Modal logics: overview

- Part I: Introduction to modal and multimodal logics
  1. Motivation and introduction
  2. The basic multimodal logic *K*
  3. The basic monomodal logics
  4. Completeness of $G(k, l, m, n)$ logics, and decidability of the basic modal logics
  5. Basic multimodal logics
  6. Other modal logics
- Part II: Applications
  7. Knowledge and announcements
  8. Belief
  9. Common knowledge and common belief
  10. Action and propositional dynamic logic
  11. Goals and intentions
  12. **Ability, agency and branching time**
- Part II: Proof methods
  13. Translation method
  14. Tableau method

Logics of agency and their epistemic extensions

Andreas Herzig

Nov., 2008

Coalition logic: overview

- ‘to be agentive’:
  - relation $\subseteq$ *Agts* $\times$ *Fmls*
    (vs. *Acts* $\times$ *Fmls* as in logics of action, e.g. *PDL*)
- in particular: agent capability
- Coalition Logic
  - non-normal modal logic
  - related to Alternating-time Temporal Logic (ATL)

Coalition logic: motivation

- reason about *abilities* of autonomous agents
  - individuals
  - groups
- applications
  - multiagent systems
  - social software, mechanism design
- logical form:
  - J subset of a given set of agents Agt
  - $\langle\!\![J]\!\!\rangle \mathbf{X}\varphi$ = "group J can make φ true"
    = "coalition J can ensure that φ at next time point, whatever the agents outside J do"

Language of coalition logic

- BNF:

$$\varphi ::= p \mid \neg\varphi \mid \varphi \vee \varphi \mid \langle\![J]\!\rangle \mathbf{X}\varphi$$

  - agents $i, j, \ldots \in Agt = \{1, 2, \ldots\}$, finite
  - coalitions $J \subseteq Agt$
  - $\langle\![J]\!\rangle \mathbf{X}\varphi$ = "coalition J can ensure that φ at next time point, whatever the agents outside J do"
    = $\exists$ choices of J, such that $\forall$ next states φ holds
- $\langle\![\emptyset]\!\rangle \mathbf{X}\varphi$ = "φ cannot be avoided"
  = "φ is necessarily true at next point"
- Remarks on notation
  - 'fusion' of 3 modalities;
    - Pauly: $[J]\varphi$; here reserved for normal modal operators
  - convention: $\langle\![i, j]\!\rangle \mathbf{X}\varphi = \langle\![\{i, j\}]\!\rangle \mathbf{X}\varphi$

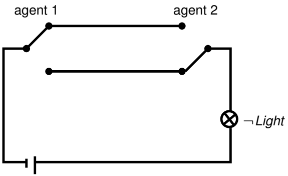
One lamp, one agent, one switch

- single agent 1 ('Ann'): $Agt = \{1\}$
- 1 has two actions: toggle the switch, and skip (= do nothing)
- light is off

$\Rightarrow$ intended conclusion: 1 controls *Light*

- 1 can make that the light is on next: $\langle\!\langle 1 \rangle\!\rangle \mathbf{X} Light$
- 1 can make that the light is off next: $\langle\!\langle 1 \rangle\!\rangle \mathbf{X} \neg Light$

One lamp, two agents, two toggle switches



- two agents 1 ('Ann') and 2 ('Bill'), and two toggle switches
- 1 has two actions: toggle1 and skip1
- 2 has two actions: toggle2 and skip2
- light is off

$\Rightarrow$ intended conclusions:

- neither 1 nor 2 alone can make that the light is on:
  $\neg\langle\!\langle 1\rangle\!\rangle \mathbf{X} \mathit{Light} \wedge \neg\langle\!\langle 2\rangle\!\rangle \mathbf{X} \mathit{Light}$
- ... but the **coalition** of 1 and 2 can:
  $\langle\!\langle 1, 2\rangle\!\rangle \mathbf{X} \mathit{Light}$

mechanism design: Bonn vs. Berlin [Pauly 2001]

