CONTEXTUAL HANDLING OF CONDITIONAL KNOWLEDGE

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ABSTRACT

This paper presents a new approach to dealing with default rules of the form "generally, if α then β" having possibly some exceptions. Our approach can be described in three steps: we first extend System P of Kraus, Lehmann and Magidor to both deal with strict rules (rules without exceptions) and default rules. Next, given an incomplete information called a *context*, we compute the set of rules for which the context does not represent an exceptional situation, and we transform these default rules into strict rules. Lastly, we use the extension of System P defined in the first step to infer plausible conclusions. We show that this local way to dealing with default information allows us to correctly handle the well-known problems of specificity, irrelevance, blocking of inheritance, ambiguity, and redundancy.

1. INTRODUCTION

One of the most important problem encountered in knowledge based systems is to both handle exceptions and generic knowledge. A rule having exceptions (called also default rule or conditional assertion) is a piece of information of the following form "generally, if α is believed then β is also believed", where α and β belong to a propositional language. A typical example of conditional assertion is "generally, birds fly". In presence of incomplete information, it is nevertheless possible to jump to some conclusions, but these conclusions are just plausible and should be revised in the light of new and complete information. For instance, given the default rules "generally, birds fly", "generally, penguins do not fly", and "all penguins are birds", then from the incomplete information Tweety is a bird (but we do not know if Tweety is a penguin or not), we would like to conclude that it plausibly flies. However, if we later learn that it is a penguin we should withdraw this conclusion.

Classical logic is not appropriate for dealing with default information since we get inconsistency each time an exceptional situation is observed. And in presence of inconsistency, classical logic infers trivial results. However, when there is no exceptions in our generic knowledge, classical logic is an efficient tool for correct reasoning.

In the last 15 years and since Reiter's paper [23] on default logic and McCarthy's work on circumscription [17], there have been several proposals to reasoning with default information (for overview works on defaults reasoning systems, see [5]; [13]). Some of these proposals are based on determining the natural properties of a "good" nonmonotonic consequence relation (*i.e.*, which allows satisfactory plausible reasoning in the presence of incomplete information). Typical of this kind of research are the works of Gabbay [8], Gärdenfors and Makinson [9] and Kraus, Lehmann and Magidor [12]. The laters define a logic of conditional assertions, called preferential or System P, which is considered as a common kernel of several nonmonotonic formalisms. The set of conclusions produced by System P is considered as a minimal set of conclusions that each default reasoning system should produce. Moreover, System P correctly addresses the *specificity problem*: results issued from sub-classes override those obtained from super-classes [25]. Unfortunately, in spite of these two advantages, System P is too cautious and suffers from the so-called *irrelevance problem*: from a rule "generally, if α then β", it is not possible to deduce that β follows from $\alpha \wedge \delta$ even if δ is irrelevant to α. A typical example of irrelevance problem is that from "generally, birds fly" it is not possible to deduce that red birds fly. This problem is due to the fact that the form of monotonicity considered in System P is weak. One way to deal with this irrelevance problem is to add to System P new postulates as the property of *rational monotony*, originally proposed by Makinson and largely discussed by Lehmann [14]. However, these solutions are not entirely satisfactory, since as far as we know, each proposal ever done to increase the set of conclusions obtained by System P leads also to undesirable conclusions (See Section 6 for some examples).

This paper presents a new approach to handle default information. It can be seen as an extension of System P but with a precise analysis of monotonicity property. Given a rule "generally, α's are β's", we are interested to know under which conditions we can infer β from $\alpha \wedge \delta$. Classical logic answers always (monotony property). In common sense reasoning, we want to use classical logic except when δ's are exceptions of the rule. Hence, it is important to verify if $\alpha \wedge \delta$ is an exception to the rule "generally, α's are β's". It turns out that System P can give a satisfactory answer to this question since it correctly treats the specificity problem. Our approach is in three steps:

- we first extend System P to both deal with *strict* rules (rules without exceptions) and default rules,
- next, given an information α and a set of defaults Δ, we transform each default rule of Δ into a strict rule if the *context* α is not an exception of this rule,
- lastly, we use the extended System P defined in the first step to infer plausible conclusions.

