Towards a logical characterisation of sentences of the kind "sentence p is about object c"

Robert Demolombe	Luis Fariñas del Cerro
ONERA/DTIM	IRIT
Toulouse, France	Toulouse, France
robert.demolombe@cert.fr	farinas@irit.fr

Abstract

When we have to remove every piece of information about an object in the representation of an application domain, or when we want to retrieve all the pieces of information about an object in an information system, we are faced into the same fundamental problem: how to characterise sentences, expressed in a formal language, that are about a given object?

In this paper a logical characterisation is proposed in the context of classical first order logic. This problem has also been investigated by some philosophers, in particular by N. Goodman. Our approach is different from Goodman's because we start from a characterisation that is semantical rather than syntactical. We give formal definition of the concept of sentence, or theory, being about an object, we analyse to what extend this concept is compositional with regard to logical connectives and quantifiers, and we present some corresponding properties. Some of these may appear counter-intuitive. We also give a syntactical characterisation of subsets of sentences that are not about an object. We have not shown that this characterisation is complete, but we conjecture that its extension by logical equivalence is complete.

The definitions and properties presented can be used as a starting point for defining automated deduction techniques to delete all the sentences in an information system that are about an object, or to compute the answer to a new type of query of the kind: *I want to know everything about this particular object.* Finally we show that there are many open problems, in particular if an equality predicate is allowed in the language.

1 Introduction

There are many dynamic applications where the set of represented objects may change. Some new objects are introduced in the representations, others are ruled out. For instance, an aircraft may be traced by a radar, and at a given time, for some technical reasons, the aircraft is no more in the scope of the radar. Another example may be found in a database which is used for personnel management in a company. When an employee leaves the company we may want to "erase" the overall information about this employee, because this information is no longer of interest. A similar situation may happen in the case of a patient who leaves a hospital. For privacy reasons, we may desire that the patient no longer "exists" in the database. In all these examples, we want to characterise a situation where there is an object that disappears, in the sense that in our representation of the world we have no more pieces of information about this object. Then, an important problem is to find a formal definition of the property that in a set of sentences that represents our knowledge about the world there is no sentence about a given object. Another important related problem, which **is not** investigated in this paper, is to determine how to change this set of sentences when an object disappears.

The same problem arises in the field of information retrieval. When we have structured data stored in a database, the standard technique to retrieve information is to ask a query which may formally be represented by an open sentence in a first order predicate language. The corresonding answer is a set of tuples of objects for which the formula is true. Another kind of query, that cannot be asked in current database systems, and that may be very useful when people have no idea about the kind of predicates that are used to represent information, is to request all the information regarding a given object. For example, queries of the kind, "tell me everything you know about John". Here again we need a formal characterisation of sentences that are "about John", or, in general, about an oject.

This problem has also been investigated by a few philosophers, like R. Carnap [4] and H. Putnam [10] , and, later, by N. Goodman in [7] . In his paper Goodman provides a syntactic characterisation of the property that a sentence is about an object or is "absolutely" about an object. He analyses several possible definitions and shows that this characterisation is far to be trivial. In particular, he shows that we cannot directly relate the fact that a sentence mentions an object name, and the fact that this sentence is about this object.

For example, the sentence *in everywhere in the province of Languedoc there are vineyards*, represented by: $\forall x(Languedoc(x) \rightarrow vineyards(x))$, does not mention the city of Carcassonne. However, since the city of Carcassonne is in the province of Languedoc, this sentence is about Carcassonne, in the sense that it informs us about Carcassonne. Indeed, from the fact *Languedoc(Carcassonne)*, and the former sentence, we can infer *vineyard(Carcassonne)*. In general, a sentence of the form $\forall x F(x)$ is about objects named by c because we can infer from it $F(c)$.

Conversely, there are sentences that mention an object and that are not about this object. A trivial example is the tautology *there are vineyards in Oslo or there are not vineyards in Oslo*, formally represented by $vineyards(Oslo) \vee \neg vineyards(Oslo)$, which is not about Oslo because it gives no information about the city of Oslo. A less trivial example is the sentence $vineyards(Carcassonne) \wedge (vineyards(Carcassonne) \vee vineyards(Oslo))$ which is not about Oslo since it is logically equivalent to the sentence $vineyards(Carcassonne)$.

An important feature of the formal analysis presented in this paper is that it directly refers to properties of propositions which are represented in the language in some syntactical form. In fact, it is based on the idea that a definition of the concept of aboutness in the semantics is more appropriate than in the syntax to give an intrinsic characteristion of this concept. From this point of view, our approach is different from Goodman, whose analysis, at least at the begining of his paper, is based on syntactical considerations.

The idea that motivates the formal definitions presented in next section is that a sentence which is not about a given object is a sentence that does not allow us to distinguish states of the world whose descriptions only differ by the truth-value assignment to atomic sentences that mention this object. Conversely, a sentence which is about an object may be true in some of these states, and false in others.

