

Positive and negative evidence¹

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Abstract : We take the opportunity of the retirement of our colleague Phil Halliday alias Smets, to suggest a possibilistic counterpart of his method of analysis and interpretation of inconsistent clues, based on confidence and diffidence components.

1. Greetings

- T(p) &T() : Hello. We are Thompson and Thomson from the famous secret police agency named IRIT (Information Recovering and Investigation from Testimonies)².

- T(p) : I am Thompson (with a 'p').

- T() : To be precise, without the absence of a 'p' ! I am Thomson, with the absence of a 'p'.

- T(p) : You mean, without the presence of a 'p' !

We are glad to be here for celebrating our colleague and old friend Bill Halliday.

- T() : To be precise, Phil Halliday. Yes Halliday is his true name. Smets is his secret police agent's name. His name is attached to the famous Peter, Paul and Mary case among others ! It was a real TBM, a Totally Baffling Mystery !

- T(p) : SMETS is just a fakir, no, a fake name.

- T() : Yes, SMETS means Sacred, no, Secret Mister...

- T(p) : To be precise Secret Master of Evidence Transmitted...

- T() : You mean 'Transferred Scientifically'

- T(p) : Yes, P. SMETS is just short for 'Pignistic and Secret Master of Evidence Transferred Scientifically' !

- T() : What do you mean by 'Big Mystic Master' ? I was thinking he is rather a 'Procrustean Master'!

- T(p) : No ! In the police, we are no longer cutting feet to get full confessions ! We have much better methods...

- T() : for confounding innocents who are guilty... no, culprits who pretend to be guilty, I mean, innocent.

¹ Provided on April 23rd, 1999.

² Formerly named 'Information Rewriting and Interpolation from Testimonies', which was too explicit.

2. Smets' method for confounding the truth

- T(p) : Indeed Smets' TBM³ [8], [9], I mean Transferable Bayesian Model, is most famous.

- T() : No Bayesians are untransferable. No hope !

- T(p) : Right ! To be precise, his Transferable Belief Model offers an appropriate framework for quantifying beliefs, ... I mean beliefs.

- T() : Yes, reasons to believe have their weights summed up. But sometimes reasons to believe may be misleading. Remember the Castafiore Jewels' affair [6] ! What about reasons NOT to believe a statement ? Do they provide a mean for avoiding wrong conclusions ?

- T(p) : To be or not to be...lieve is indeed our motto !

- T() : To be precise, or to be not precise, our motto is 'to believe or not to believe'.

- T(p) : By having reasons not to believe a statement, you mean that the opposite statement is plausible ?

- T() : Not exactly. $Pl(\text{non } A) = 1 - Bel(A)$. So it's the same information

- T(p) : I agree. So you mean $Bel(\text{not } A) > 0$?

- T() : No. That's just quantifying the reasons to believe 'non A' !

Our colleague Halliday, I mean Smets, has a nice proposal for representing the reasons NOT to believe [10].

- T(p) : Yes, he is rather a gothic, I mean agnostic (but he made a True Big Mystic transformation).

- T() : It's not the point. He starts with the Ukalvia case. Do you remember ?

- T(p) : You mean the Syldavia case ?

- T() : No, the Ukalvia case. Here is the story.

You are said that a newspaper reports that the economic situation was good last year in Ukalvia.

It's all you know about Ukalvia, so you start to believe the information, let's call it 'G', to some extent.

- T(p) : You mean $Bel(G) > 0$, where Bel stands for belief, and not for Belgium !

- T() : Exactly. But later you learn that the information is originated from the newspaper of the unique authorized party in Ukalvia.

- T(p) : So you start to believe that it is propaganda. To be precise that the economic situation was not good at all.

- T() : Not necessarily. I just want to come back to the state of total ignorance about the state of the economic situation in Ukalvia.

- T(p) : So the idea is that the reasons not to believe A can somewhat inhibit the reasons to believe A, if any.

