Ranking-based Semantics for Argumentation Frameworks

Leila Amgoud Jonathan Ben-Naim

IRIT – CNRS**

Abstract. An argumentation framework consists of a set of interacting arguments and a semantics for evaluating them. This paper proposes a new family of semantics which rank-orders arguments from the most acceptable to the weakest one(s). The new semantics enjoy two other main features: i) an attack weakens its target but does not kill it, ii) the number of attackers has a great impact on the acceptability of an argument. We start by proposing a set of postulates that such semantics should satisfy, then propose various semantics that satisfy them.

1 Introduction

Argumentation is a reasoning model based on the construction and evaluation of interacting arguments. The most popular semantics were proposed by Dung in his seminal paper [6]. Those semantics as well as their refinements (e.g. in [3, 5]) partition the powerset of the set of arguments into two classes: *extensions* and *non-extensions*. Every extension represents a coherent point of view. An *absolute* status is assigned to each argument: *accepted* (if it belongs to every extension), *rejected* (if it does not belong to any extension), and *undecided* if it is in some extensions and not in others. Those semantics are based on the following considerations:

- The impact of an attack from an argument b to an argument a is binary. Indeed, if b is accepted then the attack *kills* a, otherwise it has no effect on a.
- One attack against an argument a has the same effect on a as any number of attacks. Indeed, one attack is sufficient to kill a, several attacks cannot kill a to a greater extent.
- Arguments that have the same status are considered as "equally acceptable".

The first consideration may seem rational in applications like paraconsistent reasoning. An attack from an argument b to a is sufficient to kill a (assuming that b is non-attacked). Indeed, in this case arguments are formulas and attacks correspond to contradictions. It makes sense to consider that one contradiction is lethal. However, in other applications like decision making, an attack does not necessarily kill its target but just weakens it. Suppose that the two arguments a and b are exchanged by two doctors.

a**:** The patient should have a surgery since he has cancer.
b**:** But, the statistics show that the probability that a surgery will improve the state of the patient is low.

** 118, route de Narbonne, 31062, Toulouse Cedex 09, FRANCE

In this case, the attack from b only weakens a but does not kill it. The doctor may still choose to do the surgery since it gives (a small) chance for the patient to survive.

The second consideration is debatable in situations like decision making. Suppose a seller who provides the argument a in favor of a given car.

a**:** This car is certainly powerful since it is made by Peugeot.
b_1**:** The engines of Peugeot cars break down before 300000km.
b_2**:** The airbags of Peugeot cars are not reliable.
b_3**:** The spare part is very expensive.

If the buyer receives the argument b_1 against Peugeot (thus against a), then he accepts less a. The situation becomes worse if he receives b_2 and b_3. Indeed, the more arguments he receives against a, the less his confidence in a.

The third consideration is also debatable. Consider the case of grounded extension [6]. It contains the non-attacked arguments of a framework and the ones defended by them. Both types are assumed to be equally acceptable due to the first consideration which assumes that attacks kill their targets. However, being attacked weakens an argument. Thus, non-attacked arguments are more acceptable than attacked ones.

To sum up, existing semantics may be well suited for reasoning but not for applications like decision making. In the present paper, we propose a new family of semantics that are based on the following *graded* considerations:

- *Weakening:* Arguments cannot be killed. However they can be weakened to an extreme extent. An attack from an argument b to an argument a always causes a decrease in the acceptability of a (possibly only by an infinitesimal amount). The greater the acceptability of b, the greater the decrease in the acceptability of a.
- *Counting:* the more numerous the attacks against a, the greater the decrease in the acceptability of a.
- *Graduality:* Arguments are not necessarily equally acceptable.

In our approach, a semantics is a function that transforms any *argumentation framework* into a *ranking* on its set of arguments: from the most accepted to the weakest one(s). Our first step consists of proposing postulates, each of which is an intuitive and desirable property that a semantics may enjoy. Such an axiomatic approach allows a better understanding of semantics and a more precise comparison between different proposals. We investigate the dependencies and the compatibilities between postulates. In a second step, we construct two semantics based on our graded considerations that satisfy certain postulates.

2 Ranking-based semantics

An argumentation framework consists of a set of arguments and a set of attacks between them. Arguments represent reasons to believe in statements, doing actions, etc. Attacks express conflicts between pairs of arguments. In what follows, both components are assumed to be abstract entities.

Definition 1 (Argumentation framework) *An* argumentation framework *is an ordered pair* $\mathbf{A} = \langle \mathcal{A}, \mathcal{R} \rangle$*, where* $\mathcal{A}$ *is a finite set of* arguments *and* $\mathcal{R}$ *a binary relation on* $\mathcal{A}$ *We call* $\mathcal{R}$ *an* attack relation. *Intuitively,* $a\mathcal{R}b$ *(or* $\langle a, b \rangle \in \mathcal{R}$*) means that* a *attacks* b*.*

Notation *For an argumentation framework* $\mathbf{A} = \langle \mathcal{A}, \mathcal{R} \rangle$*, we define that* $\mathtt{Arg}(\mathbf{A}) = \mathcal{A}$*. For* $a \in \mathcal{A}$*,* $\mathtt{Att}_{\mathbf{A}}(a) = \{b \in \mathcal{A} \mid b\mathcal{R}a\}$*. When the context is clear, we write* $\mathtt{Att}(a)$ *for short. The same goes for all notations.*

As in classical approaches for argumentation [6], since arguments may be conflicting, it is important to evaluate them and to identify the ones to rely on for inferring conclusions (in case of handling inconsistency in knowledge bases) or making decisions, ... For that purpose, we propose ranking-based semantics which rank-order the set of arguments from the most acceptable to the weakest one(s). Thus, unlike existing semantics which assign an *absolute* status (accepted, rejected, undecided) for each argument, the new approach compares pairs of arguments on the basis of their respective sets of attackers, and states which argument is more acceptable than another. A ranking is a total and transitive binary relation.

