Decision-making under ordinal preferences and uncertainty

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Abstract

This paper investigates the problem of finding a preference relation on a set of acts from the knowledge of an ordering on events (subsets of states of the world) describing the decision-maker (DM)'s uncertainty and an ordering of consequences of acts, describing the DM's preferences. However, contrary to classical approaches to decision theory, we try to do it without resorting to any numerical representation of utility nor uncertainty, and without even using any qualitative scale on which both uncertainty and preference could be mapped. It is shown that although many axioms of Savage theory can be preserved and despite the intuitive appeal of the method for constructing a preference over acts, the approach is inconsistent with a probabilistic representation of uncertainty, but leads to the kind of uncertainty theory encountered in non-monotonic reasoning (especially preferential and rational inference), closely related to possibility theory. Moreover the method turns out to be either very little decisive or to lead to very risky decisions, although its basic principles look sound. This paper raises the question of the very possibility of purely symbolic approaches to Savage-like decision-making under uncertainty and obtains preliminary negative results.

1. Introduction

In Savage decision theory (Savage, 1972), a decision problem is cast in the framework of a set of states (of the world) and a set of consequences. States represent situations or circumstances in which the decision may take place, with the idea that this situation may affect the result of the decision. Consequences are results of decisions, such as earning money or moving to another situation. An act is viewed as a mapping d from the state space S to the consequence set X, namely in each state s the result of d is $d(s) \in X$.

Savage is interested in the following problem: Starting from a user-driven preference ordering over acts, i.e., $(X^S, \geq)$, and axioms that such a preference ordering should obey if the decision-maker is "rational", construct an ordering relation on the subsets of S, representing uncertainty of the decision maker and a preference ordering on the consequences. If $(X^S, \geq)$ satisfy suitable properties then the uncertainty on S can be represented by a probability distribution p, the preference ordering on X by a utility function u, and the preference ordering on acts by expected utility:

$$d_1 \geq d_2 \Leftrightarrow \sum_{s\in S} p(s)u(d_1(s)) \geq \sum_{s\in S} p(s)u(d_2(s)).$$

In this research, we are first interested in the converse problem, namely: if an uncertainty ordering on events in 2^S is given, and a preference ordering on consequences is given as well, how can a preference ordering on acts be recovered with a purely symbolic approach? This problem is realistic because a decision maker is not necessarily capable of describing his state of uncertainty by means of a probability distribution, nor may he/she be able to quantify his/her preferences on X. We show that if we start from a comparative probability ordering on events we get a non-transitive ordering on acts. However this anomaly is avoided if we assume a possibility ordering on events. Conversely, starting from a preference ordering over acts, i.e., $(X^S,\geq)$, and axioms suggested by the previous analysis, we can derive an ordering relation on the subsets of S, representing uncertainty of the decision maker and a preference ordering on the set of consequences, X. However as shown here, the uncertainty ordering obtained in this general setting is almost possibilistic and the decision guidelines obtained in this setting are questionable.

2. From uncertainty and preference orderings to an ordering of actions

This section introduces a natural technique to compute a preference ordering on acts from a purely symbolic perspective. Let $(2^S, \geq_U)$ be an uncertainty ordering on events. The least property requested on $\geq_U$ is that it is a complete partial ordering, that it is non-trivial ($S >_U \emptyset$) and that it is faithful to deductive inference:

$$A \subseteq B \Rightarrow A \leq_U B.$$

As usual, let us define the indifference $\sim_U$ and the strict ordering $<_U$ induced from $\geq_U$ by:

$$A \sim_U B \text{ iff } A \leq_U B \text{ and } B \leq_U A$$
$$A <_U B \text{ iff } A \leq_U B \text{ and not } (B \leq_U A).$$

The preference on X is just a complete partial ordering $\leq_P$. It is assumed that X has at least two elements x_1 and x_2 s.t. $x_1 > x_2$.

If no commensurability assumption can be made, a natural way of lifting ($\leq_U$, $\leq_P$) on X^S is to assume that an act d_1 is at least as promising as an act d_2 iff the event formed by the disjunction of states in which d_1 gives results at least as good as d_2 is at least as likely as the event formed by the disjunction of states in which d_2 gives results at least as good as d_1.

