Communication Strategies in Games

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Communication strategies in games

Setting: A game of imperfect information, in which you and/or your opponent are not completely informed about all aspects of the game.

- What should you say?
- What should you not say?
- Should you believe the other players?
- Is it always good to get information?
- Etc.
For example

- **Bargaining**: say enough to make a deal, say little enough to make a good deal
- **Selling**: communicate your prices to your customers, not to your competitors.
- **Selling information**: communicate enough to show that you have the information, but not so much that you have nothing to sell anymore
- **Love**: tell your object of desire those things about yourself that make you desirable, not those that make you undesirable.
More specifically

General Idea: use *dynamic epistemic logic* to study information change in *games of imperfect information*

- Model games of imperfect information as Kripke models
- Use Dynamic Epistemic Logic to model the effect of communication.
- So: Communicative acts change the game
- The question “What should you say” becomes “With what act can I change the game to my advantage”
Running Example

- Three closed envelopes
- Ann can choose any envelope she wants
- Envelope 1 is empty
- Envelope 2 contains 3 euro for Ann, 3 for Bill
- Envelope 3 contains 6 euro for Bill
- Bill knows this
- Ann does not know which envelope contains what

What should Ann do? What should Bill say?
What should Ann do?

- She has no idea which envelope has money in it for her.
- So, she chooses at random
- Ann can expect 1 euro (on the average)
- Bill can expect 3 euro (on the average)
What should Bill say?

- Lie!
- Bill: "Envelope 3 contains 3 euro for Ann"
- Ann (if she believes him) chooses envelope 3.
- Ann expects 3, but gets 0.
- Bill gets 6.
But if Bill can only tell the truth?

- Bill: “Envelope 2 contains 3 euro for Ann”
- This does not help Bill (he still expects 3 euro)
- Better to say less
- Bill: “Envelope 1 is empty”
- Ann (if she believes him) will choose randomly between envelopes 2 and 3
- Good for both: Bill expects $4\frac{1}{2}$, Ann expects $1\frac{1}{2}$
How to model this type of situation

- *Game Theory* to model games
- *Epistemic Semantics* to model lack of information
- *Dynamic Epistemic Semantics* to model information change
Game Theory

A strategic game (of perfect information): \((N, (S_i)_{i \in N}, \pi)\)

- \(N\) is a set of *players*
- \(S_i\) is a set of *strategies* for player \(i\)
- \(\pi\) tells us what the players get for each combination of strategies
Our game

<table>
<thead>
<tr>
<th>Bill's choices \ Ann's choices</th>
<th>env. 1</th>
<th>env. 2</th>
<th>env. 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Bill has no choice)</td>
<td>0 \ 0</td>
<td>3 \ 3</td>
<td>6 \ 0</td>
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</table>
Epistemic Semantics

- Information of each agent represented by a set of possibilities (those that are consistent with her information).
- Typical representation: A Kripke model \((W, (\rightarrow_a)_{a \in \mathbb{N}}, w)\) (for a set of agents \(\mathbb{N}\))
- \(W\) is a set of possible worlds
- \(w\) is the actual world
- \(w \rightarrow_a u\) means: \(a\) thinks that \(u\) is possible in \(w\)
Game Theory + Epistemic Semantics

- Games of imperfect information: players do not know exactly which game they are playing.

- A Game of imperfect information is a Kripke model in which possible worlds are games.

- Difference with the standard model of such games: in our case, players may have false information as well

- (We really need probabilistic models...)
What Ann believes

These are Ann’s ‘possible worlds’. In each of them, Bill knows what's going on.

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<td>3 \ 3</td>
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Dynamic Epistemic Semantics

- Study of ‘epistemic actions’ that change the information of agents: private and public communication, observation, lying, suspicion, etc.

- Change of information is modeled as change in a Kripke model

- An epistemic action is an operation on Kripke models

- Plaza (1989), Gerbrandy (1997/8), Baltag, Moss, Solecki (1998), van Ditmarsch (2000), ...

- *We can (now) simply plug in dynamic epistemic semantics into game theory*
Public announcements

- Particularly simple case of an epistemic action: *public announcement*

- **definition:**
  
  $Kw$ is a model. The model $K'w'$ that results after a public announcement of $\phi$ is obtained by removing all arrows pointing to worlds where $\phi$ is false.