Definition 2 (Ranking) *A* ranking $\preceq$ *on a set* $\mathcal{A}$ *is a binary relation on* $\mathcal{A}$ *such that:*

- $\preceq$ *is total (i.e. for all* $a, b \in \mathcal{A}$*,* $a \preceq b$ *or* $b \preceq a$*);*
- $\preceq$ *is transitive (i.e. for all* $a, b, c \in \mathcal{A}$*, if* $a \preceq b$ *and* $b \preceq c$*, then* $a \preceq c$*).*

Intuitively, $a \preceq b$ means that a is *at least as acceptable* as b. So, $b \npreceq a$ means that a is *strictly more acceptable* than b.

Definition 3 (Ranking-based semantics) *A* ranking-based semantics $\mathbf{S}$ *is a function that transforms any argumentation framework* $\mathbf{A} = \langle \mathcal{A}, \mathcal{R} \rangle$ *into a ranking on* $\mathcal{A}$*.*

A ranking should not be defined in an arbitrary way, but should obey to some principles (postulates). In the next section, we discuss some intuitive and basic postulates.

3 Postulates for semantics

A ranking on a set of arguments is defined using only the attacks between arguments. Thus, the first postulate that a semantics should enjoy ensures that the evaluation of arguments does not depend on their identity but only on the attack relation. Let us illustrate our purpose on the following example.

Example 1 *Consider the two argumentation frameworks depicted in the figure below.*

The ranking relation between a *and* b *should be the same as the one between* c *and* d*.*

Before presenting the postulate, let us first define when two argumentation frameworks are equivalent.

Definition 4 (Isomorphism) *Let* $\mathbf{A} = \langle \mathcal{A}, \mathcal{R} \rangle$ *and* $\mathbf{A}' = \langle \mathcal{A}', \mathcal{R}' \rangle$ *be two argumentation frameworks. An* isomorphism *from* $\mathbf{A}$ *to* $\mathbf{A}'$ *is a bijective function* f *from* $\mathcal{A}$ *to* $\mathcal{A}'$ *such that* $\forall\, a, b \in \mathcal{A}$, $a\mathcal{R}b$ *iff* $f(a)\mathcal{R}'f(b)$.

The first postulate, called *abstraction*, is defined as follows:

Postulate 1 (Abstraction) *A ranking-based semantics* $\mathbf{S}$ *satisfies* abstraction ($\texttt{Ab}$) *iff for any two frameworks* $\mathbf{A} = \langle \mathcal{A}, \mathcal{R} \rangle$ *and* $\mathbf{A}' = \langle \mathcal{A}', \mathcal{R}' \rangle$*, for any isomorphism* f *from* $\mathbf{A}$ *to* $\mathbf{A}'$*, we have that* $\forall\, a, b \in \mathcal{A}$, $\langle a, b \rangle \in \mathbf{S}(\mathbf{A})$ *iff* $\langle f(a), f(b) \rangle \in \mathbf{S}(\mathbf{A}')$.

It is worth pointing out that extension-based semantics (i.e., Dung's semantics) obey in some sense to this postulate. For instance, both argumentation frameworks of Example 1 have one preferred extension which contains the non attacked argument (a resp. c).

The second postulate states the following: the question whether an argument a is at least as acceptable as an argument b should be independent of any argument c that is not connected to a or b, that is, there is no path from a or b to c (ignoring the direction of the edges). Let us first define the independent parts of an argumentation framework.

Definition 5 (Weak connected component) *A* weak connected component *of an argumentation framework* $\mathbf{A}$ *is a maximal subgraph of* $\mathbf{A}$ *in which any two vertices are connected to each other by a path (ignoring the direction of the edges).* $\texttt{Com}(\mathbf{A})$ *denotes the set of every argumentation framework* $\mathbf{B}$ *s.t.* $\mathbf{B}$ *is a weak connected component of* $\mathbf{A}$ *or the graph union of several weak connected components of* $\mathbf{A}$.

Let us now define the postulate, called *independence*.

Postulate 2 (Independence) *A ranking-based semantics* $\mathbf{S}$ *satisfies* independence ($\texttt{In}$) *iff for every argumentation framework* $\mathbf{A}$*,* $\forall\, \mathbf{B} \in \texttt{Com}(\mathbf{A})$*,* $\forall\, a, b \in \texttt{Arg}(\mathbf{B})$*,* $\langle a, b \rangle \in \mathbf{S}(\mathbf{A})$ *iff* $\langle a, b \rangle \in \mathbf{S}(\mathbf{B})$.

Example 1 (Cont) Assume that the two graphs of Example 1 constitutes a single argumentation framework. Then, the ranking relation between a and b (and the one between c and d) should remain the same after the fusion of the two frameworks.

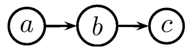
As already said, a ranking is based on the attack relation. Thus, comparing two arguments amounts to comparing their respective sets of attackers. It is natural to consider an argument which has no attackers as more acceptable (and thus ranked higher) than another argument which has attackers. The third postulate encodes this idea.

