A Qualitative Theory of Motion Based on Spatio-Temporal Primitives

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Abstract

This paper presents a formal theory for reasoning about motion of spatial entities, in a qualitative framework. Taking over a theory intended for spatial entities, we enrich it to achieve a theory whose intended models are spatio-temporal entities, an idea sometimes proposed by philosophers or AI authors but never fully exploited. We show what kind of properties usually assumed as desirable parts of any space-time theory are recovered from our model, thus giving a sound theoretical basis for a natural, qualitative representation of motion.

1 INTRODUCTION

This paper presents a theory for representing and reasoning about motion in a qualitative framework. A lot of work has been devoted to the representation of space in the past few years, as spatial concepts pervades many domains of AI. Part of these works have focused on the building of theories for reasoning about incomplete and/or imprecise information as such theories would be more easily exploited than the traditional quantitative models (widely used in robotics for instance). For that purpose, in what is called "Qualitative Spatial Reasoning" (QSR), a lot of models make use of regions of space as their ontological primitives rather than the points of traditional geometry (for a general presentation, see [Vieu, 1997]). This ontological shift imposes to build new theories for most spatial and geometrical concepts. Several studies have been devoted to the building of mereo-topological theories on that basis (for instance the well-known RCC theory [Randell *et al.*, 1992]; see also [Fleck, 1996],[Asher and Vieu, 1995] and for a survey of mereo-topology [Varzi, 1996]), as mereo-topology (essentially, the modeling of connection and part-hood) is regarded as the basis of common-sense geometry.

Very little work has been done on motion in such a qualitative framework. Even works done under the banner of so-called "qualitative physics" usually make use of the classical quantitative model of kinematics. Motion is thus usually represented in a Newtonian/Cartesian framework [Rajagopalan and Kuipers, 1994],[Hays, 1989]. Other studies focus more on related aspects, for instance [Forbus, 1983, Davis, 1989] insist more on the concept of dynamic process, and [Shanahan, 1995, Hartley, 1992] on default reasoning in metric spaces that are not really clearly characterized. Motion is nonetheless a key notion in our understanding of spatial relations; indeed, changes of state in RCC has been analyzed through transition graphs in which the relations form so-called "conceptual neighborhoods", via potential motion. Only certain changes are allowed, assuming continuous change between relations (this means spatial changes are restricted to the edges of such graphs). Continuity is the central notion here, but remains implicitly assumed without a formal definition; only the work of Galton [Galton, 1993, Galton, 1997] has begun to address what continuity implies for a common-sense theory of motion. Still, this kind of work characterizes continuity as a set of logical constraints on the transitions in a temporal framework and does not add much insight to the already existing transition graph (it is more descriptive than explanatory), and falls short of an explicit, generic characterization of spatio-temporal continuity.

We present here a mereo-topological model in which the primitive entities are spatio-temporal regions, on which we define both spatio-temporal and temporal relations. These entities can be interpreted as the trajectories of physical objects and events. Such an approach is not entirely new since Russell [Russell, 1914] or Carnap [Carnap, 1958] had already suggested it, for reasons analyzed in [Simons, 1987], and Hayes has begun to show the use that could be made of such objects for common-sense reasoning [Hayes, 1985b, Hayes, 1985a]; mainly, the homogeneity of a theory which treats events and objects at the same level has an interesting expressive power, and directly addresses the problem of objects identity through time. We show that a qualitative notion of continuity can be expressed in such a theory, which allows for a full characterization of conceptual neighborhoods of spatial relations. Section 2 presents the topological theory (of Asher & Vieu) we take as a basis, section 3 and 4 present our extension to that theory, along with the definition of continuity within the theory and a presentation of how we recover conceptual neighborhoods from this general definition. Section 5 shows how to define some common-sense concepts related to motion.

2 THE TOPOLOGICAL BASE

In this section we take over the theory of topological concepts presented in [Asher and Vieu, 1995], which is shown to be consistent and complete with respect to a certain class of models, and which has at least the same expressive power as RCC8 [Randell *et al.*, 1992]; both theories are based on the work of Clarke [Clarke, 1981]. The main difference between them is that RCC8 does not distinguish between open and closed regions whereas Asher and Vieu's theory does. It is not obvious that the results we present here could be demonstrated in RCC, because of this difference. Moreover Asher and Vieu's theory has been thought as a spatio-temporal theory from the beginning, although this aspect has never been much exploited by the authors (not even in a formal account of motion verbs in [Asher and Sablayrolles, 1995]). Objects of this theory can be interpreted as the spatio-temporal referents of physical objects or events, or regions of space. Figure 2 shows the classical intended 2D spatial interpretation of some relations that can be defined from the primitive relation C (connection), while Figure 1 shows an illustration of how a spatio-temporal relation is to be intuitively understood. Here the "spatial" dimension of an entity is a part of the horizontal axis while the temporal evolution of the entity is shown along the vertical axis. Space could of course be 2- or 3-dimensional, and this Figure is only one example of what our intended models are.

We will now present the topological axioms from [Asher and Vieu, 1995]. For the sake of clarity, in the following, universal quantifiers scoping over whole formulas are omitted. Upper case symbols stand for predicates, lower case ones for variables or constants. The symbol $\triangleq$ stands for definition. We use a first order language with equality.

A 2.1 $\mathrm{C}xx$

A 2.2 $\mathrm{C}xy \rightarrow \mathrm{C}yx$

A 2.3 $(\forall z\ (\mathrm{C}zx \leftrightarrow \mathrm{C}zy) \rightarrow x = y)$

A lot of relations can be defined on that sole basis, we will make use of the following (see [Asher and Vieu, 1995] for the formal definition): P (part of), PP, (proper part of), O

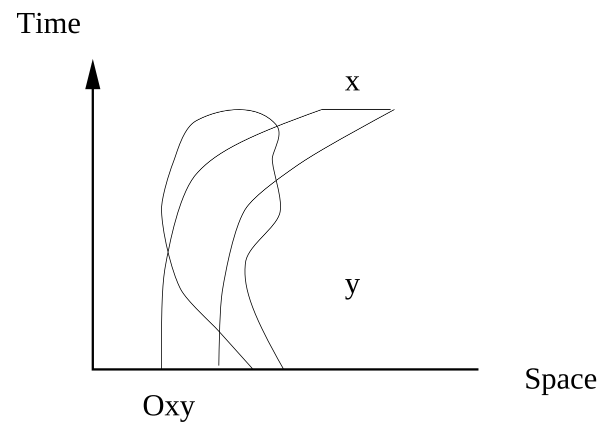

Figure 1: A Spatio-Temporal Interpretation of O(verlap)

