Delegation as a communicative act: a logical analysis

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In the context of agents’ organization:

- Agent $i$ might decide to exploit other agents in order to achieve his goal.
- $i$ delegates some elements in his multi-agent plan to agent $j$ in the organization.
  - $j$’s tasks may be changed.
  - $j$’s role will change on behalf of $i$.
  - $j$’s obligation will also change.
  - The organization’s structure becomes dynamic!
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Delegation as a request: a scenario

Agent *i*’s car is damaged and he decides to delegate to a mechanic the task of repairing the car. So:

- *i* relies on the mechanic’s action of repairing the car
- *i*’s intention to ask the mechanic to repair the car and
- *i*’s intention to communicate the mechanic that he is granting him the permission to repair the car (on his behalf)
- *i*’s in the position to grant a permission to the mechanic to repair his car
- *i* also creates the permission for the mechanic to repair the car.
Delegation as a request

Agent $i$ delegates agent $j$ to do action $\alpha$ to brings about $\varphi$

(A) $i$ intends that $j$ will perform $\alpha$ so that $\varphi$ will be achieved

(B) $i$ thinks it is possible that $j$ will perform $\alpha$

(C) $i$ intends to communicate to $j$ that $i$’s relying on $j$’s execution of $\alpha$

(D) $i$ intends to communicate to $j$ that $j$ has the permission to perform $\alpha$

(E) $i$ has the power and the authority to grant to $j$ the permission to perform $\alpha$

(F) It is forbidden for $j$ to perform $\alpha$. 
Delegation as an order: a scenario

- agent $i$ is the president of a company which must be represented at a journalist meeting
- agent $i$ delegates to agent $j$, an employee of the company, the task of participating to this meeting

→ The employee $j$ does have any refusal option!
Delegation as an order

Agent $i$ delegates agent $j$ to do action $\alpha$ to brings about $\varphi$

(A) $i$ intends that $j$ will perform $\alpha$ so that $\varphi$ will be achieved

(B) $i$ thinks it is possible that $j$ will perform $\alpha$

(C) $i$ intends to communicate to $j$ that $i$’s relying on $j$’s execution of $\alpha$

(D’) $i$ intends to communicate to $j$ that $j$’s obliged to perform $\alpha$

(E’) $i$ has the power and the authority to oblige $j$ to perform $\alpha$

(F’) $j$’s not obliged to perform $\alpha$ but $j$ has the obligation after having completed the delegation act.
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Language:
\[ \varphi ::= p \mid \neg \varphi \mid \varphi \lor \varphi \mid \text{Do}_{i: \alpha} \varphi \mid \text{After}_{i: \alpha} \varphi \mid \text{Bel}_{i} \varphi \mid \text{Choice}_{i} \varphi \mid \text{Oblig} \varphi \]

where \( p \) ranges over \( \text{ATM} \), \( \alpha \) ranges over \( \text{ACT} \) and \( i \) ranges over \( \text{AGT} \).

\text{After}_{i: \alpha} \varphi: \text{‘after agent } i \text{ does } \alpha \text{, it is the case that } \varphi\text{’}

\text{Do}_{i: \alpha} \varphi: \text{‘agent } i \text{ is going to do } \alpha \text{ and } \varphi \text{ will be true afterwards’}

\text{Bel}_{i} \varphi: \text{‘agent } i \text{ believes that } \varphi\text{’}

\text{Choice}_{i} \varphi: \text{‘agent } i \text{ has the chosen goal that } \varphi\text{’}

\text{Oblig} \varphi: \text{‘} \varphi \text{ is obligatory’}.
Syntax

Abbreviations:

\[
\begin{align*}
\text{Poss}_i \varphi & \equiv \neg \text{Bel}_i \neg \varphi \\
\text{Capable}_i(\alpha) & \equiv \neg \text{After}_{i:\alpha} \bot \\
\text{Int}(i, \varphi, \alpha) & \equiv \text{Choice}_i \text{Do}_{i:\alpha} \varphi \\
\text{Power}_i(\varphi, \alpha) & \equiv \text{Capable}_i(\alpha) \land \text{After}_{i:\alpha} \varphi \\
\text{Oblig}_i(\alpha) & \equiv \text{Oblig} \text{Do}_{i:\alpha} \top \\
\text{Forbid}_i(\alpha) & \equiv \text{Oblig} \neg \text{Do}_{i:\alpha} \top \\
\text{Perm}_i(\alpha) & \equiv \neg \text{Oblig} \neg \text{Do}_{i:\alpha} \top
\end{align*}
\]
Semantics: Frame

