Corrigendum to *Database Preference Queries - A Possibilistic Logic Approach with Symbolic Priorities*

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Abstract This note corrects a claim made in the above-mentioned paper about the exact representation of a conditional preference network by means of a possibilistic logic base with partially ordered symbolic weights. We provide a counter-example that shows that the possibilistic logic representation is indeed not always exact. This is the basis of a short discussion on the difficulty of obtaining an exact representation.

This note corrects a claim made in [6] about the representation of Conditional Preference networks (CP-nets for short) [1] by means of a possibilistic logic base [2], as well as a similar claim in [7,8].

A CP-net encodes a set of preference statements concerning the values of Boolean decision variables, conditioned on the values of other Boolean decision variables that influence the former. More formally, let $V = \{X_1, \cdots, X_n\}$ be a set of Boolean variables. We denote by $Ast(S)$ the set of interpretations of variables of S ($\subseteq V$).

Definition 1 *A CP-net $\mathcal{N}$ over $V = \{X_1, \cdots, X_n\}$ is a directed graph with nodes $X_1, \cdots, X_n$, and there is a directed edge from X_i to X_j if the preference about the value X_j depends on the value of X_i. Each node $X_i \in V$ is associated with a conditional preference table CPT_i that associates a strict preference ($x_i > \neg x_i$ or $\neg x_i > x_i$) with each possible instantiation $u_i^j \in Ast(Pa(X_i))$ of the parents $Pa(X_i)$ of X_i (if any). Each entry in a conditional preference table CPT_i is of the form $\phi = u_i^j : \star_j x_i > \star_j \neg x_i$, where $u_i^j \in Ast(Pa(X_i)), \star_j$ is blank if the preference is $x_i > \neg x_i$ and is $\neg$ otherwise.*

Each interpretation ω, i.e., an instantiation of all variables in V, is understood as a solution to the decision problem described by the preference statements in the CP-net. A CP-net induces a partial preference ordering over solutions defined as follows. Each

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conditional preference statement $u_i^j : \star_j x_i > \star_j \neg x_i$ expresses a preference between any two solutions ω_1 and ω_2 that satisfy u_i^j and only differ on variable x_i, that is, ω_1 satisfies x_i and ω_2 satisfies $\neg x_i$. Namely, the preference of x_i over $\neg x_i$ is valid in context u_i^j, all other things being equal, what is called the *ceteris paribus* assumption.

Definition 2 *A worsening flip consists in turning an interpretation ω_1 into ω_2 by flipping the truth-value of a single variable x_i, so that ω_1 is preferred to ω_2.*

In other words, a worsening flip compares two solutions differing only on one variable, according to the conditional preference table of this variable, completed by applying a ceteris paribus principle to the variables that do not appear in the table.

Definition 3 *A CP-net $\mathcal{N}$ defines a partial order $\succ_{\mathcal{N}}$ over the interpretations of $V = \{X_1, \cdots, X_n\}$ such that $\omega_1 \succ_{\mathcal{N}} \omega_2$ if and only if there is a sequence of worsening flips changing ω_1 into ω_2.*

The encoding of a CP-net in possibilistic logic is supposed to be made as follows [2,6]:

- Each entry of the form $u_i^j : \star_j x_i > \star_j \neg x_i$ in the table CPT_i for each node $X_i, i = 1, \ldots, n$ is encoded by the possibilistic logic clause $(\neg u_i^j \vee \star_j x_i, \alpha_i)$, where α_i is a symbolic weight (whose value is unspecified). This is the syntactic counterpart of the constraint $N(\neg u_i^j \vee \star_j x_i) \geq \alpha_i > 0$, where N is a necessity measure [3], α_i is a symbolic weight representing a certainty value in a necessity scale (a totally ordered set with bottom element 0).
- Since the same weight is attached to each clause built from CPT_i, the set of weighted clauses induced from CPT_i is equivalent to one weighted formula (ϕ_i, α_i), for each variable X_i, where $\phi_i = \bigwedge_{u_i^j \in Ast(Pa(X_i))} \neg u_i^j \vee \star_j x_i$, since $N(\phi \wedge \psi) = \min(N(\phi), N(\psi))$. So, each node in the CP-net is associated with a single possibilistic pair made of a propositional logic formula and a symbolic weight.
- Additional constraints over symbolic weights are added. The weight α_i attached to each node X_i, is supposed to be strictly smaller than the weight of each of its parents (thus accounting for the observed priority of father nodes over children nodes in CP-nets).