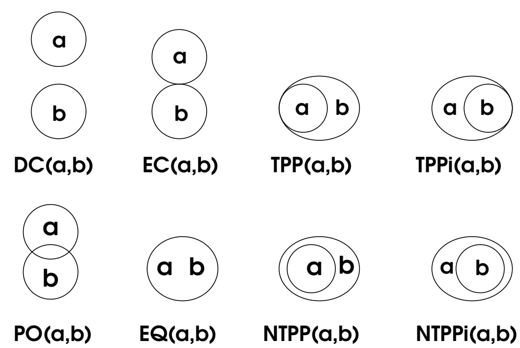

Figure 2: A 2D Spatial Interpretation of Topological Relations

(overlap), PO (partial overlap), EC (external connection), TP (tangential part), NTP (non tangential part), TPP and NTPP and the converse relations TPP^{-1} and NTPP^{-1} (also written TPPI and NTPPI), cf Figure 2. The following are existence axioms of classical operators [Asher and Vieu, 1995]:

A 2.4 $\forall x \forall y \exists z\ \forall u(\mathrm{C}uz \leftrightarrow (\mathrm{C}ux \vee \mathrm{C}uy))$
(sum, noted $x{+}y$)

A 2.5 $\forall x \exists y\ \neg \mathrm{C}yx \rightarrow \exists z\ \forall u(\mathrm{C}uz \leftrightarrow \exists v\ (\neg \mathrm{C}vx \wedge \mathrm{C}vu))$
(complement, noted $-x$)

A 2.6 $\exists x\ \forall u\ \ \mathrm{C}ux$
(existence of a universe, noted a^*)

A 2.7 $\mathrm{O}xy \rightarrow \exists z\ \forall u(\mathrm{C}uz \leftrightarrow \exists v\ (\mathrm{P}vx \wedge \mathrm{P}vy \wedge \mathrm{C}vu))$
(intersection, noted $x \cdot y$)

A 2.8 $\forall x \exists y\ \forall u\ \ (\mathrm{C}uy \leftrightarrow \exists v\ (\mathrm{NTP}vx \wedge \mathrm{C}vu))$
(interior, noted ix)

D 2.1 (closure): $cx \triangleq\ -i(-x)$

A 2.9 $c(a^*) =\ a^*$

The topological operators are constrained in a classical way:

D 2.2 $\text{OP}x \triangleq (ix = x)$ (definition of an open)

A 2.10 $(\text{OP}x \wedge \text{OP}y \wedge \text{O}xy) \rightarrow \text{OP}(x \cdot y)$

The authors also define a few useful notions:

D 2.3 $\text{SP}xy \triangleq \neg \text{C}cxcy$ (separateness[Vieu, 1991])

D 2.4 $\text{CON}x \triangleq \neg(\exists x_1 \exists x_2 (x = x_1 + x_2 \wedge \text{SP}x_1 x_2))$ (self-connectedness)

D 2.5 $\text{AT}x \triangleq \neg \exists y \text{PP}yx$ (atomicity)

We define quasi-atoms, whose only proper parts are their interiors:

D 2.6 $\text{QAT}x \triangleq \text{AT}ix \ \wedge \ \neg\text{OP}x$

and "generalized" atoms, which can be atoms or quasi-atoms:

D 2.7 $\text{GAT}x \triangleq \text{AT}x \vee \text{QAT}x$

and are conceptually close to atoms (the only difference being that they can include part of the closure of an atom).

3 TEMPORAL RELATIONS

The mere topology we have introduced is quite general and does not imply a spatio-temporal interpretation; in order to do so it is necessary to capture a notion of temporal order between entities of the theory. The kind of properties we want are related to the assumptions already made about those entities: they are extended in a primitive space-time so that temporal relations actually bear on the spatio-temporal entities *themselves*, and not on time extents. Thus our temporal relations are close to event logics (see e.g. Kamp [Kamp, 1979]), which are quite similar to interval-based temporal logics (see vanBenthem [van Benthem, 1995] or Galton [Galton, 1995] for a general presentation), with the difference that two objects can be different and still be contemporaneous. However these logics are often based on two primitives, $<$ for precedence, and either inclusion or overlap. If we want to keep the topological distinction between connection and overlap made on the spatio-temporal entities, we need to have topological concepts on the temporal level as well (and Varzi [Varzi, 1996] has shown this is not possible with only overlap or inclusion, i.e. mereology). This distinction is part of Allen's common-sense theory of time [Allen and Hayes, 1985] and Galton [Galton, 1993] has shown it is necessary to account for continuous motion. But Allen's theory assumes self-connected intervals and Galton assumes the existence of both instants and intervals. We believe however that none of these hypotheses are necessary and keep a more general model where entities can be disconnected; it is moreover an ontologically more parsimonious (assuming only extended entities) model which turns out to be quite practical. We thus introduce a relation of temporal connection $\asymp$ (interpreted in much the same way as a temporal 'C') besides the obvious $<$. For readability's sake, and to distinguish them from the spatio-temporal relations, temporal relations are infixed.

A 3.1 $x \asymp y \rightarrow y \asymp x$ (symmetry)

A 3.2 $x \asymp x$ (reflexivity)

A 3.3 $x \asymp y \rightarrow \neg x < y$ (incompatibility between $\asymp$ and $<$)

A 3.4 $x < y \rightarrow \neg y < x$ (antisymmetry of $<$)

A 3.5 $(x < y \ \wedge \ y \asymp z \ \wedge \ z < t) \rightarrow x < t$ (composition of $\asymp$ and $<$)

From this we can define the intuitive relations:

D 3.1 $x \subseteq_t y \triangleq \forall z (z \asymp x \rightarrow z \asymp y)$ (temporal inclusion)

D 3.2 $x \sigma y \triangleq \exists z (z \subseteq_t y \ \wedge \ z \subseteq_t x)$ (temporal overlap)

D 3.3 $(x \equiv_t y) \triangleq x \subseteq_t y \wedge y \subseteq_t x$ (temporal equivalence)

The transitivity of $<$ can be derived from A3.2 and A3.5 and its irreflexivity from A3.3 and A3.2. The relation $<$ is therefore a strict partial order. Figure 3 gives an illustration of the temporal relations between spatio-temporal entities.

