ON SENTENCES OF THE KIND "SENTENCE 'P' IS ABOUT TOPIC T"

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1 Introduction

In philosophy, and in the study of non-classical logics, a good deal of interest has been shown in the phenomenon of *aboutness*, and in the formal explication of the idea that a given sentence is about some particular class of topics. More recently, in computer science, it has become clear that *aboutness* is of practical interest for a number of application domains.

These trends, both theoretical and practical, are described and discussed below in Sections 3 and 4, while in Sections 1 and 2 we first present our own steps towards an analysis of *aboutness*.

Four fundamental assumptions guide our approach in this paper:

1. Sentences of the type "sentence 'p' is about topic t" are shorthand for "the proposition expressed by the sentence 'p' is about topic t", where the notion of proposition is interpreted in a way which conforms to our use of a three-valued semantics of the Bochvar kind [2]. According to this interpretation, a proposition can be viewed as a set of partially defined worlds, that is, a set of worlds in relation to which some of the members of the set of all propositional variables may be undefined, but where the given proposition itself is true.
2. We shall assume that any particular proposition concerns a set of topics. And thus we shall say that the sentence 'p' is about some topic t provided t is a member of that set of topics with which the sentence 'p' is concerned.
3. Suppose that two sentences are equivalent in classical propositional calculus, and that they contain just the same atoms. Then we shall suppose that the one is about a given topic t if and only if the other is also about t. But note the proviso concerning "contains the same atoms". For instance, although the following is a truth of classical

propositional logic:

$$p \leftrightarrow (p \wedge (q \vee \neg q))$$

we are not prepared to allow that the sentences on the left-hand side and on the right-hand side of the equivalence sign are necessarily about the same topic.

Our reason may be explained as follows: suppose that the sentence 'q' is about some topic t, but that the sentence 'p' is not about t. Given that the sentence 'q' is about t, we would want to accept that '$q \vee \neg q$' might also be about topic t, and thus that t *may be* among the topics that '$p \wedge (q \vee \neg q)$' is about. From which it follows, obviously, that although the sentences 'p' and '$p \wedge (q \vee \neg q)$' are logically equivalent, they nevertheless *need not* be about the same topics.

One objection to this argument would be the contention that tautologies are not about any topic at all. We shall have a little more to say about that issue in our discussion below of related work by Nelson Goodman and David Lewis.

4. We shall assume that the set of topics a *compound* sentence is about need not be independent of its *mode of composition.* Consider, for instance, the following two sentences: 'Maria is married to Jim', 'Maria is married to Jules'. Considered separately, and in the absence of any specific presuppositions to the contrary, neither of these two sentences is about the topic of bigamy, although their conjunction most certainly is. So the conjunctive mode of combining the two sentences may itself alter the class of topics concerned.

2 Syntax and Semantics

In this section we first present formal definitions of the language, of the models, and of the truth-conditions. We then consider several additional model-theoretic conditions which, were they to be adopted, would give a richer structure to a set of sentences that are about some topic.

2.1 Formal Ccharacterisation of the Llanguage L

Let CPC be *the language of* classical propositional calculus. Let σ be a set of names for sentences of CPC. Let τ be a set of names of topics. Let $A(-,-)$ be a sorted binary predicate whose first argument is of sort name of topic (τ), and whose second argument is of sort name of sentence (σ).

The language L is defined by the following rules:

- any atomic sentence of CPC, and any ground atom of the form $A(x, y)$ are sentences of L. (The arguments x, y must of course satisfy the restrictions as to sort mentioned above.)
- if p and q are any sentences of L, then $\neg p$ and $p \vee q$ are sentences of L;
- the logical connectives $\wedge$, $\rightarrow$ and $\leftrightarrow$, are introduced in the usual way, in terms of negation and disjunction;
- all the sentences of L are defined by the above rules.

Notation: 'p' names the sentence occurring between the quotation marks in 'p'. Intuitively, we read sentences of the form $A(t, `p\text{'})$ as: "the sentence 'p' is about the topic t".

Some illustrative examples of sentences of L:

- $A(t, `p\text{'}) \wedge (p \rightarrow q)$, "the sentence '$p$' is about t and if p then q";
- $A(t, `p \wedge q\text{'}) \rightarrow A(t, `q \wedge p\text{'})$, " if the sentence '$p \wedge q$' is about t, then the sentence '$q \wedge p$' is about t";
- $A(t, `p\text{'}) \rightarrow A(t', `p\text{'})$, "if the sentence 'p' is about t, then the sentence 'p' is about t' ".

2.2 Models

A model M of the logic is a tuple $M = \langle W, I, J, \mathcal{T}, \mathcal{S}, N, T, F \rangle$ where:

- W is a set of worlds;
- I is a function that assigns to each topic name in L an element of $\mathcal{T}$;
- J is a function that assigns to each sentence name in L an element of $\mathcal{S}$;
- $\mathcal{T}$ is a set of topics;
- $\mathcal{S}$ is the set of sentences of CPC;
- N is a function which assigns sets of topics to pairs of sets of worlds. The definition domain of N is: $2^W \times 2^W \rightarrow 2^{\mathcal{T}}$;
- T is a function which assigns to each atom in CPC a set of worlds;
- F is a function which assigns to each atom in CPC a set of worlds.

We impose on M the constraint that the name "p" is assigned, by the function J, the sentence 'p' itself.

We also impose on the models M the constraint $T(p) \cap F(p) = \emptyset$. The intuition is that, in a given world, a sentence 'p' can be true or false, or neither true nor false, but it cannot be both true and false.

The functions T and F are extended to any sentence in CPC by the following rules:

$$\begin{aligned} T(\neg p) &= F(p) \\ F(\neg p) &= T(p) \\ T(p \vee q) &= (T(p) \cap D(q)) \cup (T(q) \cap D(p)) \\ F(p \vee q) &= F(p) \cap F(q) \end{aligned}$$

where $D(p)$ is an abbreviation of $T(p) \cup F(p)$.

The functions T and F define a 3-valued logic for sentences in CPC. This logic was initially defined by Bochvar in [2]. Independently of our work the same logic has also been used by Buvač, Mason and McCarthy in [3, 16] to characterize the set of meaningful sentences in a given context.