Postulate 3 (Void Precedence) *A ranking-based semantics* $\mathbf{S}$ *satisfies* void precedence ($\texttt{VP}$) *iff for every argumentation framework* $\mathbf{A} = \langle \mathcal{A}, \mathcal{R} \rangle$*, for all* $a, b \in \mathcal{A}$*, if* $\texttt{Att}(a) = \emptyset$ *and* $\texttt{Att}(b) \neq \emptyset$*, then* $\langle b, a \rangle \notin \mathbf{S}(\mathbf{A})$.

Example 1 (Cont) This postulate ensures that a is ranked higher than b, and c is ranked higher than d.

Non-attacked arguments are also favored by extension-based semantics. They belong to any extension under grounded, complete, stable, preferred semantics. Thus, they are accepted. However, they may have the same status (accepted) as attacked arguments (which are defended). Let us consider the following example.

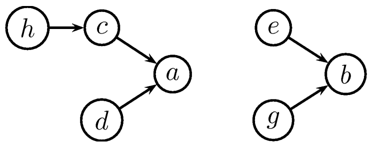
Example 2 *Assume the argumentation framework depicted in the figure below.*



The grounded extension of this framework is $\{a, c\}$. The arguments a and c are both accepted whereas b is rejected. Our approach ranks argument a higher than c since c is attacked, thus weakened. Thus, it ensures a more refined treatment of arguments.

What about two arguments that both have attackers? The idea is to privilege the one with weak attackers. Since an attack always weakens its target, then the fourth postulate states that having attacked attackers is better than having non-attacked attackers. In other words, being *defended* is better than not. The intuition here is that if an argument a is defended, then its attackers are *weakened*. However, an argument which is not defended still has strong attackers. Let us illustrate our purpose on an example.

Example 3 *Consider the argumentation framework depicted in the figure below.*



Both arguments a and b have two attackers. The two attackers of b are not attacked, thus they are strong. However, a is defended by h wrt c thus, the attacker c is weakened. To sum up, a has one strong and one weak attacker while b has two strong attackers. Thus, a should be ranked higher than b.

Notation *Let $\mathbf{A} = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $a \in \mathcal{A}$. We denote by $\mathtt{Def}_{\mathbf{A}}(a) = \{b \in \mathcal{A} \mid \exists c \in \mathcal{A}$ s.t. $c\mathcal{R}a$ and $b\mathcal{R}c\}$ the set of all defenders of a in $\mathbf{A}$.*

Postulate 4 (Defense Precedence) *A ranking-based semantics* $\mathbf{S}$ *satisfies* defense precedence ($\mathtt{DP}$) *iff for every argumentation framework $\mathbf{A} = \langle \mathcal{A}, \mathcal{R} \rangle$, $\forall a, b \in \mathcal{A}$, if $|\mathtt{Att}(a)| = |\mathtt{Att}(b)|$, $\mathtt{Def}(a) \neq \emptyset$, and $\mathtt{Def}(b) = \emptyset$, then $\langle b, a \rangle \notin \mathbf{S}(\mathbf{A})$.*

Example 3 (Cont) The ($\mathtt{DP}$) postulate says that a is strictly more acceptable than b.

Note that the previous postulate treats the case where two arguments have exactly the same number of attackers and one argument may not be defended. The next postulate called *counter-transitivity*, generalizes it in two ways: first it considers arbitrary numbers of attackers and second, it considers various strengths of arguments. The postulate says that an argument a should be ranked at least as high as an argument b, if the attackers of b are at least as *numerous* and *well-ranked* as those of a. Before defining formally the postulate, let us first introduce a relation that compares sets of arguments.

Definition 6 (Group comparison) *Let $\preceq$ be a ranking on a set $\mathcal{A}$ of arguments. For every $A, B \subseteq \mathcal{A}$, $\langle A, B \rangle \in \mathtt{Gr}(\preceq)$ iff there exists an injective function f from B to A such that $\forall\, a \in B$, $f(a) \preceq a$.*

Intuitively, $\langle A, B\rangle \in \texttt{Gr}(\preceq)$ iff the elements of the group A are at least as numerous and well-ranked as those of B. The next property follows from the previous definition.

Proposition 1 *Let $\preceq$ be a ranking on a set $\mathcal{A}$ of arguments and $A, B \subseteq \mathcal{A}$. If $\langle A, B\rangle \in \texttt{Gr}(\preceq)$, then:*

- $|A| \geq |B|$*;*
- *for all $x \in B$, $\exists y \in A$ such that $y \preceq x$.*

Postulate 5 (Counter-Transitivity) *A ranking-based semantics* $\mathbf{S}$ *satisfies the postulate* counter-transitivity ($\texttt{CT}$) *iff for every argumentation framework $\mathbf{A} = \langle \mathcal{A}, \mathcal{R}\rangle$, $\forall\, a, b \in \mathcal{A}$, if $\langle \texttt{Att}(b), \texttt{Att}(a)\rangle \in \texttt{Gr}[\mathbf{S}(\mathbf{A})]$, then $\langle a, b\rangle \in \mathbf{S}(\mathbf{A})$.*

Example 3 (Cont) The two arguments a and b have the same number of attackers. Both attackers of b are strong (since non-attacked), thus they should be ranked higher than c which is attacked (due to ($\texttt{VP}$) postulate). Thus, a should be ranked at least as high as b.

