Dependencies between players in Boolean games

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Introduction

Boolean games as introduced in Harrenstein, Van der Hoek, Meyer, Witteveen (2001, 2004)

- 2-players games with p binary decision variables
- Each decision variable is controlled by one player
- Player's utilities specified by a propositional formula
- Zero-sum games
- Static games

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1. Introduction
2. **Boolean games**
3. Dependencies between players
4. Conclusion

Boolean n-players version of prisoners' dilemma

- n prisoners (denoted by $1, \ldots, n$).
- The same proposal is made to each of them:

  "Either you cover your accomplices (C_i, $i = 1, \ldots, n$) or you denounce them ($\neg C_i$, $i = 1, \ldots, n$)."

  - Denouncing makes you freed while your partners will be sent to prison (except those who denounced you as well; these ones will be freed as well),
  - But if none of you chooses to denounce, everyone will be freed.

Boolean n-players version of prisoners' dilemma

- Normal form for $n = 3$:

$3 : C_3$		
1 \ 2	C_2	$\overline{C}_2$
C_1	(1, 1, 1)	(0, 1, 0)
$\overline{C}_1$	(1, 0, 0)	(1, 1, 0)

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- n prisoners : n-dimensional matrix, therefore 2^n n-tuples must be specified.

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- Expressed much more compactly by Boolean game $G = (N, V, \pi, \Phi)$:
  - $N = \{1, \ldots, n\}$,
  - $V = \{C_1, \ldots, C_n\}$ (propositional variables),
  - $\forall i \in \{1, \ldots, n\}, \pi_i = \{C_i\}$ (control assignment function), and
  - $\Phi = \{\varphi_1, \ldots, \varphi_n\}$, with $\forall i$, $\varphi_i = (C_1 \wedge C_2 \wedge \ldots C_n) \vee \neg C_i$ (goals).

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- $\forall i$, i has 2 possible strategies: $s_{i_1} = \{C_i\}$ and $s_{i_2} = \{\overline{C_i}\}$
- the strategy $\overline{C}_i$ is a winning strategy for i.
- S is the set of strategy profile for G ; $|S| = 8$

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- s_{-i} denotes the projection of s on $N \setminus \{i\}$
- $s = \{C_1C_2C_3\}$; $s_{-1} = (C_2, C_3)$; $s_{-2} = (C_1, C_3)$; $s_{-3} = (C_1, C_2)$

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- A pure-strategy Nash equilibrium (PNE) is a strategy profile such as each player's strategy is an optimal response to other players' strategies. $s = \{s_1, \ldots, s_n\}$ is a PNE iff $\forall i \in \{1, \ldots, n\}, \forall s_i' \in 2^{\pi_i}, u_i(s) \geq u_i(s_{-i}, s_i')$.

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- 2 pure-strategy Nash equilibria: $C_1C_2C_3$ and $\overline{C}_1\overline{C}_2\overline{C}_3$

Relevant player

Relevant variable

The set of **relevant variables** for a player i, denoted by RV_i, is the set of $v \in V$ such as v is useful to i to obtain φ_i.

Relevant player

The set of **relevant players** for a player i, denoted by RP_i, is the set of agents $j \in N$ such as j controls at least one relevant variable of i: $RP_i = \bigcup_{v \in RV_i} \pi^{-1}(v)$.

Example

- 3 friends (denoted by $(1, 2, 3)$) are invited to a party,
- $V = \{P_1, P_2, P_3\}$, where P_1 means "1 goes at the party", and the same for P_2 and P_3,
- $\pi_1 = \{P_1\}$, $\pi_2 = \{P_2\}$, $\pi_3 = \{P_3\}$,
- $\varphi_1 = P_1$, $\varphi_2 = P_1 \leftrightarrow P_2$ and $\varphi_3 = \neg P_1 \wedge P_2 \wedge P_3$.

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$$RV_1 = \{P_1\},\ RP_1 = \{1\}$$
$$RV_2 = \{P_1, P_2\},\ RP_2 = \{1,2\}$$
$$RV_3 = \{P_1, P_2, P_3\},\ RP_3 = \{1,2,3\}.$$

Dependency graph

Dependency graph

The **dependency graph of a Boolean game** G is the directed graph $\mathcal{P} = \langle N, R \rangle$ containing

- a vertex for each player, and
- an edge from i to j if j is a relevant player of i:

$$\forall i, j \in N, (i, j) \in R \text{ if } j \in RP_i$$

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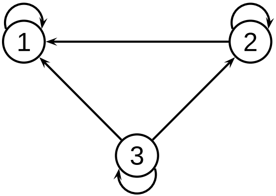

Link between dependencies and PNE

Proposition

If G is a Boolean game such that the irreflexive part of the dependency graph $\mathcal{P}$ of G is acyclic, then, G has a pure strategy Nash equilibrum.