The rest of this paper is organized as follows. The next section recalls System P. Section 3 extends this system to both deal with strict and default rules. Section 4 presents our approach and Section 5 discusses its properties. We will show that our approach gives a safe super-set of the set

of conclusions given by System P and that it correctly addresses the well-known problems of specificity, irrelevance, blocking of inheritance, ambiguity and redundancy. Section 6 compares our approach with well known Pearl's System Z and Lexicographical closure. Lastly, an application to handling evolutive systems is given.

2- SYSTEM P

In the rest of this paper, we denote by $\mathcal{L}$ a finite propositional language. Elements of $\mathcal{L}$, or *formulae*, are denoted by Greek letters $\alpha, \beta, \ldots$. An *interpretation* for $\mathcal{L}$ is an assignment of a truth value in {T, F} to each formula of $\mathcal{L}$ in accordance with the classical rules of propositional calculus; we denote by Ω the set of all such interpretations. An interpretation ω is a *model* of a formula α ($\omega \models \alpha$) iff $\omega(\alpha)$=T. A formula β is called a logical consequence of α ($\alpha \models \beta$) iff each model of α is a model of β. A formula α which is satisfied by each interpretation is called a *tautology* (denoted by $\models \alpha$). A formula α is said to be consistent iff it has at least one model and any inconsistent formula is denoted by $\bot$. A conditional information (or default rule) is a rule of the form "generally, if α then β" having possibly some exceptions. In the whole paper, these rules are denoted by "$\alpha \rightarrow \beta$" where $\rightarrow$ is a *non-classical* arrow relating two classical formulas (classical implication and classical equivalence are denoted respectively by $\Rightarrow$ and $\Leftrightarrow$). A *default base* is a multi-set $\Delta = \{\alpha_i \rightarrow \beta_i, i=1,...,n\}$ of default rules.

We recall that classical logic is *monotonic* in the sense that an inferred result is never questioned. In plausible reasoning such property is not always desirable since when we reason with conditional information we need to revise our conclusions in the light of new information. This is why we need to define nonmonotonic inference relations to reason with default information. Kraus, Lehmann and Magidor [12] have proposed a set of natural and rational properties for a nonmonotonic consequence relation. This set of properties is called System P and is defined in the following way:

Definition 1: A nonmonotonic inference relation $\vdash$ satisfies the System P iff it satisfies the following six rules :

1. Reflexivity: $\alpha \vdash \alpha$
2. Left Logical Equivalence(LLE):
   from $\models \alpha \Leftrightarrow \alpha'$ and $\alpha \vdash \beta$ deduce $\alpha' \vdash \beta$
3. Right Weakening (RW):
   from $\beta \models \beta'$ and $\alpha \vdash \beta$ deduce $\alpha \vdash \beta'$
4. OR: from $\alpha \vdash \delta$ and $\beta \vdash \delta$ deduce $\alpha \vee \beta \vdash \delta$
5. Cautious Monotony(CM):
   from $\alpha \vdash \beta$ and $\alpha \vdash \delta$ deduce $\alpha \wedge \beta \vdash \delta$
6. Cut: from $\alpha \wedge \beta \vdash \delta$ and $\alpha \vdash \beta$ deduce $\alpha \vdash \delta$

From this set of postulates, it is possible to define a syntactic notion of entailment. Let $\Delta^* = \{\alpha_i \vdash \beta_i$ such that $\alpha_i \rightarrow \beta_i \in \Delta\}$ the set obtained by interpreting each default rule $\alpha_i \rightarrow \beta_i$ of Δ by $\alpha_i \vdash \beta_i$ in System P. Then we have:

Definition 2: A formula ψ is said to be a *P-consequence* (P for preferential) of a formula ϕ given Δ, denoted by $\phi \vdash_{P,\Delta} \psi$, if and only if $\phi \vdash \psi$ has a proof from Δ^* using only rules of System P.

We will sometimes use $\vdash_P$ rather than $\vdash_{P,\Delta}$ for simplicity sake. Note that the set $\Delta_\alpha^P = \{\beta \,/\, \alpha \vdash_P \beta\}$ can be inconsistent. Indeed, in System P, from the conditional assertions $\alpha \vdash \beta$ and $\alpha \vdash \neg\beta$ we infer $\alpha \vdash \bot$. From now on, we only consider consistent default bases, namely we consider Δ such that there is no formula α where Δ_α^P is inconsistent. Handling, inconsistent default base is left for further research.