In the next section we introduce the logical framework in which a definition of the property that a sentence is not about an object is given. We start from the definition of "sentences not being about an object", because it is easier to have an intuitive understanding of this formal definition than of "being about", whose definition directly follows from the first one. Then, we analyse formal properties of this concept with regard to logical connectives and quantifiers, and also with regard to logical consequence. In section 3, we give a definition of the fact that a theory is about a given object. It is shown that the fact that a theory is about an object is not directly related to the fact it contains some sentences that are about this object. Section 4 presents suggestions to extend the definitions to languages with equality predicates. In the conclusion we present a list of open problems concerning the application and automatisation of the treatment of aboutness.

2 Sentences about an object

Definition 0. Syntactical definition of language L_c.

We define a first order predicate calculus language L_c, where c is some given constant symbol. Neither function symbols nor the equality predicate are allowed in the language. Terms are either variable symbols or constant symbols.

L_c is defined by the following rules.

1. If p is an n-ary predicate and t is a n-tuple of terms, then $p(t) \in L_c$.
2. If $F \in L_c$ and $G \in L_c$, then $(\neg F) \in L_c$ and $(F \vee G) \in L_c$.
3. if $F \in L_c$, then $(\exists x F) \in L_c$ and $(\exists x \neq c\ F) \in L_c$ [1].
4. All the sentences in L_c are defined by rules 1, 2 and 3.

As usual we adopt the following notations: $\mathrm{p} \wedge \mathrm{q} \stackrel{\text{def}}{=} \neg((\neg \mathrm{p}) \vee (\neg \mathrm{q}))$, $\mathrm{p} \rightarrow \mathrm{q} \stackrel{\text{def}}{=} (\neg \mathrm{p}) \vee \mathrm{q}$, $\mathrm{p} \leftrightarrow \mathrm{q} \stackrel{\text{def}}{=} (\mathrm{p} \rightarrow \mathrm{q}) \wedge (\mathrm{q} \rightarrow \mathrm{p})$ and $\forall x \neq c\, F \stackrel{\text{def}}{=} \neg(\exists x \neq c\, \neg F)$. Paranthesis will be omittted when there will be no risk of misinterpretation.

Quantifiers of the form $\forall x \neq c$ and $\exists x \neq c$ are called “restricted quantifiers”.

Definition 1. Interpretation.

Let's consider a language L_c as defined in Definition 0. An interpretation M of L_c is a tuple $M =< D, i >$ such that

- D is a non empty set of individuals,
- i is a function that assigns
  - to each predicate symbol of arity n a subset of D^n,

[1] Here $x \neq c$ is used as a notation to denote restricted quantifiers, it is not taken as a sentence with an occurence of equality predicate.

– to each variable symbol an element of D,

– to each constant symbol an element of D,

In the following D will be called the domain of the interpretation, and i will be called the interpretation function, or, for short, the interpretation.

Notation: the domain of M will be denoted by D_M and the interpretation function of M will be denoted by i_M.

Definition 2. Satisfiability conditions.

Let M be an interpretation of the language L_c. The fact that a formula F of L_c is true in M is denoted by $M \models F$, and is inductively defined as follows.

- If F is an atomic sentence of the form $p(t)$, where t is a tuple of constant symbols or variable symbols, we have $M \models F$ iff $i_M(t) \in i_M(p)$.
- $M \models \neg F$ and $M \models F \vee G$ are defined from $M \models F$ and $M \models G$ as usual.
- $M \models \exists x F$ iff there exists an interpretation $M_{x/d}$ that only differs from M by the interpretation of variable symbol x, such that $i_{M_{x/d}}(x)$ is the element d of $D_{M_{x/d}}$ and $M_{x/d} \models F$.
- $M \models \exists x \neq c\ F$ iff there exists an interpretation $M_{x/d}$ that only differs from M by the interpretation of variable symbol x, such that $i_{M_{x/d}}(x)$ is the element d of $D_{M_{x/d}}$ and $i_{M_{x/d}}(c)$ is not d and $M_{x/d} \models F$.

A formula F is a valid formula iff for every interpretation M we have $M \models F$. This is denoted by $\models F$.

The most important part of the paper concerns the notion of variants, which provides a foundation for our definition of aboutness.

Definition 3. Variants of an interpretation with regard to an object.

Let L_c be a language as defined in Definition 0. We call variants of M with regard to c the set M^c of interpretations M' defined from M in the following way.

- $D_{M'} = D_M$

- $i_{M'} = i_M$ for every variable symbol and constant symbol,
- $i_{M'}$ is defined from i_M for each predicate symbol as follows: if p is a predicate symbol of arity n
  - if t is a n-tuple of terms of language L_c that contain no occurence of the constant symbol c, then $i_{M'}(t) \in i_{M'}(p)$ iff $i_M(t) \in i_M(p)$,
  - if an element $< d_1, \ldots, d_n >$ of D^n is such that, for every j in [1,n], $d_j \neq i_M(c)$, then $< d_1, \ldots, d_n > \in i_{M'}(p)$ iff $< d_1, \ldots, d_n > \in i_M(p)$.

The set of variants M' of interpretation M with regard to an object named with the constant symbol c is denoted by M^c. Notice that M belongs to M^c, and that, if M' belongs to M^c, M belongs to M'^c too.

