

Multivalued logics and Topics

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Abstract

There are several practical problems of knowledge representation where it is more natural to talk in terms of the set of sentences related to a given topic than to make explicit all the sentences in this set.

A problematic feature of classical logic is that if we want to represent this set by a small set which is closed under logical consequence, we can generate sentences related to new topics. For example from p it is possible to infer $p \vee q$, even if q has no common topic with p .

In this paper we compare several multivalued logics in order to give a formal characterisation of the relationship between topics and sentences. We give a uniform presentation for the semantics of the four valued logic suggested by D. Lewis, for the D. Bochvar's three valued logic, and for classical two valued logic. We present theorems that allow to compare their possibilities in terms of characterisation of propositional variables that occur in sentences.

The main result is that Bochvar's logic is more suitable than Lewis's logic to characterise sentences that are logically equivalent, and that are formed with the same propositional variables.

Keywords: Multivalued logics, Relevance, Topics.

1 Introduction

There are several practical problems where we have to define which kind of usage can be done of sets of sentences. In these situations it may be quite heavy to characterise these sets by their extensions. Another possibility is to use the concept of topic to characterise sets of sentences.

For instance, we can use topics to define the sets of sentences stored in a knowledge base that a given user is permitted to access [CD96]. For example, in a company some users may be permitted to access any sentence that is about the topic health, while others may be permitted to access any sentence about incomes.

In the context of cooperative answering, techniques have been defined to provide users with additional information which is not explicitly requested, and which corresponds to their topics of interest [CD89]. For example, if some database user is asking a query about employees' salaries, the system could infer that the topic "income" is some user's topic of interest, and it could give other facts related to income. In other contexts it may be useful to characterise all a user knows about a given topic [Lak93] in order not to give to him information he already knows. Also, in the field of computer aided teaching, the "teacher" has to inform the student with new knowledge which is about the topic of the course [Mar91].

Another example is when data stored in data bases are not all guaranteed to be correct, in the sense that some of them may not be valid (they are stored in the data base while they are not true in the world), or they may not be complete (they are true in the world, but they are not stored in the data base) [Dem97]. To characterise sets of sentences for which data base content is guaranteed to be valid or to be complete, it can be quite convenient to use topics.

Finally, solutions to the frame problem have been proposed that use the concept of topic to define sets of sentences that are dependent or independent of a given change [Her96, ndCH94]. Here, independent is taken in the sense that beliefs in these sets are persistent after performance of an action or after performance of a belief revision.

In summary, for all these kinds of problems we need some linguistic means to define how sets of sentences have to be used. The aim here is not define which sen-

tences are true in the world. That is why these characterisations are based on sentence meaning and not on sentence truth. The consequence is that links between sentences and topics do not necessarily have the same properties as links between sentences and their truth values.

For instance in classical logic if a sentence p is true we can infer that sentence $p \vee q$ is also true. However, it is not clear that from the fact that sentence p is about topic t we can infer that sentence $p \vee q$ is also about topic t . Take, for instance, for p the sentence $(\text{John loves Mary}) \wedge (\text{Mary loves John})$, and for q the sentence $(\text{John loves Mary}) \wedge \neg(\text{Mary loves John})$. We can accept that p is about the topic happiness, but $p \vee q$, which is logically equivalent to (John loves Mary) , alone, is not necessarily about happiness. Moreover, if r is the sentence $(\text{Mary loves Peter})$, we can reject the fact that sentence $p \wedge r$, which is $(\text{John loves Mary}) \wedge (\text{Mary loves John}) \wedge (\text{Mary loves Peter})$ is also about happiness. That is, from $p \wedge r$ is about t we cannot necessarily infer that p is about t .

Let's consider now the definition of the structure of these links from a pragmatic point of view. It may be that if we want to characterise user's topics of interest to extend classical answers, the inference of the fact: $p \vee q$ is about t from the fact: p is about t , could be accepted. However, if we have in mind to define what information a user is permitted to know, that inference is definitely not acceptable. That means that inference rules for reasoning about these links should be selected with caution, and may depend on the kind of pragmatic problem we consider.