It is well known that the set Δ_α^P is considered as the minimal set of conclusions that any reasonable nonmonotonic consequence relation (for default reasoning) should generate. Moreover, the cautious monotony property garantees that System P respects the specificity principles. However, system P is too cautious and suffers from the *irrelevance* problem.

Before presenting our approach which gives a safe superset of Δ_α^P, we first need to extend System P to both deal with conditional assertions and strict rules.

3- EXTENDING SYSTEM P TO HANDLING STRICT RULES

System P deals only with default information. We propose, in this section, to extend it in order to both handle strict and default information. By strict information, we mean rules of the form "all α's are β's", denoted by $\neg\alpha \vee \beta$. We denote by Σ a set of strict rules. The extension of system P is done by adding the following two rules:

7. Recovering Classical Consequences (RCC)
   If $\Sigma \cup \{\alpha\} \models \beta$ and $\Sigma \cup \{\alpha\} \not\models \bot$ then $\alpha \vdash \beta$
8. Extended Right Weakening (ERW)
   If $\alpha \vdash \beta$ and $\Sigma \cup \{\beta\} \models \beta'$ then $\alpha \vdash \beta'$

RCC rule means that everything which can be deduced classically from Σ but not in a trivial way must be considered as plausible. ERW is very similar to the Right Weakening property of System P: it says that if β is a plausible conclusion from α, then every classical conclusion entailed from β and the set of sure information Σ is also plausible.

From now on, we denote by System P^+ the set of eight rules consisting in System P and RCC and ERW rules.

Definition 3: A formula ψ is a *P^+-consequence* of a formula ϕ given Δ and Σ, denoted by $\phi \vdash_{P+,\Delta,\Sigma} \psi$, if and only if $\phi \vdash \psi$ has a proof from Δ^* and Σ using only rules of System P^+.

When there is no ambiguity, we use $\phi \vdash_{P+} \psi$ instead of $\phi \vdash_{P+,\Delta,\Sigma} \psi$.

4- CONTEXTUAL ENTAILMENT $\vdash_{PC}$

In default reasoning, inconsistency is always due to the use of rules having exceptions. However, exceptions to a given rule depend on the context (facts or actual observations) in

which it is considered. It is hence important to identify which rules have exceptions in a given context and which have not. We use System P to realize such task:

Definition 4: Let $\mathcal{C} \in \mathfrak{L}$ be a context and Δ a set of default information. We say that a default $\alpha_i \rightarrow \beta_i$ of Δ *has an exception* in $\mathcal{C}$ if and only if one of the two conditions is satisfied:

i) $\alpha_i \wedge \mathcal{C} \wedge \beta_i$ is inconsistent, or

ii) $\exists \psi$ such that $\mathcal{C} \models \psi$ and $\alpha_i \wedge \psi \vdash_{P,\Delta} \neg\beta_i$

Intuitively, from the previous definition a rule $\alpha_i \rightarrow \beta_i$ is considered as having exceptions if and only if either it is directly in contradiction with the context $\mathcal{C}$; or it is possible, from the rule premisse α_i and the context $\mathcal{C}$, to deduce the contrary of its conclusion $\neg\beta_i$ using System P. This is a safe way to detecting rules having exceptions.

Remark: in our approach, the set of exceptional rules is computed for each given context. One may think to compute this set of rules once only from the database and independently of the context. However, this is not always possible. Indeed, consider the following example : $\Delta = \{a \rightarrow b, b \rightarrow c\}$, then, if we do not take inot account the context, the whole database could be transformed into $\Sigma = \{\neg a \vee b, \neg b \vee c\}$. Now, from $a \wedge \neg c$, it is not possible to infer b, since $\{a \wedge \neg c\} \cup \Sigma$ is inconsistant. If we take into account the context, we have $\Sigma_{a \wedge \neg c} = \{\neg a \vee b\}$, therefore we can safely conclude b (which is an intuitive result).

