Towards a Principle-based Approach for Case-based Reasoning

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Abstract. Case-based reasoning (CBR) is an experience-based approach to solving problems; it adapts previously successful cases to new problems following the key assumption: *the more similar the cases, the more similar their solutions*. Despite its popularity, there are few works on foundations, or properties, that may underlie CBR models.
This paper bridges this gap by defining various notions capturing the above assumption, and proposing a set of principles that a CBR system would satisfy. We discuss their properties and show that the principles that are founded on the CBR assumption are incompatible with some axioms underlying non-monotonic reasoning (NMR). This shows that CBR and NMR are different forms of reasoning, and sheds light on the reasons behind their disagreements.

Keywords: Case-based Reasoning · Non-Monotonic Reasoning · Principles.

1 Introduction

Case-based reasoning (CBR) is an experience-based approach to solving problems. It uses stored cases describing similar prior problem-solving episodes and adapts their solutions to fit new needs (or new cases). For example, a car dealer would guess the price of a given car by comparing its characteristics with those of cars that have been sold. This form of reasoning has been used in the literature for solving various practical problems including some in the medical (eg. [10, 14, 13]) and legal (eg. [3, 4, 15, 2]) domains.

Several works have been devoted to modeling CBR, and various approaches can be distinguished including logic-based [16, 5, 6] and argumentation-based [9, 11] approaches (see [12, 7, 1] for surveys). However, despite its popularity, there are few works on foundations, or properties, that may underlie CBR models. Foundations are important not only for a better understanding of case-based reasoning in general, but also for clarifying the basic assumptions underlying models, comparing different models, and also for comparing case-based reasoning with other kinds of reasoning like defeasible reasoning.

This paper bridges this gap. It starts by analysing the basic assumption behind case-based reasoning, namely "*the more similar the cases, the more similar their outcomes*". It discusses three independent notions that capture (in different ways) the assumption. Then, the paper proposes principles that a case-based reasoning model would satisfy and analyses their properties. Some principles ensure the three forms of the CBR assumption, and we show that they are incompatible with some axioms underlying non-monotonic reasoning (NMR) [8], namely cautious monotonicity. This shows that CBR and NMR are different forms of reasoning, and sheds light on the reasons behind their differences.

The paper is organized as follows: Section 2 introduces CBR problems, Section 3 discusses various formalizations of the key assumption behind CBR, Section 4 introduces basic principles that a model would satisfy. The last section concludes.

2 Background

Before introducing formally the basic notions of a CBR problem, let us consider the following illustrative example borrowed from [5].

Example 1. Consider the problem of identifying the price of second-hand cars. A car is described with five attributes, namely years old, power, mileage, the state of equipment, and shape. Knowing the characteristics and the prices of four cars (C_1, C_2, C_3, C_4) (summarized in the table below), the problem is to identify the price of the new car (C_n) whose characteristics are also known.

Cases	Years old	Power	Mileage	Equipment	Shape	Price
C_1	1	1300	20 000	poor	good	8000
C_2	2	1600	30 000	excellent	poor	7000
C_3	2	1600	40 000	good	good	5000
C_4	3	1500	60 000	excellent	poor	5000
C_n	2	1600	50 000	poor	good	?

To identify the price of C_n, any CBR model would compare the characteristics of cars as well as their prices. Hence, it would use two similarity measures: one for comparing prices $(\mathbf{S}^o)$ and another for comparing attributes-values $(\mathbf{S}^i)$. In [5], $\mathbf{S}^o$ is defined as follows:

$$\mathbf{S}^o(u,v) = \begin{cases} 1 & \textit{if } |u-v| \leq 500 \\ 0 & \textit{if } |u-v| \geq 2000 \\ 1 - \frac{1}{1500} * (|u-v| - 500) & \textit{if } 500 < |u-v| < 2000 \end{cases}$$

It is easy to check that $\mathbf{S}^o(x,x) = 1$, $\mathbf{S}^o(5000, 7000) = \mathbf{S}^o(5000, 8000) = 0$ and $\mathbf{S}^o(7000, 8000) = \frac{2}{3}$.