Strict counter-transitivity is another mandatory postulate in our approach. It says that an argument a should be ranked higher than an argument b, if the attackers of b are at least as numerous and well-ranked as those of a. Moreover, the attackers of b are more numerous or well-ranked. This postulate is defined in two steps.

Definition 7 (Strict group comparison) *Let $\preceq$ be a ranking on a set $\mathcal{A}$ of arguments. For every $A, B \subseteq \mathcal{A}$, $\langle A, B\rangle \in \texttt{Sgr}(\preceq)$ iff there exists an injective function f from B to A satisfying the two following conditions:*

- *$\forall\, a \in B$, $f(a) \preceq a$;*
- *$|B| < |A|$ or $\exists\, a \in B$, $a \not\preceq f(a)$.*

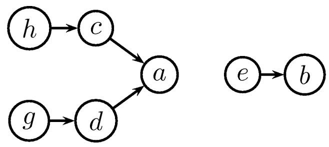
Intuitively, $\langle A, B\rangle \in \texttt{Sgr}(\preceq)$ iff the elements of A are better than those of B based on cardinality and acceptability.

Postulate 6 (Strict Counter-Transitivity) *A ranking-based semantics* $\mathbf{S}$ *satisfies* strict counter-transitivity ($\texttt{SCT}$) *iff for every argumentation framework $\mathbf{A} = \langle \mathcal{A}, \mathcal{R}\rangle$, $\forall\, a, b \in \mathcal{A}$, if $\langle \texttt{Att}(b), \texttt{Att}(a)\rangle \in \texttt{Sgr}[\mathbf{S}(\mathbf{A})]$, then $\langle b, a\rangle \notin \mathbf{S}(\mathbf{A})$.*

Example 3 (Cont) The argument b cannot be ranked as a since its attackers are ranked higher than those of a. Thus, according to the ($\texttt{SCT}$) postulate, a should be strictly more acceptable than b.

There are situations where the cardinality of the attackers and their quality are opposed as discussed in the following example.

Example 4 *Consider the argumentation framework depicted in the figure below.*



If one non-attacked attacker is sufficient to kill an argument (which is the case in most approaches to argumentation), then the argument a should naturally be ranked higher than b. But, in our approach, as explained in the introduction, no number of attacked or non-attacked attackers can kill an argument. They can just weaken it. Consequently, in this example, a is attacked by two weakened arguments, while b is attacked by one strong argument. As usual, we have to make a choice: give precedence to cardinality over quality (i.e. two weakened attackers is worse for the target than one strong attacker), or give precedence to quality over cardinality. In certain applications such as decision-making, both options are reasonable. For example, suppose we have to buy a car and we are considering a red one and a blue one. In addition:

b = The red car has got 5 stars out of 5 in our favorite car magazine;
e = The magazine does not take into account the fact that the red car is 1000 euros more expensive than the blue one;
a = The blue car has got 5 stars out of 5 in our favorite car magazine;
c = The magazine does not take into account the fact that there is a probability of 0.5 that the blue cars engine breaks down before 300000km. The reparations would cost 2000 euros;
h = A friend of ours is a mechanic. He would offer us a 10% discount on engine reparation;
d = The magazine does not take into account the fact that there is a probability of 0.5 that the blue car will be stolen from us before 10 years. The insurance will pay for another blue car, but there is a deductibility provision of 2000 euros;
g = In our neighborhood, the rate of motor vehicle theft is 10% lower than the average.

In this example, it is intuitive to consider that b is more acceptable than a. Moreover, in decision making, a rational principle is that the values of the arguments sum up, and the greatest sum wins. More precisely, in the absence of h and g, it is intuitive that e, c, and d have the same value. Thus, the value of $\{c, d\}$ is 2 times the value of e. So, in the presence of h and g, the value of $\{c, d\}$ is approximately 0.9 times 2 times the value of e, which is strictly greater than the latter. So, according to the sum-up principle, a is more seriously attacked than b. Now, suppose that the argument e is replaced by the following one:

e = The magazine does not take into account the fact that the red car is 4000 euros more expensive than the blue one.

This time it is intuitive to consider that a is more acceptable than b.

To summarize, the outcome of Example 4 is debatable. We can give precedence to cardinality over quality (i.e. b is more acceptable than a) or give precedence to quality over cardinality (i.e. a is more acceptable than b). Both options are rational and it is impossible to make a general and non-arbitrary choice. We turn to two axioms that represent these two choices.

Postulate 7 (Cardinality Precedence) *A ranking-based semantics* $\mathbf{S}$ *satisfies* cardinality preference ($\mathtt{CP}$) *iff for every argumentation framework* $\mathbf{A} = \langle \mathcal{A}, \mathcal{R} \rangle$, $\forall\, a, b \in \mathcal{A}$, *if* $|\mathtt{Att}(a)| < |\mathtt{Att}(b)|$, *then* $\langle b, a \rangle \notin \mathbf{S}(\mathbf{A})$.

On the contrary, *quality precedence* says that an argument a should be ranked higher than an argument b, if one attacker of b is ranked higher than any attacker of a.

