, \Psi_{k+1,k})) \wedge Val(e_{k+1}, i_k, Val(i_{k+1}, i_k, \Psi_{k+1,k}))$

we have:

$\vdash (Inf_{i_1,i}\Psi_{1,0}) \wedge TrustVal(i, i_1, \Psi_{1,0}) \rightarrow Bel_i\phi$

Proof. *The proof is similar as the proof of Theorem 4.*

Let us call (H) the formula $(Inf_{i_1,i}\Psi_{1,0}) \wedge TrustVal(i_1, i, \Psi_{1,0})$.

We prove by induction on l *that for every* l *in* $[1, n]$ *(H) entails* $Bel_i\Psi_{l,l-1}$.

For $l = 1$, *from* $Bel_iVal(i_1, i, \Psi_{1,0})$ *and* $Inf_{i_1,i}\Psi_{1,0}$, *by Lemma 1 we have:* $Bel_i\Psi_{1,0}$.

Induction hypothesis: (H) entails $Bel_i\Psi_{l,l-1}$.

From (H) and $\Psi_{l,l-1}$ *definition we infer: (1)* $Bel_i(Inf_{e_{l+1},i_l}Val(i_{l+1}, i_k, \Psi_{l+1,l}))$ *and (2)* $Bel_iVal(e_{l+1}, i_l, Val(i_{l+1}, i_l, \Psi_{l+1,l}))$.

Then, from (1) and (2), we infer: (3) $Bel_i(Val(i_{l+1}, i_l, \Psi_{l+1,l}))$.

From (H) and $\Psi_{l,l-1}$ *definition we also infer: (4)* $Bel_i(Inf_{i_{l+1},i_l}\Psi_{l+1,l})$.

Then, from (3) and (4), we infer: $Bel_i\Psi_{l+1,l}$. *Therefore, (H) entails* $Bel_i\Psi_{l+1,l}$.

Then, by definition of $\Psi_{n,n-1}$, *(H) entails* $Bel_i\phi$. **QED**.

The following Theorem 7 is the dual of Theorem 6 in the sense that the evaluator e_{k+1} is an evaluator of i_{k+1}'s completeness instead of i_{k+1}'s validity.

Theorem 7 *If* ϕ *is in* L *and* j, i, i_1, i_2, ... ,i_n *are in AGENT and we adopt the following notations:*

$\Psi_{n,n-1} \stackrel{def}{=} \phi$ *and*

for k *in* $[1, n-1]$: $\Psi_{k,k-1} \stackrel{def}{=} ((Inf_{e_{k+1},i_k}Cmp(i_{k+1}, i_k, \Psi_{k+1,k})) \wedge Val(e_{k+1}, i_k, Cmp(i_{k+1}, i_k, \Psi_{k+1,k}))) \rightarrow (Inf_{i_{k+1},i_k}\Psi_{k+1,k})$

we have:

$\vdash (\neg Inf_{i_1,i}\Psi_{1,0}) \wedge TrustCmp(i, i_1, \Psi_{1,0}) \rightarrow Bel_i\neg\phi$

Proof. *The proof is similar as the proof of Theorem 6.*

Here (H) is $(\neg Inf_{i_1,i}\Psi_{1,0}) \wedge TrustCmp(i, i_1, \Psi_{1,0})$ *and we prove by induction that (H) entails* $Bel_i\neg\Psi_{n,n-1}$.

Indeed, if it assumed that (H) entails $Bel_i\neg\Psi_{l,l-1}$, *from* $\Psi_{l,l-1}$ *definition* $\neg\Psi_{l,l-1}$ *is equivalent to (1)* $(Inf_{e_{l+1},i_l}Cmp(i_{l+1}, i_l, \Psi_{l+1,l})) \wedge Val(e_{l+1}, i_l, Cmp(i_{l+1}, i_l, \Psi_{l+1,l})) \wedge \neg(Inf_{i_{l+1},i_l}\Psi_{l+1,l})$. *We can easily show that (1) entails: (2)* $Cmp(i_{l+1}, i_l, \Psi_{l+1,l})$ *and (3)* $\neg(Inf_{i_{l+1},i_l}\Psi_{l+1,l})$. *Since (2) and (3) entail (4)* $\neg\Psi_{l+1,l}$, *we have* $Bel_i\neg\Psi_{l+1,l}$. **QED**.