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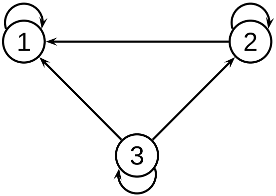

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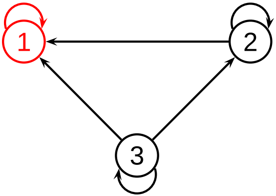

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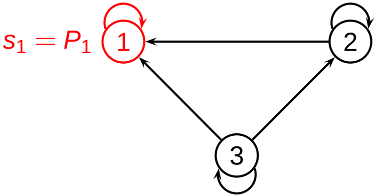

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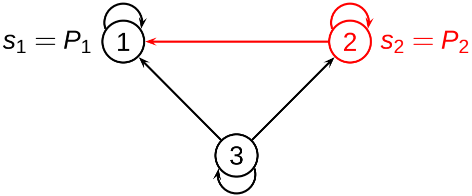

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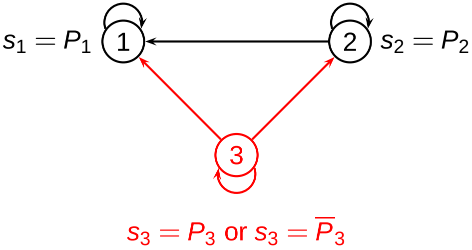

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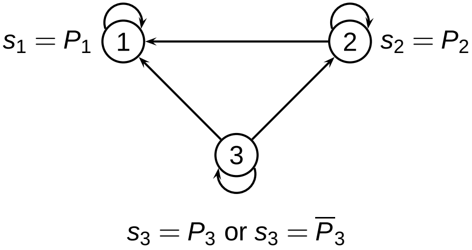

G has 2 PNEs: $\{P_1P_2P_3, P_1P_2\overline{P}_3\}$

Stable set

Stable set

$B \subseteq N$ is **stable** for R iff $R(B) \subseteq B$, ie $\forall j \in B$, $\forall i$ such that $i \in R(j)$, then $i \in B$.

Stable set

Projection

If $B \subseteq N$ is a stable set for R, the **projection** of G on B is defined by $G_B = (B, V_B, \pi_B, \Phi_B)$, where

- $V_B = \cup_{i \in B} \pi_i$,
- $\pi_B : B \rightarrow V_B$ such that $\pi_B(i) = \{v | v \in \pi_i\}$, and
- $\Phi_B = \{\varphi_i | i \in B\}$.

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Proposition

If B is a stable set, $G_B = (B, V_B, \pi_B, \Phi_B)$ is a Boolean game.

Example

- $N = (1,2,3)$, $V = \{a,b,c\}$,
- $\pi_1 = \{a\}$, $\pi_2 = \{b\}$, $\pi_3 = \{c\}$,
- $\varphi_1 = a \leftrightarrow b$, $\varphi_2 = a \leftrightarrow \neg b$ and $\varphi_3 = \neg c$,
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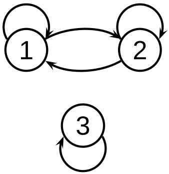

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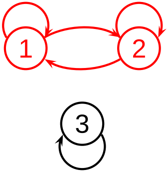

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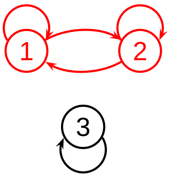

- $G_A = (A, V_A, \pi_A, \Phi_A)$, with $A = \{1,2\}$, $V_A = \{a,b\}$, $\pi_1 = a$, $\pi_2 = b$, $\varphi_1 = a \leftrightarrow b$, $\varphi_2 = a \leftrightarrow \neg b$.

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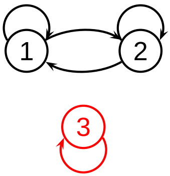

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- $G_B = (B, V_B, \pi_B, \Phi_B)$, with $B = \{3\}$, $V_B = \{c\}$, $\pi_3 = c$, $\varphi_3 = \neg c$.

Stable set

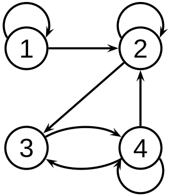
Proposition

If B is a stable set and s a PNE for G, then s_B is a PNE for G_B.