Define s as a more promising state for d_1 than for d_2 iff $d_1(s) \geq_P d_2(s)$. Let $A(d_1,d_2) = \{s, d_1(s) \geq_P d_2(s)\}$ be the set of states where d_2 does not outperform d_1, then define the preference between acts $\geq$ by:

$$d_1 \geq d_2 \text{ iff } A(d_1,d_2) \geq_U A(d_2,d_1). \quad \text{(A)}$$

The indifference relation and the strict ordering are then given by:

$$d_1 \sim d_2 \text{ iff } A(d_1,d_2) \sim_U A(d_2,d_1),$$
$$d_1 > d_2 \text{ iff } A(d_1,d_2) >_U A(d_2,d_1).$$

This approach seems to be very natural, and is the first one that comes to mind when information is only available under the form of an ordering of events and an ordering of consequences that have no relationship with each other, i.e., when the preference and uncertainty scales are not commensurable.

The properties of the relations $\geq$, $\sim$, and $>$ on X^S will depend on the properties of $\geq_U$. The most obvious choice for $\geq_U$ is a qualitative probability, i.e., an ordering such that:

i) $\geq_U$ is complete and transitive
ii) $S >_U \emptyset$, $\forall\, A \geq_U \emptyset$
iii) $A \geq 0$, $A \cap (B \cup C) = \emptyset \Rightarrow$
$B \geq_U C \Leftrightarrow A \cup B \geq_U A \cup C$.

A first negative result is that if $\geq_U$ is a qualitative probability ordering then relation $>$ in X^S is not necessarily transitive, that is it may happen that $d_1 > d_2$, $d_2 > d_3$ and $d_3 > d_1$. So we cannot agree with Savage decision theory with our ordinal approach.

Example

States	a	b	c	d	e	f
Prob.	2/9	1/9	1/9	2/9	1/9	2/9
d	5	100	0	0	-10	-10
d'	0	-15	100	-10	0	10
d"	-25	0	-40	20	40	0

It is easy to verify that
P(A(d,d'))=P({a,b,d})=5/9 > P(A(d',d))=P({c,e,f})=4/9
P(A(d',d"))=P({a,c,f})=5/9 > P(A(d",d'))=P({b,d,e})=4/9
P(A(d,d"))=P({a,b,c})=4/9 > P(A(d",d))=P({d,e,f})=5/9.
Hence d > d' >d" but d" > d'. Note that the respective utilities of the acts depend on the figures of utility in the table, but the intransitivity result does not insofar as the sign of the utility values is respected.

Thinking about a more qualitative approach, the simplest type of qualitative uncertainty ordering are those induced by necessity measures, and for which (iii) is changed into (Dubois, 1986):

$$B \geq_N C \Rightarrow A \cap B \geq_N A \cap C$$

(they are also epistemic entrenchments in the sense of Gärdenfors (Dubois,Prade 1991)). Let us study the ordering of acts induced by necessity orderings. Remember that any necessity ordering can be represented by a necessity function on an ordinal scale, i.e., $\exists N$ such that:

$$B \geq_N C \Leftrightarrow N(B) \geq N(C)$$

where $N(F) \in [0,1]$ and $N(F \cap G) = \min(N(F), N(G))$ whatever F and G. The simplicity of necessity orderings lies in the fact that they can be derived from a complete partial ordering on states only. This ordering can be encoded as a possibility distribution. In the following, we will use $\geq_N$ for $\geq_U$.

We can now write in accordance with (A)

$$d_1 \geq d_2 \Leftrightarrow N(A(d_1,d_2)) \geq N(A(d_2,d_1)) \quad (1)$$
$$\Leftrightarrow N(d_1 \geq_P d_2) \geq N(d_2 \geq_P d_1)$$

denoting $A(d_1,d_2)$ as $d_1 \geq_P d_2$, as usual in probability. $d_1 \geq d_2$ then means that d_1 will almost certainly outperform d_2 since the agent is at least as sure to be in a state where d_1 outperforms d_2 as to be in a state where d_2 outperforms d_1. In terms of possibility measures (Zadeh, 1978), it holds that:

$$d_1 \geq d_2 \Leftrightarrow \prod(d_1 >_P d_2) \geq \prod(d_2 >_P d_1)$$

since $\prod(A) = 1 - N(\bar{A})$. Hence the ordering on acts can as well be represented in terms of the dual possibility ordering $B \geq_{\prod} C$ iff $\bar{C} \geq_N \bar{B}$.