A problematic feature of classical logic is that it is possible to derive consequences from a set of assumptions that have no topic in common with the assumptions. This problem has been extensively investigated by researchers in the field of relevant logics [A.R75].

In [Eps90] (p. 120), Epstein defines a non standard semantics for dependent implication, and he introduces a function s that assigns to a sentence p a set of topics. In [L. 94], Fariñas del Cerro and Lugardon, use the same technique.

Demolombe and Jones, in [DJar], have defined a logic for reasoning about topics of sentences that does not require some extra feature, in the definition of the semantics, like this function s . They introduce a predicate $A(t, "p")$, whose meaning is that the proposition which is represented by sentence "p" is about topic t . If we denote by $\text{Var}(p)$ the set of propositional variables in sentence p , the basic property of their logic is that, if p is logically equivalent to q , in classical logic, and $\text{Var}(p) = \text{Var}(q)$, then we can infer that $A(t, "p")$ is logically equivalent, in a classical sense, to $A(t, "q")$. To de-

fine a sound semantics for this inference rule, they make use of Bochvar's three valued logic (see [Boc72, GG84]). Independently, for the formalisation of contexts, J.McCarthy and S. Buvač have also used in [BBM95, MB97] Bochvar's three valued logic to define the semantics of the predicate $\text{ist}(c,p)$, which means that p holds in context c .

Lewis in [Lew88] (p.173), has suggested, to define relevant implication, to consider a four valued logic, where, in a given world, a sentence may be true, false, true and false, or neither true nor false. The intuitive idea was that the fourth truth value "inconsistent" might be used to characterise inconsistent sentences, in the sense of classical logic, that are formed with the same propositional variables.

The objective of this paper is to compare, in a uniform logical framework, several multivalued logics, in order to select the most appropriate one for the definition of the links between sentences and topics. We consider the classical two valued logic, the four valued logic suggested by Lewis, and the Bochvar's three valued logic.

2 Four valued logic

In this section we define structures for the four valued logic. The same kind of structures will be defined in the next sections, with additional constraints for the three and two valued logic. We investigate properties of the four valued logic, and we compare this logic with classical propositional calculus (CPC).

Definition 1: Propositional Calculus Language.

Let VAR be a set of propositional variables, the associated propositional calculus language is defined as usual from VAR using the logical connectives \neg , for negation, and \vee for disjunction.

The connectives \wedge and \rightarrow are defined as usual from negation and disjunction.

Definition 2: Structure. A structure is a tuple $S = \langle W, T, F \rangle$ such that:

- W is a set of worlds.
- T is a function from VAR to 2^W .
- F is a function from VAR to 2^W .

From an intuitive point of view, if v is a propositional variable, T (resp. F) assigns to v the set of worlds where v is true (resp. false). The functions T and F are extended to compound sentences by the following rules:

$$\begin{aligned} T(\neg p) &= F(p) \\ F(\neg p) &= T(p) \end{aligned}$$

$$\begin{aligned} T(p \vee q) &= (T(p) \cap (T(q) \cup F(q))) \cup \\ & \cup (T(q) \cap (T(p) \cup F(p))) \\ F(p \vee q) &= F(p) \cap F(q) \end{aligned}$$

The truth values “undefined” and “inconsistent” are defined from “true” and “false”. The set of worlds where a sentence p is undefined (resp. inconsistent) is denoted by $U(p)$ (resp. $I(p)$). These functions are defined by:

$$U(p) \stackrel{\text{def}}{=} W - (T(p) \cup F(p)) \quad \text{and} \quad I(p) \stackrel{\text{def}}{=} T(p) \cap F(p)$$

According to these definitions we have:

$$\begin{aligned} U(\neg p) &= U(p) \\ I(\neg p) &= I(p) \\ U(p \vee q) &= U(p) \cup U(q) \\ I(p \vee q) &= (I(p) \cap I(q)) \cup (I(p) \cap F(q)) \cup \\ & (I(q) \cap F(p)) \end{aligned}$$

In the case where several structures are under consideration we adopt the notation $T_S(p)$ to denote the set of worlds where p is true in the structure S . Similar notations are adopted for $F(p)$, $U(p)$ and $I(p)$.