Regarding $\mathbf{S}^i$, it combines five measures, each of which compares the values of an attribute. $\mathbf{S}^1$ compares years old (respectively mileage) as follows: $\mathbf{S}^1(u,v) = \frac{\min(u,v)}{\max(u,v)}$. For instance, $\mathbf{S}^1(1,2) = \frac{1}{2}$ and $\mathbf{S}^1(20000, 30000) = \frac{2}{3}$. The measure that compares the powers of two cars is defined as follows: $\mathbf{S}^2(u,v) = 1 - (\frac{|u-v|}{1000})$. For instance, $\mathbf{S}^2(1300, 1600) = \frac{7}{10}$. Finally, equipment and shape are compared using the measure $\mathbf{S}^3$, which assumes the ordering bad < poor < good < excellent.

$$\mathbf{S}^3(v,v') \quad = \begin{cases} 1 & \text{if} \quad v = v' \\ \frac{2}{3} & \text{if} \quad \text{v and v' are consecutive} \\ \frac{1}{3} & \text{if} \quad \text{there is exactly one element between v and v'} \\ 0 & \text{otherwise.} \end{cases}$$

The similarity between (the characteristics of) two cars is the minimal similarity of the characteristics. For instance, $\mathbf{S}^i(C_1, C_2) = \min(\mathbf{S}^1(1,2),\ \mathbf{S}^2(1300,1600),\ \mathbf{S}^1(20000,30000),\ \mathbf{S}^3(\text{poor},\text{excellent}),\ \mathbf{S}^3(\text{good},\text{poor})) = \min(\frac{1}{2}, \frac{7}{10}, \frac{2}{3}, \frac{1}{3}, \frac{2}{3}) = \frac{1}{3}$. The table below summarises the values returned by $\mathbf{S}^i$ for each pair of cars.

Cases	C_1	C_2	C_3	C_4	C_n
C_1	1	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{5}$
C_2	$\frac{1}{3}$	1	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}$
C_3	$\frac{1}{2}$	$\frac{2}{3}$	1	$\frac{2}{3}$	$\frac{2}{3}$
C_4	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	1	$\frac{1}{3}$
C_n	$\frac{2}{5}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	1

Throughout the paper, we assume a finite and non-empty set $\mathcal{F} = \{f_1, \ldots, f_n, \mathtt{f}\}$ of *features*, where $f_1, \ldots, f_n$ describe the cases (eg. Power, Mileage, Shape) and $\mathtt{f}$ is the feature being solved (price in the example). Let $\mathtt{dom}$ be a function on $\mathcal{F}$ which returns the domain of every $f \in \mathcal{F}$. Hence, $\mathtt{dom}(\mathtt{f})$ is the set of possible outcomes of a CBR problem, which is finite in classification tasks. In addition to this set, we assume the special symbols ? and $\mathtt{Und}$, which denote respectively that the value of $\mathtt{f}$ is *pending* and *undecided* by a CBR model. We call *literal* every pair (f, v) such that $f \in \mathcal{F} \setminus \{\mathtt{f}\}$ and $v \in \mathtt{dom}(f)$, and *instance* every set of literals, where each feature $f_1, \ldots, f_n$ appears exactly once. We denote by $\mathtt{Inst}$ the set of all possible instances, and call it *input space*. The latter is endowed with a similarity measure $\mathbf{S}^i$, which assesses how close are instances. The set $\mathtt{dom}(\mathtt{f})$ is endowed with a similarity measure $\mathbf{S}^o$, which compares outcomes. Recall that a similarity measure $\mathbf{S}$ on a set X is a function $\mathbf{S} : X \times X \to [0,1]$ where:

- $\forall x \in X$, $\mathbf{S}(x,x) = 1$
- $\forall x, y \in X$, $\mathbf{S}(x,y) = \mathbf{S}(y,x)$

We consider two additional parameters $0 < \delta^i \leq 1$ and $0 < \delta^o \leq 1$, which represent the *thresholds* for considering respectively two instances and two outcomes as somewhat similar. More precisely, for $x, y \in \mathtt{Inst}$, x is dissimilar to y iff $\mathbf{S}^i(x,y) < \delta^o$ and for $v, v' \in \mathtt{dom}(\mathtt{f})$, v is dissimilar to v' iff $\mathbf{S}^o(v,v') < \delta^o$.