After the "German Question" had been solved, on June 20th, 1991, the German parliament was faced with the Berlin question: Should the German parliament and the seat of government move to Berlin or stay in Bonn? In this historic debate, the parliament was very divided and this division ran through all parties. About 100 speeches were made while another 100 speeches were placed on record. [. . .]
In its debate, the German parliament considered 5 different motions, the 3 central ones being:

- p_1 *= parliament and government move to Berlin*
- p_2 *= parliament moves to Berlin but government remains in Bonn*
- p_3 *= both parliament and government remain in Bonn*

[. . .] Since there were more than 2 motions up for vote, the parliamentary council of elders first had to decide on a voting procedure.

Bonn vs. Berlin (2) [Pauly 2001]

- initially: $p_1 \wedge p_2 \wedge p_3$
- aim: design a procedure warranting a democratic decision
  - effective procedure: no eternal voting
  - fair procedure: moves are brought about by majorities
  - ...
- requirements engineering:
  - use majority voting
  - eliminate step by step some alternative(s) until only a single remains

Bonn vs. Berlin (3) [Pauly 2001]

- list of requirements:
  1. alternatives which have been eliminated remain eliminated:
     $$\neg p \rightarrow \langle\!\![\emptyset]\!\!\rangle \mathbf{X} \neg p \text{ for all atomic } p$$
  2. at every stage at least one alternative has to be eliminated:
     $$\bigwedge_{\delta \in Sit} (\delta \rightarrow \langle\!\![\emptyset]\!\!\rangle \mathbf{X} \neg \delta)$$
     - ★ where $Sit = \{l_1 \wedge l_2 \wedge l_3 : l_i = p_i \text{ or } l_i = \neg p_i\} \setminus \{\neg p_1 \wedge \neg p_2 \wedge \neg p_3\}$
  3. each vote must be a democratic majority vote:
     $$\langle\!\![Agt]\!\!\rangle \mathbf{X} \varphi \leftrightarrow \langle\!\![J]\!\!\rangle \mathbf{X} \varphi \text{ for every } J \text{ with } |J| > \frac{1}{2} \times |Agt|$$
     - ★ simplify: suppose $|Agt|$ is odd
     - ★ formula schema
  4. each vote between two alternatives only:
     $$(\langle\!\![Agt]\!\!\rangle \mathbf{X}(\varphi \wedge \psi) \wedge \langle\!\![Agt]\!\!\rangle \mathbf{X}(\neg\varphi \wedge \psi)) \rightarrow \langle\!\![\emptyset]\!\!\rangle \mathbf{X} \psi$$
     - ★ if s has two different successors then these are the only ones
     - ★ formula schema

Bonn vs. Berlin (4) [Pauly 2001]

- once we have completed the list of requirements:
  - are the requirements consistent?
  - is there a model M where
    - requirements (1)-(4) are valid, and
    - there is a state where $p_1 \wedge p_2 \wedge p_3$ holds?
- solution 1 (adopted by council of elders)
  1. majority vote whether p_2 or $\neg p_2$
  2. if $\neg p_2$ then majority vote whether p_1 or p_2

  $\Rightarrow$ not all 3 alternatives considered equally
  - p_2 stands alone against p_1 and p_3
- solution 2
  1. ...
  2. ...
  3. ...
  4. ...

  satisfies requirement "eliminate only one option per step"

$$(p \wedge q) \rightarrow \langle\!\langle \emptyset \rangle\!\rangle \mathbf{X}(p \vee q) \quad \text{for atomic } p \neq q$$

Coalition logic in a nutshell

semantics? axiomatics? decidability? complexity?

Coalition logic in a nutshell

semantics? axiomatics? decidability? complexity?

