On structural analysis of extension-based argumentation semantics

Yuming XU,
Lidong XU,
Claudette CAYROL

IRIT, Université de Toulouse, CNRS, Toulouse, France

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Abstract

Dung's abstract argumentation provides us with a general framework to deal with argumentation. For extension-based semantics, the central issue is how to determine the extensions wrt. various semantics.

Motivated by the acceptability and reinstatement criterion, we propose the notions of J-acceptability and J-reinstatement. Correspondingly, we introduce the J-complete semantics which fills the gap between complete semantics and preferred semantics.

It is shown that acceptability together with J-acceptability forms the foundation of extension-based semantics. For example, any admissible set can be built starting from a conflict-free collection of initial sets by iteratively applying some functions based on acceptability and J-acceptability. In fact, this novel idea has a powerful ability in picturing the structure of various extensions and can be expected to play an important role in the study of various extension-based semantics.

Contents

1 Introduction

In recent years, the area of argumentation begins to become increasingly central as a core study within Artificial Intelligence. Starting from the work of Dung [10], a number of papers investigated and compared the properties of different semantics which have been proposed for abstract argumentation frameworks [4, 5, 8, 11]. Recently, more excellent work has been done in extension-based semantics and dynamic argumentation [1, 2, 3, 6, 9, 12]. For further notations and techniques of argumentation, we refer the reader to [7, 13].

As is known, the concept of acceptability plays a fundamental role in the extension-based semantics. Each traditional semantics satisfies the acceptability principle, that is, each extension under the traditional semantics is an admissible set. The reinstatement principle and the characteristic function are developed from the acceptability. But, acceptability is not enough when we extend a known admissible set to a new admissible set which has more arguments. There exists the case when an admissible set is combined with some additional arguments to form a new admissible set, whereas each of the additional arguments is not accepted wrt. the given admissible set. This case motivated us to propose the notion of joint acceptability, J-acceptability for short. Based on this novel concept, we can define the J-characteristic function and the J-reinstatement criterion which play a similar role as the characteristic function and reinstatement criterion do in the theory of extension-based semantics. Furthermore, we define a new semantics called J-complete semantics which exactly reveals the gap between complete semantics and preferred semantics.

The notion of initial set was first introduced by [15]. It is a generalization of the notion of initial argument and can be seen as the most basic germ of the building of various extensions. Based on this idea, our aim is to introduce a new fundamental criterion called J-acceptability and to combine it with the acceptability and initial sets to set up a procedure, by which any admissible set, complete extension and preferred extension can be built and described. Comparing with the known methods of finding extensions, we make it clear that initial sets should be seen as the starting points and discover that the J-acceptability is an essential supplement of acceptability. And so, our work provides a more workable method than the known approachs to construct all admissible sets.

The paper is organized as follows. Section 2 recalls the basic notions of argumentation frameworks. Section 3 introduces the J-acceptability criterion, J-complete semantics and studies their properties. Section 4 discusses our method for building and describing admissible sets, complete extensions and preferred ex-

tensions by the acceptability, J-acceptability and initial sets. Section 5 is devoted to concluding remarks and perspectives. The proofs can be found in the Appendix.

2 The basic notions of argumentation frameworks

For each argumentation framework, the arguments are produced by agents and the attack relation between them is set up according to some specific rules. We do not consider the origin and structure of arguments and the practical interaction of them.

Definition 1 *An argumentation framework is a pair $AF = (A, R)$, where A is a finite set of arguments and $R \subseteq A \times A$ represents the attack relation.*
Let $AF = (A, R)$ be an argumentation framework, $a, b \in A$ and $S \subseteq A$. a is attacked by b if $(b, a) \in R$, denoted by $b \to a$; a is called initial if a is not attacked; a is attacked by S if there is some $b \in S$ such that $(b, a) \in R$, denoted by $S \to a$; $R^+(S)$ denotes the set of arguments attacked by S; a attacks S if there is some $b \in S$ such that $(a, b) \in R$, denoted by $a \to S$.

Usually, an argumentation framework $AF = (A, R)$ can be represented by a directed graph. Nodes are used to stand for the arguments and edges represent the attack relation between arguments.

An extension is a set of arguments which can stand together. The basic requirement for any extension is conflict-freeness. That is, if an argument a attacks another argument b, then they can not stand together. Another requirement is known as admissibility and lies at the heart at all traditional extension-based semantics. It is based on the notions of acceptable argument and admissible set.

Definition 2 *Let $AF = (A, R)$ be an argumentation framework, $S, T \subseteq A$, $a \in A$.*
- S is conflict-free *if there are no $a, b \in S$ such that $(a, b) \in R$. Furthermore, S is conflict-free with T if $S \cup T$ is a conflict-free set.*
- By extension, a set $\mathcal{B}$ of subsets of A is said to be conflict-free if the union of the elements of $\mathcal{B}$, denoted by $\cup\mathcal{B}$, is a conflict-free subset of A.
- a is acceptable *$wrt.$ S (or a is defended by S) if each attacker b of a is attacked by S.*
- S is an admissible *set if S is conflict-free and each $a \in S$ is defended by S. For convenience, we denote the collection of all admissible sets of AF by $\mathcal{AS}(AF)$.*

In the literature, the reinstatement principle [3] is regarded as the converse of admissibility principle. Both principles lead to the following semantics.

Definition 3 *Let $AF = (A, R)$ be an argumentation framework and $S \subseteq A$.*
- S is a complete *extension if $S \in \mathcal{AS}(AF)$ and for each $a \in A$ defended by S, $a \in S$. The collection of all complete extensions is denoted by $\mathcal{CO}(AF)$.*
- S is the grounded *extension if it is the least element (wrt. set inclusion) of $\mathcal{CO}(AF)$. The grounded extension of AF is unique and denoted by $GE(AF)$.*
- S is a preferred *extension of AF if it is a maximal element (wrt. set inclusion) of $\mathcal{CO}(AF)$. The collection of all preferred extensions is denoted by $\mathcal{PR}(AF)$.*

The complete and grounded extensions can also be defined using the characteristic function. Let $AF = (A, R)$, the function $\mathcal{F} : 2^A \rightarrow 2^A$ which, given a set $S \subseteq A$, returns the set of the acceptable arguments wrt. S, is called the characteristic function[1] of AF. Complete extensions are exactly conflict-free fixed points of $\mathcal{F}$ and the grounded extension of AF is the least fixed point of $\mathcal{F}$.

In the following, we sometimes need to restrict to a subset of an argumentation framework.

Definition 4 *Let $AF = (A, R)$ be an argumentation framework, and $S \subseteq A$. The restriction of AF to S, denoted by $AF\ |_S$, is the sub-argumentation framework $(S, R \cap (S \times S))$.*

We also recall the I-maximality and directionality principles first introduced in [3]. The directionality principle is based on the sets of arguments which do not receive any attack from outside.