³ Don't confuse the W(eighted) TBM which does not exist with a T(rue) BMW which is just faster !

- T() : I see. But take the Ukalvia example. There are two possibilities. Either the situation is good (G) or it is good, I mean bad.

- T(p) : You mean not good ($\neg G$). Let X be the frame of discernement. $X = \{G, \neg G\}$. So $\text{Bel}(G) > 0$ is represented by the mass function

$$m(G) = \alpha ; m(X) = 1 - \alpha$$

- T() : Right. It's a simple support function, and even a possibility distribution, but we shall come back to that later. The state of total ignorance is represented by

$$m^\circ(X) = 1.$$

- T(p) : So we are apparently looking for a mass function such that $m \oplus m' = m^\circ$

where \oplus is the Dempster rule of combination.

- T() : But the solution is $m'(\neg G) = \alpha ; m'(X) = 1 - \alpha$,

which means $\text{Bel}'(\neg G) = \alpha > 0$. We are back to reasons to believe $\neg G$, rather than to representing reasons not to believe G.

- T(p) : True. Note also that we should use the normalized version of Dempster rule of combination [7], enforcing that the mass of the empty set remains zero ; otherwise the equation $m \oplus m' = m^\circ$ has no solution.

- T() : Our distinguished colleague Smets has proposed to deal separately with the confidence component and with the dissidence, I mean the diffidence component, for representing belief states [10].

- T(p) : Is it what he calls a patent, to be precise, a latent belief structure made of a pair of belief functions, one for the confidence part and the other for the diffidence part ?

- T() : Exactly, and it cannot be summarized into a unique structure of 'apparent' beliefs, without loss. Indeed apparent beliefs are obtained by subtracting the diffidence component from the confidence component, using the operation inverse of \oplus for agnostic, I mean non-dogmatic⁴ belief functions. However, something is not completely clear to me. Are the two information components of the same nature ?

- T(p) : You mean that « the reasons not to believe » do not constitute by themselves a belief structure (although they are also represented by a belief function in Smets' model).

- T() : Yes. Since we are mainly using simple support functions, let us consider what this proposal would mean in the framework of fossil, ... I mean possibility theory.

⁴ Non-dogmatic belief functions are such that $\text{Pl}(A) = 0 \Leftrightarrow A = \emptyset$ so that nothing is implausible, and nothing, except tautologies, is fully certain : $\text{Bel}(A) = 1 \Leftrightarrow A = X$ (since $\text{Pl}(A) = 1 - \text{Bel}(\neg A)$).

3. Reasons not be certain

- T(p) : So let us look at the reasons not to believe in possibility theory.

- T() : What do you say ! We do believe in possibility theory !

- T(p) : Indeed ! I was just saying let us look at our problem in the possibilistic framework [1].

- T() : Let us start with a two-valued possibility model. Let π be the characteristic function of the set of interpretations (in our universe of discourse) which restricts the possible location of the true state of the world according to the information we have.

- T(p) : Based on π we have a two-valued necessity measure N which is defined by

$$N(A) = 1 \text{ if } \{x, \pi(x) = 1\} \subseteq A ;$$

$$N(A) = 0 \text{ otherwise.}$$

- T() : Our beliefs are the A's such that $N(A) = 1$. What about reasons not to believe A ?

- T(p) : Such reasons come from observations, reports, things we have experienced that we know FOR SURE as being possible,

- T() : as we know for sure that it is possible that the information contained in a politically biased newspaper is wrong.

- T(p) : Let δ be the characteristic function of the set of observed interpretations in the universe of discourse. $\delta(x) = 1$ means that x is guaranteed to be possible. $\delta(x) = 0$ does not mean that x is impossible, but just it that that has not been (yet) observed.

- T() : What should be pointed out it that observations accumulate, so the more observations we have, the larger the set represented by δ [2].