Once we have distinguished between rules having and having not exceptions, we simply propose to select each rule $\alpha_i \rightarrow \beta_i$ which has no exception in the context $\mathcal{C}$, in order to change it into a strict rule. So, we define the following set:

$\Sigma_{\mathcal{C}} = \{\neg\alpha_i \vee \beta_i / \alpha_i \rightarrow \beta_i \in \Delta$, and $\alpha_i \rightarrow \beta_i$ has no exception in $\mathcal{C}\}$

Our consequence relation, called *contextual entailment*, is defined in a natural way:

Definition 5: A formula ψ is said to be a *PC-consequence* (P for preferential and C for context) of Δ given $\mathcal{C}$, denoted by $\mathcal{C} \vdash_{PC,\Delta} \psi$, iff ψ is a P^+-consequence of $\mathcal{C}$ given Δ and $\Sigma_{\mathcal{C}}$, namely:

$$\mathcal{C} \vdash_{PC,\Delta} \psi \quad \text{iff} \quad \mathcal{C} \vdash_{P+,\Delta,\Sigma_{\mathcal{C}}} \psi$$

For sake of simplicity, we sometimes replace $\vdash_{PC,\Delta}$ by $\vdash_{PC}$. The following section studies the properties of our inference relation.

5- PROPERTIES OF $\vdash_{PC}$

It is clear that $\vdash_{PC}$ is nonmonotonic. The following proposition shows that augmenting context reduces the set of transformed rules $\Sigma_{\mathcal{C}}$:

Proposition 1: Let $\mathcal{C}$ and $\mathcal{C}'$ be two contexts such that $\mathcal{C} \models \mathcal{C}'$, then $\Sigma_{\mathcal{C}} \subseteq \Sigma_{\mathcal{C}'}$.

Proof:

Indeed, if $\exists \alpha \rightarrow \beta \notin \Sigma_{\mathcal{C}'}$ then:

i) either $\alpha \wedge \mathcal{C}' \wedge \beta$ is inconsistent and since $\mathcal{C} \models \mathcal{C}'$ then $\alpha \wedge \mathcal{C} \wedge \beta$ is also inconsistent, or

ii) $\exists \delta$ such that $\mathcal{C}' \models \delta$ (hence $\mathcal{C} \models \delta$) and $\alpha \wedge \delta \vdash_P \neg\beta$.

Therefore $\alpha \rightarrow \beta \notin \Sigma_{\mathcal{C}}$, hence the result.

Super-set of preferential conclusions

Since the set of conclusions given by System P are incontestable, it is desirable that the inference relation $\vdash_{PC}$ recovers results of System P.

Proposition 2 : If $\mathcal{C} \vdash_P \psi$ then $\mathcal{C} \vdash_{PC} \psi$.

The proof is trivial considering the definition of $\vdash_{PC}$.

Specificity:

Specificity is at the core of default reasoning and it consists in claiming that results issued from specific classes should override those obtained from general classes [25]. However there is no formal definition of specificity even if there are general principles to delimit this notion [19]. System P seems to treat correctly the specificity problem since Cautious Monotonicity reflects this idea. As a consequence of Proposition 2 our approach handles the specificity problem:

Example 1:

Consider the following set of default rules : generally birds fly, generally penguins are birds and penguins do not fly. We have $\Delta = \{b \rightarrow f, p \rightarrow b, p \rightarrow \neg f\}$ where p,f and b mean respectively penguins, fly and birds. The followings assertions are valid:

- $b \vdash_P f$ (birds fly)
- $p \wedge b \vdash_P \neg f$ (birds which are penguins do not fly).

Indeed, applying the Cautious monotonicity to the rules $p \vdash b$ and $p \vdash \neg f$ leads to $p \wedge b \vdash \neg f$. Hence,

$$p \wedge b \vdash_{PC} \neg f.$$

Irrelevance:

The converse of Proposition 2 is not true, since $\vdash_{PC}$ addresses the irrelevance problem while $\vdash_P$ fails. Let us again consider the default base given in Example 1: $\Delta = \{b \rightarrow f, p \rightarrow b, p \rightarrow \neg f\}$. In System P, it is not possible to deduce the intuitive result that green birds fly. While, in the context $b \wedge g$ (g for green), there is no rule of Δ which is exceptional, this means that $\Sigma_{b \wedge g} = \{\neg b \vee f, \neg p \vee b, \neg p \vee \neg f\}$. Hence, using the property RCC of System P+, with our approach, we deduce that green birds fly.

In general, we can show that if $\alpha \vdash_P \beta$ is valid, then, for any formula δ, composed only by propositional symbols which do not appear in Σ, $\alpha \wedge \delta \vdash_{PC} \beta$ is also valid.