Let $\Sigma_{\mathcal{N}}$ be the possibilistic base that encodes the CP-net $\mathcal{N}$. At the semantic level, we can associate to each interpretation ω of the propositional language generated by $V = \{X_1, \cdots, X_n\}$ a vector $\boldsymbol{\omega}(\Sigma_{\mathcal{N}})$ with as many components as formulas in $\Sigma_{\mathcal{N}}$. In the ith component of the vector $\boldsymbol{\omega}(\Sigma_{\mathcal{N}})$ associated to the weighted formula $(\phi_i, \alpha_i) \in \Sigma_{\mathcal{N}}$, we put 1 if ω satisfies ϕ_i and $1 - \alpha_i$ if not, in agreement with possibilistic logic semantics [3]. Here, $1 - (.)$ just denotes the order-reversing map of the necessity scale, so that $1 > 1 - \alpha_i, \forall i$ (due to $\alpha_i > 0$). Vectors $\boldsymbol{\omega}(\Sigma_{\mathcal{N}})$ associated with each interpretation ω, have a specific format. Namely their component v_i (one per CP-net node) lies in $\{1, 1 - \alpha_i\}$ for $i = 1, \ldots, n$. The comparison between these vectors is solely dictated by the constraints relating the weights of father and children nodes together with the assumption $1 > 1 - \alpha_i, \forall i$.

Let $\mathbf{v}$ and $\mathbf{v}'$ be two vectors having the same number n of components that lie in a partially ordered set. Let $C(\mathbf{v}) = \{v_i : i = 1, \ldots n\}$ be the set of distinct components appearing in $\mathbf{v}$, and $\min C(\mathbf{v})$ denote the set of least elements in $C(\mathbf{v})$. Here, the components of the vectors $\boldsymbol{\omega}(\Sigma_{\mathcal{N}})$ lie in the set $\{1, 1 - \alpha_i : i = 1, \ldots, n\}$, where the α_i's are distinct and positive. These vectors can then be compared by means of one of the following ordering relations.

Definition 4 (min) $\mathbf{v} \succ_{min} \mathbf{v}'$ *iff* $\min(C(\mathbf{v}) \cup C(\mathbf{v}')) \subseteq C(\mathbf{v}')$.

This is the usual ordering on interpretations induced by a possibilistic knowledge base, here extended to partial orders. It can be refined as follows:

Definition 5 (discrimin) *Delete all pairs* (v_i, v'_i) *such that* $v_i = v'_i$ *in* $\mathbf{v}$ *and* $\mathbf{v}'$. *Thus, we get two sets* $R(\mathbf{v})$ *and* $R(\mathbf{v}')$ *of remaining components. Then,* $\mathbf{v} \succ_{discrimin} \mathbf{v}'$ *iff* $\min(R(\mathbf{v}) \cup R(\mathbf{v}')) \subseteq R(\mathbf{v}')$.

This partial ordering refines the previous one and can be further refined as follows:

Definition 6 (leximin) *Let* $\mathbf{v}^\sigma$ *be the reordered vector* $\mathbf{v}$ *by permutation* σ *of its components, i.e.,* $\mathbf{v}_i^\sigma = \mathbf{v}_{\sigma(i)}$. *Then* $\mathbf{v} \succ_{leximin} \mathbf{v}'$ *iff* $\exists \sigma, \mathbf{v}^\sigma \succ_{discrimin} \mathbf{v}'$.

The leximin comparison comes down to deleting all pairs (v_i, v'_j) such that $v_i = v'_j$ in $\mathbf{v}$ and $\mathbf{v}'$ (each deleted component can be used only one time in the deletion process). Thus, we get two minimal non overlapping sets $R^*(\mathbf{v})$ and $R^*(\mathbf{v}')$ of remaining components, namely $R^*(\mathbf{v}) \cap R^*(\mathbf{v}') = \emptyset$. Then, the leximin ordering comes down to $v \succ_{leximin} v'$ iff $\min(R^*(\mathbf{v}) \cup R^*(\mathbf{v}')) \subseteq R^*(\mathbf{v}')$. As shown in [5], for vectors of the form $\boldsymbol{\omega}(\Sigma_\mathcal{N})$, the discrimin and leximin orderings coincide, because the coefficients $1 - \alpha_i$ in different components always differ.

It was claimed in [6] that the above possibilistic logic encoding of a CP-net can exactly capture the CP-net ordering (defined in terms of worsening flips, as recalled at the beginning of this note) using the leximin (or discrimin) order for comparing vectors associated to interpretations of the corresponding possibilistic base. Indeed, each vector reflects the preference constraints that are satisfied or are violated by the considered solution. Unfortunately, this is true only for particular CP-nets. In actual fact, results in [5] suggest it may only provide a refinement of the CP-net ordering of solutions, namely, let $\mathcal{N}$ be an acyclic CP-net and $\succ_\mathcal{N}$ its induced partial preference ordering on interpretations. Then, it can be conjectured that

$$\forall \omega, \omega' \in \Omega, \omega \succ_\mathcal{N} \omega' \Rightarrow \boldsymbol{\omega}(\Sigma_\mathcal{N}) \succ_{leximin} \boldsymbol{\omega}'(\Sigma_\mathcal{N})$$

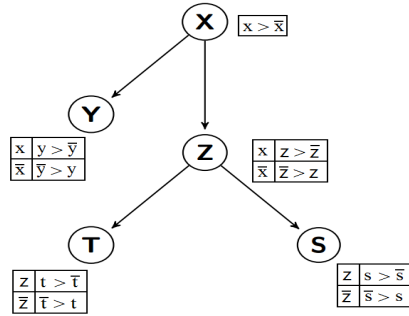The following counterexample shows that the possibilistic logic representation using