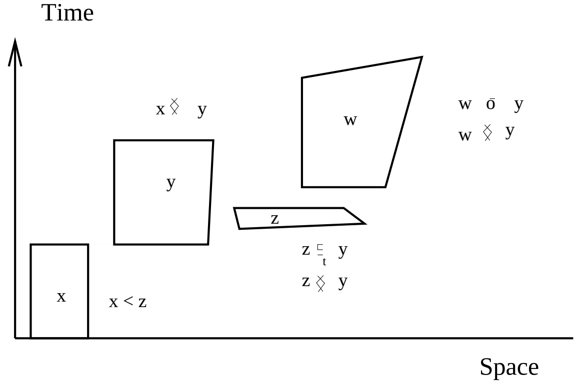

Figure 3: Temporal Relations Illustrated

We can derive all the axioms of other systems based on $<$ and ($\subseteq_t$ or σ) (cf. [van Benthem, 1995, Aurnague and Vieu, 1993, Kamp, 1979]) from the previous axioms, which are much inspired by [Aurnague and Vieu, 1993]. We can also define a notion of temporal connectedness:

D 3.4
$\text{CON}_t x \triangleq \neg(\exists x_1 \exists x_2 (x = x_1 + x_2 \ \wedge \ \neg(c x_1 \asymp c x_2)))$

We can moreover impose the linearity of the underlying temporal order by stating that there must be a relation ($<$ or $\asymp$) between two temporally self-connected entities (and only in that case since the theory allows for the sum of arbitrary entities, therefore not necessarily self-connected). The following axiom indeed eliminates branching time models.

A 3.6 $(\text{CON}_t x \wedge \text{CON}_t y) \rightarrow (x < y \ \vee \ x \asymp y \ \vee \ y < x)$

4 SPATIO-TEMPORAL INTERACTIONS

4.1 LINKS BETWEEN TIME AND SPACE-TIME

The links between spatio-temporal relations and temporal relations have yet to be defined. Some axioms were defined in the same perspective in [Vieu, 1991], using a "temporal overlap"relation, but were not part of a larger motion theory[1].

A 4.1 $\text{C}xy \rightarrow x \asymp y$ (two connected entities are also time-connected)

The models must not be temporal only, so C and $\asymp$ are different (4.2) and there is at least two temporally ordered entities (4.3).

A 4.2 $\exists x \exists y \ x \asymp y \ \wedge \ \neg \text{C}xy$

A 4.3 $\exists x \exists y \ x < y$

The interaction between + and $<$ and $\asymp$ need to be specified:

A 4.4 $(x < y \wedge z < y) \leftrightarrow (x + z) < y$

A 4.5 $(x + y) \asymp z \leftrightarrow \ x \asymp z \ \vee \ y \asymp z$

In order to define relations between parts of trajectories that can vary through time, we now define a notion of "temporal slice", i.e. the maximal part corresponding to a certain time extent (Figure 4).

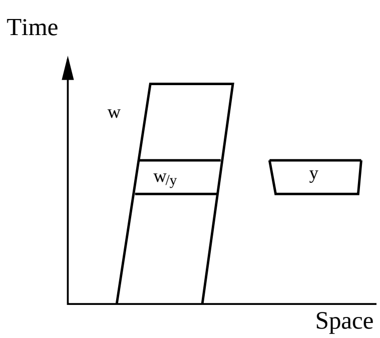

Figure 4: Illustration of a Temporal Slice

[1]The axioms of [Vieu, 1991] are theorems of our theory.

D 4.1 $\text{TS}xy \triangleq \text{P}xy \wedge \forall z \ ((\text{P}zy \wedge z \subseteq_t x) \rightarrow \text{P}zx)$

In order to prevent the proliferation of arbitrary entities, we only impose the existence of a slice of an entity when it temporally intersects another (so that we state only the existence of *meaningful* temporal parts, corresponding to temporal interaction between other entities). Thus we want:

$$\forall w, y (w \sigma y \rightarrow \exists u (TSuw \ \wedge \ u \subseteq_t y)) \qquad (1)$$

Since $(w \sigma y \rightarrow \exists u (u \subseteq_t w \ \wedge \ u \subseteq_t y))$, it is enough to state the following axiom:

A 4.6 $y \subseteq_t w \rightarrow \exists u \ (\text{TS}uw \ \wedge \ u \equiv_t y)$

and (1) then is a theorem, along with the following ones:

Th 4.1 $\text{P}xy \rightarrow \exists z (\text{TS}zy \ \wedge \ z \equiv_t x)$
(there is a temporally equivalent slice for all parts of an entity)

Th 4.2 $(\text{TS}xy \wedge \text{TS}zy \ \wedge \ x \equiv_t z) \rightarrow x = z$
(this t-equivalent slice is unique)

We will note $w_{/y}$ the part of w corresponding to the "lifetime" of y when it exists (that is when $y \subseteq_t w$).

Models for the base theory of Asher & Vieu are presented in [Asher and Vieu, 1995]. We present here the properties that a sub-part of $\mathcal{P}(\mathbb{R}^2)$ needs to be a model of our theory; lack of space precludes the presentation of the proof of the consistency of each axiom with respect to this model, which in most cases is rather straightforward. The construction can be easily generalized to show that equivalent parts of $\mathcal{P}(\mathbb{R}^3)$ or $\mathcal{P}(\mathbb{R}^4)$ are models of the theory. At this stage we do not show the kinds of model with respect to which there is semantic completeness, we merely introduce a model that shows the consistency of our theory.
Let's consider a subset $\mathcal{F} \subset \mathcal{P}(\mathbb{R}^2)$. Asher and Vieu have shown in [Asher and Vieu, 1995] the consistency and completeness of their theory with respect to a certain kind of structure. Models for our own theory will have some of these properties, which we present now.
Consider a topological space $\mathcal{E}$. Objects of the theory denote elements of a set $\mathcal{F} \subset \mathcal{P}(\mathcal{E})$. The interior and closure operators are noted "int" and "cl" (in our case $\mathcal{E}$ will be $\mathbb{R}^2$ with a classical topology). The operator $\cup^*$ and $\cap^*$ are union and intersection operators preserving the "regularity" of interiors and closures:
$x \cup^* y = x \cup y \cup int(cl(x \cup y))$
$x \cap^* y = x \cap y \cap cl(int(x \cap y))$
In order to satisfy the axioms of the topological part of the theory, elements of $\mathcal{F}$ must verify the following conditions:

i. if $X \in \mathcal{F}$ and $Y \in \mathcal{F}$ and $int(X \cap Y) \neq \emptyset$, then $X \cap^* Y \in \mathcal{F}$. (The intersection $\cap^*$ of two elements of $\mathcal{F}$ is in $\mathcal{F}$ if it has a non empty interior).

ii. if $X \in \mathcal{F}$ and $Y \in \mathcal{F}$, $X \cup^* Y \in \mathcal{F}$. (The union of two elements of $\mathcal{F}$ is in $\mathcal{F}$)
iii. $\mathcal{E} \in \mathcal{F}$.
iv. if $X \in \mathcal{F}$ and $X \neq \mathcal{E}$ then $(\mathcal{C}(X)/_{\mathcal{E}}) \in \mathcal{F}$ (if the complement of X is not empty, it is in $\mathcal{F}$)
v. Regularity: the interior of an element of $\mathcal{F}$ is not empty and "full" ($int(cl(X)) = int(X)$) and is in $\mathcal{F}$; its closure is regular ($cl(int(X)) = cl(X)$) and belongs to $\mathcal{F}$.