Ambiguity preservation:

A desirable property for a consequence relation is ambiguity preservation. In a situation where we have an argument in favor of a proposition, and an independent argument in favor of its negation, we should not conclude anything about this proposition. Note that this is different from inconsistency where an argument can support both a conclusion and its contrary.

The most popular example of ambiguity is called Nixon-diamond: knowing that "Quakers are pacifists", "Republican are not pacifists", and "Nixon is both a Quaker and a Republican", one should not deduce that Nixon is a pacifist, nor that he is not. System P preserves ambiguities, our approach too (since it is based on System P) as it is shown by the following example :

Example 2 : (Ambiguity preservation)

> Let $\Delta = \{R \rightarrow \neg P,\ Q \rightarrow P\}$, where R stands for Republican, P for pacifists and Q for Quakers. In the context $R \wedge Q$, none of these two rules have exceptions. So $\sum_{R \wedge Q} = \{\neg R \vee \neg P,\ \neg Q \vee P\}$. Since $\sum_{R \wedge Q} \cup \{R \wedge Q\}$ is inconsistent, it is neither possible to deduce that Nixon is pacifist nor its contrary (RCC is not applicable).

The previous example shows that our approach satisfies ambiguity preservation proposed for instance by Makinson in [18]. Moreover, it shows that the "one by one" inconsistency test done in definition 4 is not able to detect inconsistency generated by interacting defaults (cf. [22] and [16]). However, this kind of inconsistency is detected later when applying RCC rule.

6- RELATED WORKS

In this Section, we compare the behaviour of our approach with two well-known systems, namely Pearl's System Z, and the Lexicographical approach.

6.1- SYSTEM Z

Pearl [20] has proposed a default reasoning system, called Z, which goes beyond System P and tries to solve irrelevance problem mentioned previously. System Z is based on a ranking of default rules that respects the notion of specificity. A default rule $(\alpha \rightarrow \beta)$ is "verified" by a world ω if $\omega \models \alpha \wedge \beta$, "satisfied" if $\omega \models \neg\alpha \vee \beta$ or "falsified" if $\omega \models \alpha \wedge \neg\beta$. Given a default base $\Delta = \{\alpha_i \rightarrow \beta_i \mid i=1,...,m\}$, Pearl gives a method to rank-order the rules in Δ such that the least specific rules (*i.e.*, with most general antecedents) get the least priority. In order to do this, he defines the notion of *tolerance*:

Definition 6: A rule $\alpha \rightarrow \beta$ is *tolerated* by a base $\Delta = \{\alpha_i \rightarrow \beta_i,\ i = 1,...,n\}$ iff $\{\alpha \wedge \beta,\ \neg\alpha_1 \vee \beta_1,\ \ldots,\ \neg\alpha_n \vee \beta_n\}$ is consistent.

Δ is partitioned into an ordered set $\{\Delta_1, \Delta_2, \ldots, \Delta_k\}$ such that rules in Δ_i are tolerated by all rules in $\Delta_i \cup \ldots \cup \Delta_k$.

Each rule $d \in \Delta_i$ is associated to its specificity level: $Z(d)=i$. The less is the rank of the default the more general is its antecedent. The rank $K(\omega)$ of a possible world ω is the rank Z of the highest-ranked rule falsified by ω, augmented by one, i.e.

$$K(\omega) = \max\{Z(d_i) + 1,\ \omega \models \alpha_i \wedge \neg\beta_i\}$$

Intuitively, $K(\omega)$ is lower for worlds that only violate rules which are in lower sub-bases (*i.e.*, that are less specific). This ranking on models induces a ranking on formulae using a function z such that $z(\phi)=\min\{K(\omega) \mid \omega \models \phi\}$. So, $z(\phi)$ can be read as a degree of "abnormality" of ϕ with respect to the rules in Δ. Then Pearl defines a nonmonotonic inference relation, denoted here by $\vdash_{Z,\Delta}$, as follows:

$$\alpha \vdash_{Z,\Delta} \beta \ \Leftrightarrow\ z(\alpha \wedge \beta) < z(\alpha \wedge \neg\beta).$$

An equivalent treatment of default information has been done in the framework of possibility theory [1] and also in the evidence theory framework [3]. Moreover, it has been shown by Goldszmidt and Pearl [10] that System Z corresponds to a particular closure of the set of preferential conclusions under the rational monotony property defined as:

9. Rational Monotony:
   From $\alpha \vdash \delta$ and $\alpha \not\vdash \neg\beta$ infer $\alpha \wedge \beta \vdash \delta$

It is well known that System Z addresses irrelevance and specificity problems. However, it is confronted to the "*blocking property inheritance*" problem, namely: if a class α contains a property β which conflicts with a property inherited from a super-class δ, then no property at all is inherited from δ (even properties unrelated to β). The following example shows that our approach allows inheritance of properties:

Example 3 : (Blocking property inheritance)

> Let $\Delta = \{b \rightarrow f,\ p \rightarrow b,\ p \rightarrow \neg f,\ b \rightarrow l\}$ ($b \rightarrow l$ means "generally, birds have legs"). From Δ, system Z cannot deduce that penguins have legs. Indeed, we have $Z(b \rightarrow f) = Z(b \rightarrow l) = 0$ and $Z(p \rightarrow b) = Z(p \rightarrow \neg f) = 1$. Hence $z(p \wedge l) = z(p \wedge \neg l) = 1$, so $p \not\vdash_{Z,\Delta} l$.
>
> By contrast, our inference relation $\vdash_{PC}$ allows this deduction. Indeed, we have $\sum p = \{\neg p \vee b,\ \neg p \vee \neg f,\ \neg b \vee l\}$ so by applying RCC, we get the desired conclusion.

Besides, there is a case of ambiguity where system Z would deduce an *undesired result*, while our approach does not.

Example 4 : (Ambiguity preservation: continued)

> Let $\Delta = \{Q \rightarrow P,\ R \rightarrow \neg P,\ R \rightarrow A,\ A \rightarrow B,\ R \rightarrow \neg B\}$ which is a variant of a Nixon-Diamond example with three new rules : "Republicans are Americans", "Americans play base-ball", and "Republicans do not play base-ball". In System Z, given a Quaker Republican, we get the arbitrary result $Q \wedge R \vdash_Z \neg P$.
>
> In contrast in our approach, we do not have neither $Q \wedge R \vdash_{PC} P$ nor $Q \wedge R \vdash_{PC} \neg P$. Since $\sum_{Q \wedge R} = \{\neg Q \vee P,\ \neg R \vee \neg P,\ \neg R \vee A,\ \neg R \vee \neg B\} \cup \{Q \wedge R\}$ is inconsistent (again RCC is not applicable).

Remark: the reason why in definition 4, we have choosen system P rather than system Z, is that, the exceptions, detected by system P are real exceptions while it is not the case with system Z. For instance, in the previous example, the rule $R \rightarrow \neg P$ has an exception in the context $Q \wedge R$ for system Z and this is intuitively incorrect. However, System P does not detect all exceptions: with the database $\Delta = \{a \wedge b \rightarrow d,\ b \rightarrow \neg d,\ c \rightarrow a\}$, system P does not detect that the context $b \wedge c$ is exceptional for the rule $a \wedge b \rightarrow d$.

The two previous examples show that System Z and our approach are incomparable. However in these examples, our inference relation behaves better since it refuses to infer

undesirable results and allows inheritance of properties. Unfortunately, there are cases of ambiguities where our approach is still cautious. Let us consider the default base of example 4: $\Delta=\{Q\rightarrow P, R\rightarrow\neg P, R\rightarrow A, A\rightarrow B, R\rightarrow\neg B\}$. With System Z, we have $Q\wedge R \mathrel{\sim}_{Z,\Delta} \neg B$. In our approach, since in presence of $Q\wedge R$ we get inconsistency, then it is not possible to deduce that $Q\wedge R \mathrel{\sim}_{PC,\Delta} \neg B$. But the first aim of our approach is to generate a superset of the set of preferential conclusions without producing *any* undesirable result. If we want to be more adventurous, we can improve our approach by using some techniques of inconsistency handling as done in [21] and [3].

6.2. LEXICOGRAPHICAL CLOSURE

We propose to compare our approach with lexicographical closure proposed in [6; 2] and [15]. This approach has been proposed to overcome the blocking property inheritance of System Z. It starts from the stratification of the set of defaults $\Delta=\Delta_1\cup\ldots\cup\Delta_n$, given by system Z, and defines an ordering between interpretations in the following way:

Definition 7: Let ω, ω' be two interpretations. The interpretation ω is *lex-preferred* (lex for lexicographical) to ω', denoted by $\omega >_{lex} \omega'$, iff there exists a positive number $1\leq j\leq n$ such that:

- for each $n\geq k>j$, we have $|[\omega]_{\Delta_k}| = |[\omega']_{\Delta_k}|$,
- and $|[\omega]_{\Delta_j}| > |[\omega']_{\Delta_j}|$.

