Cooperative interpersonal communication and relevant information

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Outline

1 Cooperativity
   - Example
   - Definition

2 Relevance
   - Formal definition
   - Properties
   - Hierarchy

3 Back to cooperativity

4 Conclusion and perspectives
**Example**

Agent *a*

\[ \text{late?} \]
\[ \text{incident } \rightarrow \text{late} \]

Agent *b*

\[ \text{late?} \]
\[ \neg \text{incident } \rightarrow \neg \text{late} \]

Agent *c*

\[ \text{incident} \]
\[ \text{rain} \]
Example

Agent a

Late?

incident → late

Agent b

Late?

¬incident → ¬late

Agent c

incident

rain
Example

Agent a
late?
incident → late

Agent b
late?
¬incident → ¬late

Agent c
incident
rain

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Example

Agent a

late?

incident → late

Agent b

late?

¬incident → ¬late

Agent c

incident

rain
Example

Agent $a$

$late?$

$incident \rightarrow late$

Agent $b$

$late?$

$\neg incident \rightarrow \neg late$

Agent $c$

$incident$

$incident$

$rain$
Example

Agent a

late?
incident → late

Agent b

late?
¬incident → ¬late

Agent c

incident
rain

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Example

Agent a

\[\text{late?} \quad \text{incident} \rightarrow \text{late}\]

Agent b

\[\text{late?} \quad \neg \text{incident} \rightarrow \neg \text{late}\]

Agent c

\[\text{incident} \quad \text{rain} \quad \text{rain} \]
Example

Agent a

late?

incident → late

Agent b

late?

¬incident → ¬late

Agent c

incident

rain

rain

Cooperative interpersonal communication and relevant information
Example

Agent $a$

late?

incident $\rightarrow$ late

Agent $b$

late?

$\neg$incident $\rightarrow$ $\neg$late

Agent $c$

incident

rain

incident $\land$ rain

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Example

Agent a
\( \text{late?} \)
\( \text{incident} \rightarrow \text{late} \)

Agent c
\( \text{incident} \)
\( \text{rain} \)

Agent b
\( \text{late?} \)
\( \neg \text{incident} \rightarrow \neg \text{late} \)

Cooperative interpersonal communication and relevant information
Example

Agent a
late?
incident → late

Agent b
late?
¬incident → ¬late

Agent c
incident
rain
Example

Agent a

\( \text{late?} \)

\( \text{incident} \rightarrow \text{late} \)

Agent b

\( \text{late?} \)

\( \neg \text{incident} \rightarrow \neg \text{late} \)

Agent c

\( \text{incident} \)

\( \text{rain} \)
Example

Agent $a$

late?

incident $\rightarrow$ late

Agent $b$

latest?

$\neg$incident $\rightarrow$ $\neg$late

Agent $c$

incident

rain
Example

Agent a

late?
incident → late

Agent b

late?
¬incident → ¬late

Agent c

incident
rain

Cooperative interpersonal communication and relevant information
Example

Agent a

late?

incident \rightarrow \text{late}

Agent b

late?

\neg \text{incident} \rightarrow \neg\text{late}

Agent c

\text{incident}

\text{rain}

\neg \text{incident}
Agent $a$

$late$?

$incident \rightarrow late$

Agent $c$

$incident$

$rain$

Agent $b$

$late$?

$\neg incident \rightarrow \neg late$

$\neg incident$
Definition for cooperativity

Intuitively, agent $c$ is **cooperative with regard to agent $a$** if and only if for all piece of information $\varphi$ :
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Intuitively, agent $c$ is **cooperative with regard to agent** $a$ if and only if for all piece of information $\varphi$:

If

- agent $c$ believes that agent $a$ has some information need $Q$
- agent $c$ believes that agent $a$ can deduce from $\varphi$ something about $Q$
- agent $c$ believes that $\varphi$ does not contain elements disconnected with $Q$ for agent $a$
- agent $c$ believes that $\varphi$ is true

Then agent $c$ informs agent $a$ about $\varphi$
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AND

agent $c$ does not inform $a$ about any $\varphi$ that does not satisfy all those conditions.
Intuitively, agent $c$ is **cooperative with regard to agent** $a$ if and only if for all piece of information $\varphi$:

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Then agent $c$ informs agent $a$ about $\varphi$

AND

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Definition for cooperativity

Intuitively, agent \( c \) is \textbf{cooperative with regard to agent} \( a \) if and only if for all piece of information \( \varphi \):