Roughly speaking, the set M^c is the set of interpretation that only differs from M by the truth assignement of atomic sentences where c appears as an argument. The M^c definition is a bit complicated, its justification is that, in informal terms, a sentence is not about an object named by c iff its truth value does not change in all the variants of a given interpretation. Examples below can help to understand definition 3.

Example 1. Let L_c be a language with the unique unary predicate symbol p, and the constant symbols a, b and c. Let M be an interpretation of L_c defined by: $D = \{d_1, d_2, d_3, d_4\}$, $i_M(a) = d_1$, $i_M(b) = d_2$, $i_M(c) = d_3$, and $i_M(p) = \{d_1, d_3, d_4\}$ (see Figure 1).

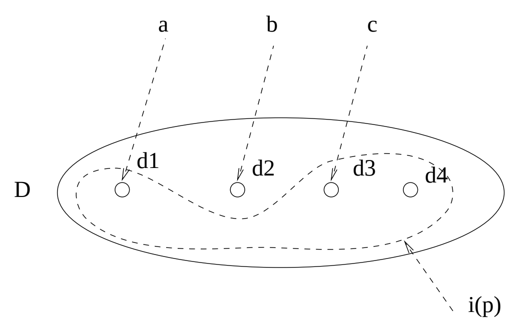

Figure 1: Example 1.

According to Definition 3, for every variant M' in M^c, $i_{M'}(p)$ contains d_1, because d_1 is the interpretation of the constant symbol a, which is different from constant symbol c. Therefore, the sentence $p(a)$ is true in every variant M'. At the opposite extreme, there are variants M' of M such that d_3 is not in $i_{M'}(p)$, because d_3 is the interpretation of c. In these variants $p(c)$ is false, although it is true in M.

Example 2. Let us consider another interpretation which is the same as in example 1, except that the constant symbols a and c are interpreted by d_3, and $i_M(p) = \{d_3, d_4\}$ (see Figure 2).

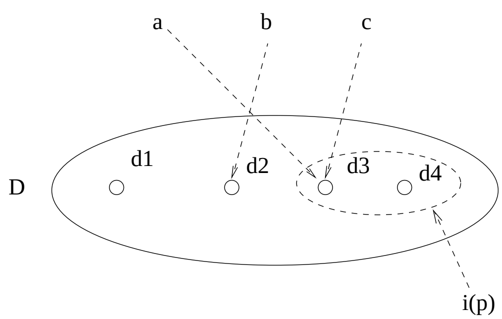

Figure 2: Example 2.

In this example, for every variant M' in M^c, d_3 is in $i_{M'}(p)$ because d_3 is the interpretation of the constant symbol a. Since d_3 is also the interpretation of c, and the interpretation of constant symbols remains unchanged in all the variants of M, the sentence $p(c)$ is true in every variant of M.

Notice that $\exists x \neq c\ p(x)$ is true in M, and it is also true in every variant of M because d_4 belongs to $i_{M'}(p)$ for every variant M' in M^c.

Example 3. Let us now consider an interpretation which is the same as in example 1, except that the interpretation of the predicate symbol p is $i_M(p) = \{d_3\}$ (see Figure 3). Then, there is a variant of M where $i_M(p) = \emptyset$. Therefore the sentence $\exists x p(x)$ is true in M, but it is false in some variant in M^c.

Definition 4. Sentences that are not about an object.

Let F be a sentence of language L_c. We say that F is not about an object named by the constant symbol c iff for every interpretation M, we have $M \models F$ iff for every interpretation M' in M^c we have $M' \models F$.

The fact that F is not about object c is denoted by $NA(F, c)$. In short we have:

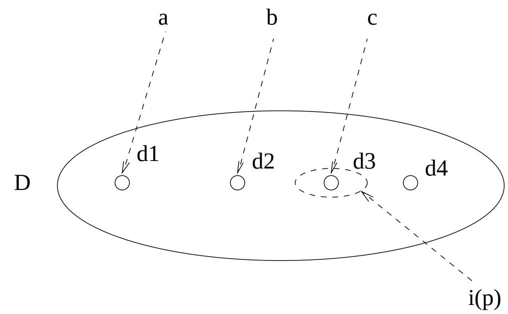

Figure 3: Example 3.

$$NA(F,c) \; holds \;\; iff \;\; \forall M(M \models F \;\; iff \;\; \forall M' \in M^c \; M' \models F)$$

We say that a formula F is about the object c, if it is not the case that $NA(F,c)$. This fact is denoted by $A(F,c)$. In short we have:

$$A(F,c) \; holds \;\; iff \;\; \exists M(\exists M' \in M^c(M \models F \; and \; M' \not\models F))$$

It can be checked that, according to Definition 4, sentence $p(a)$ is not about object c, and that sentences $p(c)$ and $\exists x p(x)$ are about object c.

Property 1. If sentences F and G are logically equivalent, then $NA(F,c)$ iff $NA(G,c)$.

In short: $\models F \leftrightarrow G \Rightarrow (NA(F,c) \Leftrightarrow NA(G,c))$.

