Postulates for logic-based argumentation systems

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Abstract

Logic-based *argumentation systems* are developed for reasoning with inconsistent information. Starting from a knowledge base encoded in a logical language, they define *arguments* and *attacks* between them using the consequence operator associated with the language. Finally, a *semantics* is used for evaluating the arguments.

In this paper, we focus on systems that are based on *deductive logics* and that use Dung's semantics. We investigate *rationality postulates* that such systems should satisfy. We define five intuitive postulates: *consistency* and *closure under the consequence operator* of the underlying logic of the set of conclusions of arguments of each extension, *closure under sub-arguments* and *exhaustiveness* of the extensions, and a *free precedence* postulate ensuring that the free formulas of the knowledge base (i.e., the ones that are not involved in inconsistency) are conclusions of arguments in every extension. We study the links between the postulates and explore conditions under which they are guaranteed or violated.

Keywords: Argumentation theory, Rationality postulates.

1. Introduction

Argumentation theory consists of a set of *arguments*, *attacks* between them and a *semantics* for identifying the acceptable arguments. An argument is a reason for believing a statement, doing an action, etc. Due to its explanatory power, argumentation has become a hot topic in Artificial Intelligence. It is used for making decisions under uncertainty (e.g., [1, 2, 3, 4, 5]), learning rules (e.g., [6, 7, 8]), modeling different types of dialogs (e.g., [9, 10, 11, 12]), and more importantly for reasoning about inconsistent information (e.g., [13, 14, 15, 16, 17]).

A *logic-based argumentation system* for reasoning with inconsistent information starts with a knowledge base encoded in a logical language, then constructs

arguments from its formulas using the consequence operator associated with the language. Attacks between the arguments are identified and a semantics is chosen for evaluating the arguments. Existing logic-based systems differ mainly in at least one of the two following aspects:

- The logical language that is used. Two families of languages are encountered: *rule-based languages* and *classical languages*. The former distinguish between strict rules (encoding rules without exceptions) and defeasible ones (encoding rules that may have exceptions). Examples of systems that use such a language are ASPIC [18], ASPIC+ [19], Delp [17] and those proposed in [20]. Classical languages like propositional or first order languages are used, for instance, in [15, 21, 22].
- The semantics that is chosen. Indeed, some systems like the ones proposed in [18, 20, 21, 23], evaluate interacting arguments with Dung's semantics [24] while others (e.g., [15, 17, 25]) use other semantics.

With the effervescence of argumentation systems, it is important to define postulates that serve as metrics for measuring their quality. Postulates are thus desirable properties that are expected from any well-defined argumentation system. They may concern either the inputs or the outputs of a system. Indeed, postulates may state which inputs and outputs are acceptable. The first work on postulates in argumentation was done by Caminada and Amgoud [26, 27]. The authors focused on rule-based systems (i.e., systems that use a rule-based language) and considered Dung's semantics for the evaluation of arguments. They proposed the following postulates for the outputs of an argumentation system:

- *Closure under strict rules:* The idea is that if a system concludes x and there is a strict rule $x \rightarrow y$[1], then the system should also conclude y.
- *Direct consistency:* the set of conclusions of arguments of each extension should be consistent. This captures the idea that extensions represent different points of view, thus should be coherent.
- *Indirect consistency:* the closure (under strict rules) of the set of conclusions of arguments of each extension should be consistent.

In [28], Caminada proposed a fourth postulate, called *non-contamination*. It ensures that two conflicting arguments cannot be used to build a third argument which will attack arbitrary arguments and prevent them from being accepted. Later,

[1] A strict rule $x \rightarrow y$ means that if x holds, then y holds with no exception whatsoever.

Amgoud and Besnard proposed in [29] a stronger version of direct consistency postulate but for argumentation systems grounded on the deductive logics of Tarski [30]. The new postulate imposes that the set of formulas that are used in the arguments of each extension should be consistent. The authors justified this choice by the fact that an extension represents a coherent point of view, thus it should only involve a consistent set of formulas. They have then shown that indirect consistency follows naturally from the new postulate.

Our aim in this paper is to define a core set of postulates on the outputs of argumentation systems. The postulates should be *compatible*, i.e., they can be satisfied all together by a system, and *independent*, i.e., none of the postulates is implied by the others. As in [29], we consider argumentation systems that are grounded on Tarski's logics. Note that this does not mean that our postulates are not relevant for rule-based systems. Their counter-parts for such systems are straightforward and interested readers can find the exact definitions in [31]. Regarding evaluation of arguments, we focus on extension-based semantics, in particular Dung's ones [24]. The core set contains five postulates: The first one generalizes direct consistency that was proposed in [26]. The second postulate replaces closure under strict rules by *closure under the consequence operator* of the underlying logic. We introduce three new postulates. The first one, *closure under sub-arguments*, says that if an extension contains an argument, then all its sub-arguments should belong to the extension as well. The basic idea is that one cannot accept an argument in an extension if at least one sub-part of the argument is considered as bad. The second postulate, called *exhaustiveness*, is somehow a dual version of the previous one. It ensures that if every formula of an argument is a conclusion of another argument in a given extension, then the argument should also belong to that extension. To put it differently, if every step of an argument is considered acceptable in an extension, then so is for the argument itself. The last postulate, called *free precedence*, says that the formulas of the knowledge base which are not involved in inconsistency are conclusions of arguments in every extension. We show that the five postulates are independent and compatible. It is worth pointing out that non-contamination is not a postulate in our setting. Indeed, the problem of contamination, as described by Caminada in [28], is due to *inconsistent* arguments (an argument is inconsistent if the set of formulas appearing in its support is inconsistent). Such arguments are self-attacking and thus prevent "interesting" arguments from being accepted. If we have to define a postulate in order to avoid this anomaly, it would be about the consistency of arguments. Indeed, since arguments are intended to justify their conclusions, then they should satisfy some minimal requirements and consistency is one of them. In this paper, we consider only consistent arguments. We show also that the strong version of direct consistency that is proposed in [29] follows

naturally from the new postulate on sub-arguments and the extended version of the initial definition of direct consistency. Thus, strong consistency does not deserve to be a separate postulate. Similarly, indirect consistency follows from this set of postulates. A second contribution of this paper consists of studying under which conditions the postulates are satisfied or violated. The satisfaction/violation of a postulate depends mainly on the attack relation. We characterize some attack relations that lead to the satisfaction of the five postulates, and some other relations that lead to the violation of consistency.

The paper is organized as follows: Section 2 defines the logic-based argumentation systems we are interested in. Section 3 introduces the five postulates and studies the links between them. Section 4 investigates conditions under which some postulates are violated. Conditions under which the postulates are satisfied are studied in Section 5. Section 6 discusses the importance of our postulates in case of weighted argumentation systems. The last section concludes.

2. Logic-based Argumentation Systems

It is well known that a logic-based argumentation system is built on an underlying logic, called *base logic* by Hunter in [32]. In the same paper, Hunter has shown that the base logics of existing systems are monotonic. This is the case even for rule-based systems. Note that in such systems a notion of *derivation* is defined. It shows how literals/formulas follow from rules. A derivation that is built from a set of rules can be built from any superset of rules. This makes the construction of arguments a monotonic process. In this paper, we do not focus on a particular logic (like propositional logic, ...), we rather consider an *abstract* logic. Such abstraction makes our study general and our results hold under any instantiation of the abstract logic. However, we consider the broad class of monotonic logics that satisfy Tarski's axioms [30].

Tarski's logics are pairs $(\mathcal{L}, \mathsf{CN})$ where $\mathcal{L}$ is a set of well-formed *formulas* and CN is a *consequence operator*. There is no particular requirement on the kind of connectors that may be used for defining formulas of $\mathcal{L}$. It is even not required to have a connector of negation. CN is a function from $2^{\mathcal{L}}$ to $2^{\mathcal{L}}$ which returns the logical consequences of a set of formulas. The exact definition of CN is not given, thus it may be instantiated in different ways. However, it should satisfy the following basic properties:

- $X \subseteq \mathsf{CN}(X)$ (Expansion)
- $\mathsf{CN}(\mathsf{CN}(X)) = \mathsf{CN}(X)$ (Idempotence)

- $\mathsf{CN}(X) = \bigcup_{Y \subseteq_f X} \mathsf{CN}(Y)$[2] (Finiteness)
- $\mathsf{CN}(\{x\}) = \mathcal{L}$ for some $x \in \mathcal{L}$ (Absurdity)
- $\mathsf{CN}(\emptyset) \neq \mathcal{L}$ (Coherence)

Any logic whose CN satisfies the above properties is *monotonic*. The associated notion of consistency is defined as follows:

Definition 1 (Consistency). *A set $X \subseteq \mathcal{L}$ is* consistent *wrt a logic $(\mathcal{L}, \mathsf{CN})$ iff $\mathsf{CN}(X) \neq \mathcal{L}$. It is* inconsistent *otherwise.*

A knowledge base is a set of formulas of $\mathcal{L}$.

Definition 2 (Knowledge base). *A* knowledge base *is any subset Σ of $\mathcal{L}$.*

Assumption: Without loss of generality, we assume that any knowledge base Σ is free of tautologies, i.e., for all $x \in \Sigma$, $x \notin \mathsf{CN}(\emptyset)$. Of course, this assumption makes sense when the logic allows tautologies, i.e., $\mathsf{CN}(\emptyset) \neq \emptyset$.

A knowledge base may be inconsistent, and thus contains *minimal conflicts*.

Definition 3 (Minimal conflicts). *A set $C \subseteq \Sigma$ is a* minimal conflict *of a knowledge base Σ iff:*

- *C is inconsistent*
- *For all $x \in C$, $C \setminus \{x\}$ is consistent.*

Notations: Let Σ be a knowledge base. $\mathcal{C}_\Sigma$ denotes the set of all minimal conflicts of Σ and $\mathtt{Free}(\Sigma) = \Sigma \setminus \bigcup_{C \in \mathcal{C}_\Sigma} C$ (i.e., $\mathtt{Free}(\Sigma)$ is a subset of Σ which contains the formulas which are not involved in any minimal conflict).

Arguments are built from a knowledge base. They represent *proofs* for conclusions. Thus, an argument has two parts: a *support* (called also *reason*) and a *conclusion*. The support satisfies three basic requirements: i) it is consistent, ii) it is minimal (for set inclusion) avoiding thus superfluous formulas, and iii) it infers with respect to the consequence operator the conclusion.

Definition 4 (Argument). *Let Σ be a knowledge base. An* argument *is a pair (X, x) such that:*

[2] $Y \subseteq_f X$ means that Y is a finite subset of X.

- $X \subseteq \Sigma$
- *X is consistent*
- $x \in \mathsf{CN}(X)$
- $\nexists X' \subset X$ *such that* $x \in \mathsf{CN}(X')$

An argument (X, x) *is a* sub-argument *of another argument* (X', x') *iff* $X \subseteq X'$.

When a logic $(\mathcal{L}, \mathsf{CN})$ allows tautologies, i.e., $\mathsf{CN}(\emptyset) \neq \emptyset$, then arguments with empty supports are built. Such arguments will be called *tautological arguments*.

Property 1. *Let* $(\mathcal{L}, \mathsf{CN})$ *be a logic. If* $\mathsf{CN}(\emptyset) \neq \emptyset$ *then for all* $x \in \mathsf{CN}(\emptyset)$*,* $(\emptyset, x)$ *is an argument.*

Notations: $\mathtt{Supp}(.)$ and $\mathtt{Conc}(.)$ are two functions that return respectively the *support* X and the *conclusion* x of an argument (X, x). For all $\mathcal{S} \subseteq \Sigma$, $\mathtt{Arg}(\mathcal{S})$ denotes the set of *all* arguments that can be built from $\mathcal{S}$ by means of Definition 4. $\mathtt{Arg}^*(\mathcal{L})$ is the set of tautological arguments that can be built from language $\mathcal{L}$. $\mathtt{Sub}(.)$ is a function that returns all the sub-arguments of a given argument. For all $\mathcal{E} \subseteq \mathtt{Arg}(\Sigma)$, $\mathtt{Concs}(\mathcal{E}) = \{\mathtt{Conc}(a) \mid a \in \mathcal{E}\}$ and $\mathtt{Base}(\mathcal{E}) = \bigcup_{a \in \mathcal{E}} \mathtt{Supp}(a)$. If $\mathcal{E} = \emptyset$, then $\mathtt{Concs}(\mathcal{E}) = \mathtt{Base}(\mathcal{E}) = \emptyset$.

An *argumentation system* is defined as follows.

Definition 5 (Argumentation system). *An* argumentation system *over a knowledge base* Σ *is a pair* $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ *where* $\mathcal{R} \subseteq \mathtt{Arg}(\Sigma) \times \mathtt{Arg}(\Sigma)$ *is an* attack *relation. For* $a, b \in \mathtt{Arg}(\Sigma)$*,* $(a, b) \in \mathcal{R}$ *(or* $a\mathcal{R}b$*) means that* a attacks b*.*

The attack relation is left *unspecified* in order to keep the system very general. Indeed, it may be instantiated in several ways (see [23] for a large number of attack relations). It is also worth mentioning that the set $\mathtt{Arg}(\Sigma)$ may be infinite even when the base Σ is finite. This is for instance the case when $(\mathcal{L}, \mathsf{CN})$ is propositional logic. This would mean that the argumentation system may be *infinite* (i.e., an argument may be attacked by an infinite number of arguments).

Arguments are evaluated using *any* semantics which is based on the notion of *admissibility* [24]. Before recalling the semantics, let us first introduce the two requirements on which they are built:

Definition 6 (Conflict-freeness – Defence). *Let* $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ *be an argumentation system and* $\mathcal{E} \subseteq \mathtt{Arg}(\Sigma)$ *and* $a \in \mathtt{Arg}(\Sigma)$*.*

- $\mathcal{E}$ *is* conflict-free *iff* $\nexists a, b \in \mathcal{E}$ *such that* $(a, b) \in \mathcal{R}$.
- $\mathcal{E}$ defends a *iff* $\forall b \in \texttt{Arg}(\Sigma)$, *if* $(b, a) \in \mathcal{R}$, *then* $\exists c \in \mathcal{E}$ *such that* $(c, b) \in \mathcal{R}$.

The following definition recalls most of the semantics that are based on admissibility, in particular those proposed in [24, 33, 34].

Definition 7 (Semantics). *Let* $\mathcal{T} = (\texttt{Arg}(\Sigma), \mathcal{R})$ *be an argumentation system and* $\mathcal{E} \subseteq \texttt{Arg}(\Sigma)$.

- $\mathcal{E}$ *is a* naive extension *iff it is a maximal (wrt set inclusion) conflict-free set.*
- $\mathcal{E}$ *is an* admissible extension *iff it is conflict-free and defends all its elements.*
- $\mathcal{E}$ *is a* complete extension *iff it is admissible and contains all the arguments it defends.*
- $\mathcal{E}$ *is a* grounded extension *iff it is the minimal (wrt set inclusion) complete extension.*
- $\mathcal{E}$ *is a* preferred extension *iff it is a maximal (wrt set inclusion) admissible extension.*
- $\mathcal{E}$ *is a* stable extension *iff it is a preferred extension that attacks any argument in* $\texttt{Arg}(\Sigma) \setminus \mathcal{E}$.
- $\mathcal{E}$ *is a* semi-stable extension *iff it is a complete extension and the union of the set* $\mathcal{E}$ *and the set of all arguments attacked by* $\mathcal{E}$ *is maximal (wrt set inclusion).*
- $\mathcal{E}$ *is an* ideal extension *iff* $\mathcal{E}$ *is a maximal (wrt set inclusion) admissible set contained in every preferred extension.*

Throughout the paper, the expression "reviewed semantics" refers to the above semantics, and $\texttt{Ext}(\mathcal{T})$ denotes the set of all extensions of $\mathcal{T}$ under a given semantics.

Let us now define the plausible conclusions that may be drawn from a knowledge base Σ by an argumentation system. The idea is to infer a formula x from Σ iff x is the conclusion of an argument in every extension of the system.