The interpretation of the relation of connection in this class of models for any variable assignation g, is as follows: $[\![Cxy]\!]_g$ =true iff $[\![x]\!]_g \cap [\![y]\!]_g \neq \emptyset$.

Now let's have a look at the added constraints on our models. Let $p_t(X)$ be the projection of $X \in \mathcal{F}$ on the temporal dimension:

$$p_t(X) = \{a \in \mathbb{R} / \exists b \in \mathbb{R},\ (b,a) \in X\}$$

We give the following interpretation to the primitives $<$ and $\asymp$. Let g be an arbitrary variable assignment:
$[\![x \asymp y]\!]_g = 1$ iff there is $a \in \mathbb{R}$, $b \in \mathbb{R}$ et $c \in \mathbb{R}$ such that $(a,c) \in [\![x]\!]_g$ and $(b,c) \in [\![y]\!]_g$.
$[\![x < y]\!]_g = 1$ iff for all real numbers a,b,c,d, $((a,b) \in [\![x]\!]_g \wedge (c,d) \in [\![y]\!]_g) \rightarrow b < d$
We add the following constraints:

α**)** for all $X \in \mathcal{F}$ and $Y \in \mathcal{F}$ such that $p_t(X) \subset p_t(Y)$ we have also $Y \cap (\mathbb{R} \times p_t(X)) \in \mathcal{F}$
This constraint is easily compatible with the others, since $Y \cap (\mathbb{R} \times p_t(X))$ has a non-empty interior since Y and X (and therefore $\mathbb{R} \times p_t(X)$) have non-empty interiors.

β**)** there are $X \in \mathcal{F}$ and $Y \in \mathcal{F}$ such that for all $(a,b) \in X$ and $(c,d) \in Y$, $b < d$.

γ**)** there are $X \in \mathcal{F}$ and $Y \in \mathcal{F}$ such that $p_t(X) \cap p_t(Y) \neq \emptyset$ et $X \cap Y = \emptyset$

The constraint α, β, γ are necessary because of (respectively) axioms A4.6, A4.3 and A4.2.

4.2 CONTINUITY

We have already mentioned the importance of continuity in our intuitive understanding of motion (motion is a perceptually continuous spatial change), and how this notion pervades a lot of work on spatial relations, the most obvious being the transition graphs (of RCC8 [Randell *et al.*, 1992], e.g.), without being given a generic formal characterization. The continuity we have in mind (along with other authors in QSR) is different from the mathematical sense where arbitrarily fine distinctions can be made about space and time. We want something closer to intuition and at the same time covering the properties shown to be necessary for a theory of motion. The continuity we propose can be defined within the theory without stating separately the possible transitions, contrarily to what Galton does in [Galton, 1993][2], and this provides a more general characterization of qualitative continuous change; it is thus moreover much easier to check the consistency of such a theory. Informally, we propose to consider a spatio-temporal individual as qualitatively continuous if it is temporally self-connected and if it doesn't make spatial "leaps" (corresponding to a sudden gain or loss of parts, or a sudden translation), that is if no part of an entity can be temporally connected to a slice of that entity but not spatio-temporally (a counter-example of this is illustrated in Figure 5, where the non-continuity of w corresponds to a qualitative "horizontal" jump; this figure also illustrates theorem 4.3 below and the entity z can be ignored at this stage of the discussion). This can be expressed

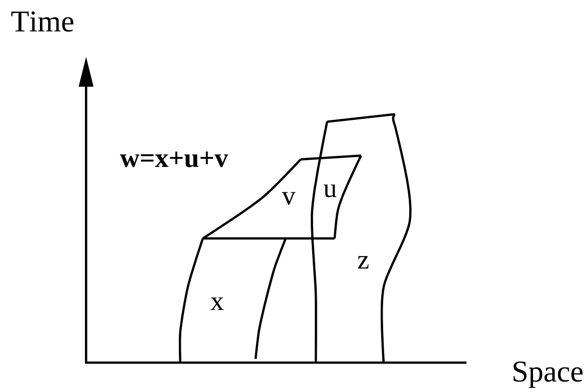

Figure 5: Entity w is not Continuous.

as follows:

D 4.2
CONTINU$w \triangleq$CON$_t w \wedge \forall x \forall u ((TSxw \wedge x \asymp u$
$\wedge$ P$uw) \rightarrow$C$xu)$

We will present now what we recover from the intuitions about continuous change. We will compare spatio-temporal relations between two contemporaneous slices of two "continuous" individuals at two different moments, in order to compare the results with RCC conceptual neighborhood graph. Figure 6 shows an example of what this means: during z_1, the corresponding slices of x and y are disconnected, later on, during z_2 the corresponding slices of x and y "spatially" overlap, i.e. every temporal slices of x and y overlap during z_2, which could be expressed as
O$_{sp}xy \triangleq$ O$xy \wedge x \subseteq_t (x \cdot y)$; this way, we can define all "spatial" relations corresponding to those definable in RCC (it is to be noted that DC is already equivalent to DC$_{sp}$). We

[2] Moreover we express the continuity of transition for any entity whereas Galton focused on rigid objects, a concept which cannot be expressed in a topological theory, and which eliminates some transitions such as NTPP to $=$.

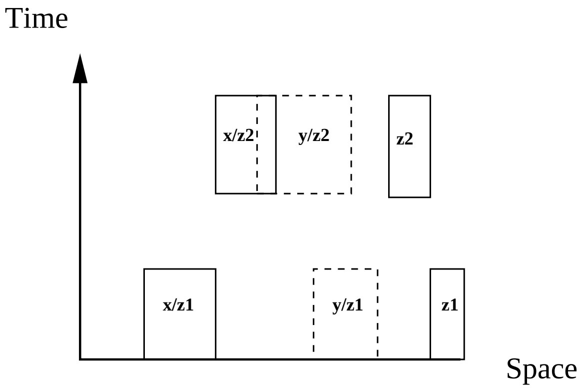

Figure 6: Purely “Spatial” Relations

found that impossible transitions correspond to the impossible transitions of the conceptual neighborhood graph of RCC8, with one exception: if the theory allows for atoms, continuity as defined here cannot apply to them since they have no parts [3]. Some transitions are moreover only possible between closed entities (a property which is assumed for RCC classical conceptual neighborhoods, since the theory cannot distinguish open and closed entities). In order to recover Figure 5, it suffices to demonstrate the following theorems (every time we mention a transition between two RCC relations they have to be understood as their purely spatial counterparts in our framework):

i) A continuous transition between DC and O is impossible (thus a continuous transition between DC and PO or TPP or NTPP or TPPI or NTPPI or “=” is impossible).

ii) A continuous transition between EC and NTP or TP is impossible (and thus NTPP and TPP and =).

iii) A continuous transition between PP and PP^{-1} is impossible (so that no transition from TPP or NTPP to TPPI or NTPPI is possible)

iv) A continuous transition between PO and NTPP is impossible. PO being symmetric, the impossibility of a PO/NTPP^{-1} is proved at the same time.