Property 2. Sentence F is not about object c iff sentence $\neg F$ is not about object c.

In short: $NA(F,c) \Leftrightarrow NA(\neg F,c)$.

Proof. We first prove that $NA(F,c) \Rightarrow NA(\neg F,c)$.

Let M be an interpretation such that $M \models \neg F$. If there exists an interpretation M' in M^c such that $M' \not\models \neg F$, we have $M' \models F$. Since we have $NA(F,c)$, from Definition 4, for every M'' in M'^c we have $M'' \models F$. From Definition 3 we have $M \in M'^c$, therefore $M \models F$.

To prove that $NA(\neg F,c) \Rightarrow NA(F,c)$, from the former result we have $NA(\neg F,c) \Rightarrow NA(\neg\neg F,c)$, and from Property 1 we have $NA(\neg\neg F,c) \leftrightarrow NA(F,c)$.

Property 3. If both sentences F and G are not about object c, then sentence $F \vee G$ is not about object c.

In short: $NA(F, c)$ and $NA(G, c) \Rightarrow NA(F \vee G, c)$.

Proof. If $M \models F \vee G$, then we have either $M \models F$ or $M \models G$. Assume we have $M \models F$. Since we have $NA(F, c)$, for every M' in M^c we have $M' \models F$ and therefore $M' \models F \vee G$. In the case where we have $M \models G$ the same conclusion follows. Then we have $NA(F \vee G, c)$.

Property 4. If both sentences F and G are not about object c, then sentence $F \wedge G$ is not about object c.

In short terms: $NA(F, c)$ and $NA(G, c) \Rightarrow NA(F \wedge G, c)$.

Proof. From $NA(F, c)$ and $NA(G, c)$, by Property 2 we have $NA(\neg F, c)$ and $NA(\neg G, c)$, and by Property 3 we have $NA(\neg F \vee \neg G, c)$. By Property 2 again we have $NA(\neg(\neg F \vee \neg G), c)$, and by Property 1 we have $NA(F \wedge G, c)$.

Property 5. If the sentence $F(c)$ is not about the object c, then $F(c)$ is logically equivalent to $\forall x F(x)$.

In short: $NA(F(c), c) \Rightarrow \models F(c) \leftrightarrow \forall x F(x)$.

Proof. Assume $NA(F(c), c)$. Let M be an interpretation such that $M \models F(c)$. Let $M_{c/d}$ be an interpretation whose only difference from M is that the constant symbol c is interpreted by some indiviual d in D_M. That is $i_{M_{c/d}}(c) = d$. There exists M' in M^c, such that for every atomic sentence of the form $p(t)$, where t is a tuple that contains at least one occurence of c, we have $i_{M'}(t) \in i_{M'}(p)$ iff $i_{M_{c/d}}(t) \in i_{M_{c/d}}(p)$. That is because M^c contains all the variants of M with regard to c.

Since we have $NA(F(c), c)$, from definition 4, $M \models F(c)$ implies $M' \models F(c)$. Then, we have $M_{c/d} \models F(c)$. The same conclusion can be drawn for any element in D_M. That is, for every d in D_M we have $M_{c/d} \models F(c)$, which from the definition of satisfiability means $M \models \forall x F(x)$.

We have shown that $\forall M(M \models F(c) \quad \Rightarrow \quad M \models \forall x F(x))$. Then we have $\models F(c) \rightarrow \forall x F(x)$. From properties of first order logic we also have $\models \forall x F(x) \rightarrow F(c)$. Then, finally, we have $\models F(c) \leftrightarrow \forall x F(x)$. □

One might intuitively explain Property 5 by the fact that sentences that contain no restricted quantifiers and are not about c are sentences where sub-formulas in the scope of quantifiers are tautologies or contradictions. However, this explanation would be wrong. Take, for example, the sentence $p(a) \wedge \exists x(p(x) \vee q(x))$, which is logically equivalent to $p(a)$.

We now present several “negative” properties, that is properties that one may expect to hold, but in fact do not hold.

Property 6. The fact that sentences F and G are both about object c does not imply that sentence $F \vee G$ is about object c.

In short: $A(F, c)$ and $A(G, c) \not\Rightarrow A(F \vee G, c)$.

Proof. Take $F = p(c)$ and $G = \neg p(c)$. It is easy to check that both F and G are about c, while $p(c) \vee \neg p(c)$ is not about c.

Property 7. The fact that sentences F and G are both about object c does not imply that sentence $F \wedge G$ is about object c.

In short: $A(F, c)$ and $A(G, c) \not\Rightarrow A(F \wedge G, c)$.

Proof. Take the same instance of F and G as in the proof of Property 6.

Using contraposition, Properties 6 and 7 could respectively be presented in the form:

$NA(F \vee G, c) \not\Rightarrow NA(F, c)$ or $NA(G, c)$

$NA(F \wedge G, c) \not\Rightarrow NA(F, c)$ or $NA(G, c)$

Property 8. If sentence F logically implies G it is not necessarily the case that $A(F, c)$ implies $A(G, c)$.

