

Belief change in the Situation Calculus:
a new proposal without plausibility levels

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Introduction

Motivations.

Belief change in the Situation Calculus.

- Scherl and Levesque (AAII 1993)

- simple solution
- no revision

- Shapiro, Pagnuco, Lespérance and Levesque (KR 2000)

- plausibility levels are assigned to situations
- problem to define the plausibility assignement in real applications

- New proposal

- no plausibility levels, revision
- plus
 - multi agents
 - actions are not necessarily public

2. A brief introduction to the Situation Calculus
3. A simple scenario
4. Formalisation
5. Conclusion

2. A brief introduction to the Situation Calculus

First (sometimes second) order classical logic with sorts and equality.

Fluents: 1 argument of the sort situation.

broken(s): in the situation s the toy is broken

position(x, s): in the situation s the toy is in the position x

Non fluents: no argument of the sort situation.

color(x): the color of the toy is x

Situations:

- constants
- terms of the form $do(a, s)$: where a is of the sort action and s is of the sort situation.

broken(do(drop, s₀))

Evolution of the world.

- *drop* causes *broken(s)* to be true

- *repair* causes *broken(s)* to be false

$$\forall s \forall a (a = \textit{drop} \rightarrow \textit{broken}(\textit{do}(a, s)))$$

$$\forall s \forall a (a = \textit{repair} \rightarrow \neg \textit{broken}(\textit{do}(a, s)))$$

If there is no other action that can change the truth value of *broken(s)* we have the

Successor State Axiom (SSA):

$$\forall s \forall a (\textit{broken}(\textit{do}(a, s)) \leftrightarrow a = \textit{drop} \vee \textit{broken}(s) \wedge \neg(a = \textit{repair}))$$

General form:

$$(S_p) \quad \forall s \forall a \forall \vec{x} (p(\vec{x}, \textit{do}(a, s)) \leftrightarrow \Gamma_p^+(\vec{x}, a, s) \vee p(\vec{x}, s) \wedge \neg \Gamma_p^-(\vec{x}, a, s))$$

Beliefs.

The situations play a similar role as possible worlds in Kripke models.

Fluent $K(s', s)$: like an accessibility relation (Scherl and Levesque).

$Knows(\phi, s)$: in the situation s it is believed that ϕ

$$Knows(\phi, s) \stackrel{\text{def}}{=} \forall s'(K(s', s) \rightarrow \phi[s'])$$

Belief change, two questions:

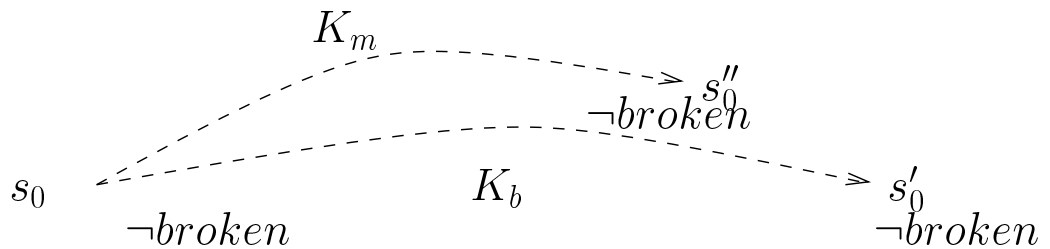
- what are the new accessible situations after performance of an action a ?
- what are the truth values of the fluents in the new accessible situations?