- We write $Kw[\phi]$
For example

After a public announcement of ‘the first envelope is empty’, these are Ann’s possibilities:

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The value of communication

- What should you say? That which has the highest value.
- **intuition:** The value of an announcement is the difference in expected utility before and after the announcement.
- **definition:** The value of a public announcement with a sentence $\phi$ in a game of imperfect information $G$ for a player $i$ is:
  
  The difference in expected utility for $i$ in $G$ and that in $G[\phi]$

- (can easily be generalized to cover all epistemic actions)
- But: *What is expected utility?*
Expected Utility

- Expected utility for player $i$ in a game of imperfect information is what $i$ gets when:
  1. he plays his strategy, and
  2. the other players do what $i$ expects them to do.
- No universal notion. Depends on: properties of $i$, properties that $i$ ascribes to the other players.
- Our example is easy: the outcome does not depend on Bill.
- A reasonable choice in our case: Ann checks for each choice of envelope what she can expect (given what she knows). She chooses randomly among those envelopes with the highest payoff.
The value of “Envelope 1 is empty”

After learning that envelope 1 is empty, Ann has two possibilities:

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- Before his announcement, Bill could expect 3
- Now, Bill expects \((3 + 6)/2 = 4\frac{1}{2}\).
- The value (to Bill) of “envelope 1 is empty” is \(1\frac{1}{2}\).
Another option for Bill

- Bill to Ann “The last envelope is not empty”
- This is true.
- If Ann believes him, she retains 4 possibilities:

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- Now, Ann's dominating strategy is “Choose envelope 3”
• Bad for Ann, good for Bill. Ann gets nothing, Bill gets 6. *This is just as good as lying.*

• (Moral: it can be bad to get true information)
Conclusions

• Sketched a formal framework for studying communication in games based on existing tools: Game Theory + Dynamic Epistemic Semantics

• Provides Game Theory with a theory of epistemic actions.

• Provides epistemic logic with a *pragmatics*
Future work

Specific 'pragmatic' questions:

- What should you believe? What is credible?
- What kind of 'Gricean' implicatures does our 'theory' imply?

General questions such as:

- Is it always valuable to communicate in cooperative games?
- Is it always costly to communicate in zero-sum games?
- (This is a series of questions, with answers depending on properties of players and on properties of games, many of which have been studied in a different setting in game theory.)
Another still open question:

- A theory of *Probabilistic Dynamic Epistemic Semantics*
Credibility

- Ann does not trust Bill to speak the truth.

- She does not believe him if he says that envelope 3 contains 3 euro for her. (And rightly so)

- Symmetrically, she does not believe him when he says (truthfully) that the second envelope has 3 for her.

- But if Bill says that envelope 1 is empty, she should believe him. Why? *he has no conceivable reason to lie*

- Bill saying that envelope 1 is empty is *credible*
Credibility

(Aumann, Farrell, Robert van Rooy)

- The speaker has a reason to say that $\phi$ iff (he knows that) $\phi$ has a positive value
- The speaker has a reason to lie iff (he knows that) $\phi$ has a positive value for him, but $\phi$ is false.
- $\phi$ is self-signaling iff the speaker has no conceivable reason to lie about $\phi$
- $\phi$ is optimal if there is no $\psi$ with a higher value
Credibility

When should the hearer believe the speaker?

- **intuition:** $\phi$ is *credible* iff
  
  1. $\phi$ is self-signaling: the speaker has no conceivable reason to lie about $\phi$
  
  [2. The speaker has a reason to say that $\phi$]

- **definition:** An announcement $\phi$ of player $i$ to player $j$ is *credible* to player $j$ in game $G$ iff it holds that in all games that $j$ considers possible:
  
  1. if $\phi$ has a positive value for $i$, then $\phi$ is true.
  
  [2. if $\phi$ is true, then $\phi$ has a positive value for $i$]
What should Ann believe

- Should Ann believe Bill if what he says is credible?
- Credible = guaranteed to be true, so yes.
- But Bill will try to mislead Ann to make the wrong choice — using the truth, if he can
- For example, “Envelope 3 is not empty” is credible
- Need something stronger: Bill has no incentives, when saying what he says, to mislead Ann