Postulate 8 (Quality Precedence) *A ranking-based semantics* $\mathbf{S}$ *satisfies* quality precedence (QP) *iff for every argumentation framework* $\mathbf{A} = \langle \mathcal{A}, \mathcal{R} \rangle$*,* $\forall\, a, b \in \mathcal{A}$*, if there exists* $c \in \mathtt{Att}(b)$ *such that* $\forall\, d \in \mathtt{Att}(a)$*,* $\langle d, c \rangle \notin \mathbf{S}(\mathbf{A})$*, then* $\langle b, a \rangle \notin \mathbf{S}(\mathbf{A})$.

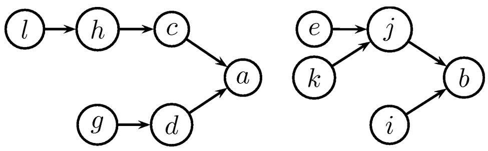
The last postulate concerns the way arguments are defended. The idea is that an argument which is defended against more attackers is ranked higher than an argument which is defended against a smaller number of attacks. Before defining formally the postulate, let us first introduce the concepts of simple and distributed defense.

Definition 8 (Simple/distributed defense) *Let* $\mathbf{A} = \langle \mathcal{A}, \mathcal{R} \rangle$ *be an argumentation framework and* $a \in \mathcal{A}$*.*

- *The defense of* a *is* simple *iff every defender of* a *attacks exactly one attacker of* a*.*
- *The defense of* a *is* distributed *iff every attacker of* a *is attacked by at least one argument.*

Let us illustrate these concepts on the following example.

Example 5 *Consider the argumentation framework depicted in the figure below.*



The two arguments a *and* b *have the same number of defenders:* $\mathtt{Def}(a) = \{h, g\}$ *and* $\mathtt{Def}(b) = \{e, k\}$*. However, the defense of* a *is simple and distributed while the defense of* b *is simple but not distributed.*

The postulate, called *distributed-defense precedence*, is defined as follows.

Postulate 9 (Distributed-Defense Precedence) *A ranking-based semantics* $\mathbf{S}$ *satisfies* distributed-defense precedence (DDP) *iff for any argumentation framework* $\mathbf{A} = \langle \mathcal{A}, \mathcal{R} \rangle$*,* $\forall\, a, b \in \mathcal{A}$ *such that* $|\mathtt{Att}(a)| = |\mathtt{Att}(b)|$ *and* $|\mathtt{Def}(a)| = |\mathtt{Def}(b)|$*, if the defense of* a *is simple and distributed and the defense of* b *is simple but not distributed, then* $\langle b, a \rangle \notin \mathbf{S}(\mathbf{A})$.

Example 5 (Cont) The postulate (DDP) ensures that a is more acceptable than b.

4 Relationships between postulates

So far we have proposed a set of postulates that are suitable for defining a ranking-based semantics in argumentation theory. In this section, we study their dependencies as well as their compatibilities (i.e., whether they can be satisfied together by a semantics). We start by showing that the postulates (CT), (SCT), (VP) and (DP) are not *independent*.

Proposition 2 *Let* $\mathbf{S}$ *be a ranking-based semantics:*

- *if* $\mathbf{S}$ *satisfies* (SCT)*, then it satisfies* (VP)*;*
- *if* $\mathbf{S}$ *satisfies both* (CT) *and* (SCT)*, then it satisfies* (DP).

The remaining postulates are independent. Let us now check the compatibility of the postulates. We show that all the postulates are compatible except (CP) and (QP) which cannot be satisfied together. Example 4 already illustrates this issue. Indeed, (QP) prefers a to b while (CP) prefers the converse.

Proposition 3 *No ranking-based semantics can satisfy both* (CP) *and* (QP).

The two postulates (CP) and (QP) can however be satisfied separately. In addition, they are compatible with the remaining postulates.

Proposition 4 *The postulates* (CP)*,* (CT)*,* (SCT)*,* (In)*,* (Ab) *and* (DDP) *are compatible.*

In the next section we provide two semantics which satisfy the six above postulates. Similarly, (QP) is compatible with some postulates.

Proposition 5 *The postulates* (QP)*,* (In) *and* (Ab) *are compatible.*

Recall that (QP) and (CP) represent two opposed directions. The extension-based semantics take one direction (QP), while graded semantics may take the other one.

5 Discussion-based and Burden-based semantics

This section introduces two semantics that satisfy the postulates, namely those that are compatible with (CP).

The first semantics, called *discussion-based semantics*, is based on a notion of linear discussion similar to 'argumentation line' in [8]. A linear discussion is a sequence of arguments such that each argument attacks the argument preceding it in the sequence.

Definition 9 (Linear discussions) *Let* $\mathbf{A} = \langle \mathcal{A}, \mathcal{R} \rangle$ *be an argumentation framework and* $a \in \mathcal{A}$*. A* linear discussion *for* a *in* $\mathbf{A}$ *is a sequence* $s = \langle a_1, \ldots, a_n \rangle$ *(where* n *is a positive integer) of elements of* $\mathcal{A}$ *such that* $a_1 = a$ *and* $\forall\, i \in \{2, 3, \ldots, n\}$ $a_i \mathcal{R} a_{i-1}$*. The length of* s *is* n*. We say that:* s *is* won *iff* n *is odd;* s *is* lost *iff* n *is even.*

Let us illustrate this notion on an example.

Example 5 (Cont) Two won linear discussions for the argument a are e.g., $s_1 = \langle a \rangle$ and $s_2 = \langle a, d, g \rangle$ and one lost linear discussion is, for instance, $s_3 = \langle a, c, h, l \rangle$. Similarly, three won linear discussions for the argument b are $s'_1 = \langle b \rangle$, $s'_2 = \langle b, j, e \rangle$ and $s'_3 = \langle b, j, k \rangle$ and one lost discussion is $s'_4 = \langle b, i \rangle$.