Definition 8 (Plausible conclusions). *Let* $\mathcal{T} = (\texttt{Arg}(\Sigma), \mathcal{R})$ *be an argumentation system over a knowledge base* Σ. *The set of* plausible conclusions *of* $\mathcal{T}$ *is* $\texttt{Output}(\mathcal{T})$ *such that:*

$$\texttt{Output}(\mathcal{T}) = \begin{cases} \{x \in \mathcal{L} \mid \forall \mathcal{E} \in \texttt{Ext}(\mathcal{T}) \exists a \in \mathcal{E} \text{ s.t. } \texttt{Conc}(a) = x\} & \text{if } \texttt{Ext}(\mathcal{T}) \neq \emptyset \\ \emptyset & \text{else} \end{cases}$$

It is easy to check that the set of plausible conclusions coincides with the set of common conclusions of the extensions, of course when extensions exist.

Property 2. *Let* $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ *be an argumentation system over a knowledge base* Σ *such that* $\mathtt{Ext}(\mathcal{T}) \neq \emptyset$*. The equality*

$$\mathtt{Output}(\mathcal{T}) = \bigcap_{\mathcal{E}_i \in \mathtt{Ext}(\mathcal{T})} \mathtt{Concs}(\mathcal{E}_i)$$

holds under any of the reviewed semantics.

It is also obvious that the plausible conclusions of an argumentation system are consequences of Σ under the consequence operator CN.

Property 3. *Let* $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ *be an argumentation system over a knowledge base* Σ*. It holds that* $\mathtt{Output}(\mathcal{T}) \subseteq \mathsf{CN}(\Sigma)$ *(under any of the reviewed semantics).*

It is easy to check that the set of plausible conclusions is empty as soon as the argumentation system has an empty extension.

Property 4. *Let* $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ *be an argumentation system and* $\mathtt{Ext}(\mathcal{T})$ *its set of extensions under a given semantics. If* $\emptyset \in \mathtt{Ext}(\mathcal{T})$*, then* $\mathtt{Output}(\mathcal{T}) = \emptyset$*.*

Under admissible semantics, the set of plausible conclusions is always empty since the empty set is admissible.

Property 5. *For all* $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$*,* $\mathtt{Output}(\mathcal{T}) = \emptyset$ *under admissible semantics.*

Thus, an argumentation system starts with a *monotonic* logic and defines a *non monotonic* one. The non monotonicity of the resulting logic is obviously due to the status of arguments which may change in light of new arguments. Note that several works in the literature studied the dynamics of argumentation systems, in particular how the extensions (under a given semantics) of a system may change when new arguments are added to the system (see for instance [35]).

3. Rationality Postulates

So far, we presented an abstract logical instantiation of Dung's argumentation framework. It can itself be instantiated by various logics satisfying Tarski's axioms and by different attack relations. The formalism is devoted for reasoning about inconsistent knowledge. Thus, it should enjoy some properties (referred to as postulates) in order to be deemed acceptable. Two families of postulates can be defined:

- postulates describing acceptable inputs (i.e., arguments/attack relations).
- postulates on the outputs. The objective is to check the "soundness" and the "completeness" of the results that are returned by the system.

As discussed in the previous section, arguments satisfy three properties which may also be considered as postulates. The first one concerns the consistency of their supports. The idea is that arguments justify conclusions in order to increase/decrease their acceptability, thus inconsistent sets of formulas cannot meet this objective. Hence, arguments should be coherent in order to be convincing. Minimality is another postulate ensuring the conciseness of the support of an argument. Indeed, a support should not contain superfluous information, i.e., formulas which are not related to the conclusion. Finally, a conclusion of an argument should follow from the support. It is worth noticing that since the seminal work by Simari and Loui in [14], the three postulates are satisfied by existing definitions of argument. There are however some few exceptions like ASPIC [18] and its extended version ASPIC+ [19] in which arguments are not necessarily consistent. Unlike the first family of postulates, most of existing rule-based systems like those proposed in [20, 36] violate the postulates proposed in [27]. Their outputs are neither complete, as they may forget conclusions, nor sound since they may be inconsistent. Thus, it is important to define postulates for guiding the well definition of systems. In what follows, we define a core set of such postulates.

The first rationality postulate that a logic-based argumentation system should satisfy concerns the completeness (called also *closure* in [26, 27]) of its output, i.e., its conclusions should be "complete". A user should not perform on her own some extra reasoning to derive statements that the system "forgot" to entail. Assume a system whose base logic is propositional logic. If x and y are both plausible conclusions of the system then one would expect $x \wedge y$ to be another plausible conclusion. In [26], closure is defined for rule-based argumentation systems. The postulate says that the set of conclusions of every extension should be closed in terms of strict rules. Indeed, if x is a conclusion of an extension and there exists a strict rule $x \rightarrow y$, then y should also be a conclusion of the same extension. The reason is that strict rules encode information which have no exceptions. In our example, whenever x holds, then y always holds. Thus, a system which returns x but not y is certainly incomplete. Note that closure under defeasible rules is not suitable since such rules may have exceptions. Let us consider the rule "generally birds fly" encoded by $b \Rightarrow f$. For a particular bird Tweety which is also a penguin, it is natural to conclude that it is a bird (b) without having f. In case of Tarskian logics, closure is defined using the consequence operator CN.

Postulate 1 (Closure under CN). *An argumentation system $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ over a knowledge base Σ is* closed under CN *iff for all $\mathcal{E} \in \mathtt{Ext}(\mathcal{T})$, $\mathtt{Concs}(\mathcal{E}) = \mathsf{CN}(\mathtt{Concs}(\mathcal{E}))$.*

Any argumentation system that has no extensions (for instance under stable semantics) satisfies the previous postulate.

Proposition 1. *For all argumentation system $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ such that $\mathtt{Ext}(\mathcal{T}) = \emptyset$, $\mathcal{T}$ is closed under CN.*

An argumentation system that satisfies this postulate does not forget tautologies, if they exist. This is true under any semantics.

Proposition 2. *Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system such that $\mathtt{Ext}(\mathcal{T}) \neq \emptyset$. If $\mathcal{T}$ is closed under CN, then for all $\mathcal{E} \in \mathtt{Ext}(\mathcal{T})$:*

- $\mathtt{Arg}^*(\mathcal{L}) \subseteq \mathcal{E}$
- $\mathsf{CN}(\emptyset) \subseteq \mathtt{Concs}(\mathcal{E})$
- *If $\mathsf{CN}(\emptyset) \neq \emptyset$, then $\mathcal{E} \neq \emptyset$.*

Proposition 3. *Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system such that $\mathtt{Ext}(\mathcal{T}) \neq \emptyset$. If $\mathcal{T}$ is closed under CN, then:*

- $\mathsf{CN}(\emptyset) \subseteq \mathtt{Output}(\mathcal{T})$.
- *If $\mathtt{Output}(\mathcal{T}) = \emptyset$, then $\mathsf{CN}(\emptyset) = \emptyset$.*

In [26], closure is imposed both on the extensions of an argumentation system and on its set of plausible conclusions. The next result shows that the closure of the output set does not deserve to be a separate postulate since it follows immediately from the closure of extensions.

Proposition 4. *Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system over a knowledge base Σ such that $\mathtt{Ext}(\mathcal{T}) \neq \emptyset$. If $\mathcal{T}$ is closed under CN, then $\mathtt{Output}(\mathcal{T}) = \mathsf{CN}(\mathtt{Output}(\mathcal{T}))$ (under any of the reviewed semantics).*

An argument may have one or several sub-arguments reflecting the different premises on which it is based. The second rationality postulate says that an argument cannot be accepted if at least one of its sub-parts is bad. A convincing argument should be based on convincing sub-arguments. Since we are interested by extension-based semantics, the postulate is expressed on the extensions of a system. It says that an argument cannot belong to a given extension if at least one

of its sub-arguments is not in the extension. It is worth mentioning that there is an interesting work in the literature [37, 38] where the authors extended Dung's framework with a new binary relation expressing the sub-argument link between arguments. However, arguments are evaluated using Dung's semantics and no additional requirements on the extensions are imposed. Thus, it may be the case that an extension contains an argument but not one or more of its sub-arguments. Thus, our postulate is suitable even for those extended systems.

Postulate 2 (Closure under sub-arguments). *An argumentation system* $\mathcal{T} = (\texttt{Args}(\Sigma), \mathcal{R})$ *over a knowledge base* Σ *is* closed under sub-arguments *iff for all* $\mathcal{E} \in \texttt{Ext}(\mathcal{T})$*, if* $a \in \mathcal{E}$*, then* $\texttt{Sub}(a) \subseteq \mathcal{E}$*.*

It can be checked that closure under sub-arguments is equivalent to closure under super-arguments. The latter means that if an argument is excluded from an extension, then all arguments built on it (its super-arguments) should also be excluded from that extension.

Property 6. *Let* $\mathcal{T} = (\texttt{Args}(\Sigma), \mathcal{R})$ *be an argumentation system over a knowledge base* Σ *such that* $\texttt{Ext}(\mathcal{T}) \neq \emptyset$*.* $\mathcal{T}$ *is closed under sub-arguments iff* $\forall \mathcal{E} \in \texttt{Ext}(\mathcal{T})$ *if* $a \notin \mathcal{E}$*, then* $\forall b \in \texttt{Args}(\Sigma)$ *such that* $a \in \texttt{Sub}(b)$*,* $b \notin \mathcal{E}$*.*

We show also that if this postulate is satisfied by an argumentation system, then every formula that is used in the support of at least one argument in an extension, is the conclusion of an argument in that extension.

Proposition 5. *Let* $\mathcal{T} = (\texttt{Args}(\Sigma), \mathcal{R})$ *be an argumentation system over a knowledge base* Σ *such that* $\texttt{Ext}(\mathcal{T}) \neq \emptyset$ *and* $\mathcal{T}$ *is closed under sub-arguments (under any of the reviewed semantics). For all* $\mathcal{E} \in \texttt{Ext}(\mathcal{T})$ *such that* $\mathcal{E} \neq \emptyset$*, it holds that:*

- *For all* $x \in \texttt{Base}(\mathcal{E})$*,* $(\{x\}, x) \in \mathcal{E}$
- $\texttt{Base}(\mathcal{E}) \subseteq \texttt{Concs}(\mathcal{E})$

The next result characterizes the extensions of argumentation systems that are closed under both CN and sub-arguments.

Proposition 6. *Let* $\mathcal{T} = (\texttt{Args}(\Sigma), \mathcal{R})$ *be an argumentation system over a knowledge base* Σ *such that* $\texttt{Ext}(\mathcal{T}) \neq \emptyset$*. If* $\mathcal{T}$ *is closed under sub-arguments and under* CN*, then for all* $\mathcal{E} \in \texttt{Ext}(\mathcal{T})$*,* $\texttt{Concs}(\mathcal{E}) = \mathsf{CN}(\texttt{Base}(\mathcal{E}))$ *(under any of the reviewed semantics).*

In argumentation literature, the extensions of an argumentation system represent coherent positions (or points of view). Thus, it is very natural to expect the set of conclusions of every extension to be consistent, i.e., the claims supported by an extension should be consistent all together. The third rationality postulate expresses this idea. In what follows, we generalize to any Tarskian logic the 'direct consistency postulate' proposed for rule-based argumentation systems in [26].

Postulate 3 (Consistency). *An argumentation system* $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ *over a knowledge base* Σ *satisfies* consistency *iff for all* $\mathcal{E} \in \mathtt{Ext}(\mathcal{T})$, $\mathtt{Concs}(\mathcal{E})$ *is consistent.*

Note that argumentation systems that have no extensions (for instance under stable semantics) satisfy consistency.

Proposition 7. *For all argumentation system* $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ *such that* $\mathtt{Ext}(\mathcal{T}) = \emptyset$, $\mathcal{T}$ *satisfies consistency.*

As for closure, in [26] a postulate imposing the consistency of the set of plausible conclusions is defined. We show next that such postulate is not necessary since an argumentation system that satisfies Postulate 3 has a consistent output.

Proposition 8. *If an argumentation system* $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ *satisfies consistency, then the set* $\mathtt{Output}(\mathcal{T})$ *is consistent (under any of the reviewed semantics).*

In [26], it was shown that some rule-based argumentation systems, like [36], violate the *indirect* consistency postulate. Recall that indirect consistency means that the closure (under strict rules) of the conclusions of each extension is consistent. When this postulate is violated, undesirable conclusions may be inferred. We show next that in case of Tarski's logics, (direct) consistency coincides with indirect consistency. Thus, this latter does not deserve to be a postulate *per se*.

Proposition 9. *Let* $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ *be an argumentation system over a knowledge base* Σ *such that* $\mathtt{Ext}(\mathcal{T}) \neq \emptyset$. $\mathcal{T}$ *satisfies* consistency *iff for all* $\mathcal{E} \in \mathtt{Ext}(\mathcal{T})$, $\mathsf{CN}(\mathtt{Concs}(\mathcal{E}))$ *is consistent (under any of the reviewed semantics).*

So far, we revisited and extended the postulates proposed by Caminada and Amgoud in [26]. We showed that three of them (the closure and the consistency of the set of plausible conclusions and indirect consistency) might not be considered as postulates since they follow naturally from more fundamental ones. The question now is: what about the strong version of consistency that is proposed by Amgoud and Besnard in [29]? Should it be considered as a postulate or not? Recall

that this postulate ensures that for each extension $\mathcal{E}$ of an argumentation system, $\texttt{Base}(\mathcal{E})$ should be consistent. This means that not only the set of conclusions of an extension should be consistent but so is even for the set of all formulas used in the extension.

Definition 9 (Strong Consistency). *An argumentation system* $\mathcal{T} = (\texttt{Arg}(\Sigma), \mathcal{R})$ *over a knowledge base* Σ *satisfies* strong consistency *iff for all* $\mathcal{E} \in \texttt{Ext}(\mathcal{T})$, $\texttt{Base}(\mathcal{E})$ *is consistent.*

This postulate is clearly stronger than Postulate 3 as shown next.

Proposition 10. *If an argumentation system satisfies strong consistency, then it also satisfies consistency (under any of the reviewed semantics).*

Strong consistency does not deserve to be a postulate *per se*. Indeed, it follows from the basic ones, namely consistency and closure under sub-arguments.

Proposition 11. *If an argumentation system satisfies consistency and closure under sub-arguments, then it also satisfies strong consistency (under any of the reviewed semantics).*

The fourth postulate is in some sense the dual of the closure under sub-arguments postulate. This latter ensures that if an argument is accepted in an extension, then all its sub-parts should also be accepted in that extension. The exhaustiveness postulate defends the idea that if each step in an argument is good enough to be in a given extension, then so is the argument itself. Note that this does not mean that the argument will be added to every extension of the argumentation system. It belongs only to those extensions which accept all its sub-parts.

Postulate 4 (Exhaustiveness). *An argumentation system* $\mathcal{T} = (\texttt{Arg}(\Sigma), \mathcal{R})$ *over a knowledge base* Σ *satisfies* exhaustiveness *iff for all* $\mathcal{E} \in \texttt{Ext}(\mathcal{T})$, *for all* $(X, x) \in \texttt{Arg}(\Sigma)$, *if* $X \cup \{x\} \subseteq \texttt{Concs}(\mathcal{E})$, *then* $(X, x) \in \mathcal{E}$.

The following result shows that when this postulate is satisfied, then extensions are closed in terms of arguments.

Proposition 12. *Let* $\mathcal{T}$ *be an argumentation system such that* $\texttt{Ext}(\mathcal{T}) \neq \emptyset$. *If* $\mathcal{T}$ *is closed under both* CN *and sub-arguments and satisfies the exhaustiveness postulate, then* $\forall \mathcal{E} \in \texttt{Ext}(\mathcal{T})$, $\mathcal{E} = \texttt{Arg}(\texttt{Base}(\mathcal{E}))$ *(under any of the reviewed semantics).*

The last postulate concerns the kind of conclusions to be drawn from a knowledge base. It states that the *free* formulas of a base (i.e., the ones that are not involved in any minimal conflict) should be inferred. Namely, any argument that is built only from this part of the base should be in every extension of an argumentation system built over the base.

Postulate 5 (Free Precedence). *An argumentation system* $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ *over a knowledge base* Σ *satisfies* free precedence *iff for all* $\mathcal{E} \in \mathtt{Ext}(\mathcal{T})$, $\mathtt{Arg}(\mathtt{Free}(\Sigma)) \subseteq \mathcal{E}$.

We show next that the free formulas are plausible conclusions of any argumentation system that satisfies Postulate 5.