Definition 5 *Let σ be a semantics and AF be an argumentation framework. $\mathcal{E}_\sigma(AF)$ denotes the set of extensions of AF under the semantics σ.*
- A set $\mathcal{E}$ of extensions is I-maximal *if and only if $\forall E_1, E_2 \in \mathcal{E}$, if $E_1 \subseteq E_2$ then $E_1 = E_2$. A semantics σ satisfies the* I-maximality *principle if and only if $\forall AF$ such that $\mathcal{E}_\sigma(AF)$ is non-empty, $\mathcal{E}_\sigma(AF)$ is I-maximal.*
- A non-empty set $S \subseteq A$ is unattacked *in AF if and only if there exists no $a \in (A \setminus S)$ such that $a \rightarrow S$.*
- A semantics σ satisfies the directionality principle *if and only if $\forall AF$ such that $\mathcal{E}_\sigma(AF)$ is non-empty, $\forall S$ unattacked in AF, $\mathcal{E}_\sigma(AF\ |_S) = \{(E \cap S) : E \in \mathcal{E}_\sigma(AF)\}$.*

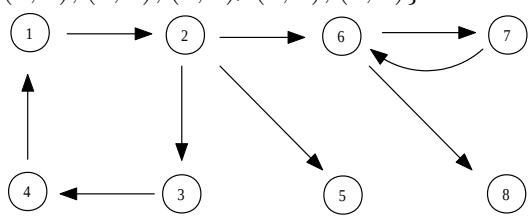
[1] Strictly speaking, this function should be denoted by $\mathcal{F}_{AF}$. The subscript will be omitted in the following.

Now, let us turn to the notion of initial set first introduced in [15].

Definition 6 *Let $AF = (A, R)$ be an argumentation framework. A non-empty admissible set I is* initial *if it has no non-empty proper subset to be admissible. The collection of all initial sets of AF is denoted by $\mathcal{IS}(AF)$.*

For any initial argument i of AF, $\{i\}$ is obviously an initial set. Two initial sets of AF may be conflicting. So, we usually consider conflict-free subsets of $\mathcal{IS}(AF)$.

Example 1 *Let $AF = (A, R)$ with $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $R = \{(1, 2), (2, 3), (3, 4), (4, 1), (2, 5), (2, 6), (6, 7), (6, 8), (7, 6)\}$. The directed graph is as follows.*



There are three initial sets: $I_1 = \{1, 3\}$, $I_2 = \{2, 4\}$ and $I_3 = \{7\}$. Obviously, I_1 and I_2 are conflicting, whereas $\{I_1, I_3\}$ is a conflict-free subset of $\mathcal{IS}(AF)$.

Due to Def.6 any non-empty admissible set contains at least one initial set. Certainly, any two initial sets contained in an admissible set are conflict-free. So, we have:

Proposition 1 *Let $AF = (A, R)$ and E be an admissible set of AF. If $\mathcal{B}$ is a collection of initial sets contained in E, then $\cup\mathcal{B}$ is an admissible set. If $\mathcal{B}$ is the collection of all initial sets contained in E, then $E \setminus (\cup\mathcal{B})$ contains no initial set.*

As is known, the grounded extension can be built starting from the set of all initial arguments by iteratively adding acceptable arguments. Namely, the initial arguments can be seen as starting points for building the grounded extension by acceptability. For an admissible extension E, the initial sets contained in E play the same role. That is, the initial sets can be regarded as the starting points for constructing an admissible set (certainly including complete and preferred extensions) by adding acceptable arguments and the so-called joint acceptable sets which we will introduce and discuss below.

3 The J-acceptability and related topics

Given an argumentation framework $AF = (A, R)$ and an admissible set E, there are usually two ways to construct a new admissible set having more arguments. If S is an admissible set which is conflict-free with E, then $E \cup S$ is an admissible set. If $S \subseteq \mathcal{F}(E)$, then $E \cup S$ is an admissible set. In fact, there is another way to construct new admissible sets. There may be some set S of arguments, which is not admissible and has no argument contained in $\mathcal{F}(E)$, such that $E \cup S$ is admissible.

Example 1 (cont'd) *For the admissible set $I_1 = \{1, 3\}$, we have that $I_3 = \{7\}$ is admissible and conflict-free with I_1 and thus $I_1 \cup I_3 = \{1, 3, 7\}$ is admissible. Since $S_1 = \{5\} \subseteq \mathcal{F}(I_1)$, $I_1 \cup S_1 = \{1, 3, 5\}$ is also admissible. Let $S_2 = \{6\}$. Although S_2 is not admissible and $S_2 \cap \mathcal{F}(I_1) = \emptyset$, we also have that $I_1 \cup S_2 = \{1, 3, 6\}$ is an admissible set.*

In the above example, S_2 is not admissible and its arguments are not acceptable with respect to I_1. But, the argument 6 can be acceptable with respect to $I_1 \cup S_2$. In words, all the arguments of S_2 are acceptable with respect to the union of I_1 and S_2. This situation leads us to propose the joint acceptability so as to distinguish from the acceptability.

Definition 7 *Given $AF = (A, R)$, a non-empty admissible set $E \subset A$ and a non-empty subset $S \subset A$. If S is not admissible, $\mathcal{F}(E) \cap S = \emptyset$ and $E \cup S$ is admissible, then we say that S is* joint acceptable wrt. E *(*J-acceptable wrt. E *for short). The collection of all J-acceptable sets wrt. E is denoted by $\mathcal{JA}(E)$.*

Although a J-acceptable set S $wrt.$ an admissible set E is not admissible, it can be shown that S is an admissible set in the modified argumentation AF' obtained by deleting the arguments of E and the arguments attacked by E.

Proposition 2 *Let $AF = (A, R)$, E be an admissible set. If S is J-acceptable wrt. E, then S is an admissible set in the framework $AF' = AF\ |_{A \setminus B}$ where $B = E \cup R^+(E)$. Conversely, each admissible set S of AF', which is conflict-free with E in AF but not admissible, is J-acceptable wrt. E in AF.*

The notion of J-acceptability induces a new principle, namely the J-reinstatement principle.

Definition 8 *A semantics σ satisfies the J-reinstatement principle if $\forall AF = (A, R)$ such that $\mathcal{E}_\sigma(AF)$ is non-empty, for each σ-extension E of AF, there is no subset of $A \setminus E$ to be J-accepted wrt. E.*

Similar as the description of complete extensions by the reinstatement criterion, we can define a new class of extensions based on the J-reinstatement criterion. This class includes the preferred extensions.

Definition 9 *Given $AF = (A, R)$ and an admissible set E. E is called a* J-complete *extension if there is no $S \subseteq A \setminus E$ to be J-acceptable with respect to E. The set of all J-complete extensions of AF will be denoted by $\mathcal{E}_{\mathcal{JCO}}(AF)$.*

Example 1 (cont'd) *$E_1 = \{1, 3, 5\}$ is complete but not J-complete. On the other hand, there is an admissible set $E_2 = \{1, 3, 6\}$ which is J-complete but not complete. Note also that the admissible set $E_3 = \{1, 3, 6, 5\}$ is both J-complete and complete. It is also a preferred extension.*

The following proposition identifies the important role of J-reinstatement criterion in the theory of extension-based semantics. In words, J-complete semantics fills the gap between complete semantics and preferred semantics.

Proposition 3 *A complete extension E of $AF = (A, R)$ is preferred if and only it is J-complete and $A \setminus E$ has no non-empty admissible subset to be conflict-free with E.*

Similar as complete semantics, the J-complete semantics does not satisfy the I-maximality criterion. On the other hand, it can be proved that the J-complete semantics satisfies the directionality criterion.

Proposition 4 *The J-complete semantics satisfies the directionality criterion.*

The notion of J-acceptability also enables to define the J-characteristic function for an argumentation framework AF.