If \textbf{agent} \( c \) \textbf{believes} that

- \( a \) has some information need \( Q \)
- \( a \) can deduce from \( \varphi \) something about \( Q \)
- \( \varphi \) does not contain elements disconnected with \( Q \) for \( a \)
- \( \varphi \) is true

Then \textbf{agent} \( c \) informs \textbf{agent} \( a \) about \( \varphi \)

AND

\( a \) does not inform \( c \) about any \( \varphi \) that does not satisfy all those conditions.
Definitions for cooperativity

Intuitively, agent $c$ is **cooperative with regard to agent** $a$ if and only if for all piece of information $\varphi$:
agent $c$ believes that $\varphi$ is relevant for $a$ concerning some $Q$
AND
agent $c$ does not inform $a$ about any $\varphi$ that does not satisfy this conditions.
Outline

1 Cooperativity

2 Relevance
   - Formal definition
   - Properties
   - Hierarchy

3 Back to cooperativity

4 Conclusion and perspectives
Let $a$ be an agent, $Q$ an objective formula, $\varphi$ a formula. We say that $\varphi$ is relevant for agent $a$ concerning $Q$ iff the following formula, denoted $R^Q_a \varphi$, is true:

$$I_a(B_a Q \lor B_a \neg Q) \land \varphi \land (B_a(\varphi \rightarrow Q) \otimes B_a(\varphi \rightarrow \neg Q))$$

- Information need: agent $a$ wants to know whether $Q$ or $\neg Q$
- Piece of information truth value: the piece of information $\varphi$ must be true
- Agent’s beliefs base: agent $a$ believes that $\varphi \rightarrow Q$ or that $\varphi \rightarrow \neg Q$
Let $a$ be an agent, $Q$ an objective formula, $\varphi$ a formula. We say that $\varphi$ is relevant for agent $a$ concerning $Q$ iff the following formula, denoted $R^Q_a \varphi$, is true:

$$l_a(B_a Q \lor B_a \neg Q) \land \varphi \land (B_a (\varphi \rightarrow Q) \otimes B_a (\varphi \rightarrow \neg Q))$$

- **Information need**: agent $a$ wants to know whether $Q$ or $\neg Q$
- **Piece of information truth value**: the piece of information $\varphi$ must be true
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Let \( a \) be an agent, \( Q \) an objective formula, \( \varphi \) a formula. We say that \( \varphi \) is relevant for agent \( a \) concerning \( Q \) iff the following formula, denoted \( R_a^{Q} \varphi \), is true:

\[
I_a (B_a Q \lor B_a \neg Q) \land \varphi \land (B_a (\varphi \rightarrow Q) \otimes B_a (\varphi \rightarrow \neg Q))
\]

- **Information need**: agent \( a \) wants to know whether \( Q \) or \( \neg Q \)
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Relevance

Let $a$ be an agent, $Q$ an objective formula, $\varphi$ a formula. We say that $\varphi$ is relevant for agent $a$ concerning $Q$ iff the following formula, denoted $R^Q_a \varphi$, is true:

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A few properties

**Proposition 1**

\[ R^Q_a \varphi \rightarrow \neg B_a \varphi \land B_a \neg \varphi \]

If \( \varphi \) is a relevant piece of information for agent \( a \), then she does not know neither \( \varphi \) nor \( \neg \varphi \).

**Proposition 2**

- \( I_a (B_a Q \lor B_a \neg Q) \rightarrow R^Q_a Q \lor R^Q_a \neg Q \)
- \( (Q_1 \leftrightarrow Q_2) \rightarrow (R^Q_{a_1} \varphi \leftrightarrow R^Q_{a_2} \varphi) \)
- \( R^Q_a \varphi \rightarrow \neg R^Q_a \neg \varphi \)
- \( \neg (\varphi_1 \land \varphi_2) \rightarrow \neg (R^Q_{a_1} \varphi_1 \land R^Q_{a_2} \varphi_2) \)
A few properties

Proposition 1

\[ R_a^Q \varphi \rightarrow \neg B_a \varphi \land B_a \neg \varphi \]

If $\varphi$ is a relevant piece of information for agent $a$, then she does not know neither $\varphi$ nor $\neg \varphi$.