It has been shown by Demolombe in [9] that for any sentences 'p' and 'q' of CPC we have $T(p) = T(q)$ and $F(p) = F(q)$ for every model M iff we have $\vdash p \leftrightarrow q$, in classical propositional calculus, and 'p' and 'q' are formed with the same atoms. However, if we have only the property $T(p) = T(q)$ for every model, then 'p' and 'q' are not necessarily formed with the same atoms. Consider, for instance, $p = a \wedge \neg a$ and $q = b \wedge \neg b$, where a and b are atoms.[1]

These results indicate that in order to characterize the set of topics of a given sentence 'p' in a way which conforms to our third initial assumption, it is not enough to consider $T(p)$ alone; an alternative which suggests itself, and which we have employed here, is to consider the pair: $\langle T(p), F(p)\rangle$.

If we give up the constraint $T(p) \cap F(p) = \emptyset$ we get a 4-valued logic of the kind suggested by David Lewis in [15]. However, it has been shown in [9] that, even in this 4-valued logic, the fact that $\vdash p \leftrightarrow q$ (where 'p' and 'q' are formed with the same atoms) is not equivalent to the fact that $T(p) = T(q)$ for every model. Consider for instance, $p = (a \wedge \neg a) \wedge b$ and $q = (a \wedge \neg a) \wedge \neg b$. In the 4-valued logic $T(p)$ and $T(q)$ are different whenever there exist a world w_1 where a is true and false and b is only true, and a world w_2 where a is true and false and b is only false.

The truth conditions for sentences in L are:

$$
\begin{array}{lll}
M, w \models p & \text{iff} & w \in T(p), \text{ if } p \text{ is an atom of CPC.} \\
M, w \models \neg p & \text{iff} & M, w \not\models p. \\
M, w \models p \vee q & \text{iff} & M, w \models p \text{ or } M, w \models q. \\
M, w \models A(t, `p\text{'}) & \text{iff} & I(t) \in N(T(J(`p\text{'})), F(J(`p\text{'}))).
\end{array}
$$

(In the following, we shall abbreviate $J($'p'$)$ to p.)

The last of the four truth conditions corresponds to our second assumption, above, and says that '$A(t,$ 'p'$)$' is true if and only if the topic t is one of the topics assigned by the function N to the proposition expressed by the sentence 'p'.

2.3 Some Interesting Schemas

We understand validity of a sentence schema in the usual way, as truth in all worlds in all models. Among the schemas which the semantical conditions given in the previous section do *not* validate is:

[1] The appendix to this paper presents a summary of some of the central results of [9].

$$A(t, \text{‘}p \wedge q\text{’}) \rightarrow (A(t, \text{‘}p\text{’}) \vee A(t, \text{‘}q\text{’})). \quad (i)$$

This (negative) result is important in relation to our fourth initial assumption, above, where we claimed that the mode of composition of a compound sentence may play a significant role in determining the class of topics the compound is about. Thus ‘$p \wedge q$’ might be about the topic of bigamy, whilst neither ‘p’ nor ‘q’ is about bigamy.

By contrast, a good case can perhaps be made for maintaining that where two or more sentences are each about the same topic, then their conjunction is also about this topic, i.e.,

$$(A(t, \text{‘}p\text{’}) \wedge A(t, \text{‘}q\text{’})) \rightarrow A(t, \text{‘}p \wedge q\text{’}). \quad (ii)$$

The validity of (ii) may be secured by imposing the following constraint on function N:

$$\begin{array}{ll} \text{If} & I(t) \in N(T(p), F(p))) \\ \text{and} & I(t) \in N(T(q), F(q))) \\ \text{then} & I(t) \in N(T(p \wedge q), F(p \wedge q))). \end{array} \quad (cii)$$

Furthermore, we have found no good reason to deny that if a sentence is about some topic t, then its negation is also about that topic[2]:

$$A(t, \text{‘}p\text{’}) \rightarrow A(t, \text{‘}\neg p\text{’}). \quad (iii)$$

The corresponding constraint on N is:

$$\begin{array}{ll} \text{If} & I(t) \in N(T(p), F(p))) \\ \text{then} & I(t) \in N(T(\neg p), F(\neg p))). \end{array} \quad (ciii)$$

As was explained above in 2.2, when two sentences containing just the same atoms are classically equivalent, then the one is about a given topic if and only if the other is about that topic. That is:

$$\begin{array}{ll} \text{If} & \models p \leftrightarrow q \text{ and } p \text{ and } q \text{ contain just the same atoms} \\ \text{then} & \models A(t, \text{‘}p\text{’}) \leftrightarrow A(t, \text{‘}q\text{’}). \end{array} \quad (iv)$$

From (iv) and (iii) it follows that:

$$A(t, \text{‘}p\text{’}) \leftrightarrow A(t, \text{‘}\neg p\text{’}) \quad (v)$$

is a valid schema.

[2]In connection with formula (iii), we should like to mention a very interesting comment made by Alice ter Meulen at the AAAI workshop *Formalizing Context* (Boston, 1995), where an earlier draft of this paper was presented. She pointed out that (iii) could be accepted only if one assumes that the presuppositions of sentence ‘p’ remain unchanged when ‘p’ is negated. For instance, if we assume that the sentence ‘Venus is a star’ is about the topic astronomy, then the sentence ‘Venus is not a star’ is also about astronomy, provided that the presupposition that Venus is an object in the sky remains unchanged. If this presupposition were to be changed, for instance if Venus were presupposed to be a film actress, then ‘Venus is not a star’ would no longer be about astronomy. The assumption that presuppositions must be kept unchanged applies, presumably, also to (ii), and quite generally in the evaluation of questions about validity/invalidity of schemas containing several occurrences of formulas of the type ‘$A(t, \text{‘}p\text{’})$’.

Given now that our fourth basic assumption above, dictates that schema (i) is not valid, a consequence is that the following schema is also not valid:

$$A(t, `p \vee q\text{'}) \rightarrow (A(t, `p\text{'}) \vee A(t, `q\text{'})). \qquad (vi)$$

For suppose $A(t, `p \wedge q\text{'})$; then by (iv) $A(t, `\neg(\neg p \vee \neg q)\text{'})$ and by (v) and (iv) $A(t, `\neg p \vee \neg q\text{'})$. Acceptance of (vi) would then yield $(A(t, `\neg p\text{'}) \vee A(t, `\neg q\text{'}))$, which by (v) gives $(A(t, `p\text{'}) \vee A(t, `q\text{'}))$, which would mean that (i) is valid.

