A Multiobjective Performance Evaluation Framework for Routing in Wireless Ad Hoc Networks

Katia Jaffrès-Runser*†, Mary R. Schurgot*, Cristina Comaniciu* and Jean-Marie Gorce†
*Dept. of Electrical and Computer Engineering, Stevens Institute of Technology, Hoboken, New Jersey 07030, USA
Email: {mschurgo, ccomanic}@stevens.edu
†Université de Lyon, INRIA, INSA-Lyon, CITI, F-69621, FRANCE
Email: {katia.runser, jean-marie.gorce}@insa-lyon.fr

***Abstract*—Wireless ad hoc networks are seldom characterized by one single performance metric, yet the current literature lacks a flexible framework to assist in characterizing the design tradeoffs in such networks. The aim of this paper is not to propose another routing strategy. Instead, we address this problem by proposing a new modeling framework for routing in ad hoc networks, which will result in a better understanding of network behavior and performance when multiple criteria are relevant. Our approach is to take a holistic view of the network that captures the cross-interactions among interference management techniques implemented at various layers of the protocol stack. The resulting framework is a complex multiobjective optimization problem that can be solved through existing multiobjective search techniques. In this contribution, we present the Pareto optimal sets for an example sensor network when delay, robustness and energy are considered.**

I. Introduction

Wireless ad hoc and sensor networks are many times operating in difficult environments and require several performance criteria to be satisfied, related to timely, reliable, and secure information transfer. To ensure information transfer across the network, one of the key elements is the routing protocol. Various constraints related to transmission delay [8], energy consumption [9] or fairness [10] are added on top of its main design goal of reliable information transmission. As a consequence, the assessment of routing protocols relies on various criteria which may be evaluated analytically or through network simulations.

To better understand the capabilities of routing on a given network topology, a pre-requisite is to know the bounds that can be achieved with respect to multiple performance criteria. These bounds can highlight the interdependence and compromises existing between the performance metrics considered. As a consequence, defining a unified framework capable of capturing the trade-offs existing between multiple performance metrics of the routing problem becomes predominant.

Most of the works proposed in the literature on the performance evaluation of wireless ad hoc networks usually consider one or sometimes two objectives at a time. Characterizing the performance of a wireless network for the sole metric of capacity [3] [2] has triggered a comprehensive work in the last decade, starting with the seminal work of Gupta and Kumar [1]. The analysis of the trade-off between capacity and delay has been investigated by Gamal et al. in [4]. Comaniciu and Poor [5] proposed to account for delay as a constraint in their capacity analysis. A tight hyperbolic bound on energy and delay for wireless networks has been provided by Brand and Molisch in [6]. Xue et al. also investigated the trade-off between delay and latency for underwater sensor networks in [7]. To the best of our knowledge, there is no work considering more than two performance objectives at a time.

Obtaining bounds on multiple benchmark objectives requires the definition of a multiobjective optimization problem whose resolution provides a Pareto set of solutions. A solution herein characterizes one possible route configuration with its resource allocation in terms of transmission rate. The Pareto set is composed of non-dominated solutions, i.e. solutions of the search space that are never dominated by any other one with respect to the evaluation criteria considered. This multiobjective optimization problem is hard to solve since it is the combination of a resource allocation problem and a routing problem.

Understanding the tradeoffs involved with various routing solutions will enable adaptive resource management across layers and nodes, leading to a more accurate "local to global performance mapping" for practical routing protocol design. In this paper, we propose a novel framework capable of providing a bound for multiple performance metrics at a time. It is composed of both a probabilistic cross-layer network model and a multiobjective optimization problem formulation. The network model proposes a novel cross-layer definition of interference where the interaction between routing decision and resource allocations is accounted for precisely. With its intrinsic probabilistic definition, it is capable of defining various routing techniques such as multi-hop single path routing, broadcast protocols or multi-path protocols. The multiobjective optimization problem is solved using the PMOTS algorithm to retrieve the set of Pareto-optimal solutions. This global cross-layer multiobjective framework is applied herein to tackle the problem of robust routing for wireless sensor networks. The following three criteria are relevant in that context: *(i)* robustness in terms of the probability of having a successful packet transmission, *(ii)* delay in terms of the average delay and *(iii)* the forwarding energy in terms of the energy spent by the network in relaying.