3. A simple scenario

2 agents: a young baby and his mother

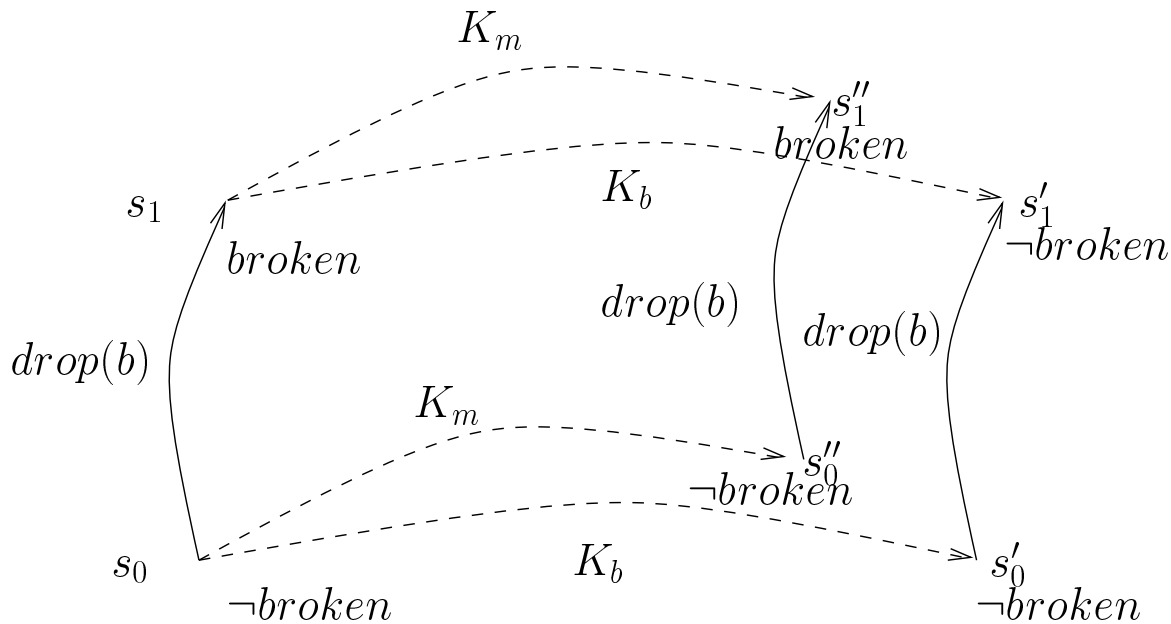
The baby observes