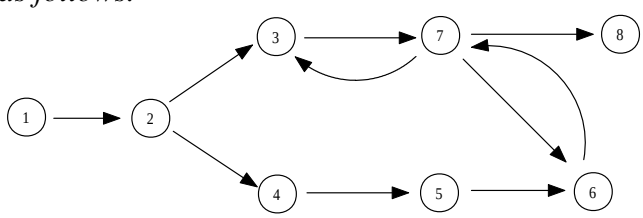
Definition 10 *Let $AF = (A, R)$. The function $\mathcal{F}_J : 2^A \to 2^{2^A}$ which, given an admissible set $E \subseteq A$, returns the collection of the J-acceptable sets wrt. E, is called the* J-characteristic function *of AF. In words, $\mathcal{F}_J(E) = \mathcal{JA}(E)$.*

Note that $\mathcal{F}_J(E)$ is usually not conflict-free, so $E \cup (\cup\mathcal{F}_J(E))$ may not be an admissible set, which in contrast, is always true for the characteristic function $\mathcal{F}$. If we select a conflict-free collection $\mathcal{B}$ of J-acceptable sets wrt. E, then it is easy to check that $E \cup (\cup(\mathcal{B}))$ is admissible. Based on these two facts, we define a selection J-function R_J on each argumentation framework as follows.

Definition 11 *Let $AF = (A, R)$, E be a non-empty admissible set. A selection function R_J assigns to E a set $R_J(E)$, which is the union of some random selected conflict-free J-acceptable sets wrt. E. That is, $R_J(E) = \cup\mathcal{C}$ where $\mathcal{C}$ is the random selected conflict-free subcollection $\mathcal{C}$ of $\mathcal{F}_J(E)$.*

Proposition 5 *Let $AF = (A, R)$, E be a non-empty admissible set. Then, $E \cup R_J(E)$ is admissible.*

Example 2 *Let $AF = (A, R)$ with $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $R = \{(1, 2), (2, 3), (2, 4), (3, 7), (4, 5), (5, 6), (6, 7), (7, 3), (7, 6), (7, 8)\}$. The directed graph is as follows.*



Obviously, $E_1 = \{1, 4\}$ is admissible and $\mathcal{F}_J(E_1) = \{J_1, J_2\}$ with $J_1 = \{3\}$ and $J_2 = \{6\}$. If $R_J(E_1) = \cup\{J_1\}$, then $E_1 \cup R_J(E_1) = \{1, 4, 3\}$ is admissible. If $R_J(E_1) = \cup\{J_2\}$, then $E_1 \cup R_J(E_1) = \{1, 4, 6\}$ is admissible. If $R_J(E_1) = \cup\{J_1, J_2\}$, then $E_1 \cup R_J(E_1) = \{1, 4, 3, 6\}$ is admissible. In particular, if we let $R_J(E_1) = \emptyset$, then $E_1 \cup R_J(E_1) = \{1, 4\} = E_1$ is obviously admissible.

4 The structure of traditional semantics

The study of extension-based semantics is a central topic in argumentation. Dung first introduced the admissible, complete, preferred and stable semantics in his outstanding work. After that, new semantics have been introduced in order to meet some special requirements or make up the drawback of known traditional semantics for specific applications.

Here, we mainly focus on the structural analysis of various traditional semantics based on the initial sets, acceptability and J-acceptability.

4.1 The structure of admissible semantics

Admissibility is a common feature for the traditional argumentation semantics. We first classify the admissible extensions into three types based on initial sets, acceptability and J-acceptability.

4.1.1 I-type admissible sets

For any admissible set E, $\mathcal{F}(E)$ is obviously admissible and can be expressed as $E \cup (\mathcal{F}(E) \setminus E)$. Note that, each argument of $\mathcal{F}(E) \setminus E$ satisfies: being accepted by E and not belonging to E. This leads us to define a subclass of admissible sets.

Definition 12 *Given $AF = (A, R)$ and $S \subseteq A$. An argument i is* regularly accepted *by S if $i \notin S$ and i is defended by S.*

If there exists an argument regularly accepted by the admissible set S, then S is a proper subset of $\mathcal{F}(S)$. And, any initial argument is regularly accepted by the empty set.

Definition 13 *Let E be an admissible set of $AF = (A, R)$ and $\mathcal{B}$ the collection of all initial sets contained in E. If each $i \in E \setminus \cup\mathcal{B}$ is regularly accepted by some admissible set $B \subseteq E$, then we say that E is a* I-type admissible *set.*

It is easy to check that the union of conflict-free initial sets is a I-type admissible set.

Example 1 (cont'd) *For the admissible set $I_1 = \{1, 3\}$, as $5 \in \mathcal{F}(I_1) \setminus I_1$ the argument 5 is regularly accepted by I_1. Let us consider the admissible set $E_1 = \{1, 3, 5\}$ and the collection $\mathcal{B} = \{I_1\}$ of initial sets contained in E_1. Obviously, the only argument 5 of $E_1 \setminus \cup\mathcal{B}$ is regularly accepted by the admissible set $I_1 \subseteq E_1$. So, E_1 is a I-type admissible set.*

The grounded extension is also a I-type admissible set. More generally, we have:

Proposition 6 *Any admissible set $\mathcal{F}^k(\emptyset)$ is a I-type admissible set.*

Next, we will analyze the structure of I-type admissible sets from two different points of view. One is to construct a I-type admissible set starting from initial sets, another one is to describe a given I-type admissible set starting from initial sets contained in it.

From any admissible set the characteristic function $\mathcal{F}$ can be iteratively applied to obtain a complete extension. We try to replace $\mathcal{F}$ by some specific operator related to the acceptability so as to obtain a larger admissible set. This goal can be reached by defining a selection function as follows.

Definition 14 *Given an admissible set E of $AF = (A, R)$. A selection function R_A assigns to E a random selected subset $R_A(E)$ of $\mathcal{F}(E) \setminus E$.*

It is easy to check that $E \cup R_A(E)$ is certainly a I-type admissible set whenever E is a I-type admissible set.

Example 2 (cont'd) *It is easy to check that $E_2 = \{1, 3\}$ is a I-type admissible set and $\mathcal{F}(E_2) \setminus E_2 = \{4, 8\}$. If $R_A(E_2) = \{4\}$, then $E_2 \cup R_A(E_2) = \{1, 3, 4\}$ is a I-type admissible set. If $R_A(E_2) = \{8\}$, then $E_2 \cup R_A(E_2) = \{1, 3, 8\}$ is a I-type admissible set. If $R_A(E_2) = \{4, 8\}$, then $E_2 \cup R_A(E_2) = \{1, 3, 4, 8\}$ is a I-type admissible set.*

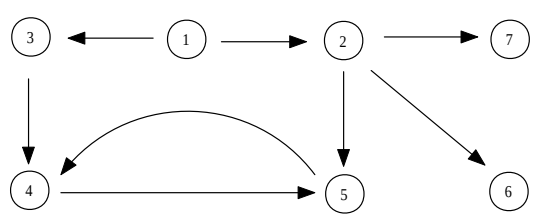
Now, let us start from initial sets to construct I-type admissible sets by using the selection function R_A.