Proposition 13. *Let* $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ *be an argumentation system over a knowledge base* Σ *such that* $\mathtt{Ext}(\mathcal{T}) \neq \emptyset$. *If* $\mathcal{T}$ *satisfies free precedence, then* $\mathtt{Free}(\Sigma) \subseteq \mathtt{Output}(\mathcal{T})$ *(under any of the reviewed semantics).*

An axiomatic approach should obey an important feature: the postulates should be *compatible*, i.e., they may be satisfied all together by an argumentation system. Hopefully, our five postulates are compatible.

Proposition 14. *The five postulates are compatible.*

The five postulates are independent in the general case. However, when the attack relation originates from the inconsistency of the knowledge base, then some postulates may follow from others under some semantics. Before presenting the formal results, let us first introduce conflict-dependent attack relations.

Definition 10 (Conflict-dependent). *An attack relation* $\mathcal{R}$ *is* conflict-dependent *iff* $\forall a, b \in \mathtt{Arg}(\Sigma)$, *if* $a\mathcal{R}b$ *then* $\mathtt{Supp}(a) \cup \mathtt{Supp}(b)$ *is inconsistent.*

Note that all the attack relations that are used in existing structured argumentation systems are conflict-dependent (see [23] for a summary of those relations). We show that, under naive semantics, strong consistency implies closure under sub-arguments.

Proposition 15. *Let* $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ *be an argumentation system over a knowledge base* Σ *such that* $\mathcal{R}$ *is conflict-dependent. If* $\forall \mathcal{E} \in \mathtt{Ext}(\mathcal{T})$, $\mathtt{Base}(\mathcal{E})$ *is consistent, then* $\mathcal{T}$ *is closed under sub-arguments under naive semantics.*

The same result holds under stable semantics, especially for argumentation systems that have at least one stable extension.

Proposition 16. *Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system over a knowledge base Σ such that $\mathcal{R}$ is conflict-dependent and $\mathtt{Ext}(\mathcal{T}) \neq \emptyset$ under stable semantics. If $\forall \mathcal{E} \in \mathtt{Ext}(\mathcal{T})$, $\mathtt{Base}(\mathcal{E})$ is consistent, then $\mathcal{T}$ is closed under sub-arguments (under stable semantics).*

This means that under naive (respectively stable) semantics, satisfying consistency and closure under sub-arguments amounts exactly to satisfying the strong version of consistency.

Proposition 17. *Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system over a knowledge base Σ such that $\mathcal{R}$ is conflict-dependent and $\mathtt{Ext}(\mathcal{T}) \neq \emptyset$. $\mathcal{T}$ satisfies consistency and closure under sub-arguments under naive (resp. stable) semantics iff $\forall \mathcal{E} \in \mathtt{Ext}(\mathcal{T})$, $\mathtt{Base}(\mathcal{E})$ is consistent under naive (resp. stable) semantics.*

We show next that under naive and stable semantics, closure under the consequence operator CN follows from closure under sub-arguments and consistency.

Proposition 18. *Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system over a knowledge base Σ such that $\mathcal{R}$ is conflict-dependent. If $\mathcal{T}$ satisfies consistency and closure under sub-arguments under naive (resp. stable) semantics and $\exists \mathcal{E} \in \mathtt{Ext}(\mathcal{T})$ with $\mathcal{E} \neq \emptyset$, then $\mathcal{T}$ is closed under CN under naive (resp. stable) semantics.*

These links are not true under the remaining semantics like preferred and complete semantics.

4. On the Violation of the Postulates

This section studies conditions under which some of the postulates are violated. We propose three properties of attack relations that may lead to the violation of the consistency postulate. The first one concerns the *origin* of the relation. We show that an attack relation should be grounded on inconsistency, i.e., conflict-dependent. Since the formalism is devoted to reasoning about inconsistent information, it is natural for inconsistency to be the origin of the attack relation.

Example 1. *Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system built over the propositional knowledge base $\Sigma = \{b, p\}$ where b stands for "Tweety is a bird" and p for "Tweety is a penguin". Assume that $\mathcal{R} = \{(x, y) \mid \mathtt{Supp}(x) \neq \mathtt{Supp}(y)\}$. Note that $\mathcal{R}$ is not conflict-dependent. It is easy to check that $b, p \notin \mathtt{Output}(\mathcal{T})$. This outcome is certainly not intuitive.*

In [29], it was shown that strong consistency is violated by argumentation systems that use a *symmetric* attack relation. One may think that this result is true only when considering the *strong* version of consistency. Unfortunately, it is even true for the weaker version. Indeed, it was shown in [31] that when the attack relation is symmetric, Postulate 3 is violated.

Proposition 19. *[31] Let $\mathcal{C}_\Sigma$ be such that $\exists C \in \mathcal{C}_\Sigma$ and $|C| > 2$. If $\mathcal{R}$ is conflict-dependent and symmetric, then the system $(\mathtt{Arg}(\Sigma), \mathcal{R})$ violates consistency.*

This result shows a broad class of attack relations that cannot be used in argumentation: the symmetric ones. Thus, relations like rebut or a combination of rebut and any other attack relation would lead to the violation of consistency. Note that this result is conditioned by the existence of n-ary ($n > 2$) minimal conflicts in the knowledge base. The idea is that, due to the binary character of the attack relation, this latter is unable to capture n-ary minimal conflicts. In rule-based systems like ASPIC [18] or Delp [17], there exist symmetric attack relations that may ensure the consistency postulate. The reason is that the logical language that is used is such systems is based on literals. Consequently, all the minimal conflicts are binary, i.e., contain only two literals. However, a rule-based system like ASPIC+ [19] may violate consistency since it allows rich logical languages (like propositional or first order languages) but uses a poor notion of consistency. Indeed, any ternary minimal conflict is not declared as inconsistent in ASPIC+ system.

Another mandatory property that an attack relation should fulfill is that it captures *all* the minimal conflicts of the knowledge base, i.e., each minimal conflict should be captured by at least one attack in $\mathcal{R}$. We show that forgetting one minimal conflict is fatal for the system since it will violate consistency. Indeed, from the forgotten conflict, one may build an admissible extension for the system. Note that minimal conflicts of cardinality one (i.e., which contain only one fomula each), are not considered here since they will not give birth to arguments due to the consistency condition of arguments' supports.

Definition 11 (Conflict-exhaustive). *An attack relation $\mathcal{R}$ is* conflict-exhaustive *iff $\forall C \in \mathcal{C}_\Sigma$ such that $|C| > 1$, $\exists X_1, X_2 \subset C$ such that $C = X_1 \cup X_2$ and $\exists a, b \in \mathtt{Arg}(\Sigma)$ such that $\mathtt{Supp}(a) = X_1$, $\mathtt{Supp}(b) = X_2$ and either $a\mathcal{R}b$ or $b\mathcal{R}a$.*

An attack relation that is conflict-dependent is not necessarily conflict-exhaustive and vise versa. We show that argumentation systems whose attack relations are not conflict-exhaustive violate consistency. Before presenting the result, let us first present some intermediary results. The first one shows that when the knowledge

base is a minimal conflict with more than one formula, then it is possible to build a conflict-free set of arguments.

Lemma 1. *Let* $\Sigma = \{x_1, \ldots, x_n\}$ *where* $n > 1$ *and* $\mathcal{C}_\Sigma = \{\Sigma\}$. *Let* $a_1, \ldots, a_n \in \mathtt{Arg}(\Sigma)$ *such that* $\mathtt{Supp}(a_i) = \{x_i\}$. *If* $\mathcal{R}$ *is conflict-dependent and not conflict-exhaustive, then the set* $\mathcal{E} = \{a_1, \ldots, a_n\}$ *is conflict-free.*

The previous conflict-free set of arguments defends its elements when the attack relation is not conflict-exhaustive.

Lemma 2. *Let* $\Sigma = \{x_1, \ldots, x_n\}$ *where* $n > 1$ *and* $\mathcal{C}_\Sigma = \{\Sigma\}$. *Let* $a_1, \ldots, a_n \in \mathtt{Arg}(\Sigma)$ *such that* $\mathtt{Supp}(a_i) = \{x_i\}$. *If* $\mathcal{R}$ *is conflict-dependent and not conflict-exhaustive, then the set* $\mathcal{E} = \{a_1, \ldots, a_n\}$ *defends its elements.*

From the two lemmas, it follows that the set $\{a_1, \ldots, a_n\}$ is an admissible extension.

Proposition 20. *Let* $\Sigma = \{x_1, \ldots, x_n\}$ *where* $n > 1$ *and* $\mathcal{C}_\Sigma = \{\Sigma\}$. *Let* a_1, *...,* $a_n \in \mathtt{Arg}(\Sigma)$ *such that* $\mathtt{Supp}(a_i) = \{x_i\}$. *If* $\mathcal{R}$ *is conflict-dependent and not conflict-exhaustive, then the set* $\mathcal{E} = \{a_1, \ldots, a_n\}$ *is an admissible extension.*

The next result shows that any argumentation system that is built from a knowledge base $\Sigma = \{x_1, \ldots, x_n\}$ where $n > 1$ and $\mathcal{C}_\Sigma = \{\Sigma\}$ violates consistency in case the attack relation is not conflict-exhaustive.

Proposition 21. *Let* $\Sigma = \{x_1, \ldots, x_n\}$ *where* $n > 1$ *and* $\mathcal{C}_\Sigma = \{\Sigma\}$. *Let* $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ *be an argumentation system over* Σ *such that* $\mathcal{R}$ *is conflict-dependent and not conflict-exhaustive.* $\mathcal{T}$ *violates consistency under admissible semantics.*

It is worth mentioning that the generalization of this result to any arbitrary knowledge base is not obvious.

Let us summarize: in order to satisfy consistency, an argumentation system built over a knowledge base under a Tarskian logic should use an attack relation that is conflict-dependent, captures all binary or more minimal conflicts but is not symmetric in case the base contains n-ary (with $n > 2$) minimal conflicts.

Closure under consequence operator may be violated by argumentation systems under some semantics. We show that under admissible semantics, every argumentation system violates this postulate in case the logic $(\mathcal{L}, \mathsf{CN})$ upon which it is based allows tautologies. Indeed, tautologies cannot be plausible conclusions of any system. This is true whatever the attack relation that is used in the system.

Proposition 22. *Let* $(\mathtt{Arg}(\Sigma), \mathcal{R})$ *be an argumentation system built upon logic* $(\mathcal{L}, \mathsf{CN})$. *If* $\mathsf{CN}(\emptyset) \neq \emptyset$*, then* $\mathcal{T}$ *violates closure under* CN *under admissible semantics.*

The set of plausible conclusions may not be closed under the consequence operator. This is the case for argumentation systems that do no have extensions, especially when their underlying logics allow tautologies. Note that stable semantics may be concerned since it does not guarantee the existence of extensions.

Proposition 23. *Let* $(\mathtt{Arg}(\Sigma), \mathcal{R})$ *be an argumentation system built upon logic* $(\mathcal{L}, \mathsf{CN})$. *If* $\mathtt{Ext}(\mathcal{T}) = \emptyset$ *and* $\mathsf{CN}(\emptyset) \neq \emptyset$*, then* $\mathtt{Output}(\mathcal{T}) \neq \mathsf{CN}(\mathtt{Output}(\mathcal{T}))$.

The free precedence postulate can also be violated under admissible semantics. Indeed, under this semantics the free formulas of a knowledge base will not be inferred by any argumentation system. This is mainly due to the fact that the empty set is always admissible.

Proposition 24. *For all argumentation system* $(\mathtt{Arg}(\Sigma), \mathcal{R})$ *built over* Σ*, if* $\mathtt{Free}(\Sigma) \neq \emptyset$*, then* $\mathcal{T}$ *violates free precedence under admissible semantics.*

An argumentation system that does not have extensions (like under stable semantics) misses the free formulas of the knowledge base.

Proposition 25. *For all argumentation system* $(\mathtt{Arg}(\Sigma), \mathcal{R})$ *built over* Σ*, if* $\mathtt{Free}(\Sigma) \neq \emptyset$ *and* $\mathtt{Ext}(\mathcal{T}) = \emptyset$*, then* $\mathcal{T}$ *violates free precedence.*

In the next section, we show that free precedence is satisfied under all the remaining semantics. This is in particular true for argumentation systems that use conflict-dependent attack relations.

5. When are the Postulates Satisfied?

In a previous section, we defined five rationality postulates for argumentation-based reasoning models. An important question now is: are there argumentation systems that may satisfy them? If yes, what are the characteristics of those systems? These questions are very ambitious since an argumentation system has three main parameters: an underlying monotonic logic $(\mathcal{L}, \mathsf{CN})$, an attack relation $\mathcal{R}$ and a semantics. In this paper, the three parameters are left unspecified. Thus, getting a complete answer is a real challenge. In this section, we identify one family of argumentation systems that satisfy closure under the consequence operator, three broad families of argumentation systems that satisfy closure under sub-arguments,

a broad family of systems that satisfy consistency. We show also that free precedence is satisfied under most existing semantics. The results are general in the sense that they hold under any Tarskian logic, any attack relation that fulfills the mandatory properties discussed in the previous section, and also under any of the reviewed semantics. Some of the results are even true for any semantics that is based on conflict-freeness, i.e., that defines conflict-free extensions.

Since most of reviewed semantics are based on complete semantics, then it is easy to show that any postulate that is satisfied by an argumentation system under complete semantics, is also satisfied by the same system under any of the remaining semantics except admissible one. This latter is ignored in the sequel since it is not interesting for reasoning. Indeed, we have shown that the set of plausible conclusions is always empty under this semantics.

Proposition 26. *Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system over a knowledge base Σ. If $\mathcal{T}$ satisfies Postulate x (where $x = 1, \ldots, 5$) under complete semantics, then $\mathcal{T}$ satisfies Postulate x under grounded, ideal, preferred, stable and semi-stable semantics.*

5.1. Satisfaction of Closure Under Consequence Operator

In this section, we identify a class of argumentation systems that satisfy closure under the consequence operator of the underlying logic $(\mathcal{L}, \mathsf{CN})$. We show that an argumentation system that uses an attack relation which captures *all* the minimal conflicts of the knowledge base, and whose extensions contain all the arguments that may be built from the set of formulas appearing in their arguments satisfies closure under CN.

Proposition 27. *Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system over a knowledge base Σ such that $\mathcal{R}$ is conflict-exhaustive. Let $\mathtt{Ext}(\mathcal{T})$ be its extensions under any semantics based on the notion of conflict-freeness. If $\forall \mathcal{E} \in \mathtt{Ext}(\mathcal{T})$, $\mathcal{E} = \mathtt{Arg}(\mathtt{Base}(\mathcal{E}))$, then $\mathcal{T}$ is closed under CN.*

It is worth mentioning that the above result holds under *any* acceptability semantics that is based on the notion of conflict-freeness. Thus, it is true for semantics that are not based on admissibility like the ones proposed in [39].

5.2. Satisfaction of Closure Under Sub-Arguments

The satisfaction of the sub-argument postulate by an argumentation system depends broadly on the properties of its attack relation. We show that when this relation satisfies both rules R_1 and R_2 (see Definition 12), then the system is closed under sub-arguments using any of the reviewed semantics.

Definition 12. *An attack relation $\mathcal{R}$ satisfies R_x (with $x \in \{1, 2\}$) iff:*

R_1 : $\forall a, b, c \in \texttt{Arg}(\Sigma)$ *such that* $\texttt{Supp}(a) \subseteq \texttt{Supp}(b)$*, it holds* $a\mathcal{R}c \Rightarrow b\mathcal{R}c$

R_2 : $\forall a, b, c \in \texttt{Arg}(\Sigma)$ *such that* $\texttt{Supp}(a) \subseteq \texttt{Supp}(b)$*,* $c\mathcal{R}a \Rightarrow c\mathcal{R}b$

The rule R_1 says that if an argument a attacks another argument c, then all the super-arguments of a should also attack c. The second rule says that if an argument a is attacked by an argument c, then all the super-arguments of a should also be attacked by c.

Proposition 28. *Let $\mathcal{T} = (\texttt{Args}(\Sigma), \mathcal{R})$ be an argumentation system . If $\mathcal{R}$ satisfies $R1$ and $R2$, then $\mathcal{T}$ is closed under sub-arguments under complete, ideal, grounded, preferred, stable and semi-stable semantics.*

We show next that closure under sub-arguments is less demanding under stable semantics. Indeed, in this case only property R_2 is required for the attack relation.