every action iff he has his eyes open.

Idem for the mother.



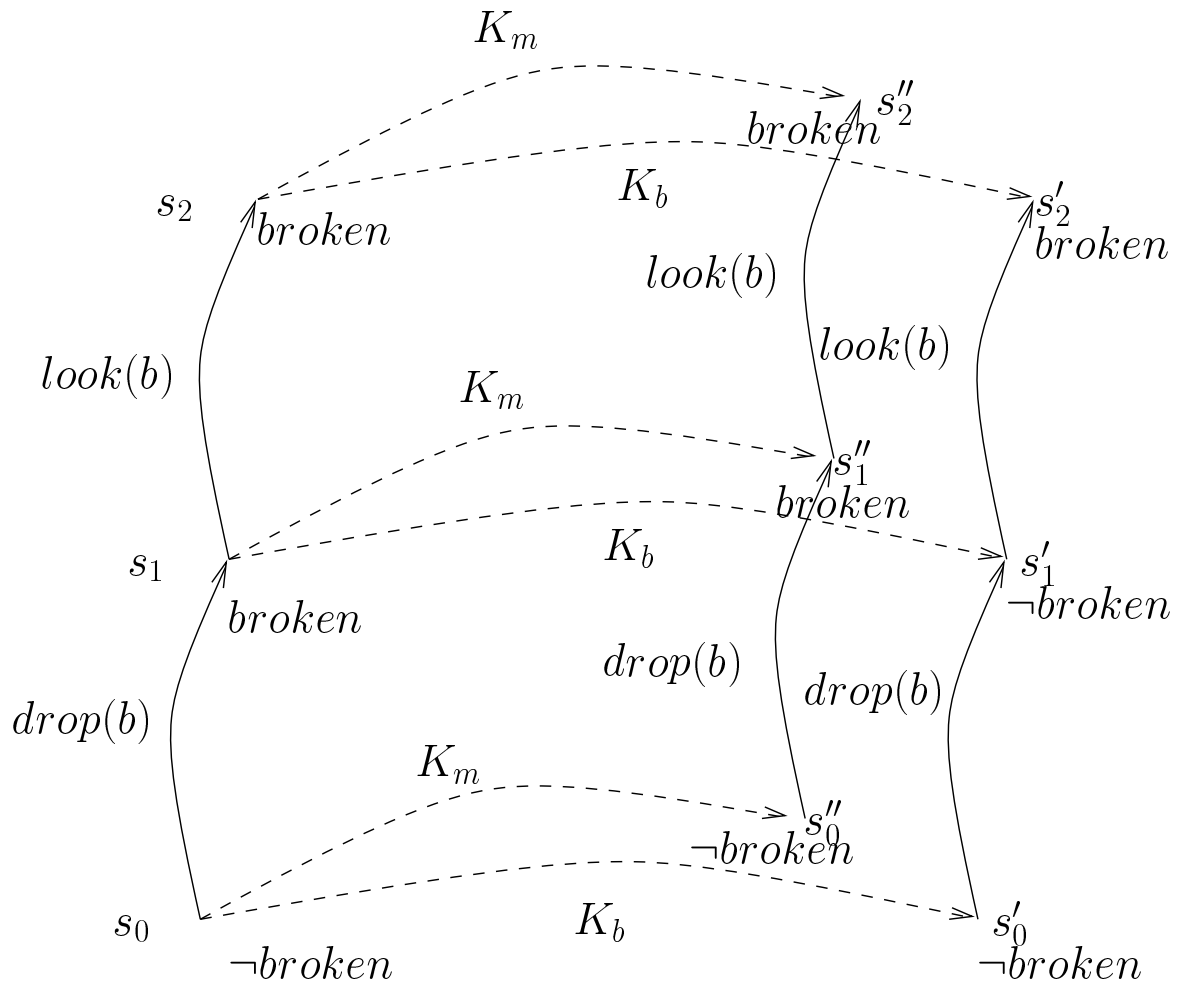
Situation s_0 .