This way, the only remaining possibilities are the transition of Figure 7.

The non-continuity of a DC↔O transition

Th 4.3 $(\text{TS}xw \wedge \text{P}yw \wedge \text{DC}xz \wedge \text{O}_{sp}yz \ \wedge \ x \between y) \rightarrow \neg\text{CONTINU}w$

[3] The problem of atoms should be dealt with separately since it is not clear what properties they should have; few authors have addressed those questions, but we think it is rather natural for atoms to be left out in what is presented here.

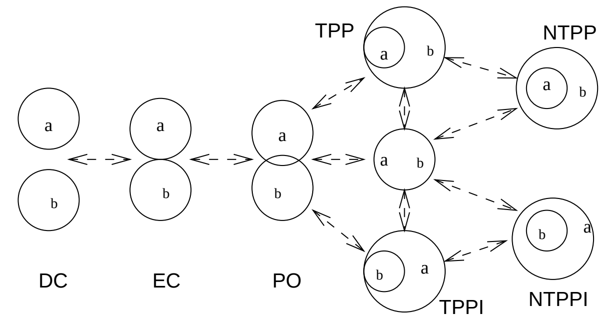

Figure 7: RCC8 Conceptual Neighborhoods

This says that a direct transition from DC (i.e. $\neg C$) and O cannot be continuous (see Figure 5). (Proof: easily, (with $u = y \cdot z$) $\neg\text{C}xu$ et $x \between u$, and this proves w’s non-continuity). The hypotheses of this theorem imply that x and y are closed (although in a non obvious way).

The non-continuity of a TP/NTP↔EC transition

This can be shown if we assume the entities involved to have only non-atomic temporal parts (that is if we assume x and y are GATs):

Th 4.4

$(\text{TS}xw \wedge \text{P}yw \wedge \text{EC}xz \wedge \text{TP}yz \ \wedge \ x \ \between \ y \wedge \neg GAT(x) \wedge \neg GAT(y)) \rightarrow \neg\text{CONTINU}w$

More easily (and more generally), we also have:

Th 4.5 $(\text{TS}xw \wedge \text{P}yw \wedge \text{EC}xz \wedge \text{NTPP}_{sp}yz \wedge x \between y) \rightarrow \neg\text{CONTINU}w$

(No continuous transition between EC and NTP).

See Figure 8 for the illustration of theorem 4.4. If x or y are generalized atoms, $x+y$ is still consistent with the definition of continuity. Note that the theorem mentions EC, which gives the intended result, given that $\text{EC}_{sp} \rightarrow$EC. The definition of $\text{NTPP}_{sp}xy$ departs a little from the other “purely” spatial relation since we may want it to hold on temporally closed intervals, and thus it cannot entail NTPPxy. We get around this by stating

$\text{NTPP}_{sp}xy \triangleq \forall z(z \subseteq_t (ix) \rightarrow \text{NTPP}x_{/z}y_{/z}$.

The non-continuity of a PP↔PP^{-1} transition

Observing that: $\text{PP}xz \ \leftrightarrow \ \neg\text{C}x(-z)$ and $\text{PP}^{-1}yz \ \leftrightarrow$ $(\text{O}y(-z) \vee \text{EC}y(-z))$ and using the previous theorems with $-z$ or $c(-z)$ in place of z, we show that there cannot be a direct continuous transition between PP and PP^{-1} (see Figure 9).

The non-continuity of a PO↔NTPP transition

We have (see Figure 10):

Th 4.6 $(z = z_1 + z_2 \wedge \text{TS}yw \wedge \text{NTPP}_{sp}yz_1 \wedge \text{P}xw \wedge \text{PO}_{sp}xz_2 \ \wedge \ x \between y) \rightarrow \neg\text{CONTINU}(w)$

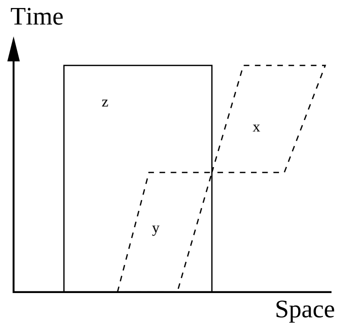

Figure 8: PP/EC Transition

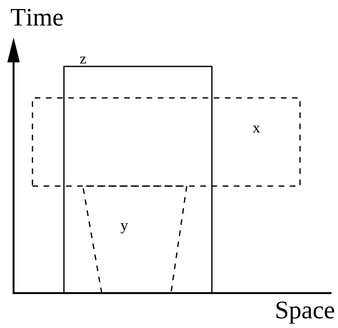

Figure 9: PP/PP^{-1} Transition

Indeed, considering $u = x - z$, we have $u \lessgtr y$ and $\neg \mathrm{C}uy$.

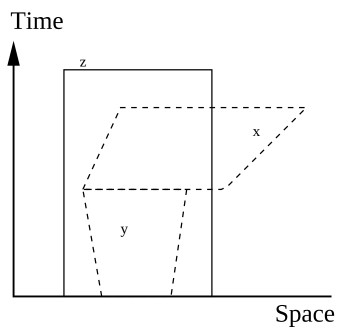

Figure 10: NTPP/PO Transition

To sum up this section, we have given a formal characterization of the conceptual neighborhood graph that is usually taken as an intuitive reality for RCC-like theories. We go further than Galton who had only listed extensively (in a different spatio-temporal setting) what this transitions should be in a limited case. We did this by defining spatial relations corresponding to the standard intended interpretations, even though theories differ as to what their exact models are. We have also shown the necessity of the distinction between open and closed regions and why we have chosen Asher and Vieu's topological framework.