Frame $F = \langle W, B, C, O, A, D \rangle$:

- $W$ is a nonempty set of possible worlds or states.
- $B : AGT \rightarrow W \times W$ maps every agent $i$ to a serial, transitive and Euclidean relation $B_i$ between possible worlds in $W$.
- $C : AGT \rightarrow W \times W$ maps every agent $i$ to a serial relation $C_i$ between possible worlds in $W$.
- $O$ is a serial relation between possible worlds in $W$.
- $A : AGT \times ACT \rightarrow W \times W$ maps every agent $i$ and action $\alpha$ to a relation $A_{i:\alpha}$ between possible worlds in $W$.
- $D : AGT \times ACT \rightarrow W \times W$ maps every agent $i$ and action $\alpha$ to a deterministic relation $D_{i:\alpha}$ between possible worlds in $W$. 
Semantics: Truth conditions

- \( M, w \models \text{After}_{i:\alpha} \varphi \) iff \( M, w' \models \varphi \) for all \( w' \) such that \((w, w') \in A_{i:\alpha}\).
- \( M, w \models \text{Do}_{i:\alpha} \varphi \) iff there is \( w' \in D_{i:\alpha}(w) \) such that \( M, w' \models \varphi \).
- \( M, w \models \text{Bel}_{i} \varphi \) iff \( M, w' \models \varphi \) for all \( w' \) such that \((w, w') \in B_i\).
- \( M, w \models \text{Choice}_{i} \varphi \) iff \( M, w' \models \varphi \) for all \( w' \) such that \((w, w') \in C_i\).
- \( M, w \models \text{Oblig} \varphi \) iff \( M, w' \models \varphi \) for all \( w' \) such that \((w, w') \in O\).
Axiomatization

All principles of modal logic KD45 for every $Bel_i$
All principles of modal logic KD for every $Choice_i$ and $Oblig_i$
All principles of modal logic K for every $After_i:\alpha$ and $Do_i:\alpha$

$$Do_i:\alpha \varphi \rightarrow \neg Do_j: \beta \neg \varphi$$
$$\bigvee_{i \in AGT, \alpha \in ACT} Do_i: \alpha \top$$

$$Do_i: \alpha \varphi \rightarrow \neg After_i: \alpha \neg \varphi$$

$$(Choice_i Do_i: \alpha \top \land Capable_i(\alpha)) \rightarrow Do_i: \alpha \top$$

$$Do_i: \alpha \top \rightarrow Choice_i Do_i: \alpha \top$$

$$Bel_i \varphi \rightarrow \neg Choice_i \neg \varphi$$

$$Choice_i \varphi \rightarrow Bel_i Choice_i \varphi$$

$$\neg Choice_i \varphi \rightarrow Bel_i \neg Choice_i \varphi$$
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Communicative intention & Rely

**Communicative intention**

\[ \text{CommInt}(i,j,\varphi,\alpha) \overset{\text{def}}{=} \text{Int}(i,\text{Bel}_j\text{Choice}_i\text{Bel}_j\varphi,\alpha) \]

**Rely**

\[ \text{Rely}(i,j,\alpha) \overset{\text{def}}{=} \text{Choice}_iX\text{Do}_j:\alpha \top \]
Formalization of delegation as a request

\[
\text{ReqDel}(i, j, \alpha, \beta) \overset{\text{def}}{=} \text{Do}_{i: \beta} \top \land \text{Rely}(i, j, \alpha) \\
\land \text{CommInt}(i, j, \text{Rely}(i, j, \alpha) \land \text{Perm}_j(\alpha), \beta) \\
\land \text{Power}_i(\text{Perm}_j(\alpha), \beta) \land \text{Forbid}_j(\alpha)
\]
Formalization of delegation as an order

\[ \text{OrdDel}(i, j, \alpha, \beta) \overset{\text{def}}{=} \text{Do}_{i, \beta} \top \land \text{Rely}(i, j, \alpha) \]
\[ \land \text{CommInt}(i, j, \text{Rely}(i, j, \alpha) \land \text{Oblig}_j(\alpha), \beta) \]
\[ \land \text{Power}_i(\text{Oblig}_j(\alpha), \beta) \land \neg \text{Oblig}_j(\alpha) \]
Summary

- A logic for delegation act
- Delegation as a request
- Delegation as an order
Thanks for your attention!
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