- the toy is not broken
- the mother and the baby believe that the toy is not broken
- they have their eyes open



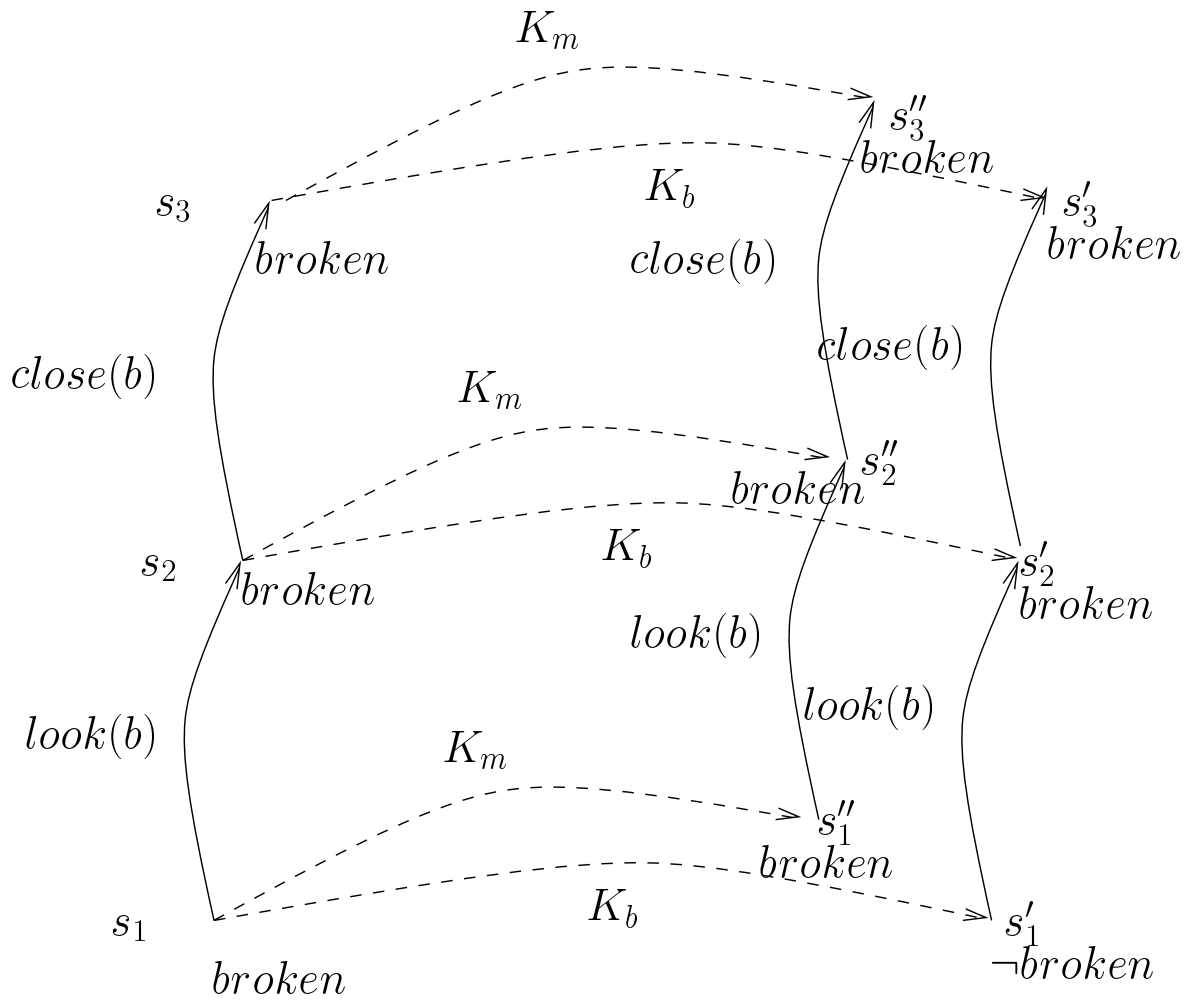
Situation s_1 .

- the baby has dropped the toy (action $drop(b)$)
- the mother believes that a fragile toy will break after it is dropped
 \Rightarrow in s_1 she believes that the toy is broken
- the baby does not believe that dropping a fragile toy will cause it to break
 \Rightarrow in s_1 he believes that the toy is broken