In short: $\models F \rightarrow G \not\Rightarrow (A(F, c) \Rightarrow A(G, c))$.

Proof. Take $F = p(a) \wedge p(c)$ and $G = p(a)$. We have $\models F \rightarrow G$. We can also check that we have $A(p(a) \wedge p(c), c)$but we do not have $A(p(a), c)$.

Property 9. If sentence F logically implies G it is not necessarily the case that $A(G, c)$ implies $A(F, c)$.

In short: $\models F \rightarrow G \not\Rightarrow (A(G, c) \Rightarrow A(F, c))$.

Proof. Take $F = p(a)$ and $G = p(a) \vee p(c)$. We have $A(p(a) \vee p(c), c)$ because there is an intepretation M where $p(a) \vee p(c)$ is true, because $p(a)$ is false and $p(c)$ is true, and M has a variant where $p(a)$ is false and $p(c)$ is false, that is, where $p(a) \vee p(c)$ is false. Moreover we do not have $A(p(a), c)$.

Using contraposition, Properties 8 and 9 could respectively be presented in the form:

$\models F \rightarrow G \not\Rightarrow (NA(G, c) \Rightarrow NA(F, c))$

$\models F \rightarrow G \not\Rightarrow (NA(F, c) \Rightarrow NA(G, c))$

The most important properties are listed below.

$$
\begin{array}{l}
\models F \leftrightarrow G \Rightarrow (NA(F,c) \Leftrightarrow NA(G,c)) \\
NA(F,c) \Leftrightarrow NA(\neg F,c) \\
NA(F,c) \text{ and } NA(G,c) \Rightarrow NA(F \vee G,c) \\
NA(F,c) \text{ and } NA(G,c) \Rightarrow NA(F \wedge G,c) \\
\models F \leftrightarrow G \Rightarrow (A(F,c) \Leftrightarrow A(G,c)) \\
A(F \vee G,c) \Rightarrow A(F,c) \text{ or } A(G,c) \\
A(F \wedge G,c) \Rightarrow A(F,c) \text{ or } A(G,c)
\end{array}
$$

$$
\begin{array}{l}
\models F \rightarrow G \not\Rightarrow (NA(F,c) \Rightarrow NA(G,c)) \\
\models F \rightarrow G \not\Rightarrow (NA(G,c) \Rightarrow NA(F,c)) \\
\models F \rightarrow G \not\Rightarrow (A(F,c) \Rightarrow A(G,c)) \\
\models F \rightarrow G \not\Rightarrow (A(G,c) \Rightarrow A(F,c))
\end{array}
$$

So far, in this paper, the property of aboutness has only been defined in the semantics. An important issue is to find a corresponding definition in the syntax. That is to find a syntactical characterisation of the set of sentences that are, or are not, about an object c. Unfortunately, at the present time we do not have such a complete characterisation. However, we shall present in the following a syntactical characterisation and we conjecture that its extension by logical equivalence is complete,

Definition 5. Syntactical definition of language L'_c.

We define a sub set L'_c of the first order language L_c as follows:

1. If p is an n-ary predicate and t is a n-tuple of terms with no occurence of the constant symbol c, then $p(t) \in L'_c$.
2. If $F \in L'_c$ and $G \in L'_c$, then $(\neg F) \in L'_c$ and $(F \vee G) \in L'_c$.
3. if $F \in L'_c$, then $(\exists x \neq c\ F) \in L'_c$.
4. All the sentences in L'_c are defined by rules 1, 2 and 3.

Property 10. If F is a sentence in L'_c then F is not about the object c.

In short: $F \in L_c \Rightarrow NA(F,c)$.

Proof. The proof is by induction on the number n of logical connectives or quantifiers in sentence F.

For $n = 0$ we have F of the form $p(t)$. If F is in L'_c there is no occurence of constant c in t. Then, if $p(t)$ is true in an interpretation M, from Definition

3 of variants of M, $p(t)$ is true in all the variants in M^c, which means, from Definition 4, that $p(t)$ is not about object c.

For $n > 0$.

If F is of the form $F = \neg G$, from the definition of L'_c, G is in L'_c. Then, by the induction hypothesis, we have $NA(G, c)$, and, from property 2, we have $NA(\neg G, c)$.

If F is of the form $F = G \vee H$, from the definition of L'_c, G and H are in L'_c. Then, by the induction hypothesis, we have $NA(G, c)$ and $NA(H, c)$, and by Property 3, we have $NA(G \vee H, c)$.

If F is of the form $F = \exists x \neq c\ G$, if M is an interpretation such that $M \models \exists x \neq c\ G$, from the definition of satisfiability conditions there exists an interpretation $M_{x/d}$ such that (1) $i_{M_{x/d}}(x) \neq i_{M_{x/d}}(c)$ and (2) $M_{x/d} \models G$.

From the definition of L'_c, G is in L'_c, and by the induction hypothesis we have $NA(G, c)$. Then, from (2), in every variant M' in $M^c_{x/d}$ we have $M' \models G$.

Since the interpretation of variable symbols and constant symbols is not changed in the variants of an interpretation, for every M' in $M^c_{x/d}$ we have $i_{M'}(x) \neq i_{M'}(c)$.