Constructing the I-type admissible sets: Let $\mathcal{B}$ be a conflict-free collection of initial sets, define $E_0 = \cup\mathcal{B}$ and $E_{k+1} = E_k \cup R_A(E_k)$ for each natural number k. Then, E_0 is the union of some conflict-free initial sets, E_1 is the set obtained by adding to E_0 some arguments regularly accepted by E_0, and so on.

The following theorem indicates that starting from a conflict-free collection of initial sets we can construct many different I-type admissible sets by iteratively applying the selection function R_A.

Theorem 1 *Let $\mathcal{B}$ be a conflict-free collection of initial sets and E_k as above, then E_k is a I-type admissible set for each natural number k.*

Example 3 *Let $AF = (A, R)$ with $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $R = \{(1, 2), (1, 3), (2, 5), (2, 6), (2, 7), (3, 4), (4, 5), (5, 4)\}$. The directed graph is as follows.*



Obviously, $E = \{1\}$ is the unique initial set of AF and $\mathcal{F}(E) = \{1, 6, 7\}$. If $R_A(E) = \{6\}$, then $E \cup R_A(E) = \{1, 6\}$ is a I-type admissible set. If $R_A(E) = \{7\}$, then $E \cup R_A(E) = \{1, 7\}$ is a I-type admissible set. If $R_A(E) = \{6, 7\}$, then $E \cup R_A(E) = \{1, 6, 7\}$ is a I-type admissible set.

In order to describe a given I-type admissible set E, we need to define another operation also based on acceptability but restricted to E.

Definition 15 *Let S and E be two admissible sets of AF such that $S \subseteq E$. An operation P on S wrt. E assigns to S a subset of E: $P(S, E) = E \cap (\mathcal{F}(S) \setminus S)$.*

Example 2 (cont'd) *For the admissible set $E_2 = \{1, 3\}$, we have that $\mathcal{F}(E_2) \setminus E_2 = \{4, 8\}$. With the admissible set $E_3 = \{1, 3, 8\}$, $P(E_2, E_3) = \{8\}$. With the admissible set $E_4 = \{1, 3, 4\}$, $P(E_2, E_4) = \{4\}$. With the admissible set $E_5 = \{1, 3, 4, 8\}$, $P(E_2, E_5) = \{4, 8\}$.*

In the above definition, if S is a I-type admissible set, then $S \cup P(S, E)$ is certainly a I-type admissible set. And thus, we can describe a given admissible set from the initial sets contained in it by using the operation $P(\cdot, \cdot)$.

Describing a given I-type admissible set: Let E be a I-type admissible set and $\mathcal{B}$ the collection of all initial sets contained in E, define $E_0 = \cup\mathcal{B}$ and $E_{k+1} = E_k \cup P(E_k, E)$ for each natural number k. Then, E_0 is the union of all initial sets contained in E, E_1 is the set obtained by adding to E_0 all the arguments which are regularly accepted by E_0 and contained in E, and so on.

The following theorem indicates that any I-type admissible set E can be described as starting from the collection $\mathcal{B}$ of all initial sets contained in E and iteratively applying the function $P(\cdot, E)$.

Theorem 2 *Let E be a I-type admissible set and $\mathcal{B}$ the collection of all initial sets contained in E, then E_k defined above is a I-type admissible set contained in E for each natural number k. Furthermore, there is some natural number m such that $E = E_{m+1} = E_{m+2}$ and $E_{m+1} = E_0 \cup P(E_0, E) \cup P(E_1, E) ... \cup P(E_m, E)$.*

As a consequence of the above results, we can provide a way for building strongly admissible sets, first introduced in [3]. An argument $i \in A$ is strongly defended by a set $S \subseteq A$ iff each attacker j of i is attacked by some $k \in S \setminus \{i\}$ such that k is strongly defended by $S \setminus \{i\}$. Then, S is said strongly admissible iff it strongly defends each of its arguments. An equivalent definition has been proposed by [8]. $S \subseteq A$ is strongly admissible iff S consists of some initial arguments, or every $i \in S$ is defended by some subset $T \subseteq (S \setminus \{i\})$ which in its turn is again strongly admissible.
Following definitions, it is easy to see that every strongly admissible set is a I-type admissible set and thus can be constructed starting from some initial arguments by applying the function P iteratively.

4.1.2 II-type admissible sets

Except for the I-type admissible sets, there is another kind of admissible sets called II-type admissible sets. They are the admissible sets which have no argument to be regularly accepted by admissible sets contained in them.

Definition 16 *Let E be an admissible set of $AF = (A, R)$ and $\mathcal{B}$ the collection of all initial sets contained in E. If each $i \in E \setminus (\cup\mathcal{B})$ belongs to a J-acceptable set $S \subseteq E$ w.r.t some admissible set $D \subseteq E$, then we say that E is a* II-type admissible *set.*

Note that, the union of a conflict-free collection of initial sets is not only a I-type admissible set but also a II-type admissible set.

Example 1 (cont'd) *Let us consider the admissible set $E_2 = \{1, 3, 6\}$ and the collection $\mathcal{B} = \{I_1\}$ of initial sets contained in E_2, where $I_1 = \{1, 3\}$.*

It is easy to check that $\{6\}$ is a J-acceptable set wrt. I_1, and thus E_2 is a II-type admissible set.

In fact, $6 \notin \mathcal{F}(I_1) \setminus I_1$ and is not regularly accepted by the admissible sets I_1 and E_2 which are the non-empty admissible sets contained in E_2.

Generally speaking, the membership of each argument of a II-type admissible set E does not only depend on other arguments of E but also on itself. This feature is exactly that the notion of J-acceptability states. Certainly, a II-type admissible set E may contain some regularly accepted arguments, whose membership depend completely on other arguments of E. The following proposition gives a description for a II-type admissible set having no regularly accepted arguments except initial arguments.

Proposition 7 *Let E be an admissible set of AF and $\mathcal{B}$ the collection of all initial sets contained in E. Then $E \setminus (\cup\mathcal{B})$ has no regularly accepted arguments if and only if for each admissible set $D \subset E$ containing $\cup\mathcal{B}$ as a subset, $E \setminus D$ is J-acceptable wrt. D.*

By the above proposition, an admissible set with no regularly accepted arguments except initial arguments must be a II-type admissible set. Next, we will analyze the structure of II-type admissible sets from two different points of view. One is to construct a II-type admissible set starting from initial sets, another is to describe a given II-type admissible set starting from the initial sets contained in it.

As for I-type admissible sets, in constructing II-type admissible sets we need to use a selection function related to the J-acceptability. Interestingly, the selection function R_J in Definition 11 is exactly the one we need. Furthermore, Prop.5 is also true if we substitute "admissible" by "II-type admissible". This result gives us the theoretical support for constructing II-type admissible sets by applying the selection function R_J.

Constructing the II-type admissible sets: Let $\mathcal{B}$ be a conflict-free collection of initial sets, define $E_0 = \cup\mathcal{B}$ and $E_{k+1} = E_k \cup R_J(E_k)$ for each natural number k. Then, E_0 is the union of some conflict-free initial sets, E_1 is the set obtained by adding to E_0 some conflict-free J-acceptable sets wrt. E_0, and so on.

The following theorem indicates that starting from a conflict-free collection of initial sets we can construct many different II-type admissible sets by iteratively applying the selection function R_J.