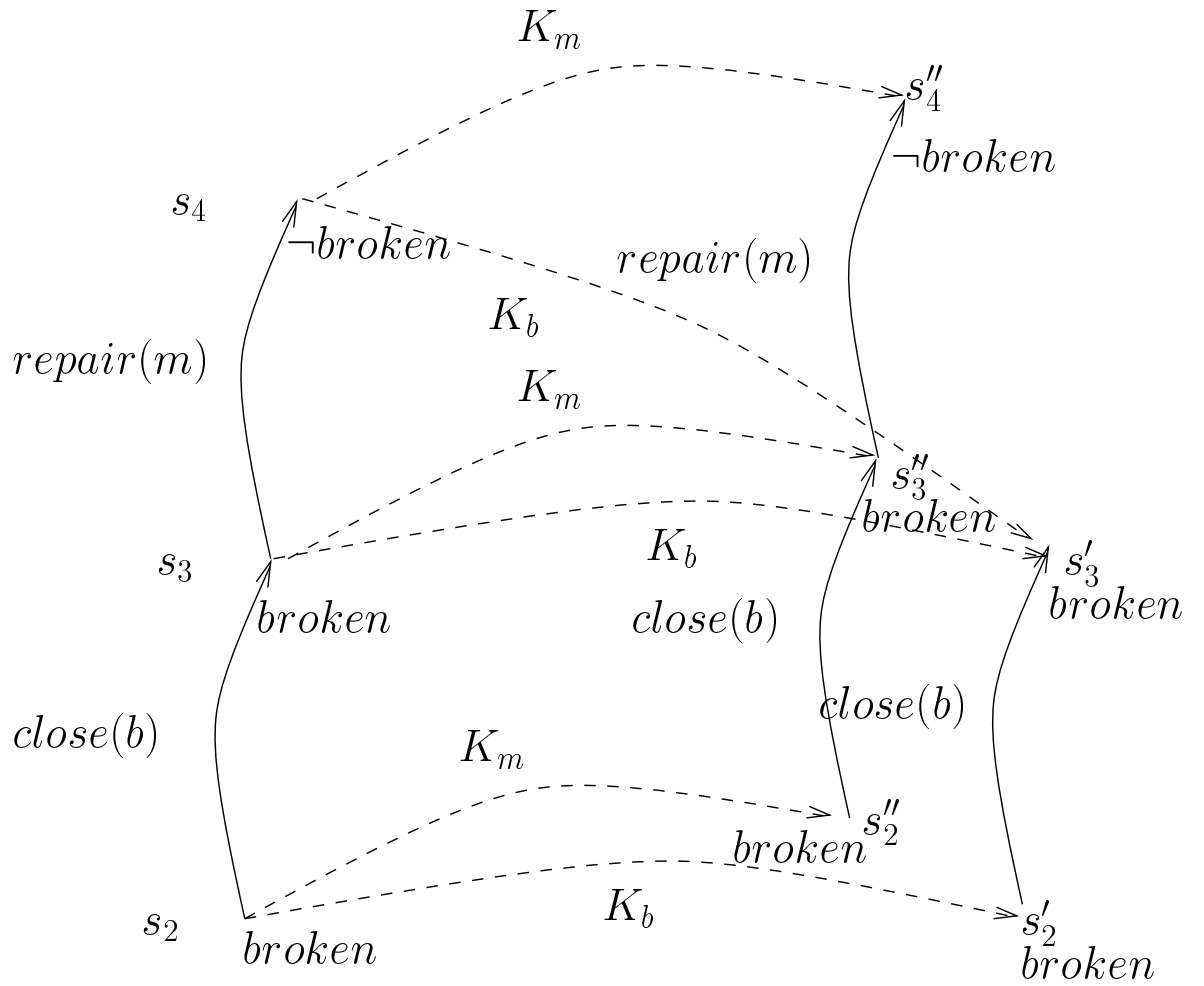
Situation s_2 .

- the baby has looked at the toy (action $look(b)$)
 \Rightarrow in s_2 he believes that the toy is broken
- in s_2 the mother still believes that the toy is broken



Situation s_3 .

- the baby has closed his eyes (action $close(b)$)
- \Rightarrow babys' and mothers' beliefs remain unchanged, except about the fact that $close(b)$ has been performed



Situation s_4 .

- the mother has repaired the toy
- in s_4 the mother believes that the toy is not broken
- in s_4 the baby believes that the toy is still broken
(because he ignores that $repair(m)$ has been performed)

Formalisation.

open(*i*): the agent *i* opens his eyes.

close(*i*): the agent *i* closes his eyes.

drop(*i*): the agent *i* drops the toy.

look(*i*): the agent *i* looks whether the toy is broken.

repair(*i*): the agent *i* repairs the toy.

broken(*s*): in the situation *s* the toy is broken.

real(*s*): *s* is a real situation.

observe(*i*, *a*, *s*): in the situation *s*, if the action *a* is performed, then after performance of *a*, the agent *i* will be informed that *a* has been performed.

K(*i*, *s'*, *s*): in the situation *s* the agent *i* believes that *s'* is a situation where he might be in.

B(*i*, $\phi(s', s)$, *s'*, *s*): $\phi(s', s)$ holds in every situation *s'* related with *i* and *s* by *K*.

$B(i, \phi(s', s), s', s) \stackrel{\text{def}}{=} \forall s'(K(i, s', s) \rightarrow \phi(s', s))$

Evolution of the imaginary situations.