For a given propositional calculus language the set of all the possible structures for the four valued logic is denoted by Σ_4 .

Definition 3: Two valued logic associated to a four valued logic. Let $S = \langle W, T, F \rangle$ be a structure in Σ_4 . The associated two valued structure s is the tuple $s = \langle W, t, f \rangle$ where t and f are functions from VAR to 2^W such that:

- For a propositional variable v : $t(v) = T(v)$.
- $t(\neg p) = W - t(p)$.
- $t(p \vee q) = t(p) \cup t(q)$.

The set of worlds, $f(p)$, where a proposition p is false is defined by: $f(p) \stackrel{\text{def}}{=} W - t(p)$

Notation: the fact that a sentence p is a tautology of classical propositional calculus (CPC) is denoted by: $\models_{\text{CPC}} p$.

Lemma 1. If for every structure S in Σ_4 we have $t_S(p) = W$ then we have $\models_{\text{CPC}} p$.

Proof. The lemma is a direct consequence of the fact that Σ_4 contains all the possible assignments for t .

Lemma 2. If for every structure S in Σ_4 we have $t_S(p) \subseteq t_S(q)$ then we have $\models_{\text{CPC}} p \rightarrow q$.

Proof. The fact $t_S(p) \subseteq t_S(q)$ holds iff the fact $t_S(p \rightarrow q) = W$ holds.

Theorem 1. Let $S = \langle W, T, F \rangle$ be a given structure in Σ_4 , we define the structure S^+ in function of S by: $W^+ = W$ and for every propositional variable v :

$$\begin{aligned} T_{S^+}(v) &= T_S(v) \\ F_{S^+}(v) &= W - T_S(v). \end{aligned}$$

Then, for every sentence p we have: $T_{S^+}(p) = t_S(p)$ and $F_{S^+}(p) = f_S(p)$.

Proof. The proof is by induction on the complexity of sentences.

Theorem 2. If for every structure S in Σ_4 we have $T_S(p) \subseteq T_S(q)$ then for every structure S in Σ_4 we have $t_S(p) \subseteq t_S(q)$.

Proof. Proof is based on Theorem 1, and uses the technique of contraposition.

Theorem 3. For every sentence p there exists a structure S in Σ_4 such that $T_S(p) \neq \emptyset$.

Proof. The proof is by induction on the complexity of sentences.

Let us denote by $\text{Var}(p)$ the set of propositional variables in p .

Theorem 4. For every sentence p , if for every structure S in Σ_4 we have $T_S(p) \subseteq T_S(q)$, then we have $\text{Var}(q) \subseteq \text{Var}(p)$.

Proof. By contraposition, Theorem 4 is equivalent to: $\text{Var}(q) \not\subseteq \text{Var}(p)$ implies that there exists a structure S in Σ_4 such that $T_S(p) \not\subseteq T_S(q)$.

Let v be a propositional variable such that $v \in \text{Var}(q)$ and $v \notin \text{Var}(p)$.

Let S_0 and w_0 be a structure and a world defined in the same way as in the proof of Theorem 3. The world w_1 in S_0 is defined as follows:

For every propositional variable u different of v we have:
 $w_1 \in T_{S_0}(u)$ and $w_1 \in F_{S_0}(u)$.
 For the variable v we have: $w_1 \notin T_{S_0}(v)$ and $w_1 \notin F_{S_0}(v)$
 (that is $w_1 \in U(v)$).