The existing literature (see Section 4) hardly reflects a consensus regarding the logical properties of 'aboutness'; so, no doubt, some will disagree with our claims regarding which schemas are valid, and which are not. Our *firmest* intuitions lead us to accept (iv) and our inclination is to accept (ii) and to reject (i); this means that (iii) and (vi) cannot *both* be accepted. These are just preliminary conclusions. Much remains to be done by way of further investigation of possible constraints on the function N. However, as will become clear from Section 3 below, a final decision about which axiom schemas to adopt can be made only in relation to consideration of a particular domain of application.

2.4 Axiomatics

The axiomatic characterization of the logic is the result of a rather straightforward translation of the schemas which are retained for the semantics.

In the axiomatics, in addition to the axiom schemas of classical propositional calculus and the inference rule Modus Ponens, we have the following weakened rule of equivalence:

$$\text{(REA)} \ \frac{\vdash p \leftrightarrow q \text{ and } p \text{ and } q \text{ contain the same atoms}}{A(t, `p\text{'}) \leftrightarrow A(t, `q\text{'})}.$$

If we accept schemas (ii) and (iii) we have the corresponding axiom schemas:

$$(A(t, `p\text{'}) \wedge A(t, `q\text{'})) \rightarrow A(t, `p \wedge q\text{'}) \qquad (aii)$$
$$A(t, `p\text{'}) \rightarrow A(t, `\neg p\text{'}). \qquad (aiii)$$

A proof of completeness of the logic similar to the proof of completeness given in [5, 4], and based on results in [9], should be forthcoming without any particular difficulties. The proof of soundness is quite easy.

3 Application Domains for Topics

We present below three application domains where we have to characterize sets of sentences in terms of their meaning, and where topics seem to be appropriate for this purpose.

When an agent puts a query to a database system, the system can help the agent by providing him with additional information relevant to the query. This kind of system behaviour is usually called 'cooperative answering'. In [6, 7] Cuppens and Demolombe have developed a method for cooperative answering based on the use of topics. Roughly speaking, if the query is the sentence 'p', and if 'p' is about the topic t, then t is identified as a topic of interest for the agent, and the system returns, in addition to the answer to 'p', other sentences that are about the agent's topics of interest, and which are consequences of the database. Here, topics permit the characterization of the set of sentences an agent is interested in. The idea that, if an agent a is interested in a topic t he is interested in all the sentences 'p' about this topic, could be represented by an axiom schema of the form: (1) $IT_a(t) \wedge A(t, `p') \rightarrow I_a(p)$, where $IT_a(t)$ means that a is interested in all the sentences about t, and $I_a(p)$ means that a is interested in 'p'.[3]

In this context, if it happens that the database system concerned cannot answer the query 'p', but it can answer the query '$p \vee q$', it will be useful for the agent to know at least whether '$p \vee q$' is true, provided that '$p \vee q$' is informative in relation to 'p' in the context of this database.[4] Then, we would like to be able to infer from the fact that agent a is in interested in 'p' that he is also interested in '$p \vee q$'. This requirement could be satisfied in two different ways. We could add to the axiom schema (1) either the axiom schema: (2) $I_a(p) \rightarrow I_a(p \vee q)$ or the axiom schema: (vii) $A(t, `p') \rightarrow A(t, `p \vee q')$. Indeed, from the assumptions that agent a is interested in topic t, and that sentence 'p' is about t, represented by $IT_a(t)$ and $A(t, `p')$, we can infer from (1) and (vii) that $I_a(p \vee q)$. We see that the structure of the set of sentences about the topic $t$ is imposed on the set of sentences agent $a$ is interested in. More formally from (1) and (vii) we can infer: $IT_a(t) \rightarrow ((A(t, `p') \rightarrow A(t, `p \vee q')) \rightarrow (I_a(p) \rightarrow I_a(p \vee q)))$.

Another application domain for topics is the characterization of the reliability of agents who insert data in a database. Informally, Demolombe and Jones in [10] define an agent to be reliable for the sentence 'p' iff it is guaranteed that, if he inserts 'p' in the database, then 'p' is true of the world. It might be much more convenient to define reliability of agents in terms of topics rather than in terms of sentences. For instance, in a company, one agent may be known to be reliable for all sentences about the topic 'accounting', another one for all sentences about the topic 'health'. The idea that if an agent a is reliable for a topic t and a sentence 'p' is about topic t, then he is reliable for the sentence 'p', could be represented by the axiom schema: (4) $RT_a(t) \wedge A(t, `p') \rightarrow R_a(p)$. Here, if we accept that $A(t, `p')$

[3] These comments anticipate future planned work. At this stage $IT_a(t)$ and $I_a(p)$ have not been formally defined, they should be considered just as convenient notations.

[4] We shall not go into the question of how to give a formal definition of 'informative' here, but some proposals are to be found in the database literature.

implies $A(t, `p \vee q')$, that is (vii), it follows that if an agent is reliable for t, he is reliable for 'p' and also for '$p \vee q$', that is: $R_a(p) \rightarrow R_a(p \vee q)$. This consequence in general is not acceptable because the reason why an agent believes p (if does so), and the reason why he believes $p \vee q$ (if he does so), may be completely independent. Then, in this application domain, the axiom schema (vii) should be rejected.

Finally, a third possible application domain of topics is the characterization of the set of sentences an agent is permitted or prohibited to access in a database. For example, an agent may be permitted or prohibited to access all the data which are about the topic 'nuclear energy'. Permissions to know and prohibitions to know, in terms of topics or in terms of sentences, could be represented by the axiom schema: (5) $PKT_a(t) \wedge A(t, `p') \rightarrow PK_a(p)$, where $PKT_a(t)$ means that agent $a$ is permitted to know all the sentences about topic $t$, and $PK_a(p)$ means that agent $a$ is permitted to know $p$; a similar axiom schema could be accepted for prohibition: (6) $FKT_a(t) \wedge A(t, `p') \rightarrow FK_a(p)$. In [8] Cuppens and Demolombe have shown that if an agent is permitted to know p, he is also permitted to know all the consequences of p, and, in particular $p \vee q$, that is: $PK_a(p) \rightarrow PK_a(p \vee q)$. Then the structure induced by the axiom schema (vii) on $PK_a(p)$ is acceptable. However, it has also been shown that if an agent is forbidden to know p he is not necessarily forbidden to know the consequences of p, and in particular $p \vee q$. That means that the structure induced by (vii) is not accepable for prohibitions. Notice also that the axiom schema (ii) $(A(t, `p') \wedge A(t, `q')) \rightarrow A(t, `p \wedge q')$, that we above indicated a willingness to accept, would induce on $PK_a(p)$ the axiom schema $PK_a(p) \wedge PK_a(q) \rightarrow PK_a(p \wedge q)$, and it has been argued in [8] that in general this result would not be acceptable.