- the accessible situations from $do(a, s)$ are successors of the situations accessible from s , **if the action a has been observed by i** : $observe(i, a, s) \wedge s'' = do(a, s')$

- the accessible situations from $do(a, s)$ are the situations accessible from s , **if the action a has not been observed by i** : $\neg observe(i, a, s) \wedge s'' = s'$

$$(EK) \quad \forall s \forall s'' \forall a \forall i (K(i, s'', do(a, s)) \leftrightarrow \exists s' (K(i, s', s) \wedge ((observe(i, a, s) \wedge s'' = do(a, s')) \vee (\neg observe(i, a, s) \wedge s'' = s'))))$$

The Successor State Axiom of the fluent $observe(i, a, s)$ depends on each application domain.

$$(OBS) \quad \forall s \forall a \forall a' \forall i (observe(i, a, do(a', s)) \leftrightarrow a' = open(i) \vee observe(i, a, s) \wedge \neg(a' = close(i)))$$

The evolution of a fluent (SSA) depends on the **context of the situation** where we are.

Context of real situations.

$$(SSA_r) \quad \forall s \forall a (real(s) \rightarrow (broken(do(a, s)) \leftrightarrow \exists i (a = drop(i)) \vee broken(s) \wedge \neg(\exists i (a = repair(i))))))$$

Context of a mother's imaginary situation.

The toy is broken in $do(a, s')$ iff

- it has been dropped by some agent i , **or**
- the mother has looked at the toy, and the toy is broken in the real situation s , **or**
- the toy was broken and it has not been repaired and it has not been observed that it is not broken.

$$(SSA_m) \quad \forall s \forall s' \forall a (real(s) \rightarrow (K(m, s', s) \rightarrow (broken(do(a, s')) \leftrightarrow \exists i (a = drop(i)) \vee (a = look(m) \wedge broken(s)) \vee broken(s') \wedge \neg(\exists i (a = repair(i)) \vee (a = look(m) \wedge \neg broken(s))))))$$

Context of a baby's imaginary situation.

The SSA is different of the mother's SSA.

$$(SSA_b) \quad \forall s \forall s' \forall a (real(s) \rightarrow (K(b, s', s) \rightarrow broken(do(a, s')) \leftrightarrow (a = look(b) \wedge broken(s)) \vee broken(s') \wedge \neg(\exists i(a = repair(i)) \vee (a = look(b) \wedge \neg broken(s)))))$$

Notations.

$$ssa(b, s', s) \stackrel{\text{def}}{=} broken(do(a, s')) \leftrightarrow (a = look(b) \wedge broken(s)) \vee broken(s') \wedge \neg(\exists i(a = repair(i)) \vee (a = look(b) \wedge \neg broken(s)))$$

$$ssa(m, s', s) \stackrel{\text{def}}{=} broken(do(a, s')) \leftrightarrow \exists i(a = drop(i)) \vee (a = look(m) \wedge broken(s)) \vee broken(s') \wedge \neg(\exists i(a = repair(i)) \vee (a = look(m) \wedge \neg broken(s)))$$

The axioms SSA_b and SSA_m can be reformulated as:

$$(SSA_b) \quad \forall s (real(s) \rightarrow B(b, ssa(b, s', s), s', s))$$

$$(SSA_m) \quad \forall s (real(s) \rightarrow B(m, ssa(m, s', s), s', s))$$

We can have contexts that correspond to nested beliefs.

The mother believes that the baby believes that the toy will not break if it dropped (SSA_{mb}), while she believes that it will break (SSA_m).

$$(SSA_{mb}) \quad \forall s (real(s) \rightarrow B(m, B(b, ssa(b, s'', s'), s'', s'), s', s))$$

Conclusion.

We have given general answers to the two questions:

- what are the new accessible situations after performance of an action a ?
- what are the truth values of the fluents in the new accessible situations?

Observation effects are incorporated into the SSAs.

No plausibility levels.

The SSAs depend on the context (agents' subjective view).

The actions that can be observed by each agent are defined by the SSA of $observe(i, a, s)$.

We have started to investigate progression and regression.

Price to pay:

- integration of observation actions into the SSAs make them more complex
- more SSAs to model an application domain.