Therefore we have (3) $\forall M' \in M^c_{x/d}$ ($M' \models G$ and $i_{M'}(x) \neq i_{M'}(c)$).

Let M'' be a variant of M, that is $M'' \in M^c$. The interpretation $M''_{x/d}$ which only differs from M'' by the interpretation of the variable symbol x (that is $i_{M''_{x/d}}(x) = d$) is a variant of $M_{x/d}$. That is $M''_{x/d} \in M^c_{x/d}$. Then, from (3), we have $M''_{x/d} \models G$ and $i_{M''_{x/d}}(x) \neq i_{M''_{x/d}}(c)$, and, by the definition of satisfiability, we have $M'' \models \exists x \neq c\ G$.

Since $\exists x \neq c\ G$ is true in all the variants M'' in M^c, $\exists x \neq c\ G$ is not about object c. $\Box$

A trivial consequence of Properties 1 and 10 is that any sentence in L_c logically equivalent to some sentence in L'_c is not about object c. In short:

($\models F \leftrightarrow G$ and $G \in L'_c$) $\Rightarrow NA(F, c)$

We have not found any counter example showing that the implication in the other direction does not hold.

As pointed out by Goodman in [7] the fact that a sentence mentions object c is not necessarily related to the fact that this sentence is about object c.

First, the fact that c does not occur in a sentence F does not imply that F is not about c. For instance, we have $A(\exists x p(x), c)$ and $A(\forall x p(x), c)$. This can be intuitively understood if we think of $\exists x p(x)$ (resp. $\forall x p(x)$) as an infinite disjunction (resp. conjunction) whose one disjunct (resp. conjunct) is $p(c)$. Notice that for restricted quantifiers we have $NA(\exists x \neq c\ p(x), c)$ and $NA(\forall x \neq c\ p(x), c)$.

Second, the fact that c occurs in a sentence F does not imply that F is about object c. Examples that come in mind are tautologies of the form $G(c) \vee \neg G(c)$ where c occurs in G. But these are not the only examples. Since $\models F \rightarrow G$ implies $\models F \leftrightarrow F \wedge G$, if we have $NA(F, c)$ and c occurs in G, then, by Property 1, we have $NA(F \wedge G, c)$ and c occurs in $F \wedge G$. Take, for example, $F = p(a)$ and $G = p(a) \vee p(c)$; we have $NA(p(a) \wedge (p(a) \vee p(c)), c)$.

3 Theories about an object

In the previous section we have defined the notion of aboutness for a sentence alone. Properties 7 and 8 have shown that aboutness is not stable with regard to conjunction with other sentences, or with regard to logical consequence. That means that we also have to consider aboutness of a sentence in a particular "context". That is, if we understand the fact that sentence F is about object c in the sense that, if we know that F is true then we know something about the object named c, then it may be that when we learn that F is true we learn or we do not learn something about c, depending on the context of the overall set of knowledge in which the information that F is true has been acquired.

For instance, if we acquire the fact that $p(a) \vee p(c)$ is true in an empty context, that is, in a context where we **only know** that $p(a) \vee p(c)$ is true, then we learn something about object c. Indeed, if in addition we learn that $p(a)$ is false then we know that $p(c)$ is true. However, if we are in a context where we know that $p(a)$ is true, then the fact that $p(a) \vee p(c)$ is true tell us nothing about the object c, because there is no situation compatible with this context where $p(a) \vee p(c)$ may be false.

The intuitive idea is that when we know that a set of sentences is true, to determine whether this set of sentences informs us about c, we cannot consider each sentence independently.

For instance, if we know that $murder(John) \vee murder(Peter)$ is true, to determine whether this sentence tells us something about $Peter$, we have to distinguish, on the one hand, situations where we also know that $murder(John)$ is true, since in that situations $murder(John) \vee murder(Peter)$ gives no information about $Peter$, and, on the other hand, situations where we only know that $murder(John) \vee murder(Peter)$ is true, since in those situations we do know something about $Peter$.

These comments suggest that it is useful to define the notion of aboutness for a set of sentences which is closed under logical consequence, that is, for what we call a "theory". The idea is to define a theory T that is about object c as a theory that informs us about c, in the sense that there are models of T who have variants with regard to c which are not models of T.

Let $\mathcal{F}$ be a set of sentences in L_c. We say that $\mathcal{F}$ is a base for a theory T iff T is the closure of $\mathcal{F}$ under logical consequence. This is denoted by $Base(T) = \mathcal{F}$.

Let us consider, for instance, a theory T_1 such that $Base(T_1) = \{p(a)\}$, and T_2 such that $Base(T_2) = \{p(a) \vee p(c)\}$. It is assumed that both theories are defined on the same language. The theory T_2 is about object c, or informs about c, because we can find a model M of T_2, and a variant of M with regard to c, which is not a model of T_2. In the other case, though sentence $p(a) \vee p(c)$ belongs to theory T_1, every variant of every model of T_1, is a model of T_1, and sentence $p(a) \vee p(c)$ is true in every variant, because $p(a)$ is true in all these variants. That means that theory T_1 is not about object c though it contains a sentence which, if it is considered alone, is about object c.