Example

- $N = (1,2,3,4)$, $V = \{a,b,c,d\}$,
- $\pi_1 = \{a\}$, $\pi_2 = \{b\}$, $\pi_3 = \{c\}$, $\pi_4 = \{d\}$,
- $\varphi_1 = a \leftrightarrow b$, $\varphi_2 = b \leftrightarrow c$, $\varphi_3 = \neg d$, and $\varphi_4 = d \leftrightarrow (b \wedge c)$.

G has 2 PNEs : $\{abcd, \overline{a}\overline{b}\overline{c}\overline{d}\}$.



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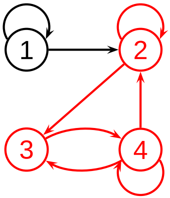
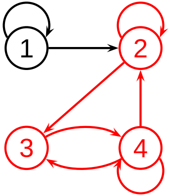


$B = \{2,3,4\}$ is a stable set. G_B is a Boolean game, with $V_B = \{b,c,d\}$, $\pi_2 = b$, $\pi_3 = c$, $\pi_4 = d$, $\varphi_2 = b \leftrightarrow c$, $\varphi_3 = \neg d$, and $\varphi_4 = d \leftrightarrow (b \wedge c)$.

Example

- $N = (1,2,3,4)$, $V = \{a,b,c,d\}$,
- $\pi_1 = \{a\}$, $\pi_2 = \{b\}$, $\pi_3 = \{c\}$, $\pi_4 = \{d\}$,
- $\varphi_1 = a \leftrightarrow b$, $\varphi_2 = b \leftrightarrow c$, $\varphi_3 = \neg d$, and $\varphi_4 = d \leftrightarrow (b \wedge c)$.

G has 2 PNEs : $\{abcd, \overline{a}\overline{b}\overline{c}\overline{d}\}$.



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$\{bcd, \overline{b}\overline{c}\overline{d}\}$ are 2 PNEs of G_B.

Stable set

Proposition

Let A and B be two stable sets of players.
If s_A is a PNE for G_A and s_B is a PNE for G_B such that $\forall i \in A \cap B$, $s_{A,i} = s_{B,i}$, then, $s_{A \cup B}$ is a PNE for $G_{A \cup B}$.

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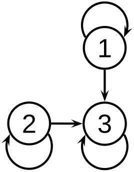
This proposition can be easily generalized with p stable sets covering the set of players.

Example

- $N = (1, 2, 3)$, $V = \{a, b, c\}$,
- $\pi_1 = \{a\}$, $\pi_2 = \{b\}$, $\pi_3 = \{c\}$,
- $\varphi_1 = a \leftrightarrow c$, $\varphi_2 = b \leftrightarrow \neg c$, and $\varphi_3 = c$.



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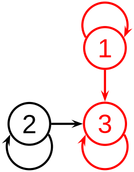

- $G_A = (A, V_A, \pi_A, \Phi_A)$, with $A = \{1,3\}$, $V_A = \{a,c\}$, $\pi_1 = a$, $\pi_3 = c$, $\varphi_1 = a \leftrightarrow c$ and $\varphi_3 = c$. G_A has one PNE : $\{ac\}$ (denoted by $s_A = (s_{A,1}, s_{A,3})$).

Example

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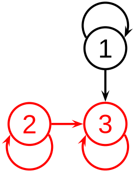

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- $G_B = (B, V_B, \pi_B, \Phi_B)$, with $B = \{2,3\}$, $V_B = \{b,c\}$, $\pi_2 = b$, $\pi_3 = c$, $\varphi_2 = b \leftrightarrow \neg c$, $\varphi_3 = c$. G_B has one PNE : $\{\overline{b}c\}$ (denoted by $s_B = (s_{B,2}, s_{B,3})$).

Example

- $N = (1,2,3)$, $V = \{a,b,c\}$,
- $\pi_1 = \{a\}$, $\pi_2 = \{b\}$, $\pi_3 = \{c\}$,
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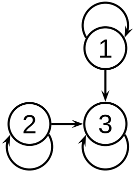

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$A \cap B = \{3\}$ and we have $s_{A,3} = s_{B,3} = c \Rightarrow G_{A \cup B}$ has one PNE: $\{a\bar{b}c\}$.

Other issues

- ECAI'06: simple characterizations of Nash equilibria and dominated strategies for Boolean games, and investigate the computational complexity of the related problems;
- PRICAI'06: extended Boolean games with ordinal preferences represented by prioritized goals and CP-nets with binary variables;
- Almost all properties presented here hold also for Boolean games with non dichotomous preferences;
- Use of the dependency graph for computing efficient coalitions
- Further issues:
  - Defining and studying dynamic Boolean games
  - Defining and studying Boolean games with incomplete information