4 Related Works

There are many works in the area of relevance logic (see [1, 11] and the collection of papers in [17]) which seem, at first glance, to have connections with our work. In fact there are many relevance logics and there are significant differences in their objectives. Nevertheless, roughly speaking, all of them have the objective of restricting derivations that can be made in classical logic to those derivations where consequences are relevant to antecedents. For example, in [12], Epstein defines several logics (called relatedness logics or dependence logics) in an attempt to formalize the notion of 'topic' or 'subject matter' or 'referential content'. For each logic two possible formalizations are proposed.

The first one is based on a 'relatedness' relation on sentences. Two sentences 'p' and 'q' are related, and this is denoted by $R(p, q)$, if their subject matters have something in common. For instance the subject matter of 'Ralph is a dog' is related to the subject matter of 'Dogs are faithful', or to that of 'George is a duck', but it is not related to the subject matter of the sentence '$2 + 2 = 4$'. It is assumed that one can say, for each pair of sentences, whether $R(p, q)$ holds or not. The structure of the relatedness relation is defined by the following properties (which are equivalent to properties described at p. 67 in [12]): R1: $R(p, q)$ iff $R(p, \neg q)$, R2: $R(p, q \wedge r)$ iff $R(p, q)$ or $R(p, r)$, R3: $R(p, q \rightarrow r)$ iff $R(p, q)$ or $R(p, r)$, R4: $R(p, p)$, R5: $R(p, q)$ iff $R(q, p)$. It is shown in [12] that two sentences are related iff they contain a common atom. A very important feature of the relatedness relation is that it is independent of the logical connectives that appear in the sentences and depends only on the relatedness of their atoms. "The subject matter of a proposition is independent of its truth value. As I see it, a virtual consequence of this assumption is that the logical connectives are neutral with respect to what a proposition is related to...", Epstein says. But he gives no justification for this statement.

Then he says: "one question that may strike you is: why don't we take 'relatedness' as a primitive in the language? The answer is because the binary relation of subject matter relatedness, call it R, is not a connective." And then, to reinforce his answer he adds: "It makes no sense to iterate it." However, Epstein does not consider the possibility of defining a language that extends classical propositional calculus with a predicate $R(x, y)$ in a way similar to our introduction of the predicate $A(x, y)$. The relation R is used in the definition of the semantics only to assign to the connective $\rightarrow$ a more restrictive meaning. According to his definition $p \rightarrow q$ is true iff it is true in the classical sense and $R(p, q)$ holds.

The second formalization is based on a function $s(p)$ which assigns to each sentence 'p' a set of topics. It is assumed that a set of topics is assigned to each atom, and that the set of topics of a compound sentence is just the union of the sets of topics assigned to each component. Here again we see that logical connectives play no role in the determination of the topics a sentence is about. It is possible, from a given relatedness relation, to define the set assignment function s, and vice versa.

Here there are significant differences from our approach. The first is that the notion of topic is not explicitly represented in the language; it is implicitly represented in the form of appropriate axiom schemas, and this observation holds for all the relevance logics we know. Therefore, it is not possible to reason about the consequences of assumptions concerning the fact that such and such a sentence is about such and such a topic. For instance, if we assume p, we can know which consequences of 'p' are relevant to 'p', or related to 'p'. But, what we cannot do in these relevant logics is to assume that a sentence 'p' is about a given topic t, and to infer from this assumption which other sentences are also about t. The second difference is that the role played by logical connectives in the determination of the subject matter of a sentence is ignored in relevant logics.

Buvač *et. al.* in [3] define the notion of meaningful sentence in a given context in the following way. A vocabulary (a set of atoms) is assigned to each context and all the sentences formed with the vocabulary of a given context are meaningful in this context. That means that the fact that a sentence is, or is not, meaningful is independent of the logical connectives that appear in this sentence. In this respect Buvač *et.al* have a different approach to the definition of meaningfulness from the one we have to aboutness.

However, an interesting common feature is that the notion of meaningfulness is formalized with the same 3-valued logic of Bochvar. They have introduced in the language a binary predicate $ist(c, p)$ whose meaning is: 'p' is meaningful in the context c, and 'p' holds in the context c. That means that the predicate $ist(c, p)$ represents in fact two notions: truth and meaningfulness. If some component of the the sentence 'p' is meaningless in the context c, then the overall sentence 'p' is meaningless in this context. Notice that an important difference between $ist(c, p)$ and $A(t, `p')$ is that $A(t, `p')$ holds independently of whether 'p' is in fact true.

Another difference is that the logic they have defined validates the properties: $ist(c, p \wedge q) \to ist(c, p) \vee ist(c, q)$ and $ist(c, p) \to \neg ist(c, \neg p)$. The first one corresponds to the property (i) we are not inclined to accept, and the second one is incompatible with the property (iii) that we are ready to accept. Also, the rule of substitutivity of equivalent sentences is restricted to sentences which are formed with the vocabulary of the given context. That is, we can infer $ist(c, p) \leftrightarrow ist(c, q)$ from $\vdash p \leftrightarrow q$, provided '$p$' and '$q$' are formed with the vocabulary of c. But that does not mean that 'p' and 'q' are formed with the same atoms. This situation arises, for example, if 'p' is '$a \vee \neg a$' and q is '$b \vee \neg b$', and a and b are two atoms in the vocabulary of c.

One may notice that, as in the dependence logic presented by Epstein, logical connectives play no role in the determination of meaningfulness. But, an important difference from Epstein, and a similarity to the logic we have presented, is that the predicate $ist(c, p)$ allows the indirect representation in the language of the notion of meaningfulness.

If we want to formalize the part of the meaning of $ist(c, p)$ that refers to meaningfulness by using the predicate $A(t, `p')$, we may take a context to be a topic and assume, for every atom $a$ in the vocabulary of $c$, that we have $A(c, `a')$. Then, if we accept for the predicate A the axiom schemas: $A(c, `p \vee q') \leftrightarrow A(c, `p') \vee A(c, `q')$ and $A(c, `\neg p') \leftrightarrow A(c, `p')$, we have $A(c, `p') \leftrightarrow A(c, `q')$ iff '$p$' and '$q$' are formed with the vocabulary of $c$; and we would expect to be able to prove the inference rule: from $\vdash p \leftrightarrow q$ and $A(c, `p') \leftrightarrow A(c, `q')$ infer $ist(c, p) \leftrightarrow ist(c, q)$.