In the following Theorem 8 the assumptions are similar as the assumptions in Theorem 6. The first difference is that here each information source i_k trusts the evaluator e_{k+1}. That is, the information transmitted by i_k to i_{k-1} expresses i_k's opinion. The second one is that agent i trusts all the information sources in their competence about the information they have transmitted to the other information sources.

Theorem 8 *If* ϕ *is in* L *and* j, i, i_1, i_2, ... ,i_n *are in AGENT and we adopt the following notations:*

$\Psi_{n,n-1} \stackrel{def}{=} \phi$ *and*

for k *in* $[1,n-1]$: $\Psi_{k,k-1} \stackrel{def}{=} (Inf_{i_{k+1},i_k}\Psi_{k+1,k}) \wedge (Inf_{e_{k+1},i_k}Val(i_{k+1},i_k,\Psi_{k+1,k})) \wedge TrustVal(i_k, e_{k+1}, Val(i_{k+1}, i_k, \Psi_{k+1,k}))$

$TrustCompAll_i \stackrel{def}{=} \bigwedge_{k\in[1,n-1]} TrustComp(i, i_k, \Psi_{k+1,k})$

we have:

$\vdash TrustVal(i, i_1, \Psi_{1,0}) \wedge (Inf_{i_1,i}\Psi_{1,0}) \wedge TrustCompAll_i \rightarrow Bel_i\phi$

Proof. *The proof is similar as the proof of Theorem 6.*

Let us call (H) the formula: $TrustVal(i,i_1,\Psi_{1,0}) \wedge (Inf_{i_1,i}\Psi_{1,0}) \wedge TrustCompAll_i$.

We prove by induction on l *that for every* l *in* $[1,n]$ *(H) entails* $Bel_i\Psi_{l,l-1}$.

For $l = 1$, *from (H) we have: (1)* $TrustVal(i, i_1, \Psi_{1,0}) \wedge (Inf_{i_1,i}\Psi_{1,0})$.

From (1) and Lemma 1 we have: $Bel_i\Psi_{1,0}$.

Induction hypothesis: (H) entails $Bel_i\Psi_{l,l-1}$.

From (H) and $\Psi_{l,l-1}$ *definition we have:*
(2) $Bel_i(Inf_{e_{l+1},i_l}Val(i_{l+1}, i_l, \Psi_{l+1,l})) \wedge TrustVal(i_l, e_{l+1}, Val(i_{l+1}, i_l, \Psi_{l+1,l}))$.

From (2) and Lemma 1 we have: (3) $Bel_iBel_{i_l}Val(i_{l+1}, i_l, \Psi_{l+1,l})$.

From $Bel_i\Psi_{l,l-1}$ *and* $\Psi_{l,l-1}$ *definition we also have: (4)* $Bel_iInf_{i_{l+1},i_l}\Psi_{l+1,l}$.

Then, from (3), (4) and Lemma 1 we have: (5) $Bel_iBel_{i_l}\Psi_{l+1,l}$.

From (H) and $TrustCompAll_i$ *definition we have:* $TrustComp(i, i_l, \Psi_{l+1,l})$, *and by* $TrustComp$ *definition we have: (6)* $Bel_i(Bel_{i_l}\Psi_{l+1,l} \rightarrow \Psi_{l+1,l})$.

Therefore, from (5) and (6) we have: $Bel_i\Psi_{l+1,l}$.