Theory T_1 is more informative than theory T_2, in the sense that it has less models than T_2, but it is less informative than T_2 about c. Intuitively, when we know that T_1 is true, to know whether $p(c)$ is true we have to know that $p(c)$ iteslf is true, while when we know that T_2 is true, to know whether $p(c)$ is true we only have to know that $p(a)$ is false.

Now, we give a formal definition of aboutness for a theory.

Definition 6. Theories that are not about an object.

Let T be a theory formed with sentences of language L_c. We say that T is not about an object named with the constant symbol c iff for every interpretation M, M is a model of T iff every interpretation M' in M^c is a model of T.

The fact theory T is not about object c is denoted by $NA'(T, c)$ and the fact M is a model of T is denoted by $M(T)$. In short:

$$NA'(T,c) \text{ holds iff } \forall M(M(T) \text{ iff } \forall M' \in M^c\ M'(T))$$

We say that T is about the object c, if it is not the case that we have $NA'(T,c)$. This fact is denoted by $A'(T,c)$. In short terms we have:

$$A'(F,c) \text{ holds iff } \exists M(\exists M' \in M^c(M(T) \text{ and not } M'(T))$$

The set of models of a theory T is denoted by $\mathcal{M}(T)$, and the set of variants of models of T, that is, the set of interpretations M' such that there exists M in $\mathcal{M}(T)$ such that $M' \in M^c$, is denoted by $\mathcal{M}^c(T)$.

Definition 7. Restriction of a theory with regard to an object.

Let T be a theory, we say that the theory T^c is the restriction of theory T with regard to object c iff T^c is the theory whose set of models is the set of variants with regard to c of the models of T.

In short T^c is the theory such that $\mathcal{M}(T^c) = \mathcal{M}^c(T)$.

Property 11. For any theory T, the theory T^c defined in Definition 7 is not about object c.

In short we have $NA'(T^c, c)$.

Proof. Let M be a model of T^c. By definition of $\mathcal{M}(T^c)$, M is a variant of a model of T. The variants of M are also variants of some model of T. Then, they are in $\mathcal{M}(T^c)$, and therefore they are models of T^c.

Notice that since $\mathcal{M}(T)$ is included in $\mathcal{M}(T^c)$, T^c is included in T. T^c is the largest theory included in T which is not about object c, because T^c contains all the sentences which are true in every model in $\mathcal{M}^c(T)$.

4 Research directions for extension to equality

An interesting possible extension of the work presented in this paper is to consider a first order language with an equality predicate.

In the definition of $NA(F,c)$ we have considered variants M' of a given interpretation M, where the interpretation of variable symbols and constant symbols are the same in M' and in M, and where the only changes are about the predicate interpretations with regard to the domain element d which interprets the constant symbol c, that is such that $d = i_M(c)$.

However, in cases where d is also the interpretation of another constant symbol, for instance when $i_M(a) = i_M(c)$, predicate interpretations are not changed with regard to d. The reason is that we wanted to know whether modifications of the truth value of properties about c, like $p(..., c, ...)$, can change the truth value of a given sentence F, but we did not want to modify the truth values of properties about a like $p(..., a, ...)$.

Now, if we extend the language with an equality predicate, the first question that comes in mind is: "*do we have to change, in the variants of M, the interpretation of the equality predicate, as we did for other predicates?*". A possible answer is "*no*".

An argument in this sense is that, if we change the interpretation of equality, we also have to change the interpretation of constant symbols. For example, if in M we have $i_M(a) = i_M(c)$ and $i_M(c) = d$, $a = c$ is true in M, and, if there is a variant M' where $a = c$ is false, in M' we have $i_{M'}(a) \neq i_{M'}(c)$, that is $i_{M'}(a) \neq d$ or $i_{M'}(c) \neq d$. That would lead to a change in the meaning of the constant symbols in the variants of M. If we change the meaning of constant symbols, we change the objects named by the constant symbols in the variants of M, and it does not make sense any more to say that a sentence informs about an object named by the constant symbol c if the meaning of c changes from variants to variants.

Then, another question is : "*do we have to change the definition of variants?*". We think that a natural answer is "*yes*". In particular in the situations where **we know** that $a = c$. Inded, if we know, for instance, that $p(a) \wedge a = c$ is true, we know that $p(c)$ is true. So, it is quite obvious that sentence $p(a) \wedge a = c$ is about c. If we do not change the definitions presented in section 2, and if equality is treated like any other predicate, we face the conclusion that $p(a) \wedge a = c$ is not about c.

Therefore, we propose to change the definition of variants in Definition 3 in such a way that, **in a context where we know** that, for instance, $a = c$, the interpretation of predicates is also changed for the domain element which is is the interpretation of c, even if this element is also the interpretation of a constant symbol different of c. According to this new definition, though we have rigid interpretation of constant symbols, if $p(a)$ is true in M and we know that $a = c$, the truth value of $p(a)$ will change in the variants of M.