Fig. 1 CP-net associated to Example 1

the leximin order may compare solutions that the CP-net leaves incomparable:

Example 1 *Let us consider the CP-net of Fig. 1 with variables $V = \{X, Y, Z, S, T\}$, where $X \in \{x, \bar{x}\}$, etc., and the interpretations $\omega = xy\bar{z}st$ and $\omega' = \bar{x}\bar{y}\bar{z}\bar{s}t$. It can be checked that ω and ω' are incomparable by the CP-net ordering, since there is no sequence of worsening flips between these two interpretations. However, the leximin order can compare them, namely $\omega \succ_{leximin} \omega'$: indeed $\boldsymbol{\omega}(\Sigma_{\mathcal{N}}) = (1, 1, 1 - \alpha_3, 1, 1 - \alpha_5)$ and $\boldsymbol{\omega}'(\Sigma_{\mathcal{N}}) = (1 - \alpha_1, 1, 1, 1, 1)$, where $1 - \alpha_1 < 1 - \alpha_3 < 1 - \alpha_5$, with the convention $X = X_1$, $Y = X_2$, $Z = X_3$, $S = X_4$, and $T = X_5$.*

Yet another preference relation on interpretations of a possibilistic logic base can be obtained using the *symmetric Pareto* ordering, denoted by $\succ_{SP}$, and defined as follows:

Definition 7 (symmetric Pareto) *Let $\mathbf{v}$ and $\mathbf{v}'$ be two vectors having the same number of components, then $\mathbf{v} \succ_{SP} \mathbf{v}'$ if and only if there exists a permutation σ of the components of $\mathbf{v}'$, yielding vector $\mathbf{v}'^{\sigma}$, such that $\mathbf{v} \succ_{Pareto} \mathbf{v}'^{\sigma}$ (where as usual, $\mathbf{v} \succ_{Pareto} \mathbf{v}'$ if and only if $\forall i, v_i \geq v_i'$ and $\exists j, v_j > v_j'$).*

It is obvious that the leximin ordering refines the symmetric Pareto ordering:

$$\mathbf{v} \succ_{SP} \mathbf{v}' \Rightarrow \mathbf{v} \succ_{leximin} \mathbf{v}'.$$

It was also claimed in [7,8] that using the symmetric Pareto order of interpretations of the possibilistic logic encoding of a CP-net exactly captures the CP-net ordering. In fact this result seems to be true only for a special subclass of CP-nets where each node has at most one child node, (as claimed by Proposition 4 in [5]). The following counterexample shows that an exact representation of a CP-net is indeed not obtained using the *symmetric Pareto* order on networks where nodes can have more than one child node:

Example 2 *Let us consider the CP-net of Fig. 1, together with the interpretations $\omega_1 = xyz\bar{s}\bar{t}$ and $\omega_2 = xy\bar{z}st$. We notice that ω_1 is preferred to ω_2 according to the CP-net order. But the symmetric Pareto order leaves them incomparable. Indeed $\boldsymbol{\omega_1}(\Sigma_{\mathcal{N}}) = (1, 1, 1, 1 - \alpha_4, 1 - \alpha_5)$ and $\boldsymbol{\omega_2}(\Sigma_{\mathcal{N}}) = (1, 1, 1 - \alpha_3, 1, 1)$, with $1 - \alpha_4 < 1$ and $1 - \alpha_5 < 1$, while $1 - \alpha_3 < 1 - \alpha_4$ and $1 - \alpha_3 < 1 - \alpha_5$.*

In this example, the CP-net ordering proves more discriminant than the symmetric Pareto ordering. In contrast, the ordering $\succ_{SP}$ agrees with CP-net ordering $\succ_{\mathcal{N}}$ on the interpretations considered in Example 1, while the $\succ_{leximin}$ is in turn more discriminant than the CP-ordering.

In the general case, there are arguments to conjecture that

$$\boldsymbol{\omega}(\Sigma_{\mathcal{N}}) \succ_{SP} \boldsymbol{\omega}'(\Sigma_{\mathcal{N}}) \text{ implies } \omega \succ_{\mathcal{N}} \omega'.$$

See [5] for a proof assuming that in a CP-net, an interpretation ω that violates more preference tables than another interpretation ω' (in the sense of inclusion) is strictly less preferred, i.e. $\omega' \succ_{\mathcal{N}} \omega$ (a claim that however does seem to have been proved yet).

To conclude, the question of an exact representation of any CP-net by a partially ordered set of propositional formulae remains open, but this note suggests that the discrepancies between the two representation settings look more important than expected (see [4] for additional discrepancies between CP-net and possibilistic logic, pertaining to the transitivity of priorities between father nodes and children nodes in CP-nets). All that can be expected is a formal proof that for general acyclic CP-net structures the ordering $\succ_{\mathcal{N}}$ can only be bracketed by the SP and leximin orderings induced by the associated partially ordered base $\Sigma_{\mathcal{N}}$.

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