In [13], Nelson Goodman offers a very interesting analysis of aboutness from a perspective quite different from ours. "Our sole problem" he says (p. 3), "...is to determine what a sentence is about, *given* what its terms designate". So, for instance, he considers that the sentence 'Paris is growing' is about Paris, and that 'The capital city of France is growing' is also about Paris. In his analysis of what he calls 'absolute' aboutness, his preliminary conjecture is that a sentence S is about (say) Paris, if some sentence T that mentions Paris follows logically from S (p. 4). However, Goodman then immediately points out that this preliminary idea must be refined, because it leads to unaccepatble consequences. The sentence 'Paris is growing or London is growing' mentions London and is implied by 'Paris is growing'. But we do not want to conclude that 'Paris is growing' is about London!

Goodman proposes some refinements which avoid this problem. However, as he says, the notion of 'absolute' aboutness he defines is purely extensional (p. 10). In keeping with this, tautologies are not about anything; so, although 'Paris is growing' mentions Paris and is about Paris, and 'Paris is not growing' also mentions Paris and is about Paris, the sentence 'Paris is growing or Paris is not growing' mentions Paris but is not about anything at all, Goodman claims. Likewise, self-contradictory sentences are not about anything, and Goodman criticizes Carnap's account of aboutness because it fails to "...meet the requirement that logically equivalent statements are about the same things." (p. 9, footnote 1).

Thus Goodman's approach to the analysis of aboutness is very different from our account of sentences of the kind 'sentence 'p' is about topic t'. The notion of *selectivity* is at the core of Goodman's understanding of aboutness: "...'about' behaves somewhat as 'choose' does. If I ask Johnny to choose some presents and he replies 'I choose everything', he has not chosen anything. Choosing something involves not choosing something else. That Johnny chooses every x is always false. Likewise, saying so and so about an object involves not saying so and so about some other." (p. 5).

There seem to be very strong similarities to fundamental aspects of information theory here. According to Shannon and Weaver's account of 'amount of information', a signal can carry information only if its occurrence reduces uncertainty, and it reduces uncertainty only if its occurrence eliminates some other possibilities. In other words, if a signal α was bound to happen anyway, in the sense that there was no possibility that any signal other than α could have occurred, then the occurrence of α has no surprisal value and thus, on this view, it carries no information. Compare with Goodman's claim "...that nothing can be said about every object, or about every class of objects, or about every class of classes of objects, etc." (p. 6). Nothing can be said about everything, because 'saying about' requires the possibility of contrast, but there are no other things with which to make the contrast if the subject is 'everything'.

Whether or not information-theoretic ideas influenced Goodman's approach to aboutness, it is clear that our proposals regarding the topics a sentence is about are not based on consideration of what information that sentence carries, at least not 'information' in the sense of information theory. The sentences 'It is raining or it is not raining' and 'It is snowing or it is not snowing' are, of course, indistinguishable from the purely extensional point of view, and neither of them carries any information, in the sense of information theory, because neither of them eliminates any possibilities. But we agree with David Lewis (see [14]) that there is a finer-grained level of analysis (than the extensional) at which it would be correct to say that these tautologies do not have the same meaning. And, we conjecture, it is differences in meaning of that finer-grained kind which will have to be captured, if we are to understand the logic of sentences of the kind 'sentence 'p' is about topic t'. Our main suggestion in this paper has been that the intensionality of such sentences might be articulated in terms of the semantical framework of Bochvar's 3-valued logic.

There is another difference between our proposal and Goodman's, with regard to their respective objectives. Goodman does confront the question: "What is it that determines what a given sentence is about?". And his answer, as we have indicated, takes its point of departure in a consideration of what the terms of the given sentence designate. However, we do not pretend to have supplied an answer to *Goodman's* question. What we have done is to specify truth conditions for sentences of the kind 'sentence 'p' is about topic t', and this specification just requires t to be a member of the set of topics 'p' is about. But we say no more by way of characterization of the latter set than it is the value of a function whose argument is the pair consisting of the truth-set of the sentence 'p' names, and the falsity-set of the sentence 'p' names, where the notions of truth and falsity are defined as in the semantics for a 3-valued logic of the Bochvar kind. Beyond this, we have given no further account of what it is for a proposition to be about a set of topics.

So what, then, can we be said to have achieved, as regard a contribution to the understanding of topics and aboutness? The answer is that we have supplied a formal-semantical framework within which it is possible to determine, in a systematic fashion, the consistency or inconsistency of sets of sentences which themselves make claims about which topics some given sentences are about. And thus we may also check which implication relations do, or do not, hold between sentences which make claims of these kinds. In this respect, our approach is comparable to that which has dominated applications of modal logic in the last 40 years. For instance, possible-worlds semantics for alethic modal logic does not supply an answer to the question "What is it that determines whether a given sentence is possible/necessary?" – at least, it does not supply an answer of a non-circular kind, in which no appeal is made to some prior, intuitive understanding of the concepts of possibility and necessity. But a possible world semantics for sentences of alethic modal logic nevertheless *does* provide a tool for the systematic investigation of questions about implication and consistency, in regard to sets of sentences about possibilities and necessities.

Further investigation of the question of what it is that determines what a given proposition is about remains for us as an issue for future work. One point, however, seems clear to us already: that the designations of the terms occurring in the sentence expressing a proposition need not by themselves exhaust the class of topics the proposition may be about. That the proposition expressed by 'Maria is married to Jim and Maria is married to Jules' is about the topic bigamy, is a case in point.

Finally, to close this comparison with other related works, we have to mention some other work by David Lewis. In [15] Lewis defines a 'subject matter' as 'part of the world in intension'. An equivalence relation on worlds is defined from a given subject matter as follows: two worlds are in the same equivalence class if and only if they are exactly alike for that part of the world defined by the subject matter. Moreover, a proposition corresponds to each equivalence class. This equivalence relation suggests an extensional view of subject matters.

Then Lewis informally analyses the structure of subject matters. This analysis is based on the inclusion relation on subject matters. From his point of view a subject matter M1 is included in a subject matter M2, if the part of the world described by M1 is included in the part of the world described by M2. For instance the subject matter '1680' is included in the subject matter '17th century', since the description of the 17th century contains the description of every year in the 17th century, and in particular of the year 1680.