- T(p) : It is just the converse with π . $\pi(x) = 0$ means that x is impossible for sure, while $\pi(x) = 1$ means that x is not at all ruled out. So the more information we have, the smaller the set of possible locations of the truth restricted by π .

- T() : How δ can account for the reasons not to believe A ?

- T(p) : If $\exists x_0 \delta(x_0) = 1$ and $x \notin A$, there is a conflict between belief A which rules out x_0 and the fact that x_0 has been observed.

- T() : So there are two situations in general. Either

$$\{x, \delta(x) = 1\} \subseteq \{x, \pi(x) = 1\}$$

and there is nothing in δ which can provide reasons not to believe statements which are beliefs supported by π ,

- T(p) : or the above inclusion of δ into π does not hold, and then

$$\{x, \delta(x) = 1 \text{ and } \pi(x) = 0\}$$

provide reasons not to believe any statement A such as the inclusion

$$A \supseteq \{x, \delta(x) = 1 \text{ and } \pi(x) = 0\} \text{ fails.}$$

- T() : This suggests a natural way to revise π by δ by changing π into

$$\pi'(x) = \max(\pi(x), \delta(x))$$

- T(p) : It is in fact a contrition, I mean a contraction of the belief set [5], [3] !

- T() : Right. This can be extended to possibility measures valued in a non binary scale.

- T(p) : Yes in particular we shall say that we have reasons not to believe a statement A, such as its necessity degree $N(A) \geq \alpha$ according to π ,

if $\exists x$ such as x is not a model of A, $x \notin A$ for short, and $\delta(x) > 1 - \alpha$.

- T() : Indeed $N(A) \geq \alpha$ means that $\Pi(\neg A) \leq 1 - \alpha$, and if $\exists x \notin A$ such that $\delta(x) > 1 - \alpha$, we are violating the consistency requirement between observations and the belief set, expressed in the non binary case by the inclusion

$$\forall x \delta(x) \leq \pi(x).$$

- T(p) : This suggests a nice framework for maybe reconsidering the Peter, Paul and Mary affair [9], [4] where nobody has been finally charged !

- T() : It is not sure. Introducing reasons not to believe can only enlarge the set of possible situations !

- T(p) : Anyway, 'to be or not to believe' is our motto !

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**Annex : Songs Much Enjoyable To Sing
(Très Bonne Musique):**

Que je t'aime TBM

A J. Philippe SMET's song.

Quand t'es dans l'ignorance
Quand t'as plus de croyances
Que les probabilités
Ne peuv'nt plus s'additionner.

Quand les subjectivistes
Ne trouvent plus de piste
Quand les statisticiens
N'y comprennent plus rien

Que je t'aime TBM, oh TBM (bis)

Quand les réseaux d'neurones
Deviennent monotones
Qu' les réseaux bayesiens
Ne s'calculent plus très bien

Qu'les algos génétiques
Ne rapportent plus de fric
Et que la logique floue
Ne rapporte plus un sou

Que je t'aime TBM, oh TBM (bis)

Quand Big Boss hypothèque
Tous les bijoux aztèques
Quand il faut deviner
Où il les a cachés

Qu'Peter Paul et Mary
Ont tous un alibi
Et quand tous les capteurs
Tombent en panne à toute heure

Que je t'aime TBM, oh TBM (bis)

TBM Propaganda

A special song by Peter Paul and Mary
(An unpublished addendum to their song "Day is done")

Tell me why you're a probabilist?
How could you in such a mafia enlist?
Don't you know belief degrees never add?
The pignistic transform is not bad
When you need to decide

And if you wish to solve all problems
It's quite easy, try the TBM
(bis)
TBM and the pignistic (ter)

Tell me why you like fuzzy logic
It's only some crazy kind of magic
Forget about soft computing and the like
A belief function looks better
In a published paper

And if you wish to solve all problems
It's quite easy, try the TBM
(bis)
TBM and the pignistic (ter)