- 3 different disciplines ⇒ 3 different semantics

game theory	⇒	multiplayer games
TCS	⇒	Alternating Transition Systems ATS
logic/economy	⇒	effectivity functions

- single axiomatics
  - complete for each of the 3 semantics
- satisfiability problem: PSPACE-complete

Overview

CL axiomatics

CL axiomatics [Pauly 2001, 2002]

(RE) if $\varphi \leftrightarrow \psi$ then $\langle\![J]\!\rangle \mathbf{X}\varphi \leftrightarrow \langle\![J]\!\rangle \mathbf{X}\psi$

(M) $\langle\![J]\!\rangle \mathbf{X}(\varphi \wedge \psi) \rightarrow (\langle\![J]\!\rangle \mathbf{X}\varphi \wedge \langle\![J]\!\rangle \mathbf{X}\psi)$

($\bot$) $\neg \langle\![J]\!\rangle \mathbf{X}\bot$

($\top$) $\langle\![J]\!\rangle \mathbf{X}\top$

(Agt) $\neg \langle\![\emptyset]\!\rangle \mathbf{X}\varphi \rightarrow \langle\![Agt]\!\rangle \mathbf{X}\neg\varphi$

(S) for $J_1 \cap J_2 = \emptyset$:

$$(\langle\![J_1]\!\rangle \mathbf{X}\varphi_1 \wedge \langle\![J_2]\!\rangle \mathbf{X}\varphi_2) \rightarrow \langle\![J_1 \cup J_2]\!\rangle \mathbf{X}(\varphi_1 \wedge \varphi_2)$$

- (S) = superadditivity

CL axiomatics: necessitation is derivable

- if $\vdash \varphi$ then $\vdash \langle\![J]\!\rangle \mathbf{X}\varphi$

CL axiomatics: necessitation is derivable

- if $\vdash \varphi$ then $\vdash \langle\!\langle J \rangle\!\rangle \mathbf{X}\varphi$
  1. $\vdash \varphi$ (hypothesis)
  2. $\vdash \top \leftrightarrow \varphi$ from 1.
  3. $\vdash \langle\!\langle J \rangle\!\rangle \mathbf{X}\top \leftrightarrow \langle\!\langle J \rangle\!\rangle \mathbf{X}\varphi$ from 2. by (RE)
  4. $\vdash \langle\!\langle J \rangle\!\rangle \mathbf{X}\top$ ($\top$)
  5. $\vdash \langle\!\langle J \rangle\!\rangle \mathbf{X}\varphi$ from 3. and 4.

CL axiomatics: $\langle\![\emptyset]\!\rangle\mathbf{X}$ is a normal box

- $\langle\![\emptyset]\!\rangle\mathbf{X}$ is a normal '$\Box$':
  - $\vdash (\langle\![\emptyset]\!\rangle\mathbf{X}\varphi_1 \wedge \langle\![\emptyset]\!\rangle\mathbf{X}\varphi_2) \to \langle\![\emptyset]\!\rangle\mathbf{X}(\varphi_1 \wedge \varphi_2)$

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  - proof: apply (S)
- $\langle\![Agt]\!\rangle \mathbf{X}$ is a normal '$\Diamond$':

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  - proof: apply (S)
- $\langle\![Agt]\!\rangle \mathbf{X}$ is a normal '$\Diamond$':
  - proof: use duality of $\langle\![Agt]\!\rangle \mathbf{X}$ and $\langle\![\emptyset]\!\rangle \mathbf{X}$ ...
- $\langle\![Agt]\!\rangle \mathbf{X}$ and $\langle\![\emptyset]\!\rangle \mathbf{X}$ are dual:
  - $\vdash \neg\langle\![Agt]\!\rangle \mathbf{X}\neg\varphi \leftrightarrow \langle\![\emptyset]\!\rangle \mathbf{X}\varphi$

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  - proof:
    1. $\vdash \neg\langle\!\![Agt]\!\!\rangle \mathbf{X}\neg\varphi \to \langle\!\![\emptyset]\!\!\rangle \mathbf{X}\varphi$ — by (*Agt*)
    2. $\vdash \langle\!\![\emptyset]\!\!\rangle \mathbf{X}\varphi \land \langle\!\![Agt]\!\!\rangle \mathbf{X}\neg\varphi \to \langle\!\![Agt]\!\!\rangle \mathbf{X}\bot$ — by (S)