5 MOTION CLASSES

5.1 REPRESENTING MOTION CLASSES

This section is dedicated to showing how the theory can be used to represent certain types of motion. This has been inspired from a study of some motion verbs [Muller and Sarda, 1997], and puts the expressive power of the theory to the test. We insist on the fact that we have restricted ourselves to the topological aspects of such motions and that the representations are consequently to be understood as a partial definition only of the semantics of those verbs, and as a guide for an intuitive (but formal) definition of meaningful topological motion classes.
In order to do so, we need to introduce a few preliminary notions. First, we define another spatio-temporal partial overlap as an overlap which is not spatial only (a slice of x has to be completely out of y at some time):

D 5.1 STPO$xy \triangleq$O$xy \ \wedge\ \neg(x \equiv_t x \cdot y) \ \wedge\ \neg(y \equiv_t y \cdot x)$

Th 5.1 STPO$xy \rightarrow \exists u(TSux \ \wedge\ \neg$C$uy)$

A few other definitions are needed:

D 5.2 LOC$xy \triangleq \exists z($TS$zx \wedge$ P$zy)$
(a slice of x is part of y)

D 5.3 TEMP_IN$xy \triangleq$LOC$xy \wedge$ STPOxy
(x is "temporarily" a part of y)

We define relations equivalent to Allen's relations on intervals [Allen and Hayes, 1985] (note that =,<, already exists as $\equiv_t$, <):

D 5.4 MEET$xy \triangleq ix < iy \wedge x \lessgtr y$

D 5.5
START$xy \triangleq x \subseteq_t y \wedge \forall z(z < x \rightarrow z < y) \wedge \neg y \subseteq_t x$

D 5.6
FINISH$xy \triangleq x \subseteq_t y \ \wedge\ \forall z(x < z \rightarrow y < z) \ \wedge\ \neg y \subseteq_t x$

D 5.7 O$_t xy \triangleq x \sigma y \ \wedge\ \exists u($START$uy \wedge$ FINISH$ux)$

D 5.8
DURING$xy \triangleq x \subseteq_t y \ \wedge\ \neg$FINISH$xy \ \wedge\ \neg$START$xy$

The converse relations can be defined in an obvious way, and will be noted MEET$_i$, START$_i$, FINISH$_i$, O$_{ti}$, DURING$_i$. We can now very simply express different motion classes, such as those mentioned in the beginning of the section. We assume for all those motions that the entities involved (intuitively, the arguments of a motion event) are continuous, without writing it each time in the definition.

The first argument corresponds to an entity z that "temporalize" the relation: the relation holds between $x_{/z}$ and $y_{/z}$, x and y being two other spatio-temporal regions. At a later stage, we could build a theory of physical objects where x and y would be the trajectories of two objects and z would be another entity giving the temporal extent of the relation (e.g. the spatio-temporal extension of an event). None of these interpretations are necessary to understand the following given relations, which are defined independently of any assumption about entities except that they are spatio-temporally extended. We define six classes of motion, named LEAVE, REACH, HIT, CROSS, INTERNAL and EXTERNAL; intuitively, each of these classes could be instantiated by the following motion verbs: LEAVE, REACH, HIT and CROSS correspond to the topological behavior of the corresponding verbs, INTERNAL corresponds to verbs such as "to drive around (the country)" and EXTERNAL to verbs such as "to avoid" (see Figure 11).

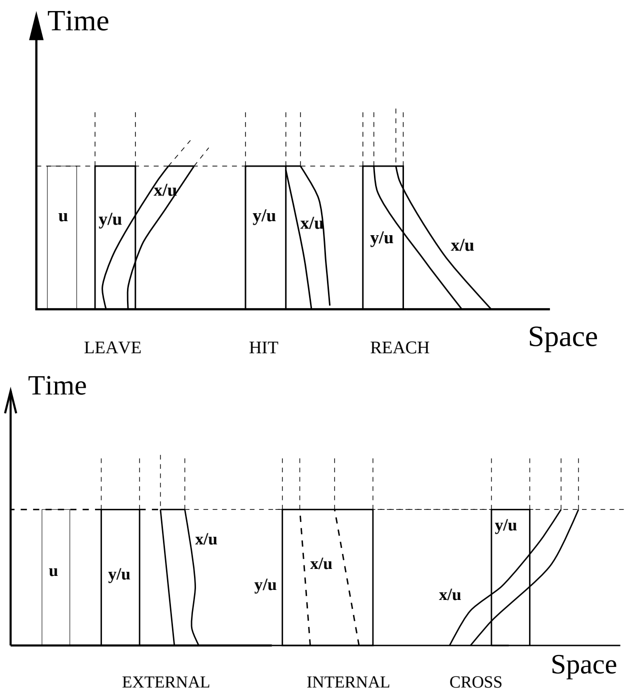

Figure 11: The Six Motion Classes

D 5.9
REACH$zxy \triangleq$TEMP_IN$x_{/z} y_{/z} \wedge$ FINISH$(x_{/z} \cdot y_{/z})z$

D 5.10
LEAVE$zxy \triangleq$TEMP_IN$x_{/z} y_{/z} \wedge$ START$(x_{/z} \cdot y_{/z})z$

D 5.11 INTERNAL$zxy \triangleq$PP$x_{/z} y_{/z}$

D 5.12
HIT$zxy \triangleq$EC$x_{/z} y_{/z} \wedge \forall x_1, y_1[(Px_1 x_{/z} \wedge$P$y_1 y_{/z} \wedge$EC$x_1 y_1) \rightarrow ($FINISH$x_1 z \wedge$FINISH$y_1 z)]$

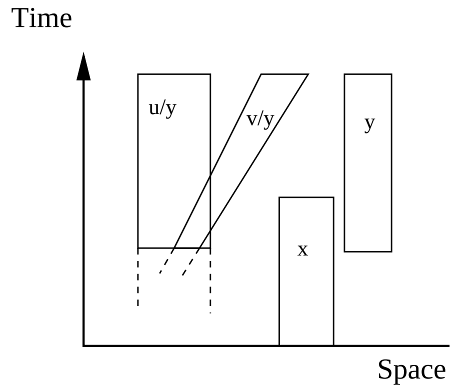

Figure 12: An Example for Time and Space-Time Composition

D 5.13 EXTERNAL$zxy \triangleq \neg$C$x_{/z} y_{/z}$

D 5.14
CROSS$zxy \triangleq \exists z_1, z_2(z = z_1 + z_2 \wedge$ MEET$z_1 z_2 \wedge$ REACH$z_1 xy \wedge$ LEAVE$z_2 xy)$

5.2 REASONING ABOUT MOTION CLASSES

5.2.1 Combining Motion and Temporal Information

To be able to reason efficiently about spatial qualitative models, it is customary to use and implement composition tables, which are dedicated to more specific deductions than generic first-order theorem provers. To reason about motion, different paths can be followed. We show in Table 1-2 a composition of temporal and spatio-temporal information between two entities, following the pattern: $(($R$_1 xy \wedge$ Motion$(y, u, v)) \rightarrow$R$_2 u_{/x} v_{/x})$, R$_1$ and Motion being the entries in the table and R$_2$ being the corresponding result. This table takes as input information about the class of motion (Motion) and a temporal relation R$_1$ between the entity that determine the temporal extent during which the relation holds and any other entity (we choose to express this second piece of information with an Allen relation). The result is a spatio-temporal relation R$_2$ between the arguments (u and v) "during" the other entity (x) [4]. Figure 12 illustrates the kind of situation corresponding to the table, where R$_1$ is O$_t$ and Motion is LEAVE.