In other contexts where we have no information about $a = c$ Definition 3 should not be changed. In more formal terms, when $M \models p(a) \wedge a = c$, the new definition should make a distinction between the case where we consider sentence $F = p(a) \wedge a = c$, and the case where we consider the sentence $G = p(a)$.

This definition would lead to the conclusion that $a = c$ is not about c. That seems to be counterintuitive, but it is not so odd if the **only** information we know is $a = c$. The reason why we may intuitively believe that $a = c$ informs about c is that, if we know some fact about a, we know the same fact about c, but if we know nothing about a, from $a = c$ we can infer nothing about c.

However, the problem requires a more detailed analysis for sentences which are not of the form $F(a) \wedge a = c$. For instance, we intuitively understand that $p(a) \wedge (a = b \vee a = c)$ should be about c, as it is the case for $p(a) \wedge (p(b) \vee p(c))$. But $p(a) \wedge (a = c \vee \neg a = c)$ should not be about c. And what about sentences of the kind $p(a) \rightarrow a = c$ or $(p(a) \wedge a = c) \vee (q(b) \wedge b = c)$?

5 Conclusion

A formal definition of the concept of aboutness has been given semantically for sentences and for theories. We have also proved properties that show that the notion of not being about an object is compositional for logical connectives, but not for quantifiers. The notion of being about is compositional only for negation. Moreover, these notions are closed neither under logical consequence nor under logical antecedents. Many of these results are not intuitive, but they are consistent with properties of the notion of being absolutely about demonstrated by Goodman in [7].

The language L'_c gives a syntactical characterisation of a sub set of sentences that are not about the object named by c. It is a basic open question to know whether the class of sentences that are about c is decidable.

We have also given the definition of a theory being, or not being, about an object. In the case where a theory is about an object we have defined the largest sub-theory that is not about this object. This may be used as a foundation for the definition of the operation of "retracting" an object from a theory.

There are many open problems related to the formal definition of aboutness. One is to investigate to what extent the concept of aboutness for topics, for instance as is introduced by Demolombe and Jones in [5] (see also [6, 8, 9]), is related to the concept of aboutness for objects. We guess they are orthogonal, because the fact that a sentence is about a given topic mainly depends on the meaning of the sentence, it is independent of its extension. In particular two logically equivalent sentences are not necessarily about the same topics, while they are about the same object.

In the area of belief revision many people have attempted to characterise parts of a theory that are invariant with respect to contraction of the theory by a sentence [3]. It is rather tempting to use the notion of aboutness for that purpose.

The definition of T^c (Definition 7) is not constructive. An interesting open problem is to find a constructive method to determine the base of a theory T^c as a function of the base of T.

To answer queries of the form "tell me every thing you know about John" we need to define inference rules to derive consequences of the basis $\mathcal{F}$ of a theory T that are about a given object c, in the context of T. That is, sentences that are true in some model M of T, and are false in some variant M' in M^c. If we regard $\mathcal{F}$ as the conjunction sentences that are in $\mathcal{F}$, from Property 9 we know that if $\mathcal{F}$ is about object c, it is not necessarily the case that any consequence G of $\mathcal{F}$ is about c. Then, a practical problem is to find additional conditions on $\mathcal{F}$ and G that allow us to restrict the generation of consequences to those that are about c.

Finally, an interesting field for future investigation is to try to extend the automated deduction method, based on the connection method, defined by Wolfang Bible [1, 2], to sentences that are about the same objects. The intuitive idea is to connect two sentences that are about at least one common object, in order to make more efficient the derivation of sentences that are about a given object.

Acknowledgements. We are very grateful to the anonymous referee who has made many valuable comments about the paper, and who suggested interesting changes or extensions. We also want to thank David Pearce for his great help in the preparation of this paper.

References

References

[1] W. Bibel. Matrices with Connections. *Journal of the ACM*, 28:633–645, 1981.

[2] W. Bibel. Mating in Matrices. *Cmmunications of the ACM*, 26:844–852, 1983.

[3] C. Alchourron P. Gardenfors and D. Makinson. On the logic of theory change : Partial meet contraction and revision functions. *The journal of symbolic logic*, 50(2), 1985.

[4] R. Carnap. The logical syntax of language. 1937.

[5] R. Demolombe and A.J.I. Jones. On sentences of the kind "sentence "p" is about topic "t": some steps toward a formal-logical analysis. In H-J. Ohlbach and U. Reyle, editor, *Logic, Language and Reasoning. Essays in Honor of Dov Gabbay.* Kluwer Academic Press, 1998.

[6] R.L. Epstein. *The Semantic Foundations of Logic, Volume1: Propositional Logic.* Kluwer Academic, 1990.

[7] N. Goodman. About. *Mind*, LXX(277), 1961.

[8] L. Fariñas del Cerro and V. Lugardon. Sequents for dependence logic. *Logique et Analyse*, 133-134:57–71, 1991.

[9] D. K. Lewis. Relevant implication. *Theoria*, LIV(3), 1988.

[10] H. Putnam. Formalization of the concept "About". *Philosophy of Science*, XXV:125–130, 1958.