Proposition 29. *Let $\mathcal{T} = (\texttt{Args}(\Sigma), \mathcal{R})$ be an argumentation system . If $\mathcal{R}$ satisfies R_2, then $\mathcal{T}$ satisfies closure under sub-arguments under stable semantics.*

The reverse is not necessarily true as shown next.

Example 2. *Let $\texttt{Arg}(\Sigma) = \{a, b, c, d\}$ be an argumentation system such that $\texttt{Sub}(b) = \{a, b\}$, $\texttt{Sub}(a) = \{a\}$, $\texttt{Sub}(c) = \{c\}$, $\texttt{Sub}(d) = \{d\}$. Assume also that $c\mathcal{R}a$ and $d\mathcal{R}b$. It is clear that R_2 is violated since c does not attack b. However, the stable extension $\{c, d\}$ is closed wrt sub-arguments.*

The second family of argumentation systems that satisfy closure under sub-arguments uses attack relations that are based on and sensible for inconsistency.

Definition 13 (Conflict-sensitive). *An attack relation $\mathcal{R}$ is* conflict-sensitive *iff $\forall a, b \in \texttt{Arg}(\Sigma)$, if $\texttt{Supp}(a) \cup \texttt{Supp}(b)$ is inconsistent, then either $a\mathcal{R}b$ or $b\mathcal{R}a$.*

When the attack relation is conflict-dependent and sensitive, closure under sub-arguments is satisfied under any of the reviewed semantics.

Proposition 30. *Let $\mathcal{T} = (\texttt{Args}(\Sigma), \mathcal{R})$ be an argumentation system . If $\mathcal{R}$ is conflict-dependent and sensitive, then $\mathcal{T}$ is closed under sub-arguments under complete, ideal, grounded, preferred, stable and semi-stable semantics.*

Notice that the attack relations in the first family of argumentation system are not necessarily based on inconsistency. Finally, we show that argumentation systems whose extensions are closed in terms of arguments enjoy closure under sub-arguments. This result is true under *any* acceptability semantics, i.e., even non reviewed ones.

Proposition 31. *Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system over a knowledge base Σ such that $\mathtt{Ext}(\mathcal{T}) \neq \emptyset$. If $\forall \mathcal{E} \in \mathtt{Ext}(\mathcal{T})$, $\mathcal{E} = \mathtt{Arg}(\mathtt{Base}(\mathcal{E}))$, then $\mathcal{T}$ is closed under sub-arguments under any semantics.*

5.3. Satisfaction of Consistency

In this section, we identify a class of argumentation systems that satisfy consistency. As for closure under sub-arguments, the result depends of the properties of the attack relations. Before that, we start with a result showing a case where consistency and its strong version coincide.

Proposition 32. *Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system. If $\forall \mathcal{E} \in \mathtt{Ext}(\mathcal{T})$, $\mathcal{E} = \mathtt{Arg}(\mathtt{Base}(\mathcal{E}))$, then $\mathcal{T}$ satisfies consistency implies $\mathcal{T}$ satisfies strong consistency (under any of the reviewed semantics).*

We now show that a system that uses an attack relation which captures *all* the minimal conflicts of the knowledge base and whose extensions contain all the arguments that may be built from the set of formulas appearing in their arguments, satisfies consistency.

Proposition 33. *Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system over a knowledge base Σ such that $\mathcal{R}$ is conflict-exhaustive. If $\forall \mathcal{E} \in \mathtt{Ext}(\mathcal{T})$, $\mathcal{E} = \mathtt{Arg}(\mathtt{Base}(\mathcal{E}))$, then $\mathcal{T}$ satisfies consistency (under any semantics based on conflict-freeness).*

This result is true under *any* acceptability semantics provided that it is based on the notion of conflict-freeness. Due to Proposition 32, this class of argumentation systems satisfies also strong consistency.

Property 7. *Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system over a knowledge base Σ such that $\mathcal{R}$ is conflict-exhaustive. If $\forall \mathcal{E} \in \mathtt{Ext}(\mathcal{T})$, $\mathcal{E} = \mathtt{Arg}(\mathtt{Base}(\mathcal{E}))$, then $\mathcal{T}$ satisfies strong consistency.*

This result is very general since, as we already said, the requirement on the attack relation is very natural and even satisfied by all the existing attack relations (see [23] for a review of those relations).

5.4. Satisfaction of Free Precedence

Unlike the other postulates, the satisfaction of free precedence postulate depends mainly on the semantics that is chosen for the evaluation of arguments. In fact, when the attack relation of an argumentation system built over a knowledge base is conflict-dependent, then the arguments built from the free part of the base are neither attacked nor attack any other argument of the system.

Proposition 34. *For all argumentation system $\mathcal{T} = (\texttt{Arg}(\Sigma), \mathcal{R})$ built over a knowledge base Σ, if $\mathcal{R}$ is conflict-dependent then $\forall a \in \texttt{Arg}(\texttt{Free}(\Sigma))$, $\nexists b \in \texttt{Arg}(\Sigma)$ such that $a\mathcal{R}b$ or $b\mathcal{R}a$.*

As a consequence of the previous result, the set of arguments built from the free part of a knowledge base is an admissible extension of any argumentation system whose attack relation is conflict-dependent.

Proposition 35. *For all argumentation system $\mathcal{T} = (\texttt{Arg}(\Sigma), \mathcal{R})$ such that $\mathcal{R}$ is conflict-dependent, $\texttt{Arg}(\texttt{Free}(\Sigma))$ is an admissible extension of $\mathcal{T}$.*

The next result shows that any argumentation system that uses a conflict-dependent attack relation satisfies free precedence under most of reviewed semantics, except stable and admissible semantics. Note that in case of stable semantics, when a system has extensions, free precedence is satisfied.

Proposition 36. *For all argumentation system $\mathcal{T} = (\texttt{Arg}(\Sigma), \mathcal{R})$ such that $\mathcal{R}$ is conflict-dependent, $\mathcal{T}$ satisfies free precedence under grounded, ideal, complete, semi-stable and preferred semantics.*

6. Postulates for Weighted Argumentation Systems

Since early nineties, arguments were assumed to have different strengths. To the best of my knowledge, the first work on preference-based argumentation systems is the one by Simari and Loui [14]. In that paper, arguments are built from a propositional knowledge base, and the ones that are based on specific information are assumed stronger than those built from general rules. In [13], arguments are built from a possibilistic knowledge base, and are compared following the weakest link principle. The idea is that an argument is better than another one if the weakest formula used in the former is more certain than the weakest formula in the latter. Besides, there is a consensus in the literature on the fact that the strengths of arguments should be taken into account in the evaluation of arguments (e.g. [14, 21, 40]).

The first *abstract* preference-based argumentation framework was proposed in [41, 21]. It takes as input a set of arguments, an attack relation, and a preference relation between arguments which is abstract and can be instantiated in different ways. This proposal was instantiated in [40] by considering a particular preference relation (issued from values promoted by arguments) and generalized in [42] in order to reason even about preferences. Thus, arguments may support preferences about arguments. The basic idea behind these frameworks is to ignore an attack if the attacked argument is stronger than its attacker. Dung's semantics are applied on the remaining attacks. Formally, a preference-based argumentation system is defined as follows:

Definition 14. *A* preference-based argumentation framework *(PAF) is a tuple* $(\mathcal{A}, \mathcal{R}, \succeq)$ *where* $\mathcal{A}$ *is a set of arguments,* $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ *is an attack relation between arguments and* $\succeq \subseteq \mathcal{A} \times \mathcal{A}$ *is a preorder (i.e., reflexive and transitive relation).*

The relation $\succeq$ is a preference relation between arguments. The notation $a \succeq b$ means that the argument a is at least as preferred as b. For the evaluation of arguments, in most existing systems (e.g., [14, 21, 40, 42, 36]) a new binary relation, called defeat, is defined among arguments. It combines the attack relation with the preference one as follows.

Definition 15 (Defeat). *Let* $(\mathcal{A}, \mathcal{R}, \succeq)$ *be a PAF. For two arguments* $a, b \in \mathcal{A}$*,* a *defeats* b*, denoted* a $\texttt{Def}$ b *iff* $a\mathcal{R}b$ *and not* $(b \succ a)$*.*

The extensions (under a given semantics) of a PAF are those of the framework $(\mathcal{A}, \texttt{Def})$. PAFs are developed for reasoning about inconsistent information, in particular when weights or priorities among formulas are available. Consequently, the five postulates we defined in this section should be satisfied by such formalisms, in particular by $(\mathcal{A}, \texttt{Def})$. Unfortunately, this is not necessarily true with some existing PAFs except the rule-based system proposed in [43]. Indeed, we have shown in [44] that PAFs that use a non symmetric attack relation do not guarantee conflict-free extensions. Let us illustrate this issue on an example.

Example 3. *Assume a PAF such that* $\mathcal{A} = \{a, b\}$*,* $\mathcal{R} = \{(a, b)\}$ *and* $b \succ a$ *(i.e.,* b *is strictly preferred to* a*). In this case,* $\texttt{Def} = \emptyset$ *and the PAF has* $\mathcal{E} = \{a, b\}$ *as its preferred extension.*

Note that the previous extension is not conflict-free. As a consequence, the PAF may violate the rationality postulate on consistency in case the attack relation is conflict-dependent as shown next.

Example 3 (Cont): Assume that the previous PAF is built over $\Sigma = \{x, \neg x\}$, $a = (\{x\}, x)$ and $b = (\{\neg x\}, \neg x)$. It is clear that $\texttt{Concs}(\mathcal{E}) = \Sigma$ is inconsistent. Thus, consistency is violated.

A new approach for preference-based argumentation was proposed in [45]. It takes into account preferences at the semantics level rather than the attack level. The idea is to extend existing acceptability semantics with preferences. In case preferences are not available or do not conflict with the attacks, the extensions of the new semantics coincide with those of the basic ones. This approach computes extensions which are conflict-free. However, as discussed in this paper, conflict-freeness is not sufficient to ensure sound results. Thus, the rationality postulates discussed in the present paper should be satisfied by instantiations of the abstract framework proposed in [45].

Another category of weighted argumentation systems was proposed in [46, 47, 48] where instead of assuming arguments with different strengths, the attacks are assumed to have varying degrees of strengths. In such systems, the extensions are not necessarily conflict-free. This is argued by the fact that a certain degree of incoherence is allowed in the systems. Consequently, the consistency postulate is not desirable in this context. It is even violated. The situation is different with the other four postulates. Indeed, free formulas should be inferred from any knowledge base since they are not involved in inconsistency. Closure under consequence operator is also mandatory even in this case since forgetting conclusions is not acceptable. Exhaustiveness remain an intuitive postulate even for weighted systems. Indeed, if every part of an argument is accepted in an extension, then there is no reason to reject the argument itself from the extension.

7. Conclusion

In this paper we tackled the important problem of defining rational logic-based argumentation systems. We focused on defining postulates that such systems should verify. For that purpose, we revisited and extended the two existing works on the topic [29, 27]. Our contributions are the following:

1. We discussed the existing postulates in the literature, and showed that some of them do not deserve to be postulates per se since they follow from more fundamental ones. This is particularly the case for: strong consistency postulate proposed in [29], output consistency, output closure and indirect consistency that are proposed in [26].

2. We defined five *independent* and *compatible* postulates under *any* Tarskian logic: closure under consequence operator, closure under sub-arguments, consistency, exhaustiveness and free precedence. Recall that two of these postulates were *only* defined under rule-based logics in [27].
3. We provided families of argumentation systems that satisfy closure under sub-arguments, one family of argumentation systems that satisfy consistency, and have shown that free precedence postulate is satisfied by any argumentation system that uses a conflict-dependent attack relation. This is particularly true under most known semantics. However, it is not guaranteed under stable semantics since this latter does not guarantee the existence of extensions. We also provided two broad families of argumentation systems that violate consistency. The results are very general since they hold under any Tarskian logic, any semantics and any attack relation which satisfies some mandatory properties.
4. We discussed the importance of the proposed postulates in preference-based argumentation frameworks.

This work provides guidelines for instantiating Dung's framework as well as its extensions with preferences. It defines the properties that should be ensured. It can also be used for evaluating existing systems. For instance, instantiating Dung's system with canonical undercut [49] as attack relation is certainly a bad choice since the resulting system will violate consistency. In [23] some examples of systems that satisfy consistency are provided. Those systems are built on propositional logic and use specified attack relations.

A lot of work still needs to be done. Our aim is to have a representation theorem that characterizes all the systems that satisfy the five postulates. However, since a system has too many parameters (underlying logic, attack relation, semantics), this objective seems not reachable. Consequently, we will investigate more classes of systems that satisfy the postulates.

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References

[1] L. Amgoud, H. Prade, Using arguments for making and explaining decisions, Artificial Intelligence Journal 173 (2009) 413–436.

[2] B. Bonet, H. Geffner, Arguing for decisions: A qualitative model of decision making, in: International Conference on Uncertainty in Artificial Intelligence, UAI'96, 1996, pp. 98–105.

[3] A. Kakas, P. Moraitis, Argumentation based decision making for autonomous agents, in: International Joint Conference on Autonomous Agents and Multi-Agents systems, AAMAS'03, 2003, pp. 883–890.

[4] J. Müller, A. Hunter, An argumentation-based approach for decision making, in: IEEE 24th International Conference on Tools with Artificial Intelligence, ICTAI'12, 2012, pp. 564–571.

[5] P. Matt, F. Toni, J. Vaccari, Dominant decisions by argumentation agents, in: International Workshop on Argumentation in Multi-Agent Systems, ArgMAS'09, 2009, pp. 42–59.

[6] L. Amgoud, M. Serrurier, Arguing and explaining classifications, in: International Joint Conference on Autonomous Agents and Multiagent Systems, AAMAS'07, 2007, pp. 979–985.

[7] M. Mozina, J. Zabkar, I. Bratko, Argument based rule learning, in: European Conference on Artificial Intelligence, ECAI'06, 2006, pp. 504–508.

[8] Y. Gao, F. Toni, R. Craven, Argumentation-based reinforcement learning for robocup keepaway, in: Computational Models of Argument, COMMA'12, 2012, pp. 519–520.

[9] L. Amgoud, S. Belabbes, H. Prade, Towards a formal framework for the search of a consensus between autonomous agents, in: Argumentation in Multi-Agent Systems, 2006, pp. 264–278.

[10] L. Amgoud, Y. Dimopoulos, P. Moraitis, A unified and general framework for argumentation-based negotiation, in: Proceedings of the 6th International Joint Conference on Autonomous Agents and Multiagent Systems, AAMAS'07, ACM Press, 2007, pp. 963–970.

[11] E. Black, A. Hunter, An inquiry dialogue system, Autonomous Agents and Multi-Agent Systems 19 (2) (2009) 173–209.

[12] K. Sycara, Persuasive argumentation in negotiation, Theory and Decision 28 (1990) 203–242.

[13] S. Benferhat, D. Dubois, H. Prade, Argumentative inference in uncertain and inconsistent knowledge bases, in: International Conference on Uncertainty in Artificial Intelligence, UAI'93, 1993, pp. 411–419.

[14] G. Simari, R. Loui, A mathematical treatment of defeasible reasoning and its implementation, Artificial Intelligence Journal 53 (1992) 125–157.

[15] P. Besnard, A. Hunter, A logic-based theory of deductive arguments, Artificial Intelligence 128 (1-2) (2001) 203–235.

[16] M. Elvang-Gøransson, J. Fox, P. Krause, Acceptability of arguments as 'logical uncertainty, in: European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty, ECSQARU'93, 1993, pp. 85–90.

[17] A. García, G. Simari, Defeasible logic programming: An argumentative approach, Theory and Practice of Logic Programming 4 (1-2) (2004) 95–138.

[18] L. Amgoud, M. Caminada, C. Cayrol, M. Lagasquie, H. Prakken, Towards a consensual formal model: inference part, Deliverable D2.2 of ASPIC project.

[19] H. Prakken, An abstract framework for argumentation with structured arguments, Journal of Argument and Computation 1 (2010) 93–124.

[20] G. Governatori, M. Maher, G. Antoniou, D. Billington, Argumentation semantics for defeasible logic, Journal of Logic and Computation 14 (5) (2004) 675–702.