Theorem 3 *Let $\mathcal{B}$ a conflict-free collection of initial sets and E_k as above, then E_k is a II-type admissible set for each natural number k.*

Example 3 (cont'd) *For the unique initial set $E = \{1\}$ of AF, we have that $\mathcal{F}_J(E) = \{J_1, J_2\}$ with $J_1 = \{4\}$ and $J_2 = \{5\}$. If $R_J(E) = J_1$, then $E \cup R_J(E) = \{1, 4\}$ is a II-type admissible set. If $R_J(E) = J_2$, then $E \cup R_J(E) = \{1, 5\}$ is a II-type admissible set. Since J_1 and J_2 are in conflict, $R_J(E) \neq J_1 \cup J_2$.*

In describing a given I-type admissible set, the main work is iteratively applying the function $P(\cdot, \cdot)$. This idea can be further extended to the case of J-acceptability so as to describe a given II-type admissible set.

Definition 17 *Let S and E be two admissible sets of AF such that $S \subseteq E$. An operation P_J on S wrt. E assigns to S a subset of E: $P_J(S, E) = \cup\{T : T$ is J-acceptable wrt. S and $T \subset E\}$.*

Example 2 (cont'd) *For the admissible set $E_1 = \{1, 4\}$, we have that $\mathcal{F}_J(E_1) = \{J_1, J_2\}$ where $J_1 = \{3\}$ and $J_2 = \{6\}$. With the admissible set $E_6 = \{1, 4, 3\}$, $P_J(E_1, E_6) = \cup\{J_1\} = \{3\}$. With the admissible set $E_7 = \{1, 4, 6\}$, $P_J(E_1, E_7) = \cup\{J_2\} = \{6\}$. With the admissible set $E_8 = \{1, 4, 3, 6\}$, $P_J(E_1, E_8) = \cup\{J_1, J_2\} = \{3, 6\}$.*

Note that, $S \cup P_J(S, E)$ is a II-type admissible set whenever S is a II-type admissible set. And so, we can describe any II-type admissible set E starting from

the initial sets contained in it by iteratively applying the function $P_J(\cdot,\cdot)$.

Describing a given II-type admissible set: Let E be a II-type admissible set and $\mathcal{B}$ the collection of all initial sets contained in E, define $E_0 = \cup\mathcal{B}$ and $E_{k+1} = E_k \cup P_J(E_k, E)$ for each natural number k. Then, E_0 is the union of some conflict-free initial sets, E_1 is the set obtained by adding to E_0 all the J-acceptable sets wrt. E_0 which are contained in E, and so on.

The following theorem indicates that any II-type admissible set E can be described as starting from the collection $\mathcal{B}$ of all initial sets contained in E and iteratively applying the function $P_J(\cdot, E)$.

Theorem 4 *Let E be a II-type admissible set and $\mathcal{B}$ the collection of all initial sets contained in E, then E_k defined above is a II-type admissible set contained in E for each natural number k. Furthermore, there is some natural number m such that $E = E_{m+1} = E_{m+2}$ and $E_{m+1} = E_0 \cup P_J(E_0, E) \cup P_J(E_1, E) ... \cup P_J(E_m, E)$.*

4.1.3 The mixed-type admissible sets

Roughly speaking, the I-type and II-type admissible extensions have onefold structure. So, we usually call them simple admissible extensions. The other admissible extensions may be given the name of mixed-type admissible sets as follows.

Definition 18 *Given $AF = (A, R)$, an admissible set E is* mixed-type *if it is neither I-type no II-type.*

Example 1 (cont'd) *Let us consider the admissible set $E_3 = \{1, 3, 5, 6\}$ and the collection $\mathcal{B} = \{I_1\}$ of initial sets contained in E_3. Obviously, the argument 5 of $E_3 \setminus \cup\mathcal{B}$ is regularly accepted by the admissible set $I_1 \subseteq E_3$. So, E_3 is not a II-type admissible set.*

We also note that the argument 6 is not regularly accepted by any admissible set S contained in E_3, where $S = \{1, 3\}$ or $\{1, 3, 5\}$. It follows that E_3 is not a I-type admissible set.

To sum up, E_3 is a mixed-type admissible set.

Starting from a conflict-free collection $\mathcal{B}$ of initial sets, many mixed-type admissible sets can be constructed by iteratively applying the functions R and R_J.

Constructing an admissible extension of mixed-type: Let $\mathcal{B}$ be a conflict-free collection of initial sets, define $S_0 = \cup\mathcal{B}$, $T_0 = S_0 \cup R_J(S_0)$. For each natural number k, let $S_{k+1} = T_k \cup R(T_k)$ and $T_{k+1} = S_{k+1} \cup R_J(S_{k+1})$. Note that, both S_k and T_k are admissible. In order to obtain a mixed-type admissible set S_m or T_m, we require that $S_{r+1} \neq T_r$ and $T_{t+1} \neq S_{t+1}$ for some $r, t < m$.

Example 3 (cont'd) *For the unique initial set $E = \{1\}$ of AF, we have that $S_0 = \{1\}$. Note that, $\mathcal{F}_J(S_0) = \{J_1, J_2\}$ with $J_1 = \{4\}$ and $J_2 = \{5\}$. If we let $R_J(S_0) = J_1$, then $T_0 = \{1, 4\}$ is a II-type admissible set. Furthermore, $\mathcal{F}(T_0) = \{6, 7\}$. If we let $R(T_0) = \{6\}$, then $S_1 = \{1, 4, 6\}$ is a mixed-type admissible set. Again, we have $\mathcal{F}(S_1) = \{J_2\}$. If we let $R_J(S_1) = J_2$, then $T_1 = \{1, 4, 6, 5\}$ is a mixed-type admissible set.*

We can describe an admissible set of mixed-type starting from the initial sets contained in it by iteratively applying the functions $P(\cdot, \cdot)$ and $P_J(\cdot, \cdot)$.

Describing a given admissible extension of mixed-type: Let E be a II-type admissible set and $\mathcal{B}$ the collection of all initial sets contained in E, define $S_0 = \cup\mathcal{B}$ and $T_0 = S_0 \cup P_J(S_0, E)$. For each natural number k, let $S_{k+1} = T_k \cup P(T_k, E)$ and $T_{k+1} = S_{k+1} \cup P_J(S_{k+1}, E)$ until $E = S_m$ or T_m for some natural number m. That is, we have $E = S_0 \cup P_J(S_0, E) \cup P(T_1, E) ... \cup P_J(S_{m-1}, E) \cup P(T_{m-1}, E)$ or $E = S_0 \cup P_J(S_0, E) \cup P(T_0, E) ... \cup P(T_{m-1}, E) \cup P_J(S_m, E)$.

4.2 The structure of complete and preferred semantics

Since both the complete extensions and the preferred extensions are all admissible sets, they also can be classified into different types just like we have done for admissible sets. Meanwhile, we have similar results for constructing them and describing them from conflict-free initial sets by iteratively applying the related functions R, R_J, P and P_J. Due to space limitation, we only list the main notions and results.

Definition 19 *Given a complete (or preferred) extension E of $AF = (A, R)$. E is said to be of I-type (resp. II-type, mixed-type) if it is a I-type (resp. II-type, mixed-type) admissible set.*

Constructing a I-type complete (or preferred) extension: Let $\mathcal{B}$ be a conflict-free collection of initial sets, define $E_0 = \cup\mathcal{B}$ and $E_{k+1} = \mathcal{F}(E_k)$ for each natural

number k. Then, E_0 is the union of conflict-free initial sets and thus a I-type admissible set. E_1 is the set obtained by adding to E_0 all arguments regularly accepted by E_0 and thus a I-type admissible set, and so on.