Proposition 2

1. $l_a(B_a Q \lor B_a \neg Q) \rightarrow R_a^Q Q \mathrm{ xor } R_a^Q \neg Q$
2. $(Q_1 \leftrightarrow Q_2) \rightarrow (R_a^{Q_1} \varphi \leftrightarrow R_a^{Q_2} \varphi)$
3. $R_a^Q \varphi \rightarrow \neg R_a^Q \neg \varphi$
4. $\neg (\varphi_1 \land \varphi_2) \rightarrow \neg (R_a^{Q_1} \varphi_1 \land R_a^{Q_2} \varphi_2)$
A few properties

**Proposition 4**

\[ R_a^Q \varphi \rightarrow \neg B_a R_a^Q \varphi \]

If \( \varphi \) is a relevant piece of information, then agent \( a \) does not believe it.

**Proposition 5**

\[ B_a(\varphi_1, \varphi_2/Q) \rightarrow (\varphi_2 \land R_a^Q \varphi_1 \rightarrow R_a^Q (\varphi_1 \land \varphi_2)) \]

et

\[ B_a(\varphi_1, \varphi_2/Q) \rightarrow (R_a^Q \varphi_1 \land R_a^Q \varphi_2 \rightarrow R_a^Q (\varphi_1 \lor \varphi_2)) \]

Too many pieces of information are relevant \( \Rightarrow \) We can define a hierarchy on relevant pieces of information, that means a characterization of most relevant pieces of information.
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Proposition 4

\[ R_Q^a \varphi \rightarrow \neg B_a R_Q^a \varphi \]

If \( \varphi \) is a relevant piece of information, then agent \( a \) does not believe it.

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\[ B_a(\varphi_1, \varphi_2/Q) \rightarrow (\varphi_2 \land R_Q^a \varphi_1 \rightarrow R_Q^a (\varphi_1 \land \varphi_2)) \]

et

\[ B_a(\varphi_1, \varphi_2/Q) \rightarrow (R_Q^a \varphi_1 \land R_Q^a \varphi_2 \rightarrow R_Q^a (\varphi_1 \lor \varphi_2)) \]

Too many pieces of information are relevant \( \Rightarrow \) We can define a hierarchy on relevant pieces of information, that means a characterization of most relevant pieces of information.
Minimal explanation

Definition: Explanation

Let $\Delta$ be a set of objective formulae and $\alpha$ and $\beta$ two objective formulae.

$\beta$ is an explanation of $\alpha$ if and only if $\vdash B\Delta \rightarrow B(\beta \rightarrow \alpha)$ and $\not\vdash B\Delta \rightarrow B(\neg \beta)$.

Intuition: Minimal explanation

- for cubes: $\alpha$ is a minimal explanation of $\beta$ iff there is no other explanation $\alpha'$ of $\beta$ such that $\alpha \rightarrow \alpha'$ and $\alpha \not\rightarrow \alpha'$ (prime implicants)
- for clauses: $\alpha$ is a minimal explanation of $\beta$ iff there is no other explanation $\alpha'$ of $\beta$ such that $\alpha' \rightarrow \alpha$ and $\alpha' \not\rightarrow \alpha$ (maxima for subsumption)
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Most relevant formulae

Let $\mathcal{R}_a^Q$ be the set of relevant formulae. For all $\varphi$ in $\mathcal{R}_a^Q$, we have $B_a(\varphi \rightarrow Q)$ or $B_a(\varphi \rightarrow \neg Q)$ and $\neg B_a(\neg \varphi)$, that means that for all $\varphi$ in $\mathcal{R}_a^Q$, $\varphi$ is an explanation of $Q$ or $\neg Q$.

**Definition**

Let $\mathcal{R}m_a^Q$ be the subset of $\mathcal{R}_a^Q$ that contains the minimal explanations of $Q$ and $\neg Q$. We will write $\mathcal{R}m_a^Q \varphi$ to express that the formula $\varphi$ belongs to $\mathcal{R}m_a^Q$.

**Example**

Let us consider the following set of relevant pieces of information to agent $a$ concerning her request $Q$:

$\mathcal{R}_a^Q = \{ inc \land rain, inc \lor strike, strike \}$. Then $\mathcal{R}m_a^Q = \{ strike, inc \land rain \}$. 
Let $\mathcal{R}^Q_a$ be the set of relevant formulae. For all $\varphi$ in $\mathcal{R}^Q_a$, we have $B_a(\varphi \rightarrow Q)$ or $B_a(\varphi \rightarrow \neg Q)$ and $\neg B_a(\neg \varphi)$, that means that for all $\varphi$ in $\mathcal{R}^Q_a$, $\varphi$ is an explanation of $Q$ or $\neg Q$.

**Definition**

Let $\mathcal{R}^Q_{m_a}$ be the subset of $\mathcal{R}^Q_a$ that contains the minimal explanations of $Q$ and $\neg Q$. We will write $\mathcal{R}^Q_{m_a} \varphi$ to express that the formula $\varphi$ belongs to $\mathcal{R}^Q_{m_a}$.