From the proof of Theorem 3 we know that $w_0 \in T_{S_0}(p)$. Since v is not in p , all the propositional variables in p have the same truth value in w_0 and in w_1 . Therefore we have $w_1 \in T_{S_0}(p)$. Since the variable v is in q and v is undefined in w_1 , from the definition of U we have $w_1 \in U_{S_0}(q)$, then we have $w_1 \notin T_{S_0}(q)$. Therefore we have $T_{S_0}(p) \not\subseteq T_{S_0}(q)$.

Corollary 1. If for every structure S in Σ_4 we have $T_S(p) = T_S(q)$ then we have $\text{Var}(p) = \text{Var}(q)$.

Proof. Trivial.

Theorem 5. If for every structure S in Σ_4 we have $T_S(p) \subseteq T_S(q)$ then we have: $\models_{\text{CPC}} p \rightarrow q$ and $\text{Var}(q) \subseteq \text{Var}(p)$.

$\text{Var}(p)$.

Proof. This theorem is a direct consequence of Theorem 2, Lemma 2 and Theorem 4.

Theorem 5 shows that the four valued logic is powerful enough to represent the same kind of implication as dependent implication defined by Epstein in [Eps90].

Theorem 6. If for every structure S in Σ_4 we have $T_S(p) = T_S(q)$ then we have: $\models_{\text{CPC}} p \leftrightarrow q$ and $\text{Var}(q) = \text{Var}(p)$.

Proof. This is a direct consequence of Theorem 5.

Theorem 7. (W. Carnielli, [Car94]) The facts $\models_{\text{CPC}} p \leftrightarrow q$ and $\text{Var}(p) = \text{Var}(q)$ do not imply that for every structure S in Σ_4 we have $T_S(p) = T_S(q)$.

Proof. The following example shows that Theorem 7 holds. Let p be the sentence $(a \wedge \neg a) \wedge b$, and q be the sentence $(a \wedge \neg a) \wedge \neg b$. We have $\models_{\text{CPC}} p \leftrightarrow q$ and $\text{Var}(p) = \text{Var}(q)$.

Let S be a structure such that there exists a one to one correspondance between the set of worlds in S and the natural numbers. Let us define T and F in the following way.

- $T(a)$ = set of worlds labeled by multiples of 2.
- $F(a)$ = set of worlds labeled by multiples of 3.
- $T(b)$ = set of worlds labeled by multiples of 5.
- $F(b)$ = set of worlds labeled by numbers that are not multiples of 5.

We have $T(p) = T(a) \cap F(a) \cap T(b)$ and $T(q) = T(a) \cap F(a) \cap F(b)$. Then the world w_{30} which corresponds to the integer 30 is in $T(p)$, but it is not in $T(q)$. Therefore we have $T_S(p) \neq T_S(q)$. Notice that we also have $w_{12} \notin T(p)$ and $w_{12} \in T(q)$.

The negative result presented by Theorem 7 is a bit surprising. Indeed, it might seem to be intuitive that two sentences, that are logically equivalent in CPC, and that are formed with the same propositional variables, have the same extensions.

Corollary 2. The facts $\models_{\text{CPC}} p \rightarrow q$ and $\text{Var}(q) \subseteq \text{Var}(p)$ do not imply that for every structure S in Σ_4 we have $T_S(p) \subseteq T_S(q)$.

Proof. The implication would hold, it would contradict Theorem 7.

The relationships between the two valued logic and the four valued logic are not obvious. Let us consider for example the following structure S in Σ_4 where we have a world w such that for the two propositional variables a and b we have: $w \in T(a)$, $w \in F(a)$, $w \notin T(b)$ and $w \notin F(b)$.

Then we have $w \in t(a)$ and $w \in F(a)$, and this shows that we may have for some structure S and for some sentence p : $t_S(p) \cap F_S(p) \neq \emptyset$. We also have $w \notin t(\neg a)$ and $w \in T(\neg a)$, and this shows that we

may have $T_S(p) \not\subseteq t_S(p)$. Finally we have $w \in t_S(\neg b)$ and $w \notin T_S(\neg b)$, and this shows that we may have $t_S(p) \not\subseteq T_S(p)$.