[21] L. Amgoud, C. Cayrol, A reasoning model based on the production of acceptable arguments, Annals of Mathematics and Artificial Intelligence 34 (2002) 197–216.

[22] P. Besnard, A. Hunter, Practical first-order argumentation, in: National Conference on Artificial Intelligence, AAAI'05, 2005, pp. 590–595.

[23] N. Gorogiannis, A. Hunter, Instantiating abstract argumentation with classical logic arguments: Postulates and properties, Artificial Intelligence Journal 175(9-10) (2011) 1479–1497.

[24] P. M. Dung, On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games, Artificial Intelligence Journal 77 (1995) 321–357.

[25] J. Pollock, How to reason defeasibly, Artificial Intelligence Journal 57 (1) (1992) 1–42.

[26] M. Caminada, L. Amgoud, An axiomatic account of formal argumentation, in: National Conference on Artificial Intelligence, AAAI'06, 2005, pp. 608–613.

[27] M. Caminada, L. Amgoud, On the evaluation of argumentation formalisms, Artificial Intelligence Journal 171 (5-6) (2007) 286–310.

[28] M. Caminada, Contamination in formal argumentation systems, in: Belgium-Netherlands Conference on Artificial Intelligence, BNAIC'05, 2005, pp. 59–65.

[29] L. Amgoud, P. Besnard, Bridging the gap between abstract argumentation systems and logic, in: International Conference on Scalable Uncertainty Management, SUM'09, 2009, pp. 12–27.

[30] A. Tarski, Logic, Semantics, Metamathematics (E. H. Woodger, editor)), Oxford Uni. Press, 1956, Ch. On Some Fundamental Concepts of Metamathematics.

[31] L. Amgoud, P. Besnard, A formal characterization of the outcomes of rule-based argumentation systems, in: International Conference on Scalable Uncertainty Management, SUM'13, 2013.

[32] A. Hunter, Base logics in argumentation, in: International Conference on Computational Models of Argument, COMMA'10, pp. 275–286.

[33] P. Dung, P. Mancarella, F. Toni, Computing ideal skeptical argumentation, Artificial Intelligence Journal 171 (2007) 642–674.

[34] M. Caminada, Semi-stable semantics, in: International Conference on Computational Models of Argument, COMMA'06, 2006, pp. 121–130.

[35] C. Cayrol, F. D. de Saint-Cyr, M. Lagasquie-Schiex, Change in abstract argumentation frameworks: Adding an argument, Journal of Artificial Intelligence Research 38 (2010) 49–84.

[36] H. Prakken, G. Sartor, Argument-based extended logic programming with defeasible priorities, Journal of Applied Non-Classical Logics 7 (1997) 25–75.

[37] D. Martínez, A. García, G. Simari, On acceptability in abstract argumentation frameworks with an extended defeat relation, in: Computational Models of Argument, COMMA'06, 2006, pp. 273–278.

[38] D. Martínez, A. García, G. Simari, Modelling well-structured argumentation lines, in: International Joint Conference on Artificial Intelligence, IJCAI'07, 2007, pp. 465–470.

[39] P. Baroni, M. Giacomin, G. Guida, Scc-recursiveness: a general schema for argumentation semantics, Artificial Intelligence Journal 168 (1-2) (2005) 162–210.

[40] T. J. M. Bench-Capon, Persuasion in practical argument using value-based argumentation frameworks, Journal of Logic and Computation 13(3) (2003) 429–448.

[41] L. Amgoud, Contribution a l'integration des préferences dans le raisonnement argumentatif, Ph.D. thesis, Université Paul Sabatier, Toulouse (July 1999).

[42] S. Modgil, Reasoning about preferences in argumentation frameworks, Artificial Intelligence Journal 173:9–10 (2009) 901–934.

[43] T. Alsinet, C. Chesñevar, L. Godo, A level-based approach to computing warranted arguments in possibilistic defeasible logic programming, in: International Conference on Computational Models of Argument, COMMA'08, 2008, pp. 1–12.

[44] L. Amgoud, S. Vesic, Repairing preference-based argumentation frameworks, in: Proceedings of the 21st International Joint Conference on Artificial Intelligence, IJCAI'09, 2009, pp. 665–670.

[45] L. Amgoud, S. Vesic, A new approach for preference-based argumentation frameworks, Annals of Mathematics and Artificial Intelligence 63 (2) (2011) 149–183.

[46] P. Dunne, A. Hunter, P. McBurney, S. Parsons, M. Wooldridge, Weighted argument systems: Basic definitions, algorithms, and complexity results, Artificial Intelligence Journal 175 (2) (2011) 457–486.

[47] P. Dunne, D. Martínez, A. García, G. Simari, Computation with varied-strength attacks in abstract argumentation frameworks, in: Computational Models of Argument, COMMA'10, 2010, pp. 207–218.

[48] C. Cayrol, C. Devred, M. Lagasquie-Schiex, Acceptability semantics accounting for strength of attacks in argumentation, in: European Conference on Artificial Intelligence, ECAI'10, 2010, pp. 995–996.

[49] P. Besnard, A. Hunter, Elements of Argumentation, MIT Press, 2008.

[50] C. Cayrol, On the relation between argumentation and non-monotonic coherence-based entailment, in: International Joint Conference on Artificial Intelligence, IJCAI'95, 1995, pp. 1443–1448.

Appendix

Proof of Property 1. Let $(\mathcal{L}, \mathsf{CN})$ be a logic such that $\mathsf{CN}(\emptyset) \neq \emptyset$. Let $x \in \mathsf{CN}(\emptyset)$. From Coherence axiom, $\mathsf{CN}(\emptyset) \neq \mathcal{L}$. Thus, $\emptyset$ is consistent. So, from Definition 4, $(\emptyset, x)$ is an argument. ■

Proof of Property 2. Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system over a knowledge base Σ and $\mathtt{Ext}(\mathcal{T})$ its set of extensions under any of the reviewed semantics. Assume that $\mathtt{Ext}(\mathcal{T}) \neq \emptyset$.

Let $x \in \mathtt{Output}(\mathcal{T})$. Thus, for all $\mathcal{E} \in \mathtt{Ext}(\mathcal{T})$, $\exists a \in \mathcal{E}$ such that $\mathtt{Conc}(a) = x$. It follows that $x \in \mathtt{Concs}(\mathcal{E}_i)$, $\forall \mathcal{E}_i \in \mathtt{Ext}(\mathcal{T})$ and hence $x \in \bigcap_{\mathcal{E}_i \in \mathtt{Ext}(\mathcal{T})} \mathtt{Concs}(\mathcal{E}_i)$.

Assume now that $x \in \bigcap_{\mathcal{E}_i \in \mathtt{Ext}(\mathcal{T})} \mathtt{Concs}(\mathcal{E}_i)$. Thus, $\forall \mathcal{E}_i$, $\exists a_i \in \mathcal{E}_i$ such that $\mathtt{Conc}(a_i) = x$. Consequently, $x \in \mathtt{Output}(\mathcal{T})$. ■

Proof of Property 3. Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system over a knowledge base Σ and $\mathtt{Ext}(\mathcal{T})$ its set of extensions under any of the reviewed semantics. Assume that $\mathtt{Ext}(\mathcal{T}) = \emptyset$. By Definition 8, $\mathtt{Output}(\mathcal{T}) = \emptyset$. Thus, $\mathtt{Output}(\mathcal{T}) \subseteq \mathsf{CN}(\Sigma)$.

Assume now that $\mathtt{Ext}(\mathcal{T}) \neq \emptyset$ and that $x \in \mathtt{Output}(\mathcal{T})$. Thus, from Definition 8, $\exists a \in \mathtt{Arg}(\Sigma)$ such that $\mathtt{Conc}(a) = x$. Since $a \in \mathtt{Arg}(\Sigma)$, then from Definition 4, $\mathtt{Supp}(a) \subseteq \Sigma$ and $x \in \mathsf{CN}(\mathtt{Supp}(a))$. By monotonicity of CN, it follows that $\mathsf{CN}(\mathtt{Supp}(a)) \subseteq \mathsf{CN}(\Sigma)$. Consequently, $x \in \mathsf{CN}(\Sigma)$. ■

Proof of Property 4. Follows immediately from Definition 8. ■

Proof of Property 5. Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system. From [24], the empty set is an admissible set of $\mathcal{T}$. Frpm Property 4, $\mathtt{Output}(\mathcal{T}) = \emptyset$ under admissible semantics. ■

Proof of Property 6. Let $\mathcal{T} = (\mathtt{Args}(\Sigma), \mathcal{R})$ be an argumentation system such that $\mathtt{Ext}(\mathcal{T}) \neq \emptyset$. Let $\mathcal{E} \in \mathtt{Ext}(\mathcal{T})$ (under a given semantics). Assume that $\mathcal{T}$ is closed under sub-arguments and that $b \in \mathtt{Args}(\Sigma)$ but $b \notin \mathcal{E}$. Assume $c \in \mathtt{Args}(\Sigma)$ such that$b \in \mathtt{Sub}(c)$ and $c \in \mathcal{E}$. Since $\mathcal{T}$ is closed under sub-arguments, then b would be in $\mathcal{E}$. Contradiction. Assume now that if $a \notin \mathcal{E}$, then $\forall b \in \mathtt{Args}(\Sigma)$ such that $a \in \mathtt{Sub}(b)$, $b \notin \mathcal{E}$. Let $a \in \mathcal{E}$ and assume that $b \in \mathtt{Sub}(a)$ and $b \notin \mathcal{E}$. From the previous property, a should not be in $\mathcal{E}$. ■

Proof of Lemma 1. Let $\Sigma = \{x_1, \ldots, x_n\}$ where $n > 1$ and $\mathcal{C}_\Sigma = \{\Sigma\}$, and let $(\mathtt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system such that $\mathcal{R}$ is conflict-dependent and not conflict-exhaustive. Let $a_1, \ldots, a_n \in \mathtt{Arg}(\Sigma)$ be such that $\mathtt{Supp}(a_i) = \{x_i\}$. Assume that the set $\mathcal{E} = \{a_1, \ldots, a_n\}$ is not conflict-free. Thus, $\exists a_i, a_j \in \mathcal{E}$

such that $a_i \mathcal{R} a_j$. Since $\mathcal{R}$ is conflict-dependent, then $\texttt{Supp}(a_i) \cup \texttt{Supp}(a_j)$ is inconsistent. If $n = 2$, then this is impossible since $\mathcal{R}$ is not conflict-exhaustive. If $n > 2$ this is again impossible since $|\texttt{Supp}(a_i) \cup \texttt{Supp}(a_j)| < n$ and thus, from the definition of a minimal conflict, $\texttt{Supp}(a_i) \cup \texttt{Supp}(a_j)$ would be consistent. ■

Proof of Lemma 2. Let $\Sigma = \{x_1, \ldots, x_n\}$ where $n > 1$ and $\mathcal{C}_\Sigma = \{\Sigma\}$, and let $(\texttt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system such that $\mathcal{R}$ is conflict-dependent and not conflict-exhaustive. Let $a_1, \ldots, a_n \in \texttt{Arg}(\Sigma)$ be such that $\texttt{Supp}(a_i) = \{x_i\}$. Assume that the set $\mathcal{E} = \{a_1, \ldots, a_n\}$ does not defend its elements. Thus, $\exists a_i \in \mathcal{E}$ such that $\exists b \in \texttt{Arg}(\Sigma)$ and $b \mathcal{R} a_i$ and $\mathcal{E}$ does not defend a_i. Since $\mathcal{R}$ is conflict-dependent, then $\texttt{Supp}(a_i) \cup \texttt{Supp}(b)$ is inconsistent. Thus, $\texttt{Supp}(a_i) \cup \texttt{Supp}(b) = \Sigma$. This is impossible since $\mathcal{R}$ is not conflict-exhaustive. ■

Lemma 3. *Let $C \in \mathcal{C}_\Sigma$. For all $X \subset C$, if $X \neq \emptyset$, then $\exists x_1 \in \mathsf{CN}(X)$ and $\exists x_2 \in \mathsf{CN}(C \setminus X)$ such that the set $\{x_1, x_2\}$ is inconsistent.*

Proof Let C be a minimal conflict. Consider $X \subset C$ such that $X \neq \emptyset$. We prove the property by induction, after we first take care to show that X is finite. By Tarski's requirements, there exists $x_0 \in \mathcal{L}$ s.t. $\mathsf{CN}(\{x_0\}) = \mathcal{L}$. Since C is a conflict, $\mathsf{CN}(C) = \mathsf{CN}(\{x_0\})$. As a consequence, $x_0 \in \mathsf{CN}(C)$. However, $\mathsf{CN}(C) = \bigcup_{C' \subseteq_f C} \mathsf{CN}(C')$ by Tarski's requirements. Thus, $x_0 \in \mathsf{CN}(C)$ means that there exists $C' \subseteq_f C$ s.t. $x_0 \in \mathsf{CN}(C')$. This says that C' is a conflict. Since C is a minimal conflict, $C = C'$ and it follows that C is finite. Of course, so is X: Let us write $X = \{x_1, \ldots, x_n\}$. *Base step:* $n = 1$. Taking x to be x_1 is enough. *Induction step:* Assume the lemma is true up to rank $n - 1$. As CN is a closure operator, $\mathsf{CN}(\{x_1, \ldots, x_n\}) = \mathsf{CN}(\mathsf{CN}(\{x_1, \ldots, x_{n-1}\}) \cup \{x_n\})$. The induction hypothesis entails $\exists x \in \mathcal{L}$ s.t. $\mathsf{CN}(\mathsf{CN}(\{x_1, \ldots, x_{n-1}\}) \cup \{x_n\}) = \mathsf{CN}(\mathsf{CN}(\{x\}) \cup \{x_n\})$. Then, $\mathsf{CN}(\{x_1, \ldots, x_n\}) = \mathsf{CN}(\{x, x_n\})$. As $\mathsf{CN}(\{x, x_n\}) \neq \mathsf{CN}(\{x_n\})$ and $\mathsf{CN}(\{x, x_n\}) \neq \mathsf{CN}(\{x\})$ (otherwise C cannot be minimal), there exists $y \in \mathcal{L}$ s.t. $\mathsf{CN}(\{x, x_n\}) = \mathsf{CN}(\{y\})$ because $(\mathcal{L}, \mathsf{CN})$ is adjunctive. Since $\mathsf{CN}(\{x_1, \ldots, x_n\}) = \mathsf{CN}(\{x, x_n\})$ was just proved, it follows that $\mathsf{CN}(\{y\}) = \mathsf{CN}(\{x_1, \ldots, x_n\})$.