The basic idea behind the semantics is: for every argument a, for every positive integer i, to count the number of won or lost linear discussions of length i for a. We positively count the lost discussions and negatively count the won discussions. So, in any case, the smaller the number calculated, the better the situation for a.

Definition 10 (Discussion counts) *Let $\mathbf{A} = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework, $a \in \mathcal{A}$, and i a positive integer. We define that:*

$$\mathtt{Dis}_{\mathbf{A}i}(a) = \begin{cases} -N & \textit{if } i \textit{ is odd;} \\ N & \textit{if } i \textit{ is even;} \end{cases}$$

where N is the number of linear discussions for a of length i.

Example 5 (Cont) The following table provides the discussion counts $\mathtt{Dis}_{\mathbf{A}i}$ of the two arguments a and b.

i	a	b
1	-1	-1
2	2	2
3	-2	-2
4	1	0

This semantics lexicographically ranks the arguments on the basis of their won and lost linear discussions.

Definition 11 (Discussion-based semantics) *The ranking-based semantics* Dbs *transforms any argumentation framework $\mathbf{A} = \langle \mathcal{A}, \mathcal{R} \rangle$ into the ranking $\mathtt{Dbs}(\mathbf{A})$ such that $\forall\, a, b \in \mathcal{A}$, $\langle a, b \rangle \in \mathtt{Dbs}(\mathbf{A})$ iff one of the two following cases holds:*

- $\forall\, i \in \{1, 2, \ldots\}$, $\mathtt{Dis}_i(a) = \mathtt{Dis}_i(b)$;
- $\exists\, i \in \{1, 2, \ldots\}$, $\mathtt{Dis}_i(a) < \mathtt{Dis}_i(b)$ *and* $\forall\, j \in \{1, 2, \ldots, i-1\}$, $\mathtt{Dis}_j(a) = \mathtt{Dis}_j(b)$.

Example 5 (Cont) Note that for all $i = 1, 2, 3, \mathtt{Dis}_i(a) = \mathtt{Dis}_i(b)$. However, $\mathtt{Dis}_4(a) > \mathtt{Dis}_4(b)$. Thus, $\langle a, b \rangle \notin \mathtt{Dbs}(\mathbf{A})$, i.e., b is strictly more acceptable than a.

At first sight, the infinite character of the set $\{1, 2, \ldots\}$ of all positive integers may look like an issue from a computational point of view. Indeed, $\mathtt{Dis}_i(a)$ may never stop getting bigger. This is due to the possible presence of cycles in the argumentation framework. But, if $\mathtt{Dis}_i(a)$ never stops evolving, it evolves cyclically. So, we strongly conjecture that there exists a threshold t such that if $\forall\, i \leq t$, $\mathtt{Dis}_i(a) = \mathtt{Dis}_i(b)$, then $\forall\, i > t$, $\mathtt{Dis}_i(a) = \mathtt{Dis}_i(b)$. Such an equality ensuring threshold would be dependent on the length of the longest simple cycle in the argumentation framework. This threshold would be useful to write a program implementing our discussion-based semantics. Note also that the computation can simply be done up to a fixed step t. The greater t, the closer the ranking obtained to the actual discussion-based ranking. This kind of approximation can be found in the ranking-system literature. Certain well-known ranking systems are theoretically defined by an infinite number of iterations, but in practice only a finite number of iterations are computed. For example, this is the case of Pagerank, the system at the basis of the Google search engine. The original version of Pagerank can be found in [10]. An exhaustive study of Pagerank, as well as alternate versions, can be found in e.g. [9].

The postulates represent a theoretical validation for our semantics.

Theorem 1 Dbs *satisfies* (CT), (SCT), (CP), (Ab), *and* (In).

From Proposition 2, it follows that Dbs satisfies also (VP) and (DP).

Corollary 1. Dbs *satisfies* (VP) *and* (DP).

Theorem 2 Dbs *does not satisfy* (DDP).

We show next that the Dbs semantics treats (odd and even length) cycles in a similar way. Before that, let us recall what is an *elementary cycle*.

Definition 12 (Elementary cycle) *Let* $\mathbf{A} = \langle \mathcal{A}, \mathcal{R} \rangle$ *be an argumentation framework. A* cycle *in* $\mathbf{A}$ *is a sequence* $\langle a_1, a_2, \ldots, a_n \rangle$ *of elements of* $\mathcal{A}$ *(where* n *is a positive integer) such that* $\forall i \in \{1, 2, \ldots, n-1\}$, $a_i \mathcal{R} a_{i+1}$ *and* $a_n \mathcal{R} a_1$. *A cycle* $\langle a_1, a_2, \ldots, a_n \rangle$ *in* $\mathbf{A}$ *is* elementary *iff there is no cycle* $\langle b_1, b_2, \ldots, b_m \rangle$ *in* $\mathbf{A}$ *such that* $m < n$ *and* $\{b_1, b_2, \ldots, b_m\} \subseteq \{a_1, a_2, \ldots, a_n\}$.