By definition of the characteristic function $\mathcal{F}$, $E_k \subseteq \mathcal{F}(E_k) = E_{k+1}$ for each natural number k. If there is no m such that $E_m = E_{m+1}$, then the cardinality of E_k will be strictly increasing. This contradicts with the fact that A is a finite set. Suppose that $E_m = E_{m+1}$ for some natural number m, that is $E_m = \mathcal{F}(E_m)$, then E_m is a fixed point of the characteristic function $\mathcal{F}$. This indicates that E_m is a complete extension.
So we have proved that there is some natural number m such that $E_m = E_{m+1}$ which is exactly the I-type complete extension we want. Furthermore, if $\mathcal{F}_J(E_m) = \emptyset$ and there is no initial set in $A \setminus E_m$ conflicting with E_m, then E_m is a I-type preferred extension according to Prop.3.

Example 1 (cont'd) *Considering the conflict-free collection $\mathcal{B} = \{I_1\}$ where $I_1 = \{1, 3\}$ is an initial set. Let $E_0 = \cup\mathcal{B} = \{1, 3\}$ and $E_1 = \mathcal{F}(E_0) = \{1, 3, 5\}$, then $E_2 = \mathcal{F}(E_1) = E_1$ is a I-type complete extension.*

Describing a I-type complete (or preferred) extension: Let E be a I-type complete (preferred) extension and $\mathcal{B}$ the collection of all initial sets contained in E, define $E_0 = \cup\mathcal{B}$ and $E_{k+1} = \mathcal{F}(E_k)$ for each natural number k. Then, $E_0 \subseteq \mathcal{F}(E) = E$ is the union of initial sets contained in E and thus a I-type admissible set. $E_1 = \mathcal{F}(E_0) \subseteq \mathcal{F}(E) = E$ is the set obtained by adding to E_0 all arguments regularly accepted by E_0 and thus a I-type admissible set, and so on.

Remark: Since $\mathcal{F}(E_k) \subseteq \mathcal{F}(E) = E$, we have that $\mathcal{F}(E_k) \setminus E_k = P(E_k, E)$ and thus $E_{k+1} = \mathcal{F}(E_k) = E_k \cup (\mathcal{F}(E_k) \setminus E_k) = E_k \cup P(E_k, E)$. This coincides with the case describing a given I-type admissible set.

The following result holds: There is some natural number m such that $E = E_m = E_{m+1}$.

The proof is as follows: By definition of the characteristic function $\mathcal{F}$, $E_k \subseteq \mathcal{F}(E_k) = E_{k+1}$ and $E_{k+1} = \mathcal{F}(E_k) \subseteq \mathcal{F}(E) = E$ for each natural number k. If there is no m such that $E_m = E_{m+1}$, then the cardinality of E_k will be strictly increasing. This contradicts with the fact that E is a finite set.

Since E is a I-type complete (preferred) extension, it is certainly a I-type admissible extension. By Theorem 2, there is some natural number r such that $E = E_{r+1} = E_{r+2}$. Next, we prove that $E_m = E_{r+1}$ and thus $E = E_m = E_{m+1}$. By definition of $\mathcal{F}$, $E_m \subseteq E_{r+1}$ or $E_{r+1} \subseteq E_m$. Without loss of generality, we suppose that $E_m \subseteq E_{r+1}$. If $E_m \neq E_{r+1}$, then $E_m \subset E_{r+1}$. Note that, $E_0 \subseteq E_1 \subseteq ... \subseteq E_m = E_{m+1} = E_{m+2} = ...$, so we have that $E_{r+1} \subset E_{r+1}$, a contradiction.

Note that we can not always obtain a II-type complete (resp. preferred) extension starting from a conflict-free collection of initial sets. In fact, there are many argumentation frameworks which have no II-type complete (resp. preferred) extension.

Example 1 (cont'd) *There are six II-type admissible sets:* $E_1 = \{1, 3\}$, $E_2 = \{2, 4\}$, $E_3 = \{7\}$, $E_4 = \{1, 3, 6\}$, $E_5 = \{1, 3, 7\}$, $E_6 = \{2, 4, 7\}$. *But, not all of them are complete extensions. In fact,* $\mathcal{F}(E_1) = \{1, 3, 5\}$, $\mathcal{F}(E_2) = \{2, 4, 8\}$, $\mathcal{F}(E_3) = \{7, 8\}$, $\mathcal{F}(E_4) = \{1, 3, 6, 5\}$, $\mathcal{F}(E_5) = \{1, 3, 7, 5, 8\}$, $\mathcal{F}(E_6) = \{2, 4, 7, 8\}$.

A further observation indicates that $\mathcal{F}(E_4)$, $\mathcal{F}(E_5)$ *and* $\mathcal{F}(E_6)$ *are the preferred extensions, but they are not II-type preferred.*

Next, we only talk about the description of II-type complete (resp. preferred) extensions.

Describing a II-type complete (or preferred) extension: Let E be a II-type complete (preferred) extension and $\mathcal{B}$ the collection of all initial sets contained in E, define $E_0 = \cup\mathcal{B}$ and $E_{k+1} = E_k \cup P_J(E_k, E)$ for each natural number k. Then, $E_0 \subseteq E$ is the union of initial sets contained in E and thus a II-type admissible set. $E_1 \subseteq E$ is the set obtained by adding to E_0 all the J-acceptable sets wrt. E_0 which are contained in E and thus a II-type admissible set, and so on.

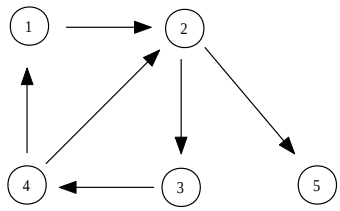
The following result holds: There is be some natural number m such that $E = E_m = E_{m+1}$.

The proof is as follows: By the definition of $P_J(E_k, E)$, $E_k \subseteq E_{k+1} = E_k \cup P_J(E_k, E) \subseteq E$ for each natural number k. If there is no m such that $E_m = E_{m+1}$, then the cardinality of E_k will be strictly increasing. This contradicts with the fact that E is a finite set. Since E is a II-type complete (preferred) extension, it is certainly a II-type admissible extension. By Theorem 4, there is some natural number r such that $E = E_{r+1} = E_{r+2}$.

Next, we prove that $E_m = E_{r+1}$ and thus $E = E_m = E_{m+1}$. By the definition of $P_J(E_k, E)$, $E_m \subseteq E_{r+1}$ or $E_{r+1} \subseteq E_m$. With out loss of generality, we suppose that $E_m \subseteq E_{r+1}$. If $E_m \neq E_{r+1}$, then $E_m \subset E_{r+1}$. Note that, $E_0 \subseteq E_1 \subseteq ... \subseteq E_m = E_{m+1} = E_{m+2} = ...$, so we have that $E_{r+1} \subset E_{r+1}$, a contradiction.

As for II-type complete (resp. preferred) extensions, the mixed complete (resp. preferred) extensions may not exist. So we only talk about the description of mixed-type complete (resp. preferred) extensions in the sequel.