3. Relationships with Savage's axiomatics

In case of necessity measure on S, the following properties of (X^S, $\geq$) can be established, where $\geq$ is defined by (1):

• P1': Restricted Savage's P1
(X^S,$\geq$) is completely related: $d_1 \geq d_2$ or $d_2 \geq d_1$ for all pairs of acts;
(X^S,$>$) is a transitive, irreflexive, partially ordered set;

the indifference relation $\sim$ is reflexive, symmetrical but *not transitive* $d_1 \sim d_2$ and $d_2 \sim d_3$ do not imply $d_1 \sim d_3$. However:

$$d_1 > d_2 \text{ and } d_2 \sim d_3 \Rightarrow d_1 \geq d_3$$

So the only difference with Savage's P1 is the lack of transitivity of the indifference relation. It also differs from

Lehmann (1996) who assumes that $\geq$ is not complete but the indifference is transitive. Non-transitive indifference is natural if $d \sim d'$ means that the merit of d is too close to the merit of d' to be told apart. Closeness is not transitive.

• Savage's P2: The preference relation $\geq$ obeys the sure thing principle

If $d_{|A}$ is the restriction of act d to states in A, we have that, whatever d_1, d_2, d and d':

$$d_{1|A} \cup d_{|\bar{A}} \geq d_{2|A} \cup d_{|\bar{A}} \Leftrightarrow d_{1|A} \cup d'_{\bar{A}} \geq d_{2|A} \cup d'_{|\bar{A}}$$

where $d_{1|A} \cup d_{|\bar{A}}$ is the act that is identical to d_1 on A and to d on its complement.

• Weak unanimity:

$A \cap B = \emptyset \Rightarrow$ if $d_{1|A} \geq d_{2|A}$ and $d'_{1|B} \geq d'_{2|B}$ then $d_{1|A} \cup d'_{1|B} \geq d_{2|A} \cup d'_{2|B}$.

This property sounds like a unanimity principle, viewing states as agents. In the classical decision theory, this property is a consequence of the sure thing principle and the transitivity of $\geq$. Although $\geq$ is not transitive here, the weak unanimity property is satisfied.

In the classical decision theory, changing $\geq$ into $>$, we get the following strict unanimity principle (which is NOT satisfied here):

$A \cap B = \emptyset \Rightarrow$ if $d_{1|A} > d_{2|A}$ and $d'_{1|B} \geq d'_{2|B}$ then $d_{1|A} \cup d'_{1|B} > d_{2|A} \cup d'_{2|B}$.

In the classical decision theory, this property is also a consequence of the sure thing principle and the transitivity of $\geq$. The difference with Savage is that strict unanimity is not satisfied here.

• Savage's P3: Compatibility with constant acts

If $A \subseteq S$, $A \neq \emptyset$, $d(s) = x, \forall s$ and $d'(s) = y, \forall s$, then $d_{|A} \geq d'_{|A} \Leftrightarrow x \geq_P y$.

• Savage's P4: Restriction to 2-consequence actions.

$(S,\geq_U)$ and $(X,\geq_P)$ induce $(X^S,\geq)$; one may now reproject X^S on 2^S by considering 2-consequence acts. For any x, y such that $y <_P x$ that we denote $d_A^{x,y}$ the act such that $d(s) = x$ for $s \in A$ and $d(s) = y$ on $\bar{A}$.

We have the following result:

$$\forall\ d_A^{x,y} \geq d_B^{x,y} \Leftrightarrow d_A^{x',y'} \geq d_B^{x',y'}.$$

This is Savage P4 that enables events A and B to be consistently compared by fixing $x > y$ arbitrarily when selecting $d_A^{x,y}$ and $d_B^{x,y}$.

Postulate P5 of Savage is trivially satisfied since it is assumed that X has as least two elements x_1 and x_2 s.t. $x_1 > x_2$.

4. Properties on $\geq$ on 2-consequence acts (events)

Using Savage's P4 to reduce $\geq$ to 2-consequence acts leads to define a new partial ordering among events. It is defined as follows:

$$A \geq B \Leftrightarrow \forall\ x > y,\ d_A^{x,y} \geq d_B^{x,y}$$

At this point it is useful to emphasize that our framework involves at the same time a procedure that lifts an uncertainty ordering on events (here an ordering of necessity) to an ordering on acts, and a procedure that, because of P4, derives a new uncertainty ordering on events from the ordering on acts by a mere restriction. Generally the new uncertainty ordering will differ from the original one, here $\geq_N$.