Then, for $l = n-1$ *we have:* $Bel_i\Psi_{n,n-1}$, *that is* $Bel_i\phi$. **QED.**

The following Theorem 9 is the dual of Theorem 8.

Theorem 9 *If* ϕ *is in* L *and* j, i, i_1, i_2, ... ,i_n *are in AGENT and we adopt the following notations:*

$\Psi_{n,n-1} \stackrel{def}{=} \phi$ *and*

for k *in* $[1,n-1]$: $\Psi_{k,k-1} \stackrel{def}{=} ((Inf_{e_{k+1},i_k}Cmp(i_{k+1}, i_k, \Psi_{k+1,k})) \wedge TrustVal(i_k, e_{k+1}, Cmp(i_{k+1}, i_k, \Psi_{k+1,k}))) \rightarrow (Inf_{i_{k+1},i_k}\Psi_{k+1,k})$

$TrustCompAll_i \stackrel{def}{=} \bigwedge_{k\in[1,n-1]} TrustComp(i, i_k, \neg\Psi_{k+1,k})$

we have:

$\vdash TrustCmp(i, i_1, \Psi_{1,0}) \wedge \neg(Inf_{i_1,i}\Psi_{1,0}) \wedge TrustCompAll_i \rightarrow Bel_i\neg\phi$

Proof. *The proof is similar as the proof of Theorem 8.*

Here (H) is the formula $TrustCmp(i,i_1,\Psi_{1,0}) \wedge \neg(Inf_{i_1,i}\Psi_{1,0}) \wedge TrustCompAll_i$ *and we prove by induction that (H) entails* $Bel_i\neg\Psi_{l,l-1}$.

It is assumed that (H) entails $Bel_i\neg\Psi_{l,l-1}$. *From* $\Psi_{l,l-1}$ *definition* $\neg\Psi_{l,l-1}$ *is equivalent to (1)* $(Inf_{e_{l+1},i_l}Cmp(i_{l+1}, i_l, \Psi_{l+1,l})) \wedge TrustVal(i_l, e_{l+1}, Cmp(i_{l+1}, i_l, \Psi_{l+1,l})) \wedge \neg(Inf_{i_{l+1},i_l}\Psi_{l+1,l})$.

We can easily show that (1) entails: (2) $Bel_{i_l}Cmp(i_{l+1}, i_l, \Psi_{l+1,l})$ *and (3)* $\neg(Inf_{i_{l+1},i_l}\Psi_{l+1,l})$. *From Lemma 1, (2) and (3) entail (4)* $Bel_{i_l}\neg\Psi_{l+1,l}$. *Therefore,* $Bel_i\neg\Psi_{l,l-1}$ *entails (5)* $Bel_iBel_{i_l}\neg\Psi_{l+1,l}$.

From $TrustCompAll_i$, *(H) entails (6)* $TrustComp(i, i_l, \neg\Psi_{l+1,l})$ *and (5) and (6) entail* $Bel_i\neg\Psi_{l+1,l}$. **QED.**

In the Theorems 2, 4, 6, 8 the information represented by ϕ is propagated under some assumptions from the first information source i_n in the chain until the agent i. It is worth noting that the proposition ϕ may be about some other information source j. For example, ϕ may be of the form: $Val(j, i, \theta)$ (resp. $Cmp(j, i, \theta)$). In that case the conclusions of these theorems express that i trusts j in his validity (resp. his completeness) about θ that is: $TrustVal(i, j, \theta)$ (resp. $TrustVal(i, j, \theta)$).

The Theorems 3, 5, 7 and 9 respectively are the dual of 2, 4, 6 and 8 and from the fact that agent i has not been informed i can infer that ϕ is false.

7 Related works

In [17] trust is represented by a probability associated to a binary relation between two agents. It is also assumed *a priori* that the trust relationship is transitive. These simplifications are assumed by the authors in order to be able to define a mathematical model to compute the "percolation" of trust in a graph of agents.