Take $X_1 = X$ and $X_2 = C \setminus X_1$. Since X is a non-empty proper subset of C, so are both X_1 and X_2. Then, the first bullet of this property can be applied to X_1 and X_2. As a result, $\exists x_1 \in \mathcal{L}$ s.t. $\mathsf{CN}(\{x_1\}) = \mathsf{CN}(X_1)$ and $\exists x_2 \in \mathcal{L}$ s.t. $\mathsf{CN}(\{x_2\}) = \mathsf{CN}(X_2)$. The expansion axiom gives $\{x_1\} \subseteq \mathsf{CN}(\{x_1\})$ and $\{x_2\} \subseteq \mathsf{CN}(\{x_2\})$. Thus, $x_1 \in \mathsf{CN}(X_1)$ and $x_2 \in \mathsf{CN}(X_2)$. Using the expansion axiom again, $X_1 \subseteq \mathsf{CN}(X_1)$ and $X_2 \subseteq \mathsf{CN}(X_2)$. Thus, $X_1 \cup X_2 \subseteq \mathsf{CN}(X_1) \cup \mathsf{CN}(X_2) = \mathsf{CN}(\{x_1\}) \cup \mathsf{CN}(\{x_2\})$. It follows that $C \subseteq \mathsf{CN}(\{x_1\}) \cup \mathsf{CN}(\{x_2\})$. Using Property 1 in [29], $\mathsf{CN}(\{x_1\}) \cup \mathsf{CN}(\{x_2\}) \subseteq \mathsf{CN}(\{x_1, x_2\})$, thus $C \subseteq \mathsf{CN}(\{x_1, x_2\})$. Since

C is inconsistent, Property 2 in [29] gives that $\mathsf{CN}(\{x_1, x_2\})$ is inconsistent as well. By the definition of inconsistency, it follows that $\mathsf{CN}(\mathsf{CN}(\{x_1, x_2\})) = \mathcal{L}$. Applying the idempotence axiom, $\mathsf{CN}(\{x_1, x_2\}) = \mathcal{L}$, thus the set $\{x_1, x_2\}$ is inconsistent. ■

Proof of Proposition 1. Follows immediately from Postulate 1. ■

Proof of Proposition 2. Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system such that $\mathtt{Ext}(\mathcal{T}) \neq \emptyset$ and $\mathcal{T}$ is closed under CN. Assume that $\mathsf{CN}(\emptyset) = \emptyset$. Thus, $\mathtt{Arg}^*(\mathcal{L}) = \emptyset$. So, for all $\mathcal{E} \in \mathtt{Ext}(\mathcal{T})$, $\mathtt{Arg}^*(\mathcal{L}) \subseteq \mathcal{E}$ and $\mathsf{CN}(\emptyset) \subseteq \mathtt{Concs}(\mathcal{E})$. Assume now that $\mathsf{CN}(\emptyset) \neq \emptyset$. For all $\mathcal{E} \in \mathtt{Ext}(\mathcal{T})$, $\emptyset \subseteq \mathtt{Concs}(\mathcal{E})$. By monotonicity of CN, $\mathsf{CN}(\emptyset) \subseteq \mathsf{CN}(\mathtt{Concs}(\mathcal{E}))$. Since $\mathcal{T}$ is closed under CN, then $\mathsf{CN}(\emptyset) \subseteq \mathtt{Concs}(\mathcal{E})$. Moreover, from the definition of the function $\mathtt{Concs}$, $\mathtt{Arg}^*(\mathcal{L}) \subseteq \mathcal{E}$. Assume that $\mathsf{CN}(\emptyset) \neq \emptyset$. Thus, $\mathtt{Arg}^*(\mathcal{L}) \neq \emptyset$. So, $\mathcal{E} \neq \emptyset$. ■

Proof of Proposition 3. Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system such that $\mathtt{Ext}(\mathcal{T}) \neq \emptyset$ and $\mathcal{T}$ is closed under CN. From Proposition 2, for all $\mathcal{E} \in \mathtt{Ext}(\mathcal{T})$, $\mathsf{CN}(\emptyset) \subseteq \mathtt{Concs}(\mathcal{E})$. Thus, $\mathsf{CN}(\emptyset) \subseteq \mathtt{Output}(\mathcal{T})$. Consequently, if $\mathtt{Output}(\mathcal{T}) = \emptyset$, then $\mathsf{CN}(\emptyset) = \emptyset$. ■

Proof of Proposition 4. Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system over a knowledge base Σ and $\mathtt{Ext}(\mathcal{T})$ its set of extensions under a given semantics with $\mathtt{Ext}(\mathcal{T}) \neq \emptyset$. Assume that $\mathcal{T}$ satisfies closure. From Expansion axiom, it follows that $\mathtt{Output}(\mathcal{T}) \subseteq \mathsf{CN}(\mathtt{Output}(\mathcal{T}))$. Assume now that $x \in \mathsf{CN}(\mathtt{Output}(\mathcal{T}))$. If $x \in \mathsf{CN}(\emptyset)$, then from Corollary 3 $x \in \mathtt{Output}(\mathcal{T})$. Assume that $x \notin \mathsf{CN}(\emptyset)$. Since CN satisfies finiteness, then there exists a finite number of formulas $x_1, \ldots, x_n \in \mathcal{L}$ such that $x_1, \ldots, x_n \in \mathtt{Output}(\mathcal{T})$ and $x \in \mathsf{CN}(\{x_1, \ldots, x_n\})$. From Property 2,

$$x_1, \ldots, x_n \in \bigcap_{\mathcal{E}_i \in \mathtt{Ext}(\mathcal{T})} \mathtt{Concs}(\mathcal{E}_i).$$

From monotonicity of CN, it holds that

$$\mathsf{CN}(\{x_1, \ldots, x_n\}) \subseteq \mathsf{CN}(\bigcap_{\mathcal{E}_i \in \mathtt{Ext}(\mathcal{T})} \mathtt{Concs}(\mathcal{E}_i)).$$

It holds also that $x \in \mathsf{CN}(\mathtt{Concs}(\mathcal{E}_1)) \cap \ldots \cap \mathsf{CN}(\mathtt{Concs}(\mathcal{E}_n))$. Since $\mathcal{T}$ satisfies closure, then for each $\mathcal{E}_i$ it holds that $\mathsf{CN}(\mathtt{Concs}(\mathcal{E}_i)) = \mathtt{Concs}(\mathcal{E}_i)$. Thus, $x \in \mathtt{Concs}(\mathcal{E}_1) \cap \ldots \cap \mathtt{Concs}(\mathcal{E}_n)$. From Property 2, it holds that $x \in \mathtt{Output}(\mathcal{T})$. ■

Proof of Proposition 5. Let $\mathcal{T} = (\texttt{Args}(\Sigma), \mathcal{R})$ be an argumentation system that is closed under sub-arguments and $\texttt{Ext}(\mathcal{T}) \neq \emptyset$ (under any of the reviewed semantics). Let $\mathcal{E} \in \texttt{Ext}(\mathcal{T})$ such that $\mathcal{E} \neq \emptyset$ and $x \in \texttt{Base}(\mathcal{E})$. Thus, $\exists a \in \mathcal{E}$ such that $x \in \texttt{Supp}(a)$. Since $\texttt{Supp}(a)$ is consistent (by definition of an argument), then the set $\{x\}$ is consistent (from Property 2 in [29]). Since $x \in \Sigma$ and $x \notin \mathsf{CN}(\emptyset)$ (from our assumption), then the pair $(\{x\}, x)$ is an argument. Moreover, $(\{x\}, x) \in \texttt{Sub}(a)$. Since $\mathcal{T}$ is closed under sub-arguments, then $(\{x\}, x) \in \mathcal{E}$. ■

Proof of Proposition 6. Let $\mathcal{T} = (\texttt{Args}(\Sigma), \mathcal{R})$ be such that $\texttt{Ext}(\mathcal{T}) \neq \emptyset$ (under a given semantics). Assume that $\mathcal{T}$ is closed under CN. There are two cases: i) $\mathsf{CN}(\emptyset) = \emptyset$. Thus, the empty set may be an extension of $\mathcal{T}$. Since $\texttt{Concs}(\emptyset) = \texttt{Base}(\emptyset) = \emptyset$, thus the property holds. ii) $\mathsf{CN}(\emptyset) \neq \emptyset$. From Proposition 2, for all $\mathcal{E} \in \texttt{Ext}(\mathcal{T})$, $\mathcal{E} \neq \emptyset$. Moreover, from Proposition 5, since $\mathcal{T}$ is closed under sub-arguments, then it follows that $\texttt{Base}(\mathcal{E}) \subseteq \texttt{Concs}(\mathcal{E})$. By monotonicity of CN, we get $\mathsf{CN}(\texttt{Base}(\mathcal{E})) \subseteq \mathsf{CN}(\texttt{Concs}(\mathcal{E}))$. Since $\mathcal{T}$ is closed under CN, then $\mathsf{CN}(\texttt{Base}(\mathcal{E})) \subseteq \texttt{Concs}(\mathcal{E})$. Besides, by definition of $\texttt{Concs}(\mathcal{E})$,

$$\texttt{Concs}(\mathcal{E}) \subseteq \bigcup_{a_i \in \mathcal{E}} \mathsf{CN}(\texttt{Supp}(a_i)).$$

From Property 1 in [29], it follows that

$$\texttt{Concs}(\mathcal{E}) \subseteq \mathsf{CN}(\bigcup_{a_i \in \mathcal{E}} \texttt{Supp}(a_i))$$

Thus, $\texttt{Concs}(\mathcal{E}) \subseteq \mathsf{CN}(\texttt{Base}(\mathcal{E}))$. ■

Proof of Proposition 7. Follows immediately from Postulate 3. ■

Proof of Proposition 8. Let $\mathcal{T} = (\texttt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system based on a knowledge base Σ and $\texttt{Ext}(\mathcal{T})$ its set of extensions under any of the reviewed semantics.

Assume that $\texttt{Ext}(\mathcal{T}) = \emptyset$ thus, $\texttt{Output}(\mathcal{T}) = \emptyset$ (from Definition 8). Besides, from Absurdity axiom, $\mathsf{CN}(\emptyset) \neq \mathcal{L}$. Thus, $\texttt{Output}(\mathcal{T})$ is consistent.

Assume that $\texttt{Ext}(\mathcal{T}) \neq \emptyset$. Two cases may raise: i) $\emptyset \in \texttt{Ext}(\mathcal{T})$, thus $\texttt{Output}(\mathcal{T}) = \emptyset$. Following the previous reasoning, we conclude that $\texttt{Output}(\mathcal{T})$ is consistent. ii) $\emptyset \notin \texttt{Ext}(\mathcal{T})$. Assume that $\mathcal{T}$ satisfies consistency. Thus, $\forall \mathcal{E}_i \in \texttt{Ext}(\mathcal{T})$, $\texttt{Concs}(\mathcal{E}_i)$ is consistent. Let $\mathcal{E}$ be a given extension in the set $\texttt{Ext}(\mathcal{T})$. Since

$$\bigcap_{\mathcal{E}_i \in \texttt{Ext}(\mathcal{T})} \texttt{Concs}(\mathcal{E}_i) \subseteq \texttt{Concs}(\mathcal{E}),$$

then the set

$$\bigcap_{\mathcal{E}_i \in \mathtt{Ext}(\mathcal{T})} \mathtt{Concs}(\mathcal{E}_i)$$

is consistent as well. Besides, from Property 2, $\mathtt{Output}(\mathcal{T}) = \bigcap_{\mathcal{E}_i \in \mathtt{Ext}(\mathcal{T})} \mathtt{Concs}(\mathcal{E}_i)$. It follows that $\mathtt{Output}(\mathcal{T})$ is consistent. ■

Proof of Proposition 9. Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system based on a knowledge base Σ such that $\mathtt{Ext}(\mathcal{T}) \neq \emptyset$ (under any of the reviewed semantics). Assume that $\mathcal{T}$ satisfies consistency. Thus, for all $\mathcal{E} \in \mathtt{Ext}(\mathcal{T})$, $\mathtt{Concs}(\mathcal{E})$ is consistent. Property 2 in [29] says that if a subset X of $\mathcal{L}$ is consistent, then $\mathsf{CN}(X)$ is consistent as well. Thus, $\mathsf{CN}(\mathtt{Concs}(\mathcal{E}))$ is consistent.

Assume now that for all $\mathcal{E} \in \mathtt{Ext}(\mathcal{T})$, $\mathsf{CN}(\mathtt{Concs}(\mathcal{E}))$ is consistent. Since by Expansion axiom $\mathtt{Concs}(\mathcal{E}) \subseteq \mathsf{CN}(\mathtt{Concs}(\mathcal{E}))$ then $\mathtt{Concs}(\mathcal{E})$ is consistent. ■

Proof of Proposition 10. Let $\mathcal{T} = (\mathtt{Args}(\Sigma), \mathcal{R})$ be an argumentation system and $\mathtt{Ext}(\mathcal{T})$ its set of extensions under any of the reviewed semantics. Assume that $\mathcal{T}$ satisfies strong consistency. If $\mathtt{Ext}(\mathcal{T}) = \emptyset$ then $\mathcal{T}$ satisfies also consistency. Assume that $\mathtt{Ext}(\mathcal{T}) \neq \emptyset$ and let $\mathcal{E} \in \mathtt{Ext}(\mathcal{T})$ such that $\mathcal{E} = \emptyset$. Thus, $\mathtt{Concs}(\mathcal{E}) = \mathtt{Base}(\mathcal{E}) = \emptyset$. From Absurdity axiom, $\mathsf{CN}(\emptyset) \neq \mathcal{L}$, thus $\mathtt{Concs}(\mathcal{E})$ is consistent. Let $\mathcal{E} \neq \emptyset$. Since $\mathcal{T}$ satisfies strong consistency, then $\bigcup_{a_i \in \mathcal{E}} \mathtt{Supp}(a_i)$ is consistent and $\mathsf{CN}(\bigcup_{a_i \in \mathcal{E}} \mathtt{Supp}(a_i))$ is consistent as well (since if X is consistent, then $\mathsf{CN}(X)$ is consistent as well). Besides, for each $a_i \in \mathcal{E}$, $\mathtt{Conc}(a_i) \in \mathsf{CN}(\mathtt{Supp}(a_i))$. Thus, $\mathtt{Concs}(\mathcal{E}) \subseteq \bigcup_{a_i \in \mathcal{E}} \mathsf{CN}(\mathtt{Supp}(a_i))$. It follows that $\mathtt{Concs}(\mathcal{E}) \subseteq \mathsf{CN}(\bigcup_{a_i \in \mathcal{E}} \mathtt{Supp}(a_i))$. Since $\mathsf{CN}(\bigcup_{a_i \in \mathcal{E}} \mathtt{Supp}(a_i))$ is consistent, then its subset $\mathtt{Concs}(\mathcal{E})$ is consistent. ■

Proof of Proposition 11. Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system over a knowledge base Σ and $\mathtt{Ext}(\mathcal{T})$ its set of extensions under any of the reviewed semantics. Assume that $\mathcal{T}$ satisfies consistency and closure under sub-arguments.

If $\mathtt{Ext}(\mathcal{T}) = \emptyset$, then $\mathcal{T}$ satisfies strong consistency (from Definition 9). Assume that $\mathtt{Ext}(\mathcal{T}) \neq \emptyset$ and let $\mathcal{E} \in \mathtt{Ext}(\mathcal{T})$. If $\mathcal{E} = \emptyset$, then $\mathtt{Base}(\mathcal{E}) = \emptyset$. Form absurdity axiom, $\mathtt{Base}(\mathcal{E})$ is consistent. Assume that $\mathcal{E} \neq \emptyset$. From closure under sub-arguments, it follows that $\mathtt{Base}(\mathcal{E}) \subseteq \mathtt{Concs}(\mathcal{E})$ (from Proposition 5). Since $\mathcal{T}$ satisfies consistency, then the set $\mathtt{Concs}(\mathcal{E})$ is consistent. From Property 2 in [29], it follows that $\mathtt{Base}(\mathcal{E})$ is consistent. ■

Proof of Proposition 12. Let $\mathcal{T}$ be an argumentation system that satisfies exhaustiveness and that is closed under both CN and sub-arguments. Let $\mathtt{Ext}(\mathcal{T})$ be its extensions under any of the reviewed semantics such that $\mathtt{Ext}(\mathcal{T}) \neq \emptyset$. Let $\mathcal{E} \in \mathtt{Ext}(\mathcal{T})$. From the definition of $\mathtt{Arg}$ and $\mathtt{Base}$, it follows that $\mathcal{E} \subseteq \mathtt{Arg}(\mathtt{Base}(\mathcal{E}))$.

If $\mathcal{E} = \emptyset$, then $\mathtt{Base}(\mathcal{E}) = \emptyset$ and $\mathtt{Output}(\mathcal{T}) = \emptyset$. From Proposition 3, $\mathsf{CN}(\emptyset) = \emptyset$, thus $\mathtt{Arg}^*(\mathcal{L}) = \emptyset$. Consequently, $\mathtt{Arg}(\mathcal{E}) = \emptyset$.