Although different from $\geq_N$, the ordering $\geq$ obtained via lifting followed by restriction can be expressed in terms of the necessity ordering $\geq_N$. Indeed, noticing that $A(d_A^{x,y}, d_B^{x,y}) = \bar{B} \cup A$, the following equivalencies holds:

$$\begin{aligned} A \geq B &\Leftrightarrow N(\bar{B} \cup A) \geq N(\bar{A} \cup B) \\ &\Leftrightarrow \Pi(A \cap \bar{B}) \geq \Pi(\bar{A} \cap B) \end{aligned}$$

using the possibility measure.

The relation $\geq$ on 2^S is closely related to the necessity and possibility orderings induced by N (and its dual Π); more specifically it is a refinement of both orderings since the following properties hold:

$$N(A) > N(B) \Rightarrow A > B$$
$$\Pi(A) > \Pi(B) \Rightarrow A > B$$

but it may happen that $N(A) = N(B)$ and $A > B$ or that $\Pi(A) = \Pi(B)$ and $A > B$.

Notice that $\sim$ on 2^S is not transitive since

$$\begin{matrix} \Pi(A \cap \bar{B}) = \Pi(\bar{A} \cap B) \\ \Pi(B \cap \bar{C}) = \Pi(\bar{B} \cap C) \end{matrix} \not\Rightarrow \Pi(A \cap \bar{C}) = \Pi(\bar{A} \cap C).$$

In fact, the indifference $\sim$ on 2^S is transitive when considering only mutually exclusive events. Remember Nahmias' notion of "unrelatedness" of events (Nahmias 1978): $A \perp B \Leftrightarrow \Pi(A \cap B) = \min(\Pi(A),\Pi(B))$ means that A and B are not mutually exclusive according to Π, and conversely A and B are said to be mutually exclusive if $\Pi(A \cap B) \neq \min(\Pi(A),\Pi(B))$ that is if $\Pi(A \cap B) < \min(\Pi(A \cap \bar{B}), \Pi(\bar{A} \cap B))$. We have that:

$$\Pi(A) \geq \Pi(B) \Leftrightarrow A \geq B \text{ iff A and B are mutually exclusive.}$$

In other terms, $\geq$ refines $\geq_N$ and the dual possibilistic ordering $\geq_\Pi$ for non-mutually exclusive events.

The partial ordering $>$ on events is a special case of the so-called "discrimax" ordering between vectors (Dubois et al., 1996; Fargier et al., 1993):

$$V \geq_{discrimax} W \Leftrightarrow \max_{i \in \mathcal{D}(V,W)} V_i \geq \max_{i \in \mathcal{D}(V,W)} W_i$$

where $\mathcal{D}(V,W) = \{i, V_i \neq W_i\}$ is the set of components of the vectors where they can be discriminated. Now if π denotes the possibility distribution associated to $\prod$ (i.e., such that $\prod(A) = \max_{s \in A} \pi(s)$) and let $\vec{a}$ be the vector $(a_1, \ldots, a_n)$ such that, denoting $S = \{s_1, s_2, \ldots, s_m\}$:

$$a_i = \pi(s_i) \quad \text{if } s_i \in A$$
$$= 0 \quad \text{otherwise}$$

then $A \geq B \Leftrightarrow \vec{a} \geq_{Discrimax} \vec{b}$.

Finally, since $\geq$ on X^S satisfies Savage's sure thing principle, we have the additivity condition (iii) of qualitative probabilities (but remember here, the indifference relation between events is not transitive):

$$A \geq 0\ A \cap (B \cup C) = \emptyset \Rightarrow (B \geq C \Leftrightarrow A \cup B \geq A \cup C).$$

In particular:

$$A \geq B \Leftrightarrow \bar{A} \leq \bar{B}.$$

However, we get properties that are NOT satisfied by qualitative probabilities:

$$A \cap (B \cup C) = \emptyset \text{ and } A \geq B \text{ and } A \geq C \Rightarrow A \geq B \cup C$$
$$A \cap (B \cup C) = \emptyset \text{ and } A > B \text{ and } A > C \Rightarrow A > B \cup C \quad \text{(Acceptance)}$$

The latter property is equivalent to a characteristic property for uncertainty orderings that represent acceptance (Dubois and Prade, 1995b; Friedman and Halpern, 1996), i.e., orderings for which for any not empty set A, the set $\{B, A \cap B > A \cap \bar{B}\}$ (the set of propositions accepted by > when A is true) is deductively closed.