Example 4 *Let $AF = (A, R)$ with $A = \{1, 2, 3, 4, 5\}$ and $R = \{(1, 2), (2, 3), (3, 4), (4, 1), (4, 2), (2, 5)\}$. The directed graph is as follows.*



It is easy to check that $I = \{1, 3\}$ is the only initial set and $E = \{1, 3, 5\}$ is the unique preferred extension, but E is not mixed-type preferred.

Describing a complete (or preferred) mixed-type extension: Let E be a mixed-type complete (preferred) extension and $\mathcal{B}$ the collection of all initial sets contained in E, define $S_0 = \cup\mathcal{B}$ and $T_0 = S_0 \cup P_J(S_0, E)$. For each natural number k, let $S_{k+1} = \mathcal{F}(T_k, E) = T_k \cup P(T_k, E)$ and $T_{k+1} = S_{k+1} \cup P_J(S_{k+1}, E)$. Then, S_0 is the union of initial sets contained in E, T_0 is the union of S_0 with all J-acceptable sets wrt. S_0 which are contained in E. S_1 is the union of T_0 with the set of arguments which are regularly defended by T_0 and contained in E, T_1 is the union of S_1 with all J-acceptable sets wrt. S_1 which are contained in E, and so on.

The following result holds: There is some natural number m such that $E = S_m$ or T_m. That is, we have $E = S_0 \cup P_J(S_0, E) \cup P(T_1, E)... \cup P_J(S_{m-1}, E) \cup P(T_{m-1}, E)$ or $E = S_0 \cup P_J(S_0, E) \cup P(T_0, E)... \cup P(T_{m-1}, E) \cup P_J(S_m, E)$.

5 Conclusion and future works

In this paper, the special type of acceptability called J-acceptability is studied. Meanwhile, the reinstatement criterion is extended to the J-reinstatement criterion and the characteristic function is extended to the J-characteristic function.

Based on the J-reinstatement criterion, we introduce the J-complete semantics which fills the gap between complete semantics and preferred semantics. It is claimed that an admissible set is preferred if and only if it is complete, J-complete and $A \setminus E$ has no non-empty admissible subset to be conflict-free with E.

With acceptability and J-acceptability, we figure out the structure of admissible sets, complete extensions and preferred extensions. That is, any admissible (resp. complete, preferred) extension can be obtained starting from a conflict-free collection of initial sets by iteratively applying some operators related to the acceptability and J-acceptability. This work suggests that for any argumentation framework, the problem of determining extensions will be completely solved if we can find out all initial sets.

J-acceptability combined with acceptability has been proved useful for giving a clear description for the extensions of the standard semantics. We plan to study the role of J-acceptability in other non-standard semantics, and more particularly for the structure analysis of their extensions.

Another direction for further research concerns the use of J-acceptability in dynamic argumentation frameworks. In literature, several works have proposed efficient ways for handling dynamics, such as [12] which introduces the division-based method, and [14] where a matrix approach allows for a decomposition of standard extensions, using unattacked sets of arguments. We are going to investigate the role of initial sets and J-acceptability in the construction of the extensions of an updated argumentation framework.

A Appendix: Proofs

Proof of Prop. 1: Suppose that $\mathcal{B} = \{B_i : 1 \leq i \leq n\}$. As $\cup\mathcal{B} \subseteq E$, $\cup\mathcal{B}$ is conflict-free. Therefore, $\mathcal{B}$ is a conflict-free set.

Let $c \in A \setminus \cup\mathcal{B}$ and $a \in \cup\mathcal{B}$ such that $(c, a) \in R$, then there is some $1 \leq i \leq n$ such that $a \in B_i$. Since B_i is initial and so admissible, there is some $b \in B_i$ such that $(b, c) \in R$. Certainly, we have $b \in \cup\mathcal{B}$. So, we prove that $\cup\mathcal{B}$ is admissible.

The second point is obvious.

□

Proof of Prop. 2: Since S is J-acceptable $wrt\ E$, $E \cup S$ is admissible. This implies that S is conflict-free in AF and thus also conflict-free in AF'.

Suppose that $i \in S$ and is attacked by j in AF', then i is attacked by j in AF. Because $E \cup S$ is admissible, there is some $k \in (E \cup S)$ such that k attacks j. On the other hand, $j \notin (E \cup R^+(E))$ means that $k \notin E$ and thus we have $k \in S$. Therefore, S is defended by itself in AF', i.e., S is admissible in AF'.

Conversely, $\mathcal{F}(E) \cap S = \emptyset$ comes from the fact that $S \subseteq (A \setminus (E \cup R^+(E)))$. And so, we only need to prove that $E \cup S$ is admissible in AF.

First, S is admissible implies that it is conflict-free in AF', and thus S is conflict-free in AF. Also note that S is conflict-free with E, so we have $E \cup S$ is conflict-free in AF. Second, let $i \in (E \cup S)$ and it is attacked by $j \in (A \setminus (E \cup S))$. If $i \in E$, then obviously there is some $k \in E$ which attacks j. Otherwise, $i \in S$. When $j \in \mathcal{F}(E)$, then there is some $k \in E$ which attacks j. When $j \notin \mathcal{F}(E)$, then $j \in (A \setminus (E \cup R^+(E)))$ and there is some $k \in S$ which attacks j according to the fact that S is admissible in AF'. By definitions, we conclude that $E \cup S$ is admissible in AF, and S is J-acceptable $wrt\ E$ in AF.

□

Proof of Prop. 3: It is an immediate consequence of Theorem 5 in [15]. □

Proof of Prop. 4: Let U be an unattacked subset of $AF = (A, R)$.

- ($\Leftarrow$) Let E be a J-complete extension of AF. If $E \cap U$ is not a J-complete extension of $AF' = AF\ |_U$, then there is some non-empty subset $S \subseteq U$ which is J-acceptable w.r.t $E \cap U$ in AF'. By definition, S is not admissible, $\mathcal{F}(E \cap U) \cap S = \emptyset$ and $S \cup (E \cap U)$ is admissible in AF'. Next, we prove that S is J-acceptable w.r.t E in AF and thus E is not J-complete in AF. This contradicts with the assumption.

  Since S is not admissible in AF', there is some $i \in S$ and $j \in (U \setminus S)$ such that $(j, i) \in R\ |_U$ but no argument of S attacks j. This means that S is not admissible in AF. Furthermore, U is an unattacked subset in AF implies that $\mathcal{F}(E \setminus (E \cap U)) \cap U = \emptyset$ and thus $\mathcal{F}(E \setminus (E \cap U)) \cap S = \emptyset$. Based on the fact $\mathcal{F}(E \cap U) \cap S = \emptyset$, we have $\mathcal{F}(E) \cap S = \emptyset$.

  Finally, $\mathcal{F}(E) \cap S = \emptyset$ indicates that no argument of E attacks the argument of S. And, S is J-acceptable w.r.t $E \cap U$ means that $S \cup$

$(E \cap U)$ is conflict-free in AF'. That is, no argument of S attacks the argument of $E \cap U$. If there is some $i \in S$ attacking an argument $j \in (E \backslash (E \cap U))$, then there must be some $k \in (E \backslash (E \cap U))$ attacking j according to the fact that E is admissible in AF. Therefore, $E \cup S$ is conflict-free.