Our aim is not to develop a new routing algorithm but to provide a larger framework capable of capturing the performance tradeoffs of a given network by computing the set of Pareto-optimal routing strategies. This characterization provides an efficient tool to:

- compare the performance of existing routing algorithms to the bound provided by the set of Pareto-optimal strategies, and
- foster the development of more efficient and flexible routing strategies, depending on the requirements an end user would put on the performance of the network.

The proposed framework encompasses various routing techniques (e.g. multi-hop, probabilistic routing, etc.) since it is based on a probabilistic network formulation.

Our main contributions in this work are two-fold:

- Propose a general cross-layer framework network model, capable of capturing the impact and interaction of a wide range of interference and resource management techniques for various channel conditions;
- Formulate a multiobjective routing optimization problem by defining appropriate evaluation functions for criteria such as: robustness of information transfer, end-to-end delay, and energy consumption.

The multiobjective routing optimization problem described in the following can be solved using existing multiobjective search techniques [11]. However, we will concentrate on the model description and only give a short description of the optimization heuristic considered for solving the problem.

The paper is organized as follows. In Section II we present our cross-layer framework based on a probabilistic network model. Section III formulates routing in an ad hoc network as a multiobjective optimization problem and Section IV provides a first formulation applied to sensor networks. Results for a simple problem instance are then given in Section V to illustrate our modeling framework and Section VI concludes the paper.

II. A Cross-layer Framework for Network Modeling

A. Probabilistic network model

Our proposed model considers a probabilistic network which is characterized by two probability measures: link and node probability. These two parameters completely characterize the network and capture cross-layer interactions.

The node probability (χ_i) captures the availability of node i for routing purposes, i.e. the probability that node i re-broadcasts a received packet. The node probability has two components ($\chi_i = \xi_i \cdot x_i$), one that is determined by the environment and protocol implementations at adjacent layers, ξ_i, (e.g. congestion models, node failures, security risks, energy levels), and one component x_i that corresponds to network routing choices, which we aim to optimize in the multiobjective routing framework.

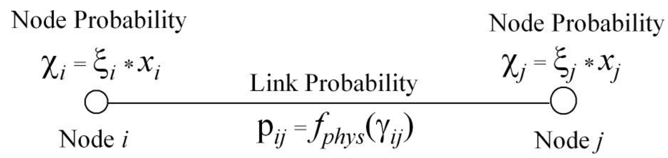

Fig. 1. Node and link probabilities on a link (i, j).

The link probability (p_{ij}) captures the link availability, i.e., the probability of a successful transmission over a link (i, j). Characterization of the link probability is impacted by impairments and enhancements at various layers of the protocol stack such as fading at the physical layer or congestion at the MAC layer. Both node and link probabilities are illustrated in Fig. 1.

Both node and link probability measures are strongly related due to the nature of the wireless channel. Hence, once the node probabilities χ_i are set, the activity of every node of the network is fixed and the interference distribution can be completely determined given the activity of the nodes on the wireless channel. As a consequence, the link probabilities can be computed as a function of the signal to interference plus noise ratio (SINR). Once link and node probabilities are available, various performance metrics such as delay, robustness or energy consumption can be calculated for various transmission schemes (unicast, multicast, broadcast, anycast, etc.).

In the following, we consider the set of node probabilities as the variables of the network optimization problem. Finding the best possible routing choices with respect to one particular criterion reduces to the problem of selecting the set of node probabilities that optimizes one particular objective of the network. Within a multiobjective perspective, solving the network optimization problem requires finding the set of Pareto-optimal solutions that concurrently optimizes several performance metrics of the network.