Let us now introduce the backbone of a CBR problem, the notion of *case*. It is an instance labelled with an outcome.

Definition 1 (Case). *A* case *is a pair* $c = \langle I, v \rangle$ *such that* $I \in \texttt{Inst}$ *and* $v \in \texttt{dom}(\texttt{f}) \cup \{?\}$. *We call* c *a* past case *when* $v \in \texttt{dom}(\texttt{f})$, *and a* new case *when* $v =?$. *A* case base *is a sample that consists of* n *past cases* $c_i = \langle I_i, v_i \rangle$ *(*$1 \leq i \leq n$*).*

In [5], a case base is said to be *consistent* if identical cases in the base have identical outcomes (i.e., for all cases $\langle I, v \rangle$ and $\langle I', v' \rangle$ in a base, if $I = I'$ then $v = v'$). In some problems like the one described in the above example, this constraint may be strong as the same instances may have different but similar outcomes. It is also possible that similar instances have the same or similar outcomes. Imagine a second-hand car $C*$ which has the same characteristics as C_1, but its price is 8400. Note that $\mathbf{S}^o(8000, 8400) = 1$, which means that the difference between the two prices is negligible. In what follows, we generalize this notion of consistency using similarity measures. The idea is that fully similar instances get fully similar outcomes.

Definition 2 (Consistency). *A case base* Σ *is* consistent *iff* $\forall \langle I, v \rangle, \langle I', v' \rangle \in \Sigma$, *if* $\mathbf{S}^i(I, I') = 1$ *then* $\mathbf{S}^o(v, v') = 1$. *It is* inconsistent *otherwise.*

It is easy to see that in a consistent case base, identical instances may receive different but fully similar outcomes.

Property 1. If a case base Σ is consistent, then $\forall \langle I, v \rangle, \langle I', v' \rangle \in \Sigma$, if $I = I'$, then $\mathbf{S}^o(v, v') = 1$.

Proof. Let Σ be a consistent case base. Assume that $\langle I, v \rangle, \langle I', v' \rangle \in \Sigma$ such that $I = I'$. Since $\mathbf{S}^i$ is a similarity measure, then $\mathbf{S}^i(I, I') = 1$. From Consistency of Σ, $\mathbf{S}^o(v, v') = 1$.

Throughout the paper, we call CBR theory, or theory for short, a tuple containing a set of attributes, their domains, two similarity measures $\mathbf{S}^i, \mathbf{S}^o$ and their thresholds.

Definition 3 (Theory). *A* theory *is a tuple* $\mathbf{T} = \langle \mathcal{F}, \texttt{dom}, \mathbf{S}^i, \mathbf{S}^o, \delta^i, \delta^o \rangle$.

3 CBR basic assumption

Case-based reasoning is heavily based on similarities between cases. It looks for the most similar past cases to the new case, then adapts their outcomes following the key rule:

> *The more similar the cases (in the sense of* $\mathbf{S}^i$*), the more similar their outcomes (in the sense of* $\mathbf{S}^o$*).*

Formalizing this rule is important for developing reasonable CBR models and also for checking whether existing models obey the rule. In [5], it has been formalized as a *fuzzy gradual* rule, which states that the similarity of two instances should be lower or equal to the similarity of their outcomes. Throughout the paper, we refer to this notion as *strong coherence.*

Definition 4 (Strong Coherence). *A case base Σ is* strongly coherent *iff* $\forall \langle I, v\rangle, \langle I', v'\rangle \in \Sigma$, $\mathbf{S}^i(I, I') \leq \mathbf{S}^o(v, v')$.