Let $i \in (E \cup S)$ be attacked by an argument $j \in (A \setminus (E \cup S))$. If $i \in E$, then it is obviously defended by E. Otherwise, $i \in S$ and thus $j \in U$ because of the facts U is an unattacked subset and $S \subseteq U$. Since $(E \cap U) \cup S$ is admissible in AF', j must be attacked by some argument of $(E \cap U) \cup S$. Therefore, $E \cup S$ is admissible in AF.

- ($\Rightarrow$) Conversely, let S be a J-complete extension of $AF' = AF \mid_U$. We have to prove that there is some J-complete extension E of AF such that $S = E \cap U$.

  If S is a J-complete extension of AF, then $E = S$ satisfies the requirement. Otherwise, S is not J-complete in AF and there is some J-complete extension E of AF such that $S \subset E$. Suppose that $(E \setminus S) \cap U \neq \emptyset$, then $S \cup ((E \setminus S) \cap U) = E \cap U$ is J-complete in AF' by the first part of the proof. This is in contradiction with S being J-complete in AF'. Therefore, we have $(E \setminus S) \cap U = \emptyset$ and thus $S = E \cap U$.

□

Proof of Prop. 5: Let $\mathcal{C} = \{S_1, S_1, ..., S_k\}$. Since S_r is J-acceptable w.r.t E for each $1 \leq r \leq k$, $E \cup S_r$ is admissible. Note that $\{S_1, S_1, ..., S_k\}$ is a conflict-free collection, so $E \cup R_J(E) = \cup\{E \cup S_r : 1 \leq r \leq k\}$ is conflict-free. Furthermore, it is easy to check that $E \cup R_J(E)$ defends itself and thus is admissible.

□

Proof of Prop. 6: Let $\mathcal{B} = \{\{i\} : i \text{ is an initial argument}\}$, then $\mathcal{F}(\emptyset) = \cup\mathcal{B}$ is obviously a I-type admissible set. Suppose $\mathcal{F}^t(\emptyset)$ is a I-type admissible set for each $t < r$, we next prove that $\mathcal{F}^r(\emptyset)$ is a I-type admissible set. And thus, the result is true for each natural number k by mathematical induction.

Let $i \in (\mathcal{F}^r(\emptyset) \setminus \mathcal{F}(\emptyset))$, then there is some $t < r$ such that $i \in \mathcal{F}^{t+1}(\emptyset) \setminus \mathcal{F}^t(\emptyset)$. So, i is regularly accepted by the admissible set $\mathcal{F}^t(\emptyset)$ contained in $\mathcal{F}^r(\emptyset)$.

□

Proof of Prop. 7:

- ($\Rightarrow$) Let D be an admissible set such that $\cup\mathcal{B} \subseteq D \subset E$, then $E \setminus D$ is not admissible. Otherwise, there is some initial set I in $E \setminus D$. This contradicts with $\cup\mathcal{B} \subseteq D$. Since $\cup\mathcal{B} \subseteq D$, each $i \in (E \setminus D)$ is not regularly accepted by D admissible. And thus, we have $\mathcal{F}(D) \cap (E \setminus D) = \emptyset$. Therefore, $E = D \cup (E \setminus D)$ is admissible implies that $E \setminus D$ is J-acceptable w.r.t D.
- ($\Leftarrow$) Suppose some $i \in (E \setminus \cup\mathcal{B})$ is regularly accepted by an admissible set $T \subseteq E$, then i is regularly accepted by the admissible set $D = T \cup (\cup\mathcal{B}) \subseteq E$. Note that $E \setminus D$ is J-acceptable w.r.t D, so $\mathcal{F}(D) \cap (E \setminus D) = \emptyset$. That is i is not accepted by D and thus by T, a contradiction.

$\square$

Proof of Theo. 1: E_0 is obviously a I-type admissible set. Suppose E_r is I-type admissible for each $r < t$, we prove that E_t is a I-type admissible set.

Let $i \in (E_t \setminus \cup\mathcal{B})$, then there is some $r < t$ such that $i \in (E_{r+1} \setminus E_r)$. That is, $i \notin E_r$ and $i \in R_A(E_r)$. So, i is regularly accepted by the admissible set E_r. By mathematical induction, we claim that E_k is I-type admissible for each natural number k.

By definition of E_k, it is obvious that $E_{k+1} = E_k \cup R_A(E_k) = (E_{k-1} \cup R_A(E_{k-1})) \cup R_A(E_k) = E_{k-1} \cup R_A(E_{k-1}) \cup R_A(E_k) = ... = E_0 \cup R_A(E_0) \cup R_A(E_1) \cup ... \cup R_A(E_k)$.

$\square$

Proof of Theo. 2: E_0 is obviously a I-type admissible set. Suppose E_r is I-type admissible for each $r < t$, we prove that E_t is a I-type admissible set.

Let $i \in (E_t \setminus \cup\mathcal{B})$, then there is some $r < t$ such that $i \in (E_{r+1} \setminus E_r)$. That is, $i \notin E_r$ and $i \in P(E_r, E)$. So, i is regularly accepted by the admissible set E_r. By mathematical induction, we claim that E_k is I-type admissible contained in E for each natural number k.

Obviously, there is some natural number m such that $E_m \subsetneq E_{m+1} = E_{m+2}$ and $E_{m+1} = E_m \cup P(E_m, E) = (E_{m-1} \cup P(E_{m-1}), E) \cup P(E_m, E) = E_{m-1} \cup P(E_{m-1}, E) \cup P(E_m, E) = ... = E_0 \cup P(E_0, E) \cup P(E_1, E) \cup ... \cup P(E_m, E)$.

$\square$

Proof of Theo. 3: E_0 is obviously a II-type admissible set. For $k \geq 1$, let $i \in (E_k \setminus (\cup\mathcal{B})$. Then, there is some $r < k$ such that $i \in (E_r \setminus E_{r-1})$. That

is, $i \in R_J(E_r)$. So, i belongs to some J-acceptable set $S \subseteq R_J(E_r)$ w.r.t E_r. By definition, we claim that E_k is a II-type admissible set.

By definition of E_k, it is obvious that $E_{k+1} = E_k \cup R_J(E_k) = (E_{k-1} \cup R_J(E_{k-1})) \cup R_J(E_k) = E_{k-1} \cup R_J(E_{k-1}) \cup R_J(E_k) = ... = E_0 \cup R_J(E_0) \cup R_J(E_1) \cup ... \cup R_J(E_k)$.

□

Proof of Theo. 4: E_0 is obviously a II-type admissible set contained in E. For $k \geq 1$, let $i \in (E_k \setminus (\cup\mathcal{B})$. Then, there is some $r < k$ such that $i \in (E_r \setminus E_{r-1})$. That is, $i \in P_J(E_r)$. So, i belongs to some J-acceptable set $S \subseteq P_J(E_r)$ w.r.t E_r. By definition, we claim that E_k is a II-type admissible set contained in E.

Obviously, there is some natural number m such that $E_m \subsetneq E_{m+1} = E_{m+2}$ and $E_{m+1} = E_m \cup P_J(E_m, E) = (E_{m-1} \cup P_J(E_{m-1}), E) \cup P_J(E_m, E) = E_{m-1} \cup P_J(E_{m-1}, E) \cup P_J(E_m, E) = ... = E_0 \cup P_J(E_0, E) \cup P_J(E_1, E) \cup ... \cup P_J(E_m, E)$.

□

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