All entries of this table are theorems of the theory; entries assuming u and v are continuous are marked with a (c); we also assume entities are all GATs (see section 2). Space doesn't allow us to present the proofs here.

[4] We consider the cases of relations similar to RCC8, for the relation holding between $u_{/x}$ and $v_{/x}$, the resulting relation is a disjunction of one of the previously defined relations.

<table>
<tr><th>R_1/Motion</th><th>LEAVE</th><th>HIT</th><th>REACH</th></tr>
<tr><td>MEET (c)</td><td rowspan="3">PO,TPP, NTPP</td><td rowspan="2">DC,EC,PO</td><td rowspan="3">DC,EC,PO</td></tr>
<tr><td>O_t</td></tr>
<tr><td>START</td><td rowspan="2">DC</td></tr>
<tr><td>DURING</td><td>All</td><td>All</td></tr>
<tr><td>$\equiv_t$</td><td>PO</td><td rowspan="2">EC</td><td>PO</td></tr>
<tr><td>FINISH</td><td rowspan="3">PO,EC,DC</td><td rowspan="3">PO,TPP, NTPP</td></tr>
<tr><td>O_{ti}</td><td rowspan="5">EC, PO</td></tr>
<tr><td>$MEET_i$ (c)</td></tr>
<tr><td>$FINISH_i$</td><td rowspan="3">PO</td><td rowspan="3">PO</td></tr>
<tr><td>$START_i$</td></tr>
<tr><td>$DURING_i$</td></tr>
</table>

Table 1: Combining Motion and Temporal Information (1).

<table>
<tr><th>R_1/Motion</th><th>CROSS</th><th>INTERN.</th><th>EXTERN.</th></tr>
<tr><td>MEET (c)</td><td rowspan="3">DC, EC, PO</td><td rowspan="2">PO, TPP, NTPP</td><td rowspan="2">DC, EC, PO</td></tr>
<tr><td>O_t</td></tr>
<tr><td>START</td><td rowspan="4">PP</td><td rowspan="4">DC</td></tr>
<tr><td>DURING</td><td>All</td></tr>
<tr><td>$\equiv_t$</td><td>PO</td></tr>
<tr><td>FINISH</td><td rowspan="3">DC, EC, PO</td></tr>
<tr><td>O_{ti}</td><td rowspan="5">PO, TPP, NTPP</td><td rowspan="5">DC, EC, PO</td></tr>
<tr><td>$MEET_i$ (c)</td></tr>
<tr><td>$FINISH_i$</td><td rowspan="3">PO</td></tr>
<tr><td>$START_i$</td></tr>
<tr><td>$DURING_i$</td></tr>
</table>

Table 2: Combining Motion and Temporal Information (2).

5.2.2 Combining Motion and "Static" Information

Obviously, even though all objects are considered as having histories, we perceive our environment with respect to objects we regard as "static" and defining some kind of frame of reference. Thus motion is often considered as the motion of an entity relative to one or more given (often implicitly), "static" entities in a frame of reference. Thus we present now the combination of information about objects of a known environment (intuitively the spatio-temporal referent of locations in a frame of reference) and motion information. We will assume that some regions (corresponding to locations) bear the same relation to each other "during" any other entity. This means, that the relation R_1 in the Tables 3-4 is to be read as $R_1 y_{/u} z_{/u} \wedge \forall v(v \subseteq_t u \rightarrow R_1 y_{/v} z_{/v})$ (this is what was meant by "purely" spatial relations, namely that the same relation holds between all respective temporal slices of those entities, cf section 4).

Thus, from $R_1 y_{/u} z_{/u}$ and $\mathrm{Motion}(u, x, y)$, we can deduce a relation R_2 such that $R_2 x_{/u} z_{/u}$. Figure 13 shows an example entry of the table, where Motion is LEAVE and R_1 is DC. The result can be DC, EC or PO. This is obtained simply by combining the entries of Tables 1-2 with a composition table for topological relations. As the temporal information makes use of the continuity hypothesis, this is also only valid assuming that hypothesis. We want to point out at this stage, that we use those relations only as a means of comparison and that they do not really express as such all the information we want to convey when dealing with motion. It can be seen from the tables that (PO, TPP, NTPP) is a common result for very different motion relations (such as LEAVE and CROSS); we mainly wanted to focus here on why this theory is a good starting point for a theory of motion.

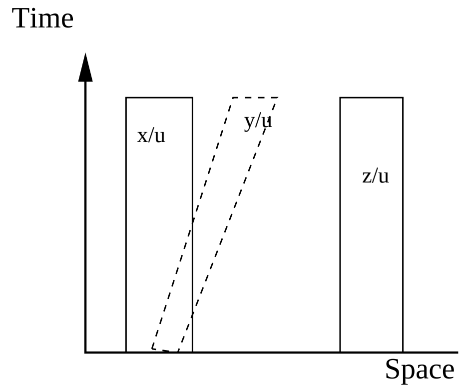

Figure 13: An Example of Location and Spatio-Temporal Combination

R_1/Motion	LEAVE	HIT	REACH
DC_{sp}	DC,EC,PO	DC,EC,PO, PP^{-1}	DC,EC,PO
EC_{sp}	DC,EC,PO	DC,EC,PO, TPP, $NTPP^{-1}$, TPP^{-1}	DC,EC, PO
PO_{sp}	DC,EC,PO, TPP,NTPP	DC,EC,PO, TPP,NTPP	DC,EC,PO, TPP, NTPP
TPP_{sp}	DC,EC,PO	DC,EC	DC,EC,PO
$NTPP_{sp}$	DC,EC,PO	DC	DC,EC,PO
=	PO	EC	PO
TPP_{sp}^{-1}	PO,TPP, NTPP	PO,TPP, NTPP	PO,TPP, NTPP
$NTPP_{sp}^{-1}$	PO,TPP, NTPP	PO,TPP, NTPP	PO,TPP, NTPP

Table 3: Composing Motion and Location Information (2).