CL axiomatics: a consequence of superadditivity

- if $J_1 \cap J_2 = \emptyset$ then:

$$\vdash (\langle\![J_1]\!\rangle \mathbf{X}\varphi \wedge \langle\![J_2]\!\rangle \mathbf{X}\neg\varphi) \to \bot$$

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- proof: suppose $J_1 \cap J_2 = \emptyset$
  1. $\vdash \bot \leftrightarrow (\varphi \land \neg\varphi)$
  2. $\vdash \langle\!\![J_1 \cup J_2]\!\!\rangle \mathbf{X}\bot \leftrightarrow \langle\!\![J_1 \cup J_2]\!\!\rangle \mathbf{X}(\varphi \land \neg\varphi)$ from 1. by (RE)
  3. $\vdash \neg\langle\!\![J_1 \cup J_2]\!\!\rangle \mathbf{X}\bot$ $(\bot)$
  4. $\vdash \neg\langle\!\![J_1 \cup J_2]\!\!\rangle \mathbf{X}(\varphi \land \neg\varphi)$ from 2. and 3.
  5. $\vdash (\langle\!\![J_1]\!\!\rangle \mathbf{X}\varphi \land \langle\!\![J_2]\!\!\rangle \mathbf{X}\neg\varphi) \to \langle\!\![J_1 \cup J_2]\!\!\rangle \mathbf{X}(\varphi \land \neg\varphi)$ (S)
  6. $\vdash (\langle\!\![J_1]\!\!\rangle \mathbf{X}\varphi \land \langle\!\![J_2]\!\!\rangle \mathbf{X}\neg\varphi) \to \bot$ from 1. and 4.

CL axiomatics: $\langle\!\![J]\!\!\rangle \mathbf{X}$ is non-normal (in general)

- $\vdash (\langle\!\![J_1]\!\!\rangle \mathbf{X}\varphi \wedge \langle\!\![J_2]\!\!\rangle \mathbf{X}\neg\varphi) \to \bot$ counterintuitive if $J_1 \cap J_2 \neq \emptyset$
  - in the “one agent, one switch” example:
    $\langle\!\![1]\!\!\rangle \mathbf{X}\mathit{Light} \wedge \langle\!\![1]\!\!\rangle \mathbf{X}\neg\mathit{Light}$ should be consistent!

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  - in the "one agent, one switch" example:
    $\langle\!\![1]\!\!\rangle \mathbf{X}\mathit{Light} \land \langle\!\![1]\!\!\rangle \mathbf{X}\neg\mathit{Light}$ should be consistent!
- more generally: $\langle\!\![J]\!\!\rangle \mathbf{X}$ is a non-normal modality
  - $\not\vdash (\langle\!\![J]\!\!\rangle \mathbf{X}\varphi_1 \land \langle\!\![J]\!\!\rangle \mathbf{X}\varphi_2) \to \langle\!\![J]\!\!\rangle \mathbf{X}(\varphi_1 \land \varphi_2)$
  - how can we prove this?
    $\Rightarrow$ semantics

Semantics 1: effectivity functions and neighborhood models

Semantics 1: effectivity functions

- S = set of states
- effectivity function $e : 2^{Agt} \longrightarrow 2^{2^S}$
- idea: $X \in e(J)$ iff ...
  - "$X \subseteq S$ is a set of possible outcomes for which J is effective"
  - "J can force the world to be in some state of X at the next step"
- $e(\emptyset)$ = set of all possible outcomes

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- lamp-examples: $S = \{s_D, s_L\}$
  - one agent, one switch:

$$e(1) = \{\{s_D\}, \{s_L\}, \{s_D, s_L\}\}$$

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  - one agent, one switch:

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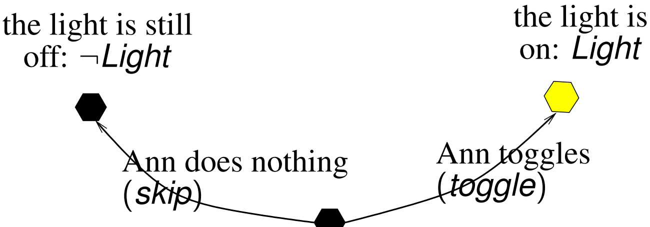
  - two agents, two toggle switches:

$$e(\emptyset) = e(\{1\}) = e(\{2\}) = \{\{s_D, s_L\}\}$$

$$e(\{1, 2\}) = \{\{s_D\}, \{s_L\}, \{s_D, s_L\}\}$$

Running example



the light is off: $\neg Light$

Ann can switch it or not: $\langle\!\langle Ann \rangle\!\rangle \mathbf{X} Light \wedge \langle\!\langle Ann \rangle\!\rangle \mathbf{X} \neg Light$

Semantics 1: *playable* effectivity functions

- e is **playable** iff
  1. $\emptyset \notin e(J)$
  2. $S \in e(J)$;
  3. if $S \setminus X \notin e(\emptyset)$ then $X \in e(Agt)$, for all $X \subseteq S$ (*Agt*-maximality)
  4. if $X \in e(J)$ then $X \cup X' \in e(J)$ (outcome monotonicity)
  5. if $J_1 \cap J_2 = \emptyset$ then

     if $X_1 \in e(J_1)$ and $X_2 \in e(J_2)$ then $X_1 \cap X_2 \in e(J_1 \cup J_2)$

     (superadditivity)

Semantics 1: neighborhood models

- idea: associate a playable effectivity function to every state

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- idea: associate a playable effectivity function to every state
- **neighborhood model** $M = ((S, E), V)$, where:
  - S = set of states;
  - $E : S \longrightarrow (2^{Agt} \longrightarrow 2^{2^S})$ such that

    every E_s is a playable effectivity function
  - $V : Atm \longrightarrow 2^S$ valuation function

Semantics 1: truth conditions

- truth conditions:

  $M, s \models p$ iff $p \in V(s)$

  $M, s \models \langle\!\![J]\!\!\rangle \mathbf{X}\varphi$ iff $\{s' \mid M, s \models \varphi\} \in E_s(J)$
- validity, satisfiability: as usual

Theorem

CL axiomatization is complete w.r.t. neighborhood semantics

Semantics 2: Alternating Transition Systems (ATS)

Semantics 2: subadditive effectivity functions

- a playable effectivity function e is **subadditive** iff for all J_1, J_2 such that $J_1 \cap J_2 = \emptyset$:

$$e(J_1 \cup J_2) = \{X_1 \cap X_2 : X_1 \in e(J_1) \text{ and } X_2 \in e(J_2)\}$$

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- subadditive effectivity functions: group effectivity determined by individual effectivity
  - if $J = \{1, \dots, n\}$:
    $e(J) = \{X_1 \cap \ldots \cap X_n : X_1 \in e(1), \dots, X_n \in e(n)\}$

Semantics 2: the client-server example

- lamp-examples satisfy subadditivity
- client-server example [Jamroga&Goranko 2004] doesn't:
  - *client*: can request *server* to set variable to 0 or to 1
  - *server*: can accept or reject *client*'s request
  - $S = \{s_0, s_1\}$
    - ★ s_0: variable is 0;
    - ★ s_1: variable is 1
  - initially in s_0
  - $e_{s_0}(\{client\}) = e_{s_0}(\{server\}) = \{\{s_0\}, \{s_0, s_1\}\}$
    $e_{s_0}(\{client, server\}) = \{\{s_0\}, \{s_1\}, \{s_0, s_1\}\}$

. . . at least in this form

Semantics 2: subadditive version of client-server example

- can the client-server example be modelled by a subadditive&playable effectivity function e?
  - $S = \{s_0, s'_0, s_1, s'_1\}$
  - $e_{s_0}(\{server\}) = e_{s'_0}(\{server\}) = \{\{s_0\}, \{s'_0, s'_1\}\}$
    $e_{s_1}(\{server\}) = e_{s'_1}(\{server\}) = \{\{s_1\}, \{s'_0, s'_1\}\}$
    $e_{s_0}(\{client\}) = e_{s'_0}(\{client\}) = \{\{s_0, s'_0\}, \{s_0, s'_1\}\}$
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Semantics 2: Alternating Transition Systems ATS