Notice that, as in our approach, Lewis accepts schema (iii) ('a proposition and its negation should be exactly alike with respect to what they are about'), and he does not accept schema (i). In order to avoid assigning the same subject matter to all the non-contingent propositions he suggests considering a four -valued logic, but only some guidelines about this logic are given.

5 Conclusion

We have presented a formal logic for sentences of the kind 'the sentence 'p' is about the topic t'. The main features of this logic are that it is based on a 3-valued logic of the Bochvar type, and that assumptions of the form 'the sentence 'p' is about the topic t' can explicitly be represented in the language by $A(t, `p')$. Several possible additional axiom schemas have been discussed, and we have argued that a decision about their acceptance or rejection must depend on the intended application domains. For some of these, it may be that none of the additional axiom schemas should be accepted, in which case consequences of assumptions of the form $A(t, `p')$ can be drawn only by using the inference rule (REA). Some might maintain that in that case, the logic is so weak that it has no practical interest. However that is not true, because in the absence of any inference rule, for the characterization of the set of sentences about a given topic, one should have to give an extensional definition of *all* the sentences in this set. For instance, if the sentence '$p \wedge \neg(q \vee r)$' is about t, one should have to say that, the following sentences, among others, also are about the topic t: '$p \wedge \neg(r \vee q)$', '$\neg(q \vee r) \wedge p$', '$\neg(r \vee q) \wedge p$', '$p \wedge \neg q \wedge \neg r$', '$p \wedge \neg r \wedge \neg q$', '$\neg q \wedge \neg r \wedge p$', '$\neg r \wedge \neg q \wedge p$', and '$\neg r \wedge p \wedge \neg q$'.

The comparison of our work with other related research has suggested to us some possible directions for future work. In particular, the analysis of Goodman's approach shows that it is interesting to consider the structure of atomic sentences, that is, in formal terms, to move from propositional calculus to predicate calculus. We could then investigate principles which determine how the topics the proposition expressed by a sentence is about are dependent upon the names and predicates occurring in that sentence, and we could try to relate such principles to the 3-valued approach to the analysis of propositions presented here. Another possible direction for future works is to investigate the definition of a structure on sets of topics. For instance, a relation of the form $t > t'$, whose meaning would be: 'the topic t is more specific than the topic t'', would permit representation of a hierarchy, and could be used to represent axiom schemas of the form: $t > t' \rightarrow (A(t, `p') \rightarrow A(t', `p'))$, whose intended meaning is that, if the topic t is more specific than the topic t', and the sentence 'p' is about t, then it is also about t'.

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Appendix

DEFINITION 5.1 (Propositional calculus language) *Let* VAR *be a set of propositional variables, the associated propositional calculus language is defined as usual from* VAR *using the logical connectives* $\neg$*, for negation, and* $\vee$ *for disjunction.*

The connectives $\wedge$ *and* $\rightarrow$ *are defined as usual from negation and disjunction.* ■

DEFINITION 5.2 (Structure) *A structure is a tuple* $S = \langle W, T, F\rangle$ *such that* W *is a set of worlds,* T *is a function from* VAR *to* 2^W *and* F *is a function from* VAR *to* 2^W*.* ■

From an intuitive point of view, if v is a propositional variable, T (resp. F) assigns to v the set of worlds where v is true (resp. false).

The functions T and F are extended to compound sentences by the following rules:

$$\begin{array}{rcl} T(\neg p) & = & F(p) \\ F(\neg p) & = & T(p) \\ T(p \vee q) & = & (T(p) \cap D(q)) \cup (T(q) \cap D(p)) \\ F(p \vee q) & = & F(p) \cap F(q). \end{array}$$

The truth values 'defined' and 'undefined' are defined from 'true' and 'false'. The set of worlds where a sentence p is defined (resp. undefined) is denoted by $D(p)$ (resp. $U(p)$). These functions are defined by:

$$D(p) \stackrel{\text{def}}{=} T(p) \cup F(p) \quad \text{and} \quad U(p) \stackrel{\text{def}}{=} W \setminus D(p).$$

In the case where several structures are under consideration we adopt the notation $T_S(p)$ to denote the set of worlds where p is true in the structure S. Similar notations are adopted for $F(p)$, $D(p)$ and $U(p)$.

For a given Popositional Calculus Language the set of all the possible structures such that:

$$T_S(v) \cap F_S(v) = \emptyset$$

is denoted by Σ_3.

DEFINITION 5.3 (Two-valued logic associated to a three-valued logic.) *Let* $S = \langle W, T, F\rangle$ *be a structure in* Σ_3*. The associated two-valued structure* s *is the tuple* $s = \langle W, t\rangle$ *where* t *is a function from* VAR *to* 2^W *such that:*

- *for a propositional variable* $v : t(v) = T(v)$*;*
- $t(\neg p) = W \setminus t(p)$*;*
- $t(p \vee q) = t(p) \cup t(q)$*.* ■

To denote the set of worlds where a proposition p is false we adopt the notation $f(p)$, and we have:

$$f(p) \stackrel{\text{def}}{=} W \setminus t(p)$$

Notation: the fact that a sentence p is a tautology of classical propositional calculus (CPC) is denoted by: $\models_{CPC} p$.

THEOREM 5.4 *Let $S = \langle W, T, F \rangle$ be a given struture in Σ_3, we define the structure S^+ in function of S by:*

$W^+ = W$
For every propositional variable v:
$T_{S^+}(v) = T_S(v)$
$F_{S^+}(v) = W \setminus T_S(v)$.

We have for all sentence p: $T_{S^+}(p) = t_S(p)$ and $F_{S^+}(p) = f_S(p)$.