Where $|[\omega]_{\Delta_k}|$ is the number of defaults $\alpha_i\rightarrow\beta_i$ in Δ_k satisfied by ω (*i.e.*, $\omega\models\neg\alpha_i\vee\beta_i$). Once the ordering $>_{lex}$ is computed the nonmonotonic inference relation is defined by the two following definitions in spirit of Shoham [24]:

Definition 8: An interpretation ω is a *lex-preferred interpretation* of Δ satisfying a formula α, called α-lex-preferred interpretation in Δ, iff $\omega\models\alpha$ and $\nexists\omega'$ s.t. $\omega' >_{lex} \omega$ and $\omega'\models\alpha$.

Definition 9: A formula β is a *lex-consequence* of α given Δ, denoted by $\alpha \mathrel{\sim}_{lex,\Delta} \beta$, if and only if $\forall\omega$ α–lex-preferred in Δ, $\omega\models\beta$.

Proposition 3 [2]: if $\alpha \mathrel{\sim}_{Z,\Delta} \beta$ then $\alpha \mathrel{\sim}_{lex,\Delta} \beta$.

The previous proposition means that $\mathrel{\sim}_{lex,\Delta}$ addresses the specificity and irrelevance problems, however, it gives also the undesirable results of System Z (see Example 4). Besides, the main drawback of the lex-consequence relation is that it is syntax-sensitive: repetitions of the same default in Δ may change the result. The following example shows that our approach is not syntax-dependent.

Example 5 : (Redundancy)

1. Consider a variant of the Quaker-Republican problem (Example 2) where the rule "Generally, Quaker are pacifists" has been duplicated: $\Delta = \{Q\rightarrow P, Q\rightarrow P, R\rightarrow\neg P\}$. Using a lexicographic approach, $\omega_1=\{R,Q,\neg P\}$ and $\omega_2=\{R,Q,P\}$ are the two interpretations satisfying $R\wedge Q$. Using System Z, Δ has only one stratum containing all the defaults. Hence, $|[\omega_1]_{\Delta_0}| =1$ and $|[\omega_2]_{\Delta_0}| =2$, so ω_2 is $R\wedge Q$-lex-preferred in Δ. Hence, $Q\wedge R \mathrel{\sim}_{lex,\Delta} P$ is valid , while we would prefer to acknowledge the ambiguity and deduce nothing. In our approach, we do not have neither $Q\wedge R \mathrel{\sim}_{PC} P$ nor $Q\wedge R \mathrel{\sim}_{PC} \neg P$.

2. Let us consider another variant of Quaker-Republican problem with the extra rule "Generally, ecologists are pacifist": $\Delta = \{Q\rightarrow P, E\rightarrow P, R\rightarrow\neg P\}$. Then the lex-consequence would deduce that $Q\wedge R\wedge E \mathrel{\sim}_{lex,\Delta} P$ while, in our approach, we deduce nothing since we are still in ambiguity situation.

In a general way, we can show that our inference relation satisfies the following property:

10. Redundancy insensitivity:
Let $\alpha_i\rightarrow\beta_i$ a default in Δ. Let $\Delta'=\Delta\cup\{\alpha_i\rightarrow\beta_i\}$. Then
$\alpha \mathrel{\sim}_{PC,\Delta} \beta$ iff $\alpha \mathrel{\sim}_{PC,\Delta} \beta$

This property can be shown by noticing that neither System P nor classical entailment are sensitive to repetition of formulae.

7- HANDLING PERSISTENCE LAWS IN EVOLUTIVE SYSTEM

Typically, default reasoning is used in knowledge based representing a static system, *i.e.*, information about the system can be improved but the system itself does not change. Here, let us consider an evolutive system, in order to describe it, we have some information about its state at given instants, general information about what are the possible states of the system (called *static laws*), and lastly general information about how it can evolve (*dynamic laws*). These laws are more or less defeasible, *i.e.*, they can admit exceptions. This is why it is interesting to represent them by default rules.