Example 1 (Cont) The case base $\Sigma_1 = \{C_i = \langle I_i, v_i\rangle, i = 1, \ldots, 4\}$ is not strongly coherent. For instance, $\mathbf{S}^i(I_1, I_3) = \frac{1}{2}$ while $\mathbf{S}^o(v_1, v_3) = 0$.

Example 2. Consider the case base $\Sigma_2 = \{C = \langle I, v\rangle, C' = \langle I', v'\rangle\}$. If $\mathbf{S}^i(I, I') = 0.7$ and $\mathbf{S}^o(v, v') \geq 0.7$, then Σ is strongly coherent. Assume now that $\mathbf{S}^i(I, I') = 0.1$ and $\mathbf{S}^o(v, v') = 1$. Again, Σ_2 is strongly coherent even if the two cases are dissimilar (let $\delta^i = 0.5$).

It is easy to show that fully similar cases in a strongly coherent case base have fully similar outcomes.

Property 2. Let Σ be a strongly coherent case base. For all $\langle I, v\rangle, \langle I', v'\rangle \in \Sigma$, if $\mathbf{S}^i(I, I') = 1$, then $\mathbf{S}^o(v, v') = 1$.

Proof. Let Σ be a strongly coherent case base. Let $\langle I, v\rangle, \langle I', v'\rangle \in \Sigma$ such that $\mathbf{S}^i(I, I') = 1$. Strong coherence of Σ implies $\mathbf{S}^i(I, I') \leq \mathbf{S}^o(v, v')$. Since $\mathbf{S}^o(v, v') \in [0, 1]$, then $\mathbf{S}^o(v, v') = 1$.

It is also easy to show that any strongly coherent case base is consistent. The converse is false as shown in Example 1 (the base Σ_1 is consistent but not strongly coherent).

Property 3. If a case base is strongly coherent, then it is consistent. The converse does not hold.

Proof. Let Σ be a case base and assume it is strongly coherent. Let $\langle I, v\rangle, \langle I', v'\rangle \in \Sigma$ such that $\mathbf{S}^i(I, I') = 1$. From Property 2, it follows that $\mathbf{S}^o(v, v') = 1$.

By directly linking the similarity of outcomes with the similarity of instances, the property of strong coherence ensures that the former is proportional to the latter. However, the similarity measures $\mathbf{S}^i$ and $\mathbf{S}^o$ as well as their corresponding thresholds (δ^i and δ^o) may be different and not necessarily commensurate. This makes the satisfaction of the property difficult in case of such measures. Let us illustrate the issue with the following example.

Example 3. Suppose we have a case base Σ_3 on student grades. There are 4 attributes corresponding to courses and which take values from the interval $[0, 20]$; the outcome is a global appreciation whose range consists of 4 qualitative levels: bad $<$ poor $<$ good $<$ excellent. Let $\mathbf{S}^o$ be the similarity measure $\mathbf{S}^3$ in Example 1. Similarity between any pair of grades obtained in a course is defined by $\mathbf{S}(u,v) = 1 - (\frac{|u-v|}{20})$. The similarity measure $\mathbf{S}^i$ takes the minimal value returned by $\mathbf{S}$ on the four courses. Assume Σ_3 contains two students who got respectively $I = \langle 20, 20, 20, 20\rangle$ and $v =$ "excellent" as global appreciation, and $I' = \langle 20, 20, 15, 15\rangle$ with appreciation $v' =$ "good". Hence, $\mathbf{S}^i(I, I') = 0.75$ and $\mathbf{S}^o(v, v') = \frac{2}{3}$. Note that the base is not strongly coherent. In order to be coherent, $\mathbf{S}^o(v, v')$ should be equal to 1, which is not reasonable in the example as the two instances are different and deserve different appreciations. Furthermore, the scale of $\mathbf{S}^o$ does not have an intermediate value between $\frac{2}{3}$ and 1.