Proposition 6 *Let* $\mathbf{A} = \langle \mathcal{A}, \mathcal{R} \rangle$ *be an argumentation framework. Suppose that* $\mathbf{A}$ *consists of one elementary cycle, i.e. there exists an elementary cycle* $\langle a_1, \ldots, a_n \rangle$ *in* $\mathbf{A}$ *such that* $\{a_1, \ldots, a_n\} = \mathcal{A}$. *Then,* $\forall\, a, b \in \mathcal{A}$, $\langle a, b \rangle \in \mathtt{Dbs}(\mathbf{A})$.

The second semantics, called *burden-based semantics*, satisfies (DDP). It follows a multiple steps process. At each step, it assigns a *burden number* to every argument. In the initial step, this number is 1 for all arguments. Then, in each step, all the burden numbers are simultaneously recomputed on the basis of the number of attackers and their burden numbers in the previous step. More precisely, for every argument a, its burden number is set back to 1, then, for every argument b attacking a, the burden number of a is increased by a quantity inversely proportional to the burden number of b in the previous step. An argument a is at least as acceptable as an argument b iff a and b have the same burden numbers or the burden numbers of b increase more rapidly.

Definition 13 (Burden numbers) *Let* $\mathbf{A} = \langle \mathcal{A}, \mathcal{R} \rangle$ *be an argumentation framework. We define by recursion the function* $\mathtt{Bur}_{\mathbf{A}}$ *on* $\{0, 1, \ldots\}$. *For every* $i \in \{0, 1, \ldots\}$, $\mathtt{Bur}_i$ *(i.e.* $\mathtt{Bur}_{\mathbf{A}}(i)$*) is the function from* $\mathcal{A}$ *to* $\mathbb{Q}$ *such that* $\forall\, a \in \mathcal{A}$,

$$\mathtt{Bur}_i(a) = \begin{cases} 1 & \textit{if } i = 0; \\ 1 + \Sigma_{b \in \mathtt{Att}(a)} 1/\mathtt{Bur}_{i-1}(b) & \textit{otherwise.} \end{cases}$$

By convention, if $\mathtt{Att}(a) = \emptyset$, then $\Sigma_{b \in \mathtt{Att}(a)} 1/\mathtt{Bur}_{i-1}(b) = 0$. $\mathtt{Bur}_i(a)$ is the burden number of a in the i^{th} step.

Let us illustrate this function on the following example.

Example 2 (Cont) The burden numbers of each argument and at each step are summarized in the table below. Note that the burden numbers of the three arguments will not change beyond step 2.

Step i	a	b	c
0	1	1	1
1	1	2	2
2	1	2	1.5
⋮	⋮	⋮	⋮

We lexicographically compare two arguments on the basis of their burden numbers.

Definition 14 (Burden-based semantics) *The ranking-based semantics* Bbs *transforms any argumentation framework* $\mathbf{A} = \langle \mathcal{A}, \mathcal{R} \rangle$ *into the ranking* $\texttt{Bbs}(\mathbf{A})$ *on* $\mathcal{A}$ *such that* $\forall\, a, b \in \mathcal{A}$*,* $\langle a, b \rangle \in \texttt{Bbs}(\mathbf{A})$ *iff one of the two following cases holds:*

- $\forall\, i \in \{0, 1, \ldots\}$, $\texttt{Bur}_i(a) = \texttt{Bur}_i(b)$*;*
- $\exists\, i \in \{0, 1, \ldots\}$, $\texttt{Bur}_i(a) < \texttt{Bur}_i(b)$ *and* $\forall\, j \in \{0, 1, \ldots, i-1\}$, $\texttt{Bur}_j(a) = \texttt{Bur}_j(b)$.

As for the discussion-based semantics, an equality-ensuring threshold probably exists for the burden-based semantics. Such a threshold would make possible an exact computation, despite the fact that $\{0, 1, \ldots\}$ is infinite.

Note that both semantics (Dbs and Bbs) do not take into account possible dependencies between an argument and one of its attackers, nor the dependencies between two attackers. Actually, Dbs and Bbs rank the arguments only on the basis of the structure obtained by "unrolling" the cycles. For example, our semantics do not distinguish between a loop (e.g. $a\mathcal{R}a$) and a cycle (e.g. $a\mathcal{R}b$, $b\mathcal{R}a$). The notion of dependence is hard to capture and is beyond the scope of this paper. Our goal in the present paper is essentially to introduce a new kind of semantics, basic postulates for it, and instances satisfying those postulates.

We show that the burden-based semantics satisfies all the postulates except (AP) which is not compatible with (CP).

Theorem 3 *The semantics* Bbs *satisfies* (CT)*,* (SCT)*,* (CP)*,* (DDP)*,* (Ab) *and* (In).

From Proposition 2, it satisfies also (VP) and (DP).

Corollary 2. Bbs *satisfies* (VP) *and* (DP).

Let us see on examples how the semantics works.

Example 2 (Cont) According to Bbs, the argument a is strictly more acceptable than c which is itself strictly more acceptable than b.

Note that Bbs returns a more refined result than Dung's semantics. Indeed, the set $\{a, c\}$ is a (preferred, grounded, stable) extension according to [6]. Our approach refines the result by ranking a higher than c since it is non attacked. This does not mean that Bbs semantics coincides with Dung's ones. The following example shows that the two approaches may return different results since they are grounded on different principles.

Example 4 (Cont) The argumentation framework has a unique extension $\{h, g, a, e\}$ which is grounded, preferred and stable. Thus, the argument b is rejected. Let us now apply the Bbs semantics on the same framework. The table below provides the burden numbers of the arguments.