In order to deal with temporal formulae, we index every formula by the time instant in which it is considered. Static laws have the form $\forall t, \varphi_t\rightarrow\psi_t$ and dynamic laws $\forall t, \varphi_t\rightarrow\psi_{t+1}$ (this representation is described in [7]). There is a particular case of dynamic laws, which are called *persistence laws* and have the form $\forall t, x_t\rightarrow x_{t+1}$ where x is a propositional variable or its negation. Persistence laws are used to answer to the frame problem (*i.e.*, when the system is evolving everything which is not concerned with this evolution (the frame) has no reason to change).

In general, each dynamic law contradicts some persistance laws. And, in this case, priority must be given to dynamic laws since they describe exceptional behaviours of the system. For instance, in the "dead battery" example (cf. [11] and [7]), we consider the following persistence law: $\neg er_t \rightarrow \neg er_{t+1}$ (generally, if the engine is not running ($\neg$er), then an instant later it will remain not running); the dynamic law $\neg er_t \wedge tk_t \rightarrow er_{t+1}$ (if you turn the ignition key (tk) when the engine was not running, then, generally, an instant later, the car engine runs (er)). These laws are in contradiction, and it is clear that the second one must be applied prioritarily.
Consider also another dynamic law: $\neg er_t \wedge tk_t \wedge db_t\rightarrow \neg er_{t+1}$ (when the battery is dead (db) the engine can not start running). Let us restrict the problem to two time points only (0 and 1). Then we have $\Delta_{dyn}=\{d_1: \neg er_0 \wedge tk_0 \rightarrow er_1, d_2: \neg er_0 \wedge tk_0 \wedge db_0\rightarrow \neg er_1\}$ is the set of the two dynamic laws and $\Delta_{pers}=\{d_3: er_0\rightarrow er_1, d_4: \neg er_0\rightarrow \neg er_1,$

d_5: $db_0 \rightarrow db_1$, d_6: $\neg db_0 \rightarrow \neg db_1\}$ the set of persistent laws, and $\Delta=\Delta_{dyn} \cup \Delta_{pers}$ (Note that tk is an action it is why it does not appear in the persistence laws).

Suppose that at time 0, the engine is not running and you turn the ignition key: this context is described by $\varphi=\neg er_0 \wedge tk_0$. What can we deduce? Only the rules d_3 and d_4 have exceptions in the context φ, so $\Sigma_\varphi = \{er_0 \vee \neg tk_0 \vee er_1, er_0 \vee \neg tk_0 \vee \neg db_0 \vee \neg er_1, \neg db_0 \vee db_1, db_0 \vee \neg db_1\}$, hence using RCC, $\neg er_0 \wedge tk_0 \vdash_{PC,\Delta} er_1$. This result is the intended solution it says that the engine will be running at time point 1. This example shows that our approach is able to handle the priority of dynamic laws under persistance laws.

Moreover, assume that the new context is $\psi=\neg er_0 \wedge tk_0 \wedge db_0$. In this case, only the rules d_1, d_3 and d_6 have exceptions, so $\Sigma_\psi = \{ er_0 \vee \neg tk_0 \vee \neg db_0 \vee \neg er_1, \neg db_0 \vee db_1, er_0 \vee \neg er_1\}$, hence using RCC, $\neg er_0 \wedge tk_0 \wedge db_0 \vdash_{PC,\Delta} \neg er_1 \wedge db_1$. This means that if the battery is dead when you turn the ignition key with your stopped engine, then in the next time point, the engine will not be running and your battery will remain dead, this is also the intuitive conclusion. This second example shows that our approach can also handle priority between dynamic laws, and furthermore is able to deal with persistency of fluents (here the battery remains dead at point time 1 : there is no frame problem!).

8- CONCLUSION

This paper describes a local way to deal with default information. We have proved that our approach delivers only a safe super-set of the set of conclusions given by system P (it correctly address the problems of irrelevance,blocking inheritance, ambiguity and redundancy).

In our approach, given a context, we transform as many default rules as possible into strict rules provided that the context is not an exception to them. The method described above can be generalised in order to improve *any* system which can detect exceptions. Here the system P is used. Note that system P is not idealistic because it is too cautious, exceptions which are detected by system P are for sure exceptions, but it does not detect all exceptions. Further research will be to find a new system more adventurous than System P and less than System Z in order to use it instead of System P.

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