In what follows, we introduce a novel notion of *weak coherence*, which makes use of the two thresholds for judging similar instances/outcomes. It states that similar cases should receive similar outcomes.

Definition 5 (Weak Coherence). *A case-base Σ is* weakly coherent *iff* $\forall \langle I, v\rangle$, $\langle I', v'\rangle \in \Sigma$, *if* $\mathbf{S}^i(I, I') \geq \delta^i$, *then* $\mathbf{S}^o(v, v') \geq \delta^o$.

Example 3 (Cont) If $\delta^i \geq 0.75$ and $\delta^o \geq \frac{2}{3}$, then Σ_2 is weakly coherent.

The above example shows that a case base may be weakly but not strongly coherent. However, weak coherence follows from the strong version when $\delta^i \geq \delta^o$.

Proposition 1. *Let Σ be a case base and $\delta^i \geq \delta^o$. If Σ is strongly coherent, then Σ is also weakly coherent.*

Proof. Assume $\delta^i \geq \delta^o$. Let Σ be a strongly coherent case base, and $\langle I, v\rangle$, $\langle I', v'\rangle \in \Sigma$. Assume $\mathbf{S}^i(I, I') \geq \delta^i$. From strong coherence, $\delta^i \leq \mathbf{S}^i(I, I') \leq \mathbf{S}^o(v, v')$. Hence, $\mathbf{S}^o(v, v') \geq \delta^o$.

It is worth mentioning that consistency does not follow from weak coherence. Indeed, it is possible to find a weakly coherent case base which contains two cases such that $\mathbf{S}^i(I, I') = 1$, thus $\mathbf{S}^i(I, I') \geq \delta^i$, while $\delta^o \leq \mathbf{S}^o(v, v') < 1$.

The two versions of coherence compare pairs of cases of a case base. Our next notion, called *regularity*, is defined on the whole set of cases and ensures that the closest instances receive the closest outcomes. Indeed, if an instance I is closer to I' than to I'', then its outcome should be closer to that of I'.

Definition 6 (Regularity). *A case-base Σ is* regular *iff* $\forall \langle I, v\rangle, \langle I', v'\rangle, \langle I'', v''\rangle \in \Sigma$, *if* $\mathbf{S}^i(I, I') \geq \mathbf{S}^i(I, I'')$ *then* $\mathbf{S}^o(v, v') \geq \mathbf{S}^o(v, v'')$.

Example 1 (Cont) The case base $\Sigma_1 = \{C_i = \langle I_i, v_i\rangle, i = 1, \ldots, 4\}$ is not regular. For instance, $\mathbf{S}^i(I_1, I_3) > \mathbf{S}^i(I_1, I_2)$ while $\mathbf{S}^o(v_1, v_3) < \mathbf{S}^o(v_1, v_2)$.

Regularity is different from the two forms of coherence, and thus it does not imply or follow from them. It is also independent from consistency.

4 Axioms for CBR

A CBR model is a function, which takes as input a theory and a new case, and returns possible outcomes of the latter. Since every instance is assigned exactly one label, one expects that a model provides a single solution. However, this is not always possible since the new case may be close to several differently labelled cases, and the model cannot discriminate between those labels. So, each of label is considered as a *candidate* outcome. It is also possible that the new case is dissimilar to all past cases of a base. Hence, instead of returning an arbitrary outcome, we assume that a model may rerun the symbol `Und` (for undecided), meaning no solution is proposed.