The following theorem shows some relationships between the two logics.

Theorem 8. For every sentence p and for all structure S in Σ_4 we have: $T_S(p) - F_S(p) \subseteq t_S(p)$ and $F_S(p) - T_S(p) \subseteq f_S(p)$.

Proof. The proof is by induction on the complexity of sentences.

The relationships between the four valued logic and the two valued logic are represented by the Figure 1.

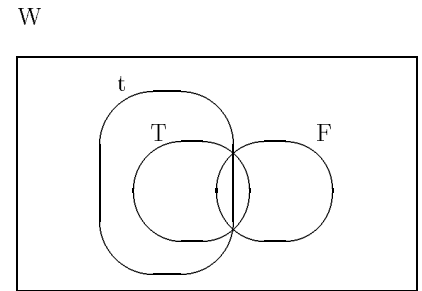


Figure 1: Relationships between the four valued logic and the two valued logic.

3 Three valued logic

In this section we consider a three valued logic. It is defined exactly in the same way as the four valued logic. The only difference is that we restrict the set of structures to those structures such that for every propositional variable v we have:

$$T_S(v) \cap F_S(v) = \emptyset$$

This set of structures is denoted by Σ_3 . The definitions of functions T , F , t and f are the same as for the four valued logic.

Lemma 3. For every sentence p and for all structure S in Σ_3 we have $T_S(p) \cap F_S(p) = \emptyset$.

Proof. The proof by induction on the complexity of sentences is trivial.

Lemma 4. The fact that for every structure S in Σ_3 we have $T_S(p) \subseteq T_S(q)$ does not imply that we have $\text{Var}(q) \subseteq \text{Var}(p)$.

Proof. Consider the two sentences $p = a \wedge \neg a$ and $q = b$.

Lemma 5. The fact that for every structure S in Σ_3 we have $T_S(p) = T_S(q)$ does not imply that we have $\text{Var}(q) = \text{Var}(p)$.

Proof. Consider the two sentences $p = a \wedge \neg a$ and $q = b \wedge \neg b$.

Theorem 9. For all sentence p and for every structure S in Σ_3 we have: $T_S(p) \subseteq t_S(p)$ and $F_S(p) \subseteq f_S(p)$.

Proof. The proof is by induction on the complexity of sentences.

Theorem 10. For every sentence p and for every structure S in Σ_3 we have:

$$t_S(p) \subseteq T_S(p) \cup U_S(p) \text{ and } f_S(p) \subseteq F_S(p) \cup U_S(p).$$

Proof. The proof is by induction on the complexity of sentences.

A direct consequence of the Theorem 10 is that for every sentence p we have $t_S(p) \cap F_S(p) = \emptyset$ and $f_S(p) \cap T_S(p) = \emptyset$. Since from the Theorem 9 we have $T_S(p) \subseteq t_S(p)$ and $F_S(p) \subseteq f_S(p)$, the relationships between the three valued logic and the two valued logic are as indicated in the figure 2.

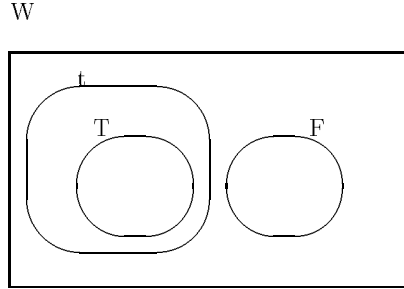


Figure 2: Relationships between the three valued logic and the two valued logic.

Theorem 11. The facts $\models_{\text{CPC}} p \rightarrow q$ and $\text{Var}(q) \subseteq \text{Var}(p)$ imply that for every structure S in Σ_3 we have $T_S(p) \subseteq T_S(q)$.

Proof. This is a consequence of Theorem 9 and Theorem 10.