Assume now that $\mathcal{E} \neq \emptyset$. Let $a \in \mathtt{Arg}(\mathtt{Base}(\mathcal{E}))$ and let $a = (X, x)$. Thus, $X \subseteq \mathtt{Base}(\mathcal{E})$. By monotonicity of CN, $\mathsf{CN}(X) \subseteq \mathsf{CN}(\mathtt{Base}(\mathcal{E}))$. From Proposition 6, since $\mathcal{T}$ is closed under both CN and sub-arguments, then $\mathtt{Concs}(\mathcal{E}) = \mathsf{CN}(\mathtt{Base}(\mathcal{E}))$. Thus, $\mathsf{CN}(X) \subseteq \mathtt{Concs}(\mathcal{E})$. Besides, $X \subseteq \mathsf{CN}(X)$ (from Expansion Axiom of CN) and $x \in \mathsf{CN}(X)$ (from the definition of an argument), thus, $X \cup \{x\} \subseteq \mathtt{Concs}(\mathcal{E})$. By exhaustiveness of $\mathcal{T}$, $a \in \mathcal{E}$. ■

Proof of Proposition 13. Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system over a knowledge base Σ, and let $\mathtt{Ext}(\mathcal{T})$ be its set of extensions under any of the reviewed semantics. Assume that $\mathcal{T}$ satisfies Postulate 5. Thus, for all $\mathcal{E} \in \mathtt{Ext}(\mathcal{T})$, $\mathtt{Arg}(\mathtt{Free}(\Sigma)) \subseteq \mathcal{E}$. Assume that $\mathtt{Free}(\Sigma) \neq \emptyset$. Thus, for all $x \in \mathtt{Free}(\Sigma)$, $(\{x\}, x) \in \mathtt{Arg}(\mathtt{Free}(\Sigma))$ (indeed, $x \notin \mathsf{CN}(\emptyset)$) and thus, $(\{x\}, x) \in \mathcal{E}$ (for all $\mathcal{E} \in \mathtt{Ext}(\mathcal{T})$). Consequently, $\mathtt{Free}(\Sigma) \subseteq \mathtt{Concs}(\mathcal{E})$ (for all $\mathcal{E} \in \mathtt{Ext}(\mathcal{T})$). So, $\mathtt{Free}(\Sigma) \subseteq \mathtt{Output}(\mathcal{T})$. ■

Proof of Proposition 14. In order to show that the five postulates are compatible, it is sufficient to define at least one argumentation system which satisfies all of them. We consider the argumentation system that was studied in [50]. This system is grounded on propositional logic (an instance of Tarski's logics) and uses the assumption attack relation defined in [16] as follows:

> An argument a attacks an argument b, denoted by $a\mathcal{R}_{as}b$, iff $\exists x \in \mathtt{Supp}(b)$ such that $\mathtt{Conc}(a) \equiv \neg x$.

The attack relation $\mathcal{R}_{as}$ is both conflict-dependent and conflict-exhaustive. Assume that $a\mathcal{R}_{as}b$. Thus, $\exists x \in \mathtt{Supp}(b)$ such that $\mathtt{Conc}(a) \equiv \neg x$. Consequently, $\mathtt{Supp}(a) \cup \mathtt{Supp}(b)$ is inconsistent.

Assume now $C \in \mathcal{C}_\Sigma$ be such that $|C| > 1$. For all $x \in C$, $C \setminus \{x\} \vdash \neg x$ and $(C \setminus \{x\}, \neg x)$ is an argument of $\mathtt{Arg}(\Sigma)$. Besides, $(\{x\}, x)$ is also an argument in $\mathtt{Arg}(\Sigma)$. By definition of $\mathcal{R}_{as}$, it follows that $(C \setminus \{x\}, \neg x)\ \mathcal{R}_{as}\ (\{x\}, x)$. Thus, $\mathcal{R}_{as}$ is conflict-exhaustive.

Let $\mathtt{Ext}(\mathcal{T})$ be the set of stable extensions of the argumentation system $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R}_{as})$ built over a propositional knowledge base Σ. It was shown in [50] that for all stable extension $\mathcal{E} \in \mathtt{Ext}(\mathcal{T})$, $\mathtt{Base}(\mathcal{E})$ is consistent. From Proposition 10, it follows that $\mathtt{Concs}(\mathcal{E})$ is consistent. Thus, $\mathcal{T}$ satisfies consistency.

In the same paper, it was shown that for all $\mathcal{E} \in \mathtt{Ext}(\mathcal{T})$, $\mathcal{E} = \mathtt{Arg}(\mathtt{Base}(\mathcal{E}))$. Since $\mathcal{R}_{as}$ is conflict-exhaustive then from Proposition 27 $\mathcal{T}$ is closed under CN.

From Proposition 31, $\mathcal{T}$ is closed under sub-arguments.

In [50], it was shown that for all stable extension $\mathcal{E} \in \texttt{Ext}(\mathcal{T})$, $\texttt{Base}(\mathcal{E})$ is a maximal (for set inclusion) consistent subbase of Σ. Thus, $\texttt{Free}(\Sigma) \subseteq \texttt{Base}(\mathcal{E})$. Consequently, $\texttt{Arg}(\texttt{Free}(\Sigma)) \subseteq \texttt{Arg}(\texttt{Base}(\mathcal{E}))$ and thus $\texttt{Arg}(\texttt{Free}(\Sigma)) \subseteq \mathcal{E}$. $\mathcal{T}$ satisfies thus free precedence.

Let us now show that $\mathcal{T}$ satisfies exhaustiveness under stable semantics. Let $\mathcal{E} \in \texttt{Ext}(\mathcal{T})$ and assume that $\exists (X, x) \in \texttt{Arg}(\Sigma)$ such that $X \cup \{x\} \subseteq \texttt{Concs}(\mathcal{E})$. Assume also that $(X, x) \notin \mathcal{E}$. Thus, $\exists a \in \mathcal{E}$ such that $a \ \mathcal{R}_{as} \ (X, x)$. So, $\exists x' \in X$ such that $\texttt{Conc}(a) \equiv \neg x'$. But, $\texttt{Conc}(a) \in \texttt{Concs}(\mathcal{E})$. Thus, $\texttt{Concs}(\mathcal{E})$ is inconsistent. This contradicts the fact that $\mathcal{T}$ satisfies consistency. ■

Proof of Proposition 15. Let $\mathcal{T} = (\texttt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system over a knowledge base Σ such that $\mathcal{R}$ is conflict-dependent. There are two cases: i) $\texttt{Ext}(\mathcal{T}) = \{\emptyset\}$. Thus, $\mathcal{T}$ is closed under sub-arguments. ii) $\texttt{Ext}(\mathcal{T}) \neq \{\emptyset\}$. Assume that $\mathcal{T}$ violates closure under sub-arguments. Thus, $\exists \mathcal{E} \in \texttt{Ext}(\mathcal{T})$ such that $\exists a \in \mathcal{E}$ and $\exists b \in \texttt{Sub}(a)$ with $b \notin \mathcal{E}$. This means that $\mathcal{E} \cup \{b\}$ is conflicting, i.e. $\exists c \in \mathcal{E}$ such that $b\mathcal{R}c$ or $c\mathcal{R}b$. Since $\mathcal{R}$ is conflict-dependent, then $\texttt{Supp}(b) \cup \texttt{Supp}(c)$ is inconsistent. However, $\texttt{Supp}(b) \subseteq \texttt{Supp}(a) \subseteq \texttt{Base}(\mathcal{E})$ and thus, $\texttt{Supp}(b) \cup \texttt{Supp}(c) \subseteq \texttt{Base}(\mathcal{E})$. This means that $\texttt{Base}(\mathcal{E})$ is inconsistent. This contradicts the assumption. ■

Proof of Proposition 16. Let $\mathcal{T} = (\texttt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system over a knowledge base Σ such that $\mathcal{R}$ is conflict-dependent and $\texttt{Ext}(\mathcal{T}) \neq \emptyset$ under stable semantics. i) $\texttt{Ext}(\mathcal{T}) = \{\emptyset\}$. Thus, $\mathcal{T}$ is closed under sub-arguments. ii) $\texttt{Ext}(\mathcal{T}) \neq \{\emptyset\}$. Assume that $\mathcal{T}$ violates closure under sub-arguments. Thus, $\exists \mathcal{E} \in \texttt{Ext}(\mathcal{T})$ such that $\exists a \in \mathcal{E}$ and $\exists b \in \texttt{Sub}(a)$ with $b \notin \mathcal{E}$. This means that $\mathcal{E} \cup \{b\}$ is not a stable extension. Thus, $\mathcal{E} \cup \{b\}$ is conflicting, i.e., $\exists c \in \mathcal{E}$ such that $b\mathcal{R}c$ or $c\mathcal{R}b$. Since $\mathcal{R}$ is conflict-dependent, then $\texttt{Supp}(b) \cup \texttt{Supp}(c)$ is inconsistent. However, $\texttt{Supp}(b) \subseteq \texttt{Supp}(a) \subseteq \texttt{Base}(\mathcal{E})$ and thus, $\texttt{Supp}(b) \cup \texttt{Supp}(c) \subseteq \texttt{Base}(\mathcal{E})$. This means that $\texttt{Base}(\mathcal{E})$ is inconsistent. This contradicts the assumption. ■

Proof of Proposition 17. Assume an argumentation system $\mathcal{T}$ which satisfies consistency and closure under sub-arguments under naive (resp. stable) semantics. From Proposition 11, it follows that $\forall \mathcal{E} \in \texttt{Ext}(\mathcal{T})$, $\texttt{Base}(\mathcal{E})$ is consistent under naive (resp. stable) semantics. Assume now that $\forall \mathcal{E} \in \texttt{Ext}(\mathcal{T})$, $\texttt{Base}(\mathcal{E})$ is consistent under naive (resp. stable) semantics. From Proposition 10, $\mathcal{T}$ satisfies consistency and from Proposition 15 $\mathcal{T}$ is closed under sub-arguments under naive semantics. Respectively, from Proposition 16 $\mathcal{T}$ is closed under sub-arguments under stable semantics. ■

Proof of Proposition 18. Let $\mathcal{T} = (\texttt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system over a knowledge base Σ such that $\mathcal{R}$ is conflict-dependent. Assume that $\mathcal{T}$ is

closed under sub-arguments and satisfies consistency under naive semantics and $\exists \mathcal{E} \in \texttt{Ext}(\mathcal{T})$ with $\mathcal{E} \neq \emptyset$. Assume also that $\mathcal{T}$ violates closure under CN. Thus, $\exists \mathcal{E} \in \texttt{Ext}(\mathcal{T})$ such that $\texttt{Concs}(\mathcal{E}) \neq \mathsf{CN}(\texttt{Concs}(\mathcal{E}))$. This means that $\exists x \in \mathsf{CN}(\texttt{Concs}(\mathcal{E}))$ such that $x \notin \texttt{Concs}(\mathcal{E})$. Besides, $\texttt{Concs}(\mathcal{E}) \subseteq \bigcup \mathsf{CN}(\texttt{Supp}(a_i))$ with $a_i \in \mathcal{E}$. From Property 1 in [31], $\bigcup \mathsf{CN}(\texttt{Supp}(a_i)) \subseteq \mathsf{CN}(\bigcup \texttt{Supp}(a_i))$ and thus, $\texttt{Concs}(\mathcal{E}) \subseteq \mathsf{CN}(\texttt{Base}(\mathcal{E}))$. By monotonity of CN, $\mathsf{CN}(\texttt{Concs}(\mathcal{E})) \subseteq \mathsf{CN}(\mathsf{CN}(\texttt{Base}(\mathcal{E})))$. By Idempotence, $\mathsf{CN}(\texttt{Concs}(\mathcal{E})) \subseteq \mathsf{CN}(\texttt{Base}(\mathcal{E}))$. Thus, $x \in \mathsf{CN}(\texttt{Base}(\mathcal{E}))$. There are two possible situations: i) $x \in \mathsf{CN}(\emptyset)$, thus $(\emptyset, x) \in \texttt{Arg}^*(\mathcal{L})$ and $(\emptyset, x) \notin \mathcal{E}$. Thus, $\exists a \in \mathcal{E}$ such that $a\mathcal{R}(\emptyset, x)$ or $(\emptyset, x)\mathcal{R}a$. Since $\mathcal{R}$ is conflict-dependent, then $\texttt{Supp}(a)$ is inconsistent. This contradicts the fact that a is an argument. ii) $x \notin \mathsf{CN}(\emptyset)$, thus since CN verifies compactness, $\exists X \subseteq \texttt{Base}(\mathcal{E})$ such that X is finite and $x \in \mathsf{CN}(X)$. Moreover, from Proposition 11, $\texttt{Base}(\mathcal{E})$ is consistent. Then, X is consistent as well. Consequently, the pair (X, x) is an argument. Besides, since $x \notin \texttt{Concs}(\mathcal{E})$ then $(X, x) \notin \mathcal{E}$. This means that $\exists a \in \mathcal{E}$ such that $a\mathcal{R}(X, x)$ or $(X, x)\mathcal{R}a$. Finally, since $\mathcal{R}$ is conflict-dependent, then $\texttt{Supp}(a) \cup X$ is inconsistent and consequently $\texttt{Base}(\mathcal{E})$ is inconsistent. This contradicts the assumption. The same proof holds under stable semantics. ■

Proof of Proposition 20. Let $\Sigma = \{x_1, \ldots, x_n\}$ where $n > 1$ and $\mathcal{C}_\Sigma = \{\Sigma\}$, and let $(\texttt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system such that $\mathcal{R}$ is conflict-dependent and not conflict-exhaustive. Let $a_1, \ldots, a_n \in \texttt{Arg}(\Sigma)$ be such that $\texttt{Supp}(a_i) = \{x_i\}$. From Lemma 1, the set $\mathcal{E} = \{a_1, \ldots, a_n\}$ is conflict-free and from Lemma 2 it defends its elements. Thus, $\mathcal{E} = \{a_1, \ldots, a_n\}$ is an admissible set. ■

Proof of Proposition 21. Let $\Sigma = \{x_1, \ldots, x_n\}$ where $n > 1$ and $\mathcal{C}_\Sigma = \{\Sigma\}$, and let $\mathcal{T} = (\texttt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system such that $\mathcal{R}$ is conflict-dependent and not conflict-exhaustive. By assumption, for all $x_i \in \Sigma$, $x_i \notin \mathsf{CN}(\emptyset)$. Thus, $(\{x_i\}, x_i)$ is an argument. Let $a_1, \ldots, a_n \in \texttt{Arg}(\Sigma)$ be such that $\texttt{Supp}(a_i) = \{x_i\}$ and $\texttt{Conc}(a_i) = x_i$. From Proposition 20, the set $\mathcal{E} = \{a_1, \ldots, a_n\}$ is an admissible set. Besides, $\texttt{Concs}(\mathcal{E}) = \{x_1, \ldots, x_n\}$, thus $\mathcal{T}$ violates consistency. ■

Proof of Proposition 22. Let $\mathcal{T} = (\texttt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system built upon a logic $(\mathcal{L}, \mathsf{CN})$ such that $\mathsf{CN}(\emptyset) \neq \emptyset$. Since the empty set is an admissible extension of every argumentation system, thus $\emptyset \in \texttt{Ext}(\mathcal{T})$ (under admissible semantics) and $\texttt{Concs}(\emptyset) = \emptyset$. Since $\mathsf{CN}(\emptyset) \neq \emptyset$ then $\texttt{Concs}(\emptyset) \neq \mathsf{CN}(\texttt{Concs}(\emptyset))$. So, $\mathcal{T}$ violates closure under CN. ■

Proof of Proposition 23. Let $\mathcal{T} = (\texttt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system built upon a logic $(\mathcal{L}, \mathsf{CN})$ such that $\mathsf{CN}(\emptyset) \neq \emptyset$. Since $\texttt{Ext}(\mathcal{T}) = \emptyset$ then $\texttt{Output}(\mathcal{T}) = \emptyset$. Then $\texttt{Output}(\mathcal{T}) \neq \mathsf{CN}(\texttt{Output}(\mathcal{T}))$. ■

Proof of Proposition 24. Let $\mathcal{T}$ = $(\mathtt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system built over Σ and $\mathtt{Free}(\Sigma) \neq \emptyset$. Since the empty set is admissible, then $\mathtt{Arg}(\mathtt{Free}(\Sigma)) \not\subseteq \emptyset$. Thus, $\mathcal{T}$ violates free precedence under admissible semantics. ■

Proof of Proposition 25. Let $\mathcal{T}$ = $(\mathtt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system built over Σ and $\mathtt{Free}(\Sigma) \neq \emptyset$. Assume that $\mathtt{Ext}(\mathcal{T}) = \emptyset$. Thus, $\mathtt{Output}(\mathcal{T}) = \emptyset$. Consequently, $\mathtt{Free}(\Sigma) \not\subseteq \mathtt{Output}(\mathcal{T})$. ■