- neighborhood models $M = ((S, E), V)$ where every E_s is a subadditive playable effectivity function
- alternative presentation:
  Alternating Transition Systems (ATS) $M = ((S, E), V)$ where
  - S and V as usual
  - $E : S \longrightarrow (Agt \longrightarrow 2^{2^S})$ such that
    - ★ $E_s(i) \neq \emptyset$
    - ★ if $X_1 \in E_s(1), \ldots, X_n \in E_s(n)$ then $X_1 \cap \ldots \cap X_n$ singleton
    - ★ $E_s(i)$ not necessarily closed under supersets
  - $E_s(J)$ defined from $\{E_s(i) : i \in J\}$
  - $M, s \models \langle\!\langle J \rangle\!\rangle \mathbf{X}\varphi$ iff there is $X \in E_s(J)$ s.th. for all $s' \in X$, $M, s' \models \varphi$
- semantics of Alternating-time Temporal Logic (ATL)
  - extension of CL by LTL operators: $\langle\!\langle J \rangle\!\rangle \mathbf{G}\varphi$, $\langle\!\langle J \rangle\!\rangle(\varphi \mathbf{Until} \psi)$
  - ATL = extension of CTL: $\exists \mathbf{X}\varphi = \langle\!\langle Agt \rangle\!\rangle \mathbf{G}\varphi$; $\forall \mathbf{X}\varphi = \langle\!\langle \emptyset \rangle\!\rangle \mathbf{G}\varphi$;

Theorem

CL axiomatization is complete w.r.t. ATS-models

Semantics 3: game frames

Semantics 3: strategic games

- **strategic game** $G = (S, \{\Sigma_i\}_{i \in Agt}, o)$ where
  - S = set of states
  - Σ_i = nonempty set of *choices* ('action tokens') of agent $i \in Agt$
  - $o : (\prod_{i \in Agt} \Sigma_i) \longrightarrow S$ *outcome function*
    - every $\sigma \in \prod_{i \in Agt} \Sigma_i$ is a *choice profile* = combination of agents' choices
    - $o(\sigma)$ = unique outcome when all agents simultaneously perform 'their' action in σ
- **game model**: associate a strategic game to every $s \in S$

Theorem

CL axiomatization is complete w.r.t. game models

Semantics 3: example of strategic game

- two agents, two toggle switches; initially dark
  - $S = \{s_D, s_L\}$
  - $\Sigma_1 = \{skip_1, toggle_1\}$
    $\Sigma_2 = \{skip_2, toggle_2\}$
  - $o(\langle skip_1, skip_2 \rangle) = s_D$
    $o(\langle skip_1, toggle_2 \rangle) = s_L$
    $o(\langle toggle_1, skip_2 \rangle) = \ldots$
    ...

Semantics 3: link with effectivity functions

- given: strategic game G
- effectivity function e_G defined by: $X \in e_G(J)$ iff

there is $\sigma_J \in \prod_{i \in J} \Sigma_i$ such that for every $\sigma_{\overline{J}} \in \prod_{i \in \overline{J}} \Sigma_i$: $o(\sigma_J \times \sigma_{\overline{J}}) \in X$

- observe: e_G is playable

$\Rightarrow$ also works the other way round:

Theorem (Pauly 2001, 2002)

An effectivity function e is playable iff it is the effectivity function of some strategic game.

Adding knowledge to CL

How to combine CL and epistemic logic?