In [3] the authors have also considered that trust is just a binary relation between the truster and the trustee and the weight associated to this relation represents trust level. Then, the objective is to define a method to evaluate this weight. The method is based on the operators: aggregation, concatenation, and selection of information coming from different sources.

The authors in [16] investigate opinion propagation in order to determine agent's reputation level instead of agent's trust. It has similar objectives as our work with respect to the analysis of propagation. However, these opinions are not analyzed in detail and they are not explicitly considered as agents' beliefs.

The work presented in [2] is the work which is the closest to our work we have found in the literature. In an informal analysis trust is decomposed into several elements: the truster, the trustee and the purpose of trust. Then, these notions are formalized in the Josiang's Subjective Logic which, roughly speaking, can be seen as a combination of probability theory and epistemic logic. However, in this work trust purpose is represented by atomic propositions and no nested modal operator is used for reasoning about agents' beliefs. For example, it is not possible to represent the fact that some agent has some beliefs about the agents' beliefs in a chain of information sources as we did. The main contribution is to propose a technique to evaluate the level of trust in a context of trust propagation.

The common feature of these works is that it is assumed that trust is transitive and the goal is to find a method to compute how the trust level is propagated along a network of agents. Also, they all implicitly assume that all the agents are informed about the trust network, which is not really the case in many real applications. In [2] the trust purpose is explicit but this purpose is not analyzed in detail in the case where the purpose is to propagate information. For example, agents' properties like sincerity or competence are ignored.

8 Conclusion

Trust in information sources has been defined in terms of truster's belief about an entailment relation about some trustee's properties. It has been formally represented by formulas of the form: $Bel_i(Ant_j \to Cons_j)$, where the antecedent Ant_j and the consequent $Cons_j$ can be a communication action $Inf_{j,i}\phi$ or a belief $Bel_j\phi$ or a fact ϕ.

These trust definitions have been used to define sufficient conditions which guarantee that information is propagated along a chain of information sources: i_n, i_{n-1}, ... , i_k, ... , i_1 until agent i. A particular case we have analyzed is when this information is about another information source j and, in that case, the effect of information propagation is that i trusts j in some properties.

The conditions that guarantee propagation can be about the fact that each information source is valid or complete or about the fact that each information source validity or completeness is evaluated by a valid evaluator or about the fact that each information source trusts the previous information source in the chain about his validity or completeness.

An original feature of these properties is that the agent i can draw the conclusion that a proposition ϕ is false from the fact that he did not receive an information about ϕ.

We have presented the proofs of the theorems because we think that they can help to understand the meaning of the information $\Phi_{k,k-1}$ or $\Psi_{k,k-1}$ transmitted between each information source in the cases where this information is represented by complex formulas. Indeed, the proofs are constructive and they are by induction on the rank of the information sources. Then, we can imagine that the agent i when he is reasoning about the information sources does the same proofs.

The results which have been presented in the theorems could be used as specifications to implement automated reasoning techniques in order to apply them to specific applications. It could be that to find efficient implementations we have to restrict the expressive power of the proposition ϕ which is propagated. That should deserve further works.

A possible direction for further works is to investigate whether the assumptions in these theorems are minimal in the sense that they are not only sufficient conditions to guarantee such or such kind of propagation, but that they also are necessary conditions.

Another direction is to consider a more general structure for information sources than a linear structure. For instance, if agent i infers that agent j is valid from the fact that i has been informed by i_1 about j's sincerity and by i_2 about j's competence, the structure of information sources is a tree.

Another direction could also be to consider graded trust instead of "yes/no" trust as we did in [11, 10]. For example, in this approach graded validity is represented by: $Bel_i^g(Inf_{j,i}\phi \Rightarrow^h \phi)$, where h is the regularity level of the fact that $Inf_{j,i}\phi$ entails ϕ and g is the uncertainty level of agent i about this entailment level. That could be relevant to express that trust level decreases a long a chain of information sources.

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