5. Link with non-monotonic reasoning

Notice that the ordering $A \geq B$ can also be related to nonmonotonic inference $A \mathrel{|\!\sim} B$ which expresses that $\prod(A \cap B) > \prod(A \cap \bar{B})$. In (Benferhat et al., 1992) this is shown to be characteristic of rational inference of Lehmann (Lehmann and Magidor, 1992), that is inference which satisfies rational monotony. Since the sets $A \cap B$ and $A \cap \bar{B}$ are disjoint, we have:

$$A \mathrel{|\!\sim} B \text{ is equivalent to } A \cap B > A \cap \bar{B}$$

Conversely, it is easy to check that $A > B$ can be written as $A \Delta B \mathrel{|\!\sim} \bar{B} \cup A$, (since it is equivalent to $A \cap \bar{B} > \bar{A} \cap B \Leftrightarrow A > B$, observing that $(A \Delta B) \cap (\bar{B} \cup A) = A \cap \bar{B}$ and $(A \Delta B) \cap (B \cap \bar{A}) = \bar{A} \cap B$) where Δ denotes the symmetric difference.

6. From axioms on preference over acts to a theory of uncertainty

Since many of Savage axioms have been retrieved in the previous investigation, it is interesting to start the other way around, namely given a preference relation $\geq$ on acts, that satisfies the above Savage-like properties, determine the ordering of events induced by $\geq$ using Savage's P4 and the ordering of consequences using Savage's P3 that enable the original ordering $\geq$ on acts to be recovered using the lifting procedure (A). Doing so, the approach is much more general: the assumption that uncertainty is represented by possibility measures is no longer necessary since as in the tradition of decision theory, the uncertainty representation now comes from the properties of the preference over acts only.

Let X^S be a set of acts equipped with a preference relation $\geq$ such that

- P'1: $(X^S,\geq)$ is complete: $d_1 \geq d_2$ or $d_2 \geq d_1$ for all pairs of acts.
  $(X^S,>)$ is a transitive, irreflexive, partially ordered set
- P2: $(X^S,\geq)$ satisfies the sure thing principle.
- P3: $(X^S,\geq)$ satisfies Savage's axiom of compatibility with constant acts
- P4: $(X^S,\geq)$ satisfies Savage's P4 axiom about 2-consequence actions.
- P5': $\exists$ x, y, z three constant acts such that $x > y > z$
- Lifting Axiom A: $d_1 \geq d_2 \Leftrightarrow d^{x,y}_{A(d_1,d_2)} \geq d^{x,y}_{A(d_2,d_1)}$.

Under these conditions we have proved that:

- The preference on X is a complete preorder (but the ordering of states in S may be partial)
- The ordering on events is coherent with set inclusion.
- The ordering on events satisfies the property that characterizes acceptance orderings ($A \cap (B \cup C) = \emptyset$ and $A > B$ and $A > C \Rightarrow A > B \cup C$).

This type of ordering is closely related to nonmonotonic inference, namely it can be proved that the inference relation $A \mathrel{|\!\sim} B$ defined by $A \cap B > A \cap \bar{B}$ is preferential iff and only if > is an acceptance ordering: we have proved that the monmonotonic inference built from the ordering projected from X^S to 2^S satisfies the properties of system P of Kraus et al. (1990)).

It is also known (Dubois and Prade, 1995a) that any such ordering of events can be represented by means of a family of possibility measures in the sense that:

For any preferential relation $A \mathrel{|\!\sim} B$ there exists a set $\mathcal{F}$ of possibility measures such that

$$A \mathrel{|\!\sim} B \text{ iff for all } \prod \in \mathcal{F}, \prod(A \cap B) > \prod(A \cap \bar{B})$$

So, starting from a general framework on acts, that respects Savage approach to a large extent, we find a representation of uncertainty which is very particular, and almost equivalent to possibility theory (up to the rational monotony axiom for A $\mid\!\sim$ B, which holds in Sections 3 and 4). The possibilistic approach of Sections 3 and 4, based on a preorder on states and a necessity measure is recovered exactly if a counterpart of rational monotony for 2-consequence acts is added to the above set of postulates.