Definition 7 (CBR Model). *Let* $\mathbf{T} = \langle \mathcal{F}, \mathtt{dom}, \mathbf{S}^i, \mathbf{S}^o, \delta^i, \delta^o \rangle$ *be a theory. A* CBR model *is a function* $\mathtt{R}$ *mapping every case base* Σ *and new case* $\langle I, ? \rangle$ *into a set* $O \subseteq \mathtt{dom(f)} \cup \{\mathtt{Und}\}$ *such that* $O \neq \emptyset$ *and either* $O = \{\mathtt{Und}\}$ *or* $O \subseteq \mathtt{dom(f)}$*. We write* $\Sigma \oplus \langle I, ? \rangle \mid\!\sim_{\mathbf{T},\mathtt{R}} O$*.*

In what follows, we assume arbitrary but fixed theory $\mathbf{T}$, case base Σ, new case $\langle I, ? \rangle$ and CBR model $\mathbf{R}$. We introduce some principles (or properties) that a reasonable CBR model would satisfy. The first two principles concern the situation where the new case is dissimilar to all the past cases of the base. There are two possibilities. The first consists of proposing outcomes of the closest cases. This may be undesirable in applications like medical diagnosis, where a CBR model looks for a diagnosis of patients of the basis of their symptoms.

Principle 1 (Strong Completeness) $\Sigma \oplus \langle I, ? \rangle \mid\!\sim_{\mathbf{T},\mathtt{R}} O$ *with* $O \subseteq \mathtt{dom(f)}$*.*

The second possibility consists of abstaining from choosing an arbitrary outcome, and ensures that the model returns the symbol `Und`.

Principle 2 (Weak Completeness) $\Sigma \oplus \langle I, ? \rangle \mid\!\sim_{\mathbf{T},\mathtt{R}} \{\mathtt{Und}\}$ *iff* $\forall \langle I, v \rangle \in \Sigma$, $\mathbf{S}^i(I_n, I) < \delta^i$*.*

Note that any model which satisfies weak completeness returns `Und` when the case base is empty. This is reasonable as arbitrariness is avoided.

Proposition 2. *If a model* $\mathbf{R}$ *satisfies weak completeness, then* $\emptyset \oplus \langle I, ? \rangle \mid\!\sim_{\mathbf{T},\mathtt{R}} \{\mathtt{Und}\}$*.*

The strong and weak versions of completeness are incompatible, i.e., there is no CBR model which can satisfies both at the same time. Indeed, they recommend different outcomes in the above mentioned particular case.

Proposition 3. *Strong completeness and weak completeness are incompatible*

The third principle ensures that the model preserves the consistency of the case base. Of course, this therefore assumes that the base is consistent.

Principle 3 (Consistency) *Let* Σ *be consistent and* $\Sigma \oplus \langle I, ? \rangle \mid\!\sim_{\mathbf{T},\mathtt{R}} O$ *such that* $O \neq \{\mathtt{Und}\}$*. For any* $v \in O$*,* $\Sigma \cup \{\langle I_n, v \rangle\}$ *is consistent.*

The three next principles are those that capture the CBR rule discussed previously. Strong coherence states that adding the new case labelled with any of its candidate outcomes to a strongly coherent base would preserve coherence.

Principle 4 (Strong Coherence) *Let Σ be strongly coherent and $\Sigma \oplus \langle I, ?\rangle \mid\!\sim_{\mathbf{T},\mathtt{R}} O$ such that $O \neq \{\mathtt{Und}\}$. For any $v \in O$, $\Sigma \cup \{\langle I_n, v\rangle\}$ is strongly coherent.*

In the same way, weak coherence ensures that a CBR model preserves the weak coherence of a case base.

Principle 5 (Weak Coherence) *Let Σ be weakly coherent and $\Sigma \oplus \langle I, ?\rangle \mid\!\sim_{\mathbf{T},\mathtt{R}} O$ such that $O \neq \{\mathtt{Und}\}$. For any $v \in O$, $\Sigma \cup \{\langle I_n, v\rangle\}$ is weakly coherent.*

Proposition 4. *Let $\mathbf{T} = \langle \mathcal{F}, \mathtt{dom}, \mathbf{S}^i, \mathbf{S}^o, \delta^i, \delta^o\rangle$ be a theory such that $\delta^i \geq \delta^o$. If a CBR model satisfies strong coherence, then it satisfies weak coherent.*

Proof. Let $\delta^i \geq \delta^o$, $\langle I, ?\rangle$ a new case, and a CBR model which satisfies strong coherence. Let $\Sigma \oplus \langle I, ?\rangle \mid\!\sim_{\mathbf{T},\mathtt{R}} O$. It holds that for any $v \in O$, $\Sigma \cup \{\langle I_n, v\rangle\}$ is strongly coherent. From Proposition 1, since $\delta^i \geq \delta^o$, then $\Sigma \cup \{\langle I_n, v\rangle\}$ is weakly coherent.