Theorem 12. The facts $\models_{\text{CPC}} p \rightarrow q$ and $\text{Var}(p) \subseteq \text{Var}(q)$ imply that for every structure S in Σ_3 we have $F_S(q) \subseteq F_S(p)$.

Proof. Similar proof as for Theorem 11.

Lemma 6. The facts $\models_{\text{CPC}} p \rightarrow q$ and $\text{Var}(q) \subseteq \text{Var}(p)$ do not imply that for every structure S in Σ_3 we have $F_S(q) \subseteq F_S(p)$.

Proof. Let us consider the two sentences $p = a \wedge b$ and $q = a$, where a and b are propositional variables. We have $\models_{\text{CPC}} p \rightarrow q$ and $\text{Var}(q) \subseteq \text{Var}(p)$. Let S be a structure in Σ_3 and w a world of S such that that $w \in F(a)$ and $w \in U(b)$. We have $w \in F(q)$ and $w \notin F(p)$.

Theorem 13. The facts $\models_{\text{CPC}} p \leftrightarrow q$ and $\text{Var}(p) = \text{Var}(q)$ imply that for every structure S in Σ_3 we have $T_S(p) = T_S(q)$ and $F_S(p) = F_S(q)$.

Proof. This theorem is a direct consequence of Theorems 11 and 12.

Theorem 14. There exist sentences p and q such that for every structure S in Σ_3 we have $T_S(p) \subseteq T_S(q)$ and we do not have $\text{Var}(q) \subseteq \text{Var}(p)$.

Proof. Consider the sentences $p = a \wedge \neg a$ and $q = b$.

Theorem 15. If for every structure S in Σ_3 we have $T_S(p) = T_S(q)$ and $F_S(p) = F_S(q)$, then we have $\text{Var}(p) = \text{Var}(q)$.

Proof. Let us assume that for every structure S in Σ_3 we have $T_S(p) = T_S(q)$ and $F_S(p) = F_S(q)$.

Let us assume that we have $\text{Var}(q) \not\subseteq \text{Var}(p)$, then there exists a propositional variable v such that $v \in \text{Var}(q)$ and $v \notin \text{Var}(p)$.

Let S be a structure in Σ_3 and w be a world in S . We have either $w \in t_S(p)$ or $w \in f_S(p)$. Let us assume first that we have $w \in t_S(p)$.

We define a world w' of a structure S' from w and S in the following way:

If a variable u is in $\text{Var}(p)$ then:
if $w \in T_S(u)$ then $w' \in T_{S'}(u)$,
if $w \notin T_S(u)$ then $w' \in F_{S'}(u)$.
If a variable u is not in $\text{Var}(p)$ then $w' \in U_{S'}(u)$.

According to this definition we have $w' \in t_{S'}(p)$, because the fact $w' \in t_{S'}(p)$ (resp. $w \in t_S(p)$) only depends on the variable u such that $w' \in T_{S'}(u)$ (resp. $w \in T_S(u)$), and for the variables u in p we have $w \in T_S(u)$ iff $w' \in T_{S'}(u)$, and we also have $w \in t_S(p)$.

From Theorem 10 we have $t_{S'}(p) \subseteq T_{S'}(p) \cup U_{S'}(p)$, then we have $w' \in T_{S'}(p)$ or $w' \in U_{S'}(p)$. From the definition of w' none of the variables in p is undefined in w' then we do not have $w' \in U_{S'}(p)$, therefore we have $w' \in T_{S'}(p)$. Since we have $T_{S'}(p) = T_{S'}(q)$, we also have $w' \in T_{S'}(q)$.

Since the variable v of q is not in p , by definition of w' , we have $w' \in U_{S'}(v)$, and, by definition of U , we have $w' \in U_{S'}(q)$, which contradicts the fact $w' \in T_{S'}(q)$. Therefore we

have $\text{Var}(q) \subseteq \text{Var}(p)$.