7. Conclusion

This result is rather negative for decision theory when only ordinal information about uncertainty and preference on consequences is available and no commensurability assumption is assumed between uncertainty and preference. Despite the presence of well-known Savage axioms, including the sure thing principle, the admissible uncertainty functions do not contain probability functions (except very special ones, laid bare by Snow (1996) (and providing a non-infinitesimal representation of System P), which are such that S is totally ordered and the probability of each state is larger that the sum of the probabilities of less probable states; see also (Benferhat et al., 1996)).

The decision theory obtained captures either very risky attitudes or is not decisive at all. As preferential inference is very cautious, the orderings on acts which do not correspond to a total ordering on states will not be very discriminating. On the contrary if the set of states is totally ordered in terms of plausibility, the decisions will be very risky because, as usual with rational inference the decision maker will always assume that the world is in the most normal state. Cautious decisions will never be preferred.

Example: Consider the omelette example of Savage (1972, pages 13 to 15). The problem is whether to add an egg to a 5-egg omelette: The set of 6 consequences is as in the following table:

STATES / ACTS	fresh egg	rotten egg
break the egg in the omelette BIO	a 6 egg omelette (6)	nothing to eat (1)
break it apart in a cup BAC	a 6 egg omelette, a cup to wash (5)	a 5 egg omelette, a cup to wash (3)
throw it away TA	a 5 egg omelette, one spoiled egg (2)	a 5 egg omelette (4)

Integers between parentheses indicate the ordering of consequences, in decreasing order of preference. The reader can easily check that he/she agrees with this ordering. If fresh egg is more likely than rotten egg then A(BIO, BAC) = A(BIO, TA) = A(BAC, TA) = {fresh} > A(BAC, BIO) = A(TA, BIO) = A(TA, BAC) = {rotten}. So the decision making attitude induced by the approach is: break the egg in the omelette if you think the egg is fresh, throw it away if you think it is rotten, and do anything you like if you have no opinion (all acts equally preferred then). Clearly this results in many starving days, and garbage cans with lots of spoiled fresh eggs.

Although the axioms on acts, described above, look innocuous, it is difficult to maintain that the decision guidelines offered by the theory are reasonable. In practice, it is advisable to act more cautiously, and to break the egg in a spare cup in case of serious doubt. In contrast the qualitative theory developed by Dubois and Prade (1995c) that justifies a pessimistic decision criterion generalizing Wald criterion looks more satisfactory. Applied to the egg example, it recommends act BAC in case of relative ignorance on the egg state; see (Dubois, Prade and Sabbadin, 1997). However that theory relies on a commensurability assumption between uncertainty and preference, which may be questioned by tenants of a purely symbolic approach. In the purely symbolic approach of this paper, the destructive property seems to be the lifting axiom (the only one not related to Savage's framework). However the lifting axiom seems difficult to avoid when only ordering relations are available on events and consequences, and one wishes to stick to a purely symbolic technique to compute an ordering on acts.

This paper leads to an open question: Is there an alternative to the lifting axiom that would enable a reasonably cautious and albeit decisive ordering on acts to be computed on the basis of ordering relations on events and consequences? In other words, how to do away with commensurability assumptions and/or numerical approaches while capturing anthropomorphic decision attitudes?

A natural alternative to the lifting principle used in this paper would rather start by considering the set of consequences which are reached at least as certainly by decision d_1 as by decision d_2, namely the set of consequence $B(d_1 \geq d_2) = \left\{x, d_1^{-1}(x) \geq_U d_2^{-1}(x)\right\}$ where $d^{-1}(x)$ is the set of states in S from which d leads to consequence x. The problem is now to compare $B(d_1 \geq d_2)$ and $B(d_2 \geq d_1)$ in terms of preferences on X. Since $B(d_1 \geq d_2)$ is not usually a singleton on X, we need to add an additional hypothesis on DM's attitude in face of risk. Namely, if the DM is rather pessimistic (cautious), he/she may consider that $d_1 \geq d_2$ iff $\min_X B(d_1 \geq d_2) \geq_P \min_X B(d_2 \geq d_1)$, while if the DM is optimistic it may act on the basis of the $d_1 \geq d_2$ iff $\max_X B(d_1 \geq d_2) \geq_P \max_X B(d_2 \geq d_1)$. We may also think of comparing $\min_X B(d_1 \geq d_2)$ with $\max_X B(d_2 \geq d_1)$, or to refine the 'min' and 'max' over X by 'discrimin' and 'discrimax'. The investigation of this approach, which supposes that we know something about DM's attitude, is a topic for further research.

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