Proof of Proposition 26. Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system. Assume that $\mathcal{T}$ satisfies closure under CN under complete semantics. Thus, $\forall \mathcal{E} \in \mathtt{Ext}_c(\mathcal{T})$[3], $\mathtt{Concs}(\mathcal{E}) = \mathsf{CN}(\mathtt{Concs}(\mathcal{E}))$. Since $\mathtt{Ext}_s(\mathcal{T}) \subseteq \mathtt{Ext}_{ss}(\mathcal{T}) \subseteq \mathtt{Ext}_p(\mathcal{T}) \subseteq \mathtt{Ext}_c(\mathcal{T})$, then $\mathcal{T}$ satisfies closure under CN under preferred, stable and semi-stable semantics. Since ideal extension is a complete one, then $\mathcal{T}$ satisfies closure under CN under ideal semantics. Finally, since the grounded extension of $\mathcal{T}$ is a complete extension, then $\mathcal{T}$ satisfies closure under CN under grounded semantics. The same reasoning holds for the four other postulates. ■

Proof of Proposition 27. Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system such that $\mathcal{R}$ is conflict-exhaustive. Let $\mathtt{Ext}(\mathcal{T})$ be its extensions under any semantics based on the notion of conflict-freeness, i.e., for all $\mathcal{E} \in \mathtt{Ext}(\mathcal{T})$, the set $\mathcal{E}$ is conflict-free. Assume that $\forall \mathcal{E} \in \mathtt{Ext}(\mathcal{T})$, $\mathcal{E} = \mathtt{Arg}(\mathtt{Base}(\mathcal{E}))$.
Let $\mathcal{E} \in \mathtt{Ext}(\mathcal{T})$ and $x \in \mathsf{CN}(\mathtt{Concs}(\mathcal{E}))$. Thus, $\exists \{x_1, \ldots, x_n\} \subseteq \mathtt{Concs}(\mathcal{E})$ such that $x \in \mathsf{CN}(\{x_1, \ldots, x_n\})$. Besides, $\forall x_i$, $\exists a_i \in \mathcal{E}$ such that $x_i \in \mathsf{CN}(\mathtt{Supp}(a_i))$. Thus, $\{x_1, \ldots, x_n\} \subseteq \bigcup_{i=1,n} \mathsf{CN}(\mathtt{Supp}(a_i))$. From Property 1 in [29],

$$\bigcup_{i=1,n} \mathsf{CN}(\mathtt{Supp}(a_i)) \subseteq \mathsf{CN}(\bigcup_{i=1,n} \mathtt{Supp}(a_i)).$$

Then, $\{x_1, \ldots, x_n\} \subseteq \mathsf{CN}(\bigcup_{i=1,n} \mathtt{Supp}(a_i))$ and $x \in \mathsf{CN}(\bigcup_{i=1,n} \mathtt{Supp}(a_i))$. From Property 7, $\mathtt{Base}(\mathcal{E})$ is consistent. Since $\bigcup_{i=1,n} \mathtt{Supp}(a_i) \subseteq \mathtt{Base}(\mathcal{E})$, then

$$\bigcup_{i=1,n} \mathtt{Supp}(a_i) \text{ is consistent}$$

(see Property 2 in [29]). Let X be a minimal (for set $\subseteq$) subset of $\bigcup_{i=1,n} \mathtt{Supp}(a_i)$ such that $x \in \mathsf{CN}(X)$. Consequently, the pair (X, x) is an argument. Hence, $(X, x) \in \mathtt{Arg}(\mathtt{Base}(\mathcal{E}))$ and thus, $(X, x) \in \mathcal{E}$. It follows that $x \in \mathtt{Concs}(\mathcal{E})$. ■

[3] $\mathtt{Ext}_c(\mathcal{T})$ (resp. $\mathtt{Ext}_p(\mathcal{T})$, $\mathtt{Ext}_s(\mathcal{T})$, $\mathtt{Ext}_{ss}(\mathcal{T})$) stands for the set of all complete (resp. preferred, stable, and semi-stable) extensions of $\mathcal{T}$.

Proof of Proposition 28. Let $\mathcal{T} = (\texttt{Args}(\Sigma), \mathcal{R})$ be an argumentation system such that $\mathcal{R}$ satisfies R_1 and R_2. Let us show that this result holds under complete semantics. Then, from Proposition 26, it follows that it holds also under the other semantics.

Let $\mathcal{E}$ be a complete extension of $\mathcal{T}$. Assume that $\mathcal{E}$ is not closed under sub-arguments. Thus, $\exists a \in \mathcal{E}$ such that $\texttt{Sub}(a) \not\subseteq \mathcal{E}$. This means that $\exists a' \in \texttt{Sub}(a)$ and $a' \notin \mathcal{E}$.

We show first that $\mathcal{E} \cup \{a'\}$ is conflict-free. Assume that $\mathcal{E} \cup \{a'\}$ is conflicting. Thus, $\exists b \in \mathcal{E}$ such that either $a'\mathcal{R}b$ or $b\mathcal{R}a'$ hold. Assume that $a'\mathcal{R}b$. Since $a' \in \texttt{Sub}(a)$ and $\mathcal{R}$ verifies R_1, then $a\mathcal{R}b$. This contradicts the fact that $\mathcal{E}$ is a complete extension. Assume now that $b\mathcal{R}a'$. Since $\mathcal{R}$ satisfies R_2, then $b\mathcal{R}a$, contradiction.

Since $a' \notin \mathcal{E}$, then $\mathcal{E}$ does not defend a'. Thus, $\exists b \notin \mathcal{E}$ such that $b\mathcal{R}a'$ and $\nexists c \in \mathcal{E}$ such that $c\mathcal{R}b$. Since $b\mathcal{R}a'$ and $\mathcal{R}$ satisfies R_2, then $b\mathcal{R}a$. Since $a \in \mathcal{E}$ and $\mathcal{E}$ is complete, this means that $\exists c \in \mathcal{E}$ such that $c\mathcal{R}b$. Contradiction. ■

Proof of Proposition 29. Let $\mathcal{T} = (\texttt{Args}(\Sigma), \mathcal{R})$ be an argumentation system such that $\mathcal{R}$ satisfies R_2. Let $\mathcal{E}$ be a stable extension of $\mathcal{T}$ which is not closed under sub-arguments. Thus, $\exists a \in \mathcal{E}$ such that $\texttt{Sub}(a) \not\subseteq \mathcal{E}$. This means that $\exists a' \in \texttt{Sub}(a)$ and $a' \notin \mathcal{E}$. Then, $\exists b \in \mathcal{E}$ such that $b\mathcal{R}a'$ (according to the definition of a stable extension). Since $\mathcal{R}$ satisfies R_2, then $b\mathcal{R}a$. This contradicts the fact that $\mathcal{E}$ is conflict-free. ■

Proof of Proposition 30. Let $\mathcal{E}$ be an complete extension of an argumentation system $\mathcal{T} = (\texttt{Args}(\Sigma), \mathcal{R})$. Assume that $\mathcal{R}$ is conflict-dependent and sensitive. Let us show that $\mathcal{T}$ is closed under sub-arguments under complete semantics. Then, from Proposition 26 $\mathcal{T}$ satisfies the same postulate under the other semantics.

Assume that $\mathcal{E}$ is not closed under sub-arguments. That is, $\exists a, a' \in \texttt{Args}(\Sigma)$ such that $a' \in \texttt{Sub}(a)$, $a \in \mathcal{E}$ and $a' \notin \mathcal{E}$.

We show that $\mathcal{E} \cup \{a'\}$ is conflict-free. Assume that it is conflicting. Thus, $\exists b \in \mathcal{E}$ such that either $a'\mathcal{R}b$ or $b\mathcal{R}a'$. Since $\mathcal{R}$ is conflict-dependent, then $\texttt{Supp}(a') \cup \texttt{Supp}(b)$ is inconsistent. Besides, $a' \in \texttt{Sub}(a)$ thus $\texttt{Supp}(a') \subseteq \texttt{Supp}(a)$. From Property 2 in [29], $\texttt{Supp}(a) \cup \texttt{Supp}(b)$ is inconsistent as well. Since $\mathcal{R}$ is conflict-sensitive, then either $a\mathcal{R}b$ or $b\mathcal{R}a$. This contradicts the fact $\mathcal{E}$ is conflict-free.

Since $a' \notin \mathcal{E}$ then $\mathcal{E}$ does not defend a'. Thus, $\exists b \in \texttt{Arg}(\Sigma)$ such that $b\mathcal{R}a'$. Since $\mathcal{R}$ is conflict-dependent, then $\texttt{Supp}(a') \cup \texttt{Supp}(b)$ is inconsistent. Besides, $a' \in \texttt{Sub}(a)$ then $\texttt{Supp}(a') \subseteq \texttt{Supp}(a)$. Thus, $\texttt{Supp}(a) \cup \texttt{Supp}(b)$ is inconsistent as well. Since $\mathcal{R}$ is conflict-sensitive, then either $a\mathcal{R}b$ or $b\mathcal{R}a$. Assume that $a\mathcal{R}b$, thus a defends a' which contradicts the fact that $\mathcal{E}$ does not defend a'. Assume now that $b\mathcal{R}a$. Since $\mathcal{E}$ is complete and $a \in \mathcal{E}$, then $\exists c \in \mathcal{E}$ such that $c\mathcal{R}b$. Thus, c defends even a', this contradicts again the fact that $\mathcal{E}$ does not defend a'. ■

Proof of Proposition 31. Let $\mathcal{T} = (\texttt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system over a knowledge base Σ such that $\texttt{Ext}(\mathcal{T}) \neq \emptyset$. Assume that $\forall \mathcal{E} \in \texttt{Ext}(\mathcal{T})$, $\mathcal{E} = \texttt{Arg}(\texttt{Base}(\mathcal{E}))$. Let $\mathcal{E} \in \texttt{Ext}(\mathcal{T})$ and $a \in \mathcal{E}$. Since $a \in \mathcal{E}$, then $\texttt{Supp}(a) \subseteq \texttt{Base}(\mathcal{E})$. Let $b \in \texttt{Sub}(a)$, thus $\texttt{Supp}(b) \subseteq \texttt{Supp}(a)$ and $\texttt{Supp}(b) \subseteq \texttt{Base}(\mathcal{E})$. It follows that $b \in \texttt{Arg}(\texttt{Base}(\mathcal{E}))$. Consequently, $b \in \mathcal{E}$. Then, $\mathcal{T}$ is closed under sub-arguments. ■

Proof of Proposition 32. Let $\mathcal{T} = (\texttt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system such that such that $\texttt{Ext}(\mathcal{T}) \neq \emptyset$. Assume that $\forall \mathcal{E} \in \texttt{Ext}(\mathcal{T})$, $\mathcal{E} = \texttt{Arg}(\texttt{Base}(\mathcal{E}))$. Assume also that $\mathcal{T}$ violates strong consistency. Thus, there exists an extension $\mathcal{E}$ of $\mathcal{T}$ (under a given semantics) such that $\texttt{Base}(\mathcal{E})$ is inconsistent. Thus, $\exists C \in \mathcal{C}_\Sigma$ such that $C \subseteq \texttt{Base}(\mathcal{E})$. Since $\texttt{Base}(\mathcal{E}) = \bigcup_{a_i \in \mathcal{E}} \texttt{Supp}(a_i)$ and $\texttt{Supp}(a_i)$ is consistent, then $|C| \geq 2$. Thus, $\exists X \subset C$ such that X and $C \setminus X$ are consistent. From Proposition 1 (in [29]), there exist two arguments a and b where $\texttt{Supp}(a) = X$ and $\texttt{Supp}(b) = C \setminus X$. From Lemma 3, $\exists x_1 \in \mathsf{CN}(X)$ and $\exists x_2 \in \mathsf{CN}(C \setminus X)$ such that the set $\{x_1, x_2\}$ in inconsistent. Let $\texttt{Conc}(a) = x_1$ and $\texttt{Conc}(b) = x_2$. Since $a, b \in \texttt{Arg}(\texttt{Base}(\mathcal{E}))$ and that $\mathcal{E} = \texttt{Arg}(\texttt{Base}(\mathcal{E}))$, then $a, b \in \mathcal{E}$. Thus, $\texttt{Concs}(\mathcal{E})$ is inconsistent. ■

Proof of Proposition 33. Let $\mathcal{T} = (\texttt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system over a knowledge base Σ such that $\mathcal{R}$ is conflict-exhaustive. Let $\texttt{Ext}(\mathcal{T})$ be its extensions such that for all $\mathcal{E} \in \texttt{Ext}(\mathcal{T})$, $\mathcal{E}$ is conflict free. Assume that for each $\mathcal{E} \in \texttt{Ext}(\mathcal{T})$, $\mathcal{E} = \texttt{Arg}(\texttt{Base}(\mathcal{E}))$. From Proposition 32, showing consistency amounts exactly to showing strong consistency since both notions coincide.

Let $\mathcal{E} \in \texttt{Ext}(\mathcal{T})$ such that $\texttt{Base}(\mathcal{E})$ is inconsistent. Thus, $\exists C \in \mathcal{C}_\Sigma$ such that $C \subseteq \texttt{Base}(\mathcal{E})$. Since $\texttt{Base}(\mathcal{E}) = \bigcup \texttt{Supp}(a_i)$ $(a_i \in \mathcal{E})$ and $\texttt{Supp}(a_i)$ is consistent (by definition of an argument), then $|C| \geq 2$. Since $\mathcal{R}$ is conflict-exhaustive, then $\exists X \subset C$ such that$\exists a, b \in \texttt{Arg}(\Sigma)$ and $\texttt{Supp}(a) = X$, $\texttt{Supp}(b) = C \setminus X$ and either $a\mathcal{R}b$ or $b\mathcal{R}a$. Besides, $\texttt{Supp}(a) \subseteq \texttt{Base}(\mathcal{E})$ (resp. $\texttt{Supp}(b) \subseteq \texttt{Base}(\mathcal{E})$), then $a, b \in \texttt{Arg}(\texttt{Base}(\mathcal{E}))$. Since $\mathcal{E} = \texttt{Arg}(\texttt{Base}(\mathcal{E}))$, then $a, b \in \mathcal{E}$. This means that the extension $\mathcal{E}$ is conflicting. Contradiction. ■

Proof of Proposition 34. Let $(\texttt{Arg}(\Sigma), \mathcal{R})$ be s.t. $\mathcal{R}$ is conflict-dependent. Let $a \in \texttt{Arg}(\texttt{Free}(\Sigma))$. Assume that $\exists b \in \texttt{Arg}(\Sigma)$ s.t. $a\mathcal{R}b$ or $b\mathcal{R}a$. Since $\mathcal{R}$ is conflict-dependent, then $\exists C \in \mathcal{C}_\Sigma$ such that $C \subseteq \texttt{Supp}(a) \cup \texttt{Supp}(b)$. By definition of an argument, both $\texttt{Supp}(a)$ and $\texttt{Supp}(b)$ are consistent. Then, $C \cap \texttt{Supp}(a) \neq \emptyset$. This contradicts the fact that $\texttt{Supp}(a) \subseteq \texttt{Free}(\Sigma)$. Thus, arguments of $\texttt{Arg}(\texttt{Free}(\Sigma))$ cannot be involved in conflicts wrt $\mathcal{R}$. ■

Proof of Proposition 35. Let $\mathcal{T}$ = $(\texttt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system s.t. $\mathcal{R}$ is conflict-dependent. From Proposition 34, the set $\texttt{Arg}(\texttt{Free}(\Sigma))$ is conflict-free and defends its elements (it is not attacked). Thus, $\texttt{Arg}(\texttt{Free}(\Sigma))$ is admissible. ■

Proof of Proposition 36. Let $\mathcal{T} = (\texttt{Arg}(\Sigma), \mathcal{R})$ be an argumentation system such that $\mathcal{R}$ is conflict-dependent. From Proposition 34, the set $\texttt{Arg}(\texttt{Free}(\Sigma))$ is not attacked by any argument of $\texttt{Arg}(\Sigma)$. Thus, $\texttt{Arg}(\texttt{Free}(\Sigma)) \subseteq G$ where G is the grounded extension of $\mathcal{T}$. Thus, $\mathcal{T}$ satisfies free precedence under grounded semantics. Besides, it was shown in [33] that G is a subset of the ideal extension, say I, of any argumentation framework. Thus, $\mathcal{T}$ satisfies free precedence under ideal semantics. Similarly, I is a subset of any preferred extension of $\mathcal{T}$. Thus, $\mathcal{T}$ satisfies free precedence under preferred semantics. Finally, every complete extension contains non attacked arguments, thus $\texttt{Arg}(\texttt{Free}(\Sigma))$. So, $\mathcal{T}$ satisfies free precedence under complete semantics. ■