To illustrate our framework, we consider here a network where the nodes are independent and randomly distributed according to a random point process of density ρ over a disk $\mathcal{D}$. The communication between any two nodes is performed in a half-duplex mode over a single to multi-hop path. The bandwidth of the channel is divided into R resources (time slots, frequencies or codes). For clarity purposes, we present this model in the context of time-multiplexing.

This paper concentrates on a single flow but our framework can be extended to multiple flows since the proposed interference model accurately accounts for all the nodes transmitting in the network. Hence, one source transmits a constant traffic in one of the R time slots. A relay does not keep track of the packets already transmitted and consequently may forward the same packet several times. However, a node relays the packets in the order they are received in one of its available resources. If several packets are received in the same frame, it can only transmit the proportion of packets its global transmission probability x_i allows. The packets that the node cannot forward are dropped. The maximum number of hops H_M a packet can travel in the network is also fixed.

B. Link probabilities

A realistic link (i, j) in time slot r is characterized by its transmission probability $p_{ij}(r)$, which is a function of the statistical distribution of the SINR at the location of the destination node j. Such a computation captures the cross-layer impact of the routing decision on the physical layer performance since the activity of all the nodes of the network are accounted for statistically in the model. The following are some preliminary definitions and notations that are needed to define the link probability:

Pathloss attenuation factor: a_{ij} reflects the attenuation due to propagation effects between nodes i and j. In our simulations, the simple isotropic propagation model is considered.

Interference: Since we consider time-multiplexed channels, interference only occurs between transmissions using the same channel at the same time. Hence, the power of interference $I_{ij}(r)$ on a link (i, j) using resource r and computed at node j is defined by:

$$I_{ij}(r) = \sum_{k=1}^{K} P_k \; a_{kj} \quad \text{for } k \neq i \tag{1}$$

where K is the number of interfering signals in resource r.

SINR: The SINR between any two nodes i and j in resource r is given by:

$$\overline{\gamma}_{ij}(r) = \frac{P_{ij}}{N_0 + I_{ij}(r)} \tag{2}$$

where P_{ij} is the power received in j, $I_{ij}(r)$ is the interference power on the link and N_0 the noise power density. We have $P_{ij} = P_i \; a_{ij}$ for a fixed nominal transmission power P_i and a pathloss attenuation factor a_{ij}.

Packet error rate (PER): For a specific value of SINR γ, the packet error rate PER can be computed according to:

$$PER(\gamma) = 1 - [1 - BER(\gamma)]^{N_b} \tag{3}$$

where N_b is the number of bits of a data packet and $BER(\gamma)$ is the bit error rate for the specified SINR per bit γ which depends on the physical layer technology and the statistics of the channel. Results are given for an AWGN channel and a BPSK modulation without coding where $BER(\gamma) = Q\left(\sqrt{2\gamma}\right) = 0.5 * \text{erfc}(\sqrt{\gamma})$.

Transmission rate: The activity of a network node in a channel $r \in [1, .., R]$ is given by its transmission rate $\tau_i(r) \in [0, 1]$ in that particular channel. This rate is defined as the percentage of time a node i transmits using resource r.

Additional Notations: A node i is said to be active in the network if $\sum_r \tau_i(r) > 0$, and

- M gives the number of active nodes of the network,
- An interfering set on a link (i, j) is a set of $K \leq M - 1$ active nodes,
- $\mathcal{L}_{-i}$ refers to the set of all possible interfering sets and has a cardinality of $L = \sum_{k=1}^{M-1} \binom{M-1}{k} + 1$.

The link probability: $p_{ij}(r)$ depends on the distribution of the SINR, and consequently on the distribution of the corresponding packet error rates. It is defined by the equation:

$$p_{ij}(r) = \sum_{l=1}^{L} \left[1 - PER_l(r)\right] . \mathbf{P}_l(r) \tag{4}$$

where the index l represents one of the L interfering sets. Consequently, $\gamma_l(r)$ is the SINR experienced because of the interfering set l on the link (i, j) for the resource r and $PER_l(r)$ is the corresponding PER. The SINR can be computed according to Eq. (2) considering the K interfering links of l and the PER according to Eq. (3).