Proof: The proof is by induction on the complexity of sentences.
If p is an atomic sentence, by definition of T_{S^+} and F_{S^+}, we have $T_{S^+}(v) = T_S(v)$ and $F_{S^+}(v) = W \setminus T_S(v)$. By definition of $t_S(p)$ and $f_S(p)$ we have $t_S(p) = T_S(p)$ and $f_S(p) = W \setminus T_S(p)$. Therefore we have $T_{S^+}(p) = t_S(p)$ and $F_{S^+}(p) = f_S(p)$.
For a sentence of the form $\neg p$ we have by definition of T: $T_{S^+}(\neg p) = F_{S^+}(p)$. By induction hypothesis we have: $F_{S^+}(p) = f_S(p)$, and by definition of f we have: $f_S(p) = t_S(\neg p)$. Therefore we have $T_{S^+}(\neg p) = t_S(\neg p)$. For similar reasons we have $F_{S^+}(\neg p) = T_{S^+}(p) = t_S(p) = f_S(\neg p)$.
For a sentence of the form $p \vee q$ by definition of T we have: $T_{S^+}(p \vee q) = (T_{S^+}(p) \cap (T_{S^+}(q) \cup F_{S^+}(q))) \cup (T_{S^+}(q) \cap (T_{S^+}(p) \cup F_{S^+}(p))$. By induction hypothesis we have $T_{S^+}(p) = t_S(p)$ and $F_{S^+}(p) = f_S(p)$. Since we have $t_S(p) \cup f_S(p) = W$ we also have $T_{S^+}(p) \cup F_{S^+}(p) = W$. For the same reasons we have $T_{S^+}(q) \cup F_{S^+}(q) = W$. Therefore we have $T_{S^+}(p \vee q) = T_{S^+}(p) \cup T_{S^+}(q)$. From the induction hypothesis we have $T_{S^+}(p) \cup T_{S^+}(q) = t_S(p) \cup t_S(q)$, and by definition of t we have $t_S(p) \cup t_S(q) = t_S(p \vee q)$, and therefore we have $T_{S^+}(p \vee q) = t_S(p \vee q)$. For similar reasons we have: $F_{S^+}(p \vee q) = F_{S^+}(p) \cap F_{S^+}(q) = f_S(p) \cap f_S(q) = f_S(p \vee q)$. ■

THEOREM 5.5 *If for all structure S in Σ_3 we have $T_S(p) \subseteq T_S(q)$ then for all structure S in Σ_3 we have $t_S(p) \subseteq t_S(q)$.*

Proof: By contraposition, Theorem 5.5 is equivalent to: if there exists a structure S in Σ_3 such that $t_S(p) \not\subseteq t_S(q)$ then there exists a structure S in Σ_3 such that $T_S(p) \not\subseteq T_S(q)$.
Let us assume $t_S(p) \not\subseteq t_S(q)$. From Theorem 5.4 we have $T_{S^+}(p) = t_S(p)$ and $F_{S^+}(p) = f_S(p)$, then we have $T_{S^+}(p) \not\subseteq T_{S^+}(q)$. Therefore there exists a structure in Σ_3, namely S^+, such that we have $T_{S^+}(p) \not\subseteq T_{S^+}(q)$. ■

THEOREM 5.6 *For all sentence p and for all structure S in Σ_3 we have: $T_S(p) \subseteq t_S(p)$ and $F_S(p) \subseteq f_S(p)$.*

Proof: The proof is by induction on the complexity of sentences.
If p is an atomic sentence, by definition of t, we have $T_S(p) = t_S(p)$, then we have $T_S(p) \subseteq t_S(p)$. From the definition of the three-valued logic we have: $T_S(p) \cap F_S(p) = \emptyset$. Then, if some world w is in $F_S(p)$, it is not in $t_S(p)$, and therefore it is in $f_S(p)$. Then we also have $F_S(p) \subseteq f_S(p)$.
For a sentence of the form $\neg p$ we have $T_S(\neg p) = F_S(p)$ and, by induction hypothesis, we have $F_S(p) \subseteq f_S(p)$. Since, by definition of f, we have $f_S(p) = t_S(\neg p)$, we finally have: $T_S(\neg p) \subseteq t_S(\neg p)$. For similar reasons we have $F_S(\neg p) = T_S(p) \subseteq t_S(p) = f_S(\neg p)$.
For a sentence of the form $p \vee q$, by definition of T, we have: $T_S(p \vee q) = (T_S(p) \cap (T_S(q) \cup F_S(q))) \cup (T_S(q) \cap (T_S(p) \vee F_S(p)))$. By induction hypothesis we have: $T_S(p) \subseteq t_S(p)$ and $F_S(p) \subseteq f_S(p)$, then we have $T_S(p \vee q) \subseteq (t_S(p) \cup t_S(q))$ and, since we have $t_S(p) \cup t_S(q) = t_S(p \vee q)$, we have $T_S(p \vee q) \subseteq t_S(p \vee q)$. From the definition of F we have $F_S(p \vee q) = F_S(p) \cap F_S(q)$, and by induction hypothesis we have $F_S(p) \subseteq f_S(p)$ and $F_S(q) \subseteq f_S(q)$, therefore we have $F_S(p \vee q) \subseteq f_S(p \vee q)$. ∎

THEOREM 5.7 *For all sentence p and for all structure S in Σ_3 we have: $t_S(p) \subseteq T_S(p) \cup U_S(p)$ and $f_S(p) \subseteq F_S(p) \cup U_S(p)$.*

Proof: The proof is by induction on the complexity of sentences.
If p is an atomic sentence, by definition of t, we have $T_S(p) = t_S(p)$, then we have $t_S(p) \subseteq T_S(p) \cup U_S(p)$. By definition of f, we have $f_S(p) = W \setminus T_S(p)$, and in the three-valued logic we have $W \setminus T_S(p) = F_S(p) \cup U_S(p)$, then we have $f_S(p) \subseteq F_S(p) \cup U_S(p)$.
For a sentence of the form $\neg p$, by definition of t, we have $t_S(\neg p) = f_S(p)$, and by induction hypothesis we have $f_S(p) \subseteq F_S(p) \cup U_S(p)$. Since we have $F_S(p) = T_S(\neg p)$ and $U_S(p) = U_S(\neg p)$, we have $t_S(\neg p) \subseteq T_S(\neg p) \cup U_S(\neg p)$. For similar reasons we have: $f_S(\neg p) = t_S(p) \subseteq T_S(p) \cup U_S(p) = F_S(\neg p) \cup U_S(\neg p)$.
For a sentence of the form $p \vee q$, by definition of t, we have $t_S(p \vee q) = t_S(p) \cup t_S(q)$, and by induction hypothesis, we have $t_S(p) \subseteq T_S(p) \cup U_S(p)$ and $t_S(q) \subseteq T_S(q) \cup U_S(q)$. From the definition of T we have: $T_S(p \vee q) = (T_S(p) \cup T_S(q)) \setminus (U_S(p) \cup U_S(q))$, then we have: $t_S(p \vee q) \subseteq T_S(p \vee q) \cup U_S(p) \cup U_S(q)$,that is $t_S(p \vee q) \subseteq T_S(p \vee q) \cup U_S(p \vee q)$.
For similar reasons we have $f_S(p \vee q) = f_S(p) \cap f_S(q)$ and $f_S(p) \subseteq F_S(p) \cup U_S(p)$ and $f_S(q) \subseteq F_S(q) \cup U_S(q)$. Then we have: $f_S(p \vee q) \subseteq (F_S(p) \cup U_S(p)) \cap (F_S(q) \cup U_S(q))$, and we have: $f_S(p \vee q) \subseteq (F_S(p) \cap F_S(q)) \cup U_S(p) \cup U_S(q)$, that is: $f_S(p \vee q) \subseteq F_S(p \vee q) \cup U_S(p \vee q)$. ∎