Regularity principle ensures that a CBR model preserves the regularity of a case base.

Principle 6 (Regularity) *Let Σ be regular and $\Sigma \oplus \langle I, ?\rangle \mid\!\sim_{\mathbf{T},\mathtt{R}} O$ such that $O \neq \{\mathtt{Und}\}$. For any $v \in O$, $\Sigma \cup \{\langle I_n, v\rangle\}$ is regular.*

In what follows, we show that case-based reasoning is non-monotonic as its conclusions can be revised when a base is extended with additional cases. Let us first define formally the principle of non-monotonicity.

Principle 7 (Non-Monotonicity)

$$\begin{cases} \Sigma \oplus \langle I, ?\rangle \mid\!\sim_{\mathbf{T},\mathtt{R}} O \\ \\ \Sigma \subseteq \Sigma' \end{cases} \not\Rightarrow \ \Sigma' \oplus \langle I, ?\rangle \mid\!\sim_{\mathbf{T},\mathtt{R}} O$$

The following result shows that non-monotonicity follows from weak completeness.

Proposition 5. *If a CBR model satisfies weak completeness, then it satisfies non-monotonicity.*

Proof. Assume a CBR model $\mathbf{R}$ which satisfies weak completeness. Let Σ be a case base and $\langle I, ?\rangle$ a new case. Assume $\Sigma = \emptyset$, then $\emptyset \oplus \langle I, ?\rangle \mid\!\sim_{\mathbf{T},\mathtt{R}} \{\mathtt{Und}\}$. Let now $\Sigma' = \{\langle I, v\rangle\}$ such that $I = I_n$. Obviously, $\mathbf{S}^i(I, I_n) = 1$ and $\mathbf{S}^i(I, I_n) \geq \delta^i$ (since $0 < \delta^i \leq 1$). So, $\Sigma' \oplus \langle I, ?\rangle \not\mid\!\sim_{\mathbf{T},\mathtt{R}} \{\mathtt{Und}\}$.

We show next that one of the principles capturing the basis assumption of CBR, namely strong coherence, is incompatible with cautious monotonicity from [8]. This means there in no CBR model which can satisfy the two properties.

Definition 8 (Cautious Monotonicity).

$$\begin{cases} \Sigma \oplus \langle I, ?\rangle \mid\!\sim_{\mathbf{T},\mathbf{R}} \{v\} \\ \\ \Sigma \oplus \langle I', ?\rangle \mid\!\sim_{\mathbf{T},\mathbf{R}} \{v'\} \end{cases} \Longrightarrow \Sigma \cup \{\langle I, v\rangle\} \oplus \langle I', ?\rangle \mid\!\sim_{\mathbf{T},\mathbf{R}} \{v'\}$$

Proposition 6. *Strong coherence and cautious monotonicity are incompatible.*

5 Conclusion

The paper presented a preliminary contribution on foundations of case-based reasoning. It started by formalizing the key rule behind this form of reasoning, then proposed a set of principles that any model would satisfy. We have shown that CBR is non-monotonic in that conclusion could be revised in light of additional information (cases). However, some of its principles are incompatible with some axioms describing nonmonotonic reasoning in [8].

This work can be extended in several ways. First, we plan to investigate the properties of the principles, their consequences, and properties of models satisfying them. We also plan to develop models satisfying the axioms. Finally, we will analyse existing CBR models against the axioms.

Acknowledgements

Support from the ANR-3IA Artificial and Natural Intelligence Toulouse Institute is gratefully acknowledged.

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