**Example**

Let us consider the following set of relevant pieces of information to agent $a$ concerning her request $Q$:
$\mathcal{R}^Q_a = \{ \text{inc} \land \text{rain}, \text{inc} \lor \text{strike}, \text{strike} \}$. Then $\mathcal{R}^Q_{m_a} = \{ \text{strike}, \text{inc} \land \text{rain} \}$. 
Most relevant formulae

Let $\mathcal{R}_a^Q$ be the set of relevant formulae. For all $\varphi$ in $\mathcal{R}_a^Q$, we have $B_a(\varphi \rightarrow Q)$ or $B_a(\varphi \rightarrow \neg Q)$ and $\neg B_a(\neg \varphi)$, that means that for all $\varphi$ in $\mathcal{R}_a^Q$, $\varphi$ is an explanation of $Q$ or $\neg Q$.

**Definition**

Let $\mathcal{R}_m_a^Q$ be the subset of $\mathcal{R}_a^Q$ that contains the minimal explanations of $Q$ and $\neg Q$. We will write $\mathcal{R}_m_a^Q \varphi$ to express that the formula $\varphi$ belongs to $\mathcal{R}_m_a^Q$.

**Example**

Let us consider the following set of relevant pieces of information to agent $a$ concerning her request $Q$:

$\mathcal{R}_a^Q = \{ inc \land rain, inc \lor strike, strike \}$. Then $\mathcal{R}_m_a^Q = \{ strike, inc \land rain \}$. 
Outline

1. Cooperativity
2. Relevance
3. Back to cooperativity
4. Conclusion and perspectives
Intuitively, agent $c$ is **cooperative with regard to agent** $a$ if and only if for all piece of information $\varphi$:

- If agent $c$ believes that
  - agent $a$ has some information need $Q$
  - agent $a$ can deduce from $\varphi$ something about $Q$
  - $\varphi$ does not contain elements disconnected with $Q$ for agent $a$
  - $\varphi$ is true

Then agent $c$ informs agent $a$ about $\varphi$

AND agent $c$ does not inform $a$ about any $\varphi$ that does not satisfy all those conditions.
Intuitively, agent $c$ is **cooperative with regard to agent $a$** if and only if for all piece of information $\varphi$:
agent $c$ believes that $\varphi$ is relevant for $a$ concerning some $Q$ AND agent $c$ does not inform $a$ about any $\varphi$ that does not satisfy this conditions.
A more formal definition

Let $a$ and $c$ be two agents. The agent $c$ is cooperative with regard to $a$ iff for all formula $\varphi$, $c$ informs $a$ about $\varphi$ if and only if there is a request $Q$ such that $c$ believes that $\varphi$ is maximal relevant for $a$ concerning $Q$. This is represented by:

$$Coop(c, a) \equiv \forall \varphi Inf_{c, a} \varphi \leftrightarrow \exists Q, B_c(Rm^Q_a \varphi)$$
Cooperative interpersonal communication and relevant information
Conclusion and perspectives

Conclusion

- A first formal definition to agent-oriented relevance
- Characterization of most relevant pieces of information for an agent
- Characterization of cooperativity between two agents
- Comparison to other definitions of cooperativity

Perspectives

- Relevance for other needs that information need
- Introduction of time
Conclusion and perspectives

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Axiomatics

- Propositional tautologies and propositional inference rules;

- KD45 for $B_a$, 
  
  (K) $\vdash B_a(\varphi \rightarrow \psi) \land B_a\varphi \rightarrow B_a\psi$
  
  (D) $\vdash B_a\varphi \rightarrow \neg B_a\neg\varphi$
  
  (4) $\vdash B_a\varphi \rightarrow B_aB_a\varphi$
  
  (5) $\vdash \neg B\varphi \rightarrow B_a\neg B_a\varphi$

- (Nec) Necessitation for $B_a$, $\vdash \varphi$ $\vdash B_a\varphi$

- (UE) Unit exclusion for $I_a$, $\vdash \neg I_a(\top)$

- BI Introspection,
  
  (BI1) $\vdash I_a\varphi \rightarrow B_aI_a\varphi$
  
  (BI2) $\vdash \neg I_a\varphi \rightarrow B_a\neg I_a\varphi$
  
  (BI3) $\vdash B_a(\varphi \leftrightarrow \psi) \rightarrow (I_a\varphi \leftrightarrow I_a\psi)$