THEOREM 5.8 *The facts $\models_{CPC} p \rightarrow q$ and $Var(q) \subseteq Var(p)$ imply that for all structure S in Σ_3 we have $T_S(p) \subseteq T_S(q)$.*

Proof: Let S be a structure in Σ_3, from Theorem 5.6 we have: $T_S(p) \subseteq t_S(p)$. Since we have $\models_{CPC} p \to q$ we have $t_S(p) \subseteq t_S(q)$. From Theorem 5.7 we have $t_S(q) \subseteq T_S(q) \cup U_S(q)$, therefore we have $T_S(p) \subseteq T_S(q) \cup U_S(q)$. Since we have $Var(q) \subseteq Var(p)$, from the definition of U, we have $U_S(q) \subseteq U_S(p)$. Then we have $T_S(p) \subseteq T_S(q) \cup U_S(p)$. From the definition of T and U, we have $T_S(p) \cap U_S(p) = \emptyset$. Therefore we have $T_S(p) \subseteq T_S(q)$. ■

THEOREM 5.9 *The facts $\models_{CPC} p \to q$ and $Var(p) \subseteq Var(q)$ imply that for all structure S in Σ_3 we have $F_S(q) \subseteq F_S(p)$.*

Proof: Since $\models_{CPC} p \to q$ implies $f_S(q) \subseteq f_S(p)$, and $Var(p) \subseteq Var(q)$ implies $U_S(p) \subseteq U_S(q)$, a similar proof as for Theorem 5.8 allows to show that we have $F_S(q) \subseteq F_S(p)$. ■

THEOREM 5.10 *The facts $\models_{CPC} p \leftrightarrow q$ and Var(p)=Var(q) imply that for all structure S in Σ_3 we have $T_S(p) = T_S(q)$ and $F_S(p) = F_S(q)$.*

Proof: This theorem is a direct consequence of Theorems 5.8 and 5.9. ■

THEOREM 5.11 *If for all structure S in Σ_3 we have $T_S(p) = T_S(q)$ and $F_S(p) = F_S(q)$, then we have Var(p)=Var(q).*

Proof: Let us assume that for all sentences p and for all structures S in Σ_3 we have $T_S(p) = T_S(q)$ and $F_S(p) = F_S(q)$.
Let us assume that we have $Var(q) \not\subseteq Var(p)$, then there exists a propositional variable v such that $v \in Var(q)$ and $v \notin Var(p)$.
Let S be a structure in Σ_3 and w be a world in S. We have either $w \in t_S(p)$ or $w \in f_S(p)$. Let us assume first that we have $w \in t_S(p)$.
We define a world w' of a structure S' from w and S in the following way:

If a variable u is in Var(p) then:
if $w \in T_S(u)$ then $w' \in T_{S'}(u)$,
if $w \notin T_S(u)$ then $w' \in F_{S'}(u)$.
If a variable u is not in $Var(p)$ then $w' \in U_{S'}(u)$.

According to this definition we have $w' \in t_{S'}(p)$, because we have $w \in t_S(p)$, and the fact $w' \in t_{S'}(p)$ (resp. $w \in t_S(p)$) only depends on the variables u such that $w' \in T_{S'}(u)$ (resp. $w \in T_S(u)$), and for the variables u in p we have $w \in T_S(u)$ iff $w' \in T_{S'}(u)$.
From Theorem 5.7 we have $t_{S'}(p) \subseteq T_{S'}(p) \cup U_{S'}(p)$, then we have $w' \in T_{S'}(p)$ or $w' \in U_{S'}(p)$. From the definition of w' none of the variables in p is undefined in w' then we do not have $w' \in U_{S'}(p)$, therefore we have $w' \in T_{S'}(p)$. Since we have $T_{S'}(p) = T_{S'}(q)$, we also have $w' \in T_{S'}(q)$.
Since the variable v of q is not in p, by definition of w', we have $w' \in U_{S'}(v)$, and, by definition of U, we have $w' \in U_{S'}(q)$, which contradicts the fact $w' \in T_{S'}(q)$. Therefore we have $Var(q) \subseteq Var(p)$.

If we assume now that we have $w \in f_S(p)$, a similar proof, based on the fact $F_{S'}(p) = F_{S'}(q)$, also allows to infer $Var(q) \subseteq Var(p)$.
Then, in both cases we have $Var(q) \subseteq Var(p)$.
Since p and q play a similar role, we can also prove that $Var(p) \subseteq Var(q)$, and finally we have $Var(p) = Var(q)$. ■

THEOREM 5.12 *If for all structure S in Σ_3 we have $T_S(p) = T_S(q)$ then we have $\models_{CPC} p \leftrightarrow q$.*

Proof: If for all structure S in Σ_3 we have $T_S(p) = T_S(q)$, then, from Theorem 5.6, we have for all structure S in Σ_3 $t_S(p) = t_S(q)$. Since in Σ_3 we have all the possible assignments for t, if for all structure S in Σ_3 we have $t_S(p) = t_S(q)$, we have $\models_{CPC} p \leftrightarrow q$. Therefore we have $\models_{CPC} p \leftrightarrow q$. ■

THEOREM 5.13 *For all sentences p, if for all structures S in Σ_3 we have $T_S(p) = T_S(q)$ and $F_S(p) = F_S(q)$, then we have $\models_{CPC} p \leftrightarrow q$ and $Var(p) = Var(q)$.*

Proof: This result collates the results of Theorems 5.11 and 5.12. ■

THEOREM 5.14 *We have $\models_{CPC} p \leftrightarrow q$ and Var(p)=Var(q) iff for all structure S in Σ_3 we have $T_S(p) = T_S(q)$ and $F_S(p) = F_S(q)$.*

Proof: This theorem is a direct consequence of Theorems 5.10 and 5.13. ■

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