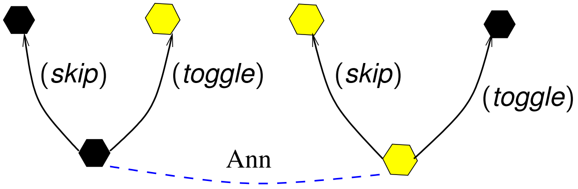
- add indistinguishability relations to neighborhood models
- cf. Alternating-time Temporal Epistemic Logic (ATEL) [v.d.Hoek&Wooldridge 2002]
- E-CL model $M = ((S, E, \mathcal{R}), V)$, where
  - $((S, E), V)$ is a neighborhood model
  - $\mathcal{R} : Agt \longrightarrow S \times S$ such that every $\mathcal{R}_i$ is an equivalence relation
- BNF:

$$\varphi ::= p \mid \neg\varphi \mid \varphi \vee \varphi \mid \langle\!\![J]\!\!\rangle \mathbf{X}\varphi \mid K_i\varphi$$

- logic E-CL = fragment of ATEL

Adding uncertainty (blind Ann)

- As previously, the light is off: $\neg Light$
- Ann is blind and cannot distinguish a world where the light is on from a world where the light is off
- $K_{Ann}\langle\!\![Ann]\!\!\rangle \mathbf{X} Light$ = "Ann knows she can achieve *Light*"



$K_{Ann}\langle\!\![Ann]\!\!\rangle \mathbf{X} li$

Uniform strategies

Definition (Uniform strategy)

A strategy is called *uniform* only if the same choice applies at every indistinguishable moment. ■

- Can we express that Ann does not know how to achieve that the light is on? Can we express that Ann has no uniform strategy? — NO!
- solution: history-semantics

History-based semantics (sketch)

- model $M = (S, \mathcal{H}, \{\Sigma_i\}_{i \in Agt}, o, \mathcal{R}, V)$ where
  - S = set of states
  - $\mathcal{H}$ set of *histories* = sequences of states $(s_1, s_2, \ldots)$
  - Σ_i = nonempty set of actions of agent $i \in Agt$
  - $o : S \times (\prod_{i \in Agt} \Sigma_i) \longrightarrow S$ outcome function
    - every $\sigma \in \prod_{i \in Agt} \Sigma_i$ is a *choice profile* = combination of agents' choices
    - $o(\sigma)$ = unique outcome when all agents simultaneously perform 'their' action in σ
  - $\mathcal{R} : Agt \longrightarrow (\mathcal{H} \times S) \times (\mathcal{H} \times S)$ such that every $\mathcal{R}_i$ is an equivalence relation
  - V valuation

History-based semantics: the lamp example

- $S = \{s_{D,t}, s_{D,s}, s_{L,t}, s_{L,s}\}$
- $M, s_{D,t} \models \neg light \wedge \mathbf{X} light$: a_1 is going to toggle in $s_{D,t}$
  $M, s_{D,s} \models \neg light \wedge \mathbf{X} \neg light$: a_1 is going to skip in $s_{D,t}$
  $M, s_{L,t} \models light \wedge \mathbf{X} \neg light$: a_1 is going to toggle in $s_{L,t}$
  ...
- equivalence classes of $\mathcal{R}_{Ann}$: $\{s_{D,t}, s_{L,t},\}$ and $\{s_{D,s}, s_{L,s}\}$
- language = STIT logic of agency (Belnap, Horty et al.):
  $[i]\varphi$ = "agent i sees to it that φ"
  - $\langle\!\langle J \rangle\!\rangle \mathbf{X}\varphi = \langle \emptyset \rangle [J] \mathbf{X}\varphi$
- papers:
  - (Herzig&Troquard AAMAS'06): sketch of solution, link to STIT
  - (Broersen, Herzig&Troquard TARK'07): 'Normal Coalition Logic'
  - (Balbiani, Herzig&Troquard JPL'08): STIT logic decidable if no coalitions (EXPTIME complete)
  - (Herzig&Schwarzentruber AiML'08): STIT logic undecidable
  - (Herzig&Lorini, JoLLI to appear): relation with PDL

Game over

Summary

- CL as a logic of ability
- 1 axiomatics and 3 different semantics
- adding epistemics to CL is not so straightforward