If we assume now that we have $w \in f_S(p)$, a similar proof, based on the fact $F_{S'}(p) = F_{S'}(q)$, also allows to infer $\text{Var}(q) \subseteq \text{Var}(p)$.

Then, in both cases we have $\text{Var}(q) \subseteq \text{Var}(p)$.

Since p and q plays a similar role, we can also prove that $\text{Var}(p) \subseteq \text{Var}(q)$, and finally we have $\text{Var}(p) = \text{Var}(q)$.

Theorem 16. If for every structure S in Σ_3 we have $T_S(p) = T_S(q)$ then we have $\models_{\text{CPC}} p \leftrightarrow q$.

Proof. We can easily see that the proofs of Theorems 1 and 2 also hold if we restrict the set of structures from Σ_4 to Σ_3 .

Theorem 17. For every sentence p , if for every structure S in Σ_3 we have $T_S(p) = T_S(q)$ and $F_S(p) = F_S(q)$, then we have $\models_{\text{CPC}} p \leftrightarrow q$ and $\text{Var}(p) = \text{Var}(q)$.

Proof. This result collates the results of Theorems 15 and 16.

Theorem 18. The fact that for every structure S in Σ_3 we have $T_S(p) = T_S(q)$ does not imply that for every structure S in Σ_3 we have $F_S(p) = F_S(q)$.

Proof. Consider the two sentences $p = a \wedge \neg a$ and $q = b \wedge \neg b$.

Theorem 19. We have $\models_{\text{CPC}} p \leftrightarrow q$ and $\text{Var}(p) = \text{Var}(q)$ iff for every structure S in Σ_3 we have $T_S(p) = T_S(q)$ and $F_S(p) = F_S(q)$.

Proof. This theorem is a direct consequence of Theorems 13 and 17.

Theorem 20. The two valued logic represented by Σ_2 has the same properties as classical propositional calculus.

Proof. We prove by induction on the complexity of sentences that for every structure S in Σ_2 we have $T_S(p) = t_S(p)$ and $F_S(p) = f_S(p)$. Moreover for every possible assignment t there exists a corresponding structure in Σ_2 such that t coincides with T .

4 Conclusion

The three logics are defined in terms of the same types of structures as defined in Definition 2. If there is no restriction on the definitions of T and F , we have the four valued logic represented by the set of structures Σ_4 . If we restrict the set Σ_4 to the structures where we have $T_S(p) \cap F_S(p) = \emptyset$, we have the Bochvar's three valued logic which is represented by the set of structures Σ_3 . Finally, if we restrict Σ_3 to the structures where we have $T_S(p) \cup F_S(p) = W$, we have a two valued logic represented by the set of structures Σ_2 which has the same properties as CPC, as it is shown by Theorem 20.

We have investigated how properties about the propositional variables in sentences can be represented by structures only in terms of truth values. That is, the only difference with classical propositional calculus is that we consider three or four different truth values. We have found a correspondance (Theorem 19) between properties about variables, and extensions of sentences, only in the case of the three valued logic: i.e. we have $\models_{\text{CPC}} p \leftrightarrow q$ and $\text{Var}(p) = \text{Var}(q)$ iff we have $\forall S \in \Sigma_3 (T_S(p) = T_S(q) \text{ and } F_S(q) \subseteq F_S(p))$. In the case of the four valued logic the fact $\forall S \in \Sigma_4 (T_S(p) = T_S(q))$ implies $\models_{\text{CPC}} p \leftrightarrow q$ and $\text{Var}(p) = \text{Var}(q)$, but the implication in the other way does not hold (Theorem 7). An open question is to found the property we have to add to $\models_{\text{CPC}} p \leftrightarrow q$ and $\text{Var}(p) = \text{Var}(q)$ in order to have the equivalence with $\forall S \in \Sigma_4 (T_S(p) = T_S(q))$.

These formal results show that Bochvar's three valued logic is the most adequate to define, in the semantics, links between topics and sentences.

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