Step i	h	g	c	d	a	e	b
0	1	1	1	1	1	1	1
1	1	1	2	2	3	1	2
2	1	1	2	2	2	1	2
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

Bbs semantics provides the following ranking: $h, g, e \preceq c, d, b \preceq a$. Thus, b is more acceptable than a since it has less attackers.

Example 5 (Cont) According to Bbs, a is strictly more acceptable than b.

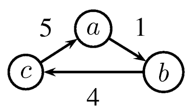
Note that in this example, the semantics Dbs returns the converse. This shows that the two semantics may return different results. This difference comes from the postulate DDP which is satisfied by Bbs but violated by Dbs.

As with Dbs, we show next that the Bbs semantics treats (odd and even length) cycles in a similar way.

Proposition 7 *Let $\mathbf{A} = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework. Suppose that $\mathbf{A}$ consists of one elementary cycle, i.e. there exists an elementary cycle $\langle a_1, \ldots, a_n \rangle$ in $\mathbf{A}$ such that $\{a_1, \ldots, a_n\} = \mathcal{A}$. Then, $\forall\, a, b \in \mathcal{A}$, $\langle a, b \rangle \in \texttt{Bbs}(\mathbf{A})$.*

6 Related work

There are three works in the literature which are somehow related to our contribution. The first attempts were done in [1, 2] where the authors identified different principles and compared existing semantics wrt them. The principles are tailored for extension-based semantics, and do not apply for ranking-based ones. The work in [4] is closer to ours. The authors defined a notion of gradual acceptability. The idea is to assign a numerical value to each argument on the basis of its attackers. The properties of the valuation function are unclear. Our approach defines, through a set of postulates, the desirable properties of a preference relation over a set of arguments. The two semantics Dbs and Bbs define formally two valuation functions that satisfy the postulates. In [7], Dung's abstract framework was extended by considering weighted attacks. The basic idea is to remove *some* attacks up to a certain degree representing the tolerated incoherence, and then to apply existing semantics on the new graph(s) by ignoring completely the weights. This leads to extensions which are not conflict-free in the sense of the attack relation. Consider the following weighted framework. If one tolerates incoherence up to degree 1 ($\beta = 1$), then the attack from a to b is ignored. Consequently, $\emptyset$ and $\{a, b\}$ are two β-grounded extensions.



This approach is different from ours for several reasons. First, it obeys to the three 'binary' considerations of Dung's framework. Indeed, weights are only used for deciding which attacks can be ignored when computing the extensions. The second difference stems from the fact that weights of attacks are inputs of a framework. In our approach, we compute the relative *importance* of arguments. The more an argument is important, the stronger the attacks emanating from it. However, this does not mean that weights of attacks are generated. In our approach, the three arguments a, b and c are equal wrt. Bbs and Dbs. The reason is that we do not consider weights on attacks. However, our semantics can easily be extended to deal with weighted attacks.

7 Conclusion

The paper developed an axiomatic approach for defining semantics for argumentation frameworks. It proposed postulates (each of which represents a criterion) that a semantics may satisfy. The approach offers thus a theoretical framework for comparing semantics. It is worth recalling that only some of the postulates (like abstraction and void precedence) are satisfied by Dung's semantics. The others are based on graded considerations which may be natural in applications like decision making. Another novelty of the approach is that it computes the acceptability of arguments without passing through multiple points of view. Its basic idea is to compute a complete ranking on the set of arguments. The paper proposed two novel semantics that satisfy the postulates but that do not necessarily return the same results. An important future work is to characterize the semantics that satisfy the postulates.

References

1. L. Amgoud and S. Vesic. A new approach for preference-based argumentation frameworks. *Annals of Mathematics and Artificial Intelligence*, 63(2):149–183, 2011.
2. P. Baroni and M. Giacomin. On principle-based evaluation of extension-based argumentation semantics. *Artificial Intelligence*, 171(10-15):675–700, 2007.
3. P. Baroni, M. Giacomin, and G. Guida. Scc-recursiveness: a general schema for argumentation semantics. *Artificial Intelligence Journal*, 168:162–210, 2005.
4. C. Cayrol and M.-C. Lagasquie-Schiex. Graduality in Argumentation. *Journal of Artificial Intelligence Research (JAIR)*, 23:245–297, 2005.
5. P. Dung, P. Mancarella, and F. Toni. Computing ideal skeptical argumentation. *Artificial Intelligence Journal*, 171:642–674, 2007.
6. P. M. Dung. On the Acceptability of Arguments and its Fundamental Role in Non-Monotonic Reasoning, Logic Programming and n-Person Games. *AIJ*, 77:321–357, 1995.
7. P. E. Dunne, A. Hunter, P. McBurney, S. Parsons, and M. Wooldridge. Weighted argument systems: Basic definitions, algorithms, and complexity results. *Artificial Intelligence*, 175(2):457–486, 2011.
8. A. García and G. Simari. Defeasible logic programming: an argumentative approach. *Theory and Practice of Logic Programming*, 4(1-2):95–138, 2004.
9. A. N. Langville and C. D. Meyer. Deeper Inside PageRank. *Internet Mathematics*, 1(3):335–400, 2005.
10. L. Page, S. Brin, R. Motwani, and T. Winograd. The PageRank Citation Ranking: Bringing Order to the Web. Technical report, Stanford Digital Library Technologies Project, 1998.