6 CONCLUSION

We have presented here an original relational theory combining space and time to achieve a theory of spatio-temporal entities and thus a qualitative theory of motion. It is possible to define a notion of qualitative spatio-temporal continuity within the theory which gives a rigorous basis to intuitions that seemed reasonable to admit about most RCC-

R_1/Motion	CROSS	INTERN.	EXTERN.
DC_{sp}	DC,EC,PO	DC	All
EC_{sp}	DC,EC,PO	DC,EC	All
PO_{sp}	All	All	All
TPP_{sp}	DC,EC,PO	All	DC
$NTPP_{sp}$	DC,EC,PO	All	DC
=	PO	TPP,NTPP	DC
TPP_{sp}^{-1}	PO,TPP, NTPP	NTPP	All
$NTPP_{sp}^{-1}$	PO,TPP, NTPP	NTPP	All

Table 4: Composing Motion and Location Information (2).

like theories of space, namely the admissible transitions between qualitative spatial relations, in a more unified and general framework than [Galton, 1993]. The expressive power of the theory allows for the definition of complex motion classes such as those expressed by motion verbs in natural language. This complexity is in itself a problem if one is to reason about such representations, so we have given a few examples of the kind of reasoning that can be of interest about motion relations and that can be demonstrated within the theory. It is to be noted however that "motion" ranges over a wide set of phenomena and it should be expected that varied courses of action can be followed in the task of automating reasoning about it, depending on the kind of task one is interested in (it can be qualitative reasoning about physical objects, such as [Hayes, 1985a, Davis, 1989] or representing natural language expressions; another area where it can be used is high-level vision systems dealing with natural language descriptions of spatial changes [Fernyhough *et al.*, 1998, Borillo and Pensec, 1995]). Besides we can now try to extend our theory to geometrical concepts (such as orientation and distance). Some work is now also needed to characterize completely the models of the theory (and then prove completeness with respect to those models).

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References

[Allen and Hayes, 1985] J. Allen and P.J. Hayes. A commonsense theory of time. IJCAI, 1985.

[Asher and Sablayrolles, 1995] N. Asher and P. Sablayrolles. A typology and discourse semantics for motion verbs and spatial PPs in French. *Journal of Semantics*, 12(1):163–209, June 1995.

[Asher and Vieu, 1995] N. Asher and L. Vieu. Towards a geometry of common sense: a semantics and a complete axiomatisation of mereotopology. In *Proceedings of IJCAI95*, 1995.

[Aurnague and Vieu, 1993] M. Aurnague and L. Vieu. A three-level approach to the semantics of space. In Zelinski-Wibbelt, editor, *Semantics of Prepositions in Natural Language Processing*, number 3 in Natural Language Processing, pages 393–439. Berlin : Mouton de Gruyter, 1993.

[Borillo and Pensec, 1995] M. Borillo and H. Pensec. From numerical observations to propositional representation: a cognitive methodology to structure hybrid spatial knowledge. Sheffield, 1995. AISB proceedings.

[Carnap, 1958] Rudolf Carnap. *Introduction to Symbolic Logic and its Applications*. Dover, New York, 1958.

[Clarke, 1981] B. Clarke. A calculus of individuals based on 'connection'. *Notre Dame Journal of Formal Logic*, 22(3):204–218, July 1981.

[Davis, 1989] E. Davis. *Representations of Commonsense Knowledge*. Morgan Kaufmann, 1989.

[Fernyhough *et al.*, 1998] J. Fernyhough, A.G. Cohn, and D.C. Hogg. Building qualitative event models automatically from visual input. In *Proc. Int. Conf. on Computer Vision (ICCV98)*, pages 350–355. IEEE, Narosa Publishing House, 1998.

[Fleck, 1996] M.M. Fleck. The topology of boundaries. *Artificial Intelligence*, 80:1–27, 1996.

[Forbus, 1983] K. Forbus. Qualitative reasoning about space and motion. In D. Gentner and A.L. Stevens, editors, *Mental models*, pages 53–73. Erlbaum, Hillsdale (NJ), 1983.

[Galton, 1993] A. Galton. Towards an integrated logics of space, time and motion. IJCAI, 1993.

[Galton, 1995] A. Galton. Time and change for IA. In Gabbay, editor, *Logics for Epistemic and Temporal Reasoning*, volume 4 of *Handbook of Logics for AI and Logic Programming*. Oxford University Press, 1995.

[Galton, 1997] A. Galton. Space, time and movement. In O. Stock, editor, *Spatial and Temporal Reasoning*. Kluwer, 1997.

[Hartley, 1992] R. Hartley. A uniform representation for time and space and their mutual constraints. *Computers Math. Applic.*, 1992.

[Hayes, 1985a] P.J. Hayes. An ontology for liquids. In J.R. Hobbs et R.C. Moore, editor, *Formal Theories of the Commonsense World*. Ablex Publishing Corporation, Norwood, 1985.

[Hayes, 1985b] P.J. Hayes. The second naive physics manifesto. In J.R. Hobbs et R.C. Moore, editor, *Formal Theories of the Commonsense World*, pages 1–36. Ablex Publishing Corporation, Norwood, 1985.

[Hays, 1989] E. Hays. On defining motion verbs and spatial prepositions. Technical report, Universitat des Saarlandes, 1989.

[Kamp, 1979] H. Kamp. Events, instants and temporal reference. In Von Stechow Bäuerle, Egli, editor, *Meaning, use and interpretation of language*, pages 376–417. de Gruyter, Berlin, 1979.

[Muller and Sarda, 1997] P. Muller and L. Sarda. The semantics of french transitive movement verbs and the ontological nature of their objects. In *Proceedings of ICCS'97*, Donostia-San Sebastian, May 1997.

[Rajagopalan and Kuipers, 1994] R. Rajagopalan and B. Kuipers. Qualitative spatial reasoning about objects in motion: application to physics problem solving. San Antonio, TX, March 1994. IEEE Conference on Artificial Intelligence for Applications (CAIA-94).

[Randell *et al.*, 1992] D. Randell, Z. Cui, and A. Cohn. A spatial logic based on regions and connection. San Mateo CA, 1992. KR'92, Morgan Kaufmann.

[Russell, 1914] Bertrand Russell. *Our Knowledge of the External World*. Routledge, London and New York, 1914.

[Shanahan, 1995] M. Shanahan. Default reasoning about spatial occupancy. *Artificial Intelligence*, 74:147–163, 1995.

[Simons, 1987] Peter Simons. *Parts - A Study in Ontology*. Oxford University Press, Oxford, 1987.

[van Benthem, 1995] J. van Benthem. Temporal logic. In Gabbay, editor, *Logics for Epistemic and Temporal Reasoning*, volume 4 of *Handbook of Logics for AI and Logic Programming*. Oxford University Press, 1995.

[Varzi, 1996] A. Varzi. Parts, wholes, and part-whole relations: The prospects of mereotopology. *Data and Knowledge Engineering*, 20(3):259–286, 1996.

[Vieu, 1991] L. Vieu. *Sémantique des relations spatiales et inférences spatio-temporelles: une contribution à l'étude des structures formelles de l'espace en langage naturel*. PhD thesis, Université Paul Sabatier, Toulouse, 1991.

[Vieu, 1997] L. Vieu. Spatial representation and reasoning in AI. In O. Stock, editor, *Spatial and Temporal Reasoning*, pages 3–40. Kluwer, 1997.