$\mathbf{P}_l(r)$ is the probability for the link (i, j) to experience the interference distribution l in resource r, i.e. the probability that the nodes of the interfering set l are transmitting concurrently and the others are not. Hence, this probability for a link (i, j) is given by:

$$\mathbf{P}_l(r) = \prod_{k=1}^{K} \tau_k(r) \; \cdot \; \prod_{m=1}^{M-K-1} (1 - \tau_m(r)) \tag{5}$$

In Eq. (5), $\prod_{k=1}^{K} \tau_k(r)$ gives the probability that the K active nodes of the interfering set l are transmitting and $\prod_{m=1}^{M-K-1} (1 - \tau_m(r))$ the probability that the $M - K - 1$ other active nodes are not.

C. Node probabilities and transmission rate

The variables of our model are the probability $\chi_i = \xi_i {\cdot} x_i$ for each node i to re-transmit a received message. In the following, we consider that $\xi_i = 1$ to simplify our model. Hence, the main variable is the 'forwarding probability' x_i. There is no notion of routing paths herein and a packet sent by a source may use one or more paths in parallel to reach the destination. For $x_i = 1$ each received packet by node i is forwarded. For $x_i < 1$ node i drops the packets with probability $1 - x_i$. Values of $x_i \in]1, R]$ are not allowed yet as they imply that node i transmits several copies of the same packet.

As stated earlier, the transmission rate $\tau_i(r)$ in resource r is a function of the node probability x_i but also depends on the amount of traffic coming into node i, which is a function of the activity of the other nodes of the network. As a consequence, computing the values of $\tau_i(r)$ knowing the x_i values is intractable since determining the $\tau_i(r)$ requires the knowledge of the link probabilities which are themselves a function of the $\tau_i(r)$ values. However, the reverse approach where the variables x are expressed as a function of the $\tau_i(r)$ can be easily derived as stated below. Hence, such a reverse approach leads to the use of the transmission rates as the variables of our multiobjective optimization problem instead of the forwarding probabilities. This reverse approach represents an important contribution of our cross-layer model since it captures an exact picture of the interference distribution at the physical layer and determines the corresponding node forwarding probability x_i at the routing level.

Relationship between x_i and the $\tau_i(r)$: Given the values of $\tau_i(r), \forall r \in [1..R], i \in [1..N]$, we can define the quantity of information coming from all the neighbors of node i (except from the destination, D) by:

$$q_i = \sum_{k \neq \{i,D\}} \sum_r p_{ki}(r).\tau_k(r).v_{ki} \tag{6}$$

where $p_{ki}(r).\tau_k(r).v_{ki}$ is the probability that a packet arrives in node i from node k in resource r.

The variable v_{ki} is introduced to represent the usefulness of the link (k,i) with respect to the maximum number of hops constraint. Hence, if no data can arrive from neighbor k because the hop count h for all the packets k received is already equal to H_M, we have $v_{ki} = 0$. On the contrary, we have $v_{ki} = 1$ if k only receives packets with a number of hops $h < H_M$. If k receives packets with both $h < H_M$ and $h = H_M$, v_{ki} represents the proportion of packets being retransmitted.

The quantity of information going out of i is given by the sum of the $\tau_i(r)$ over all the time slots. Hence, we can determine the global forwarding probability of i to be:

$$x_i = \frac{\sum_r \tau_i(r)}{\sum_{k \neq \{i,D\}} \sum_r p_{ki}(r).\tau_k(r).v_{ki}} \tag{7}$$

III. A MULTIOBJECTIVE OPTIMIZATION PROBLEM

The performance of most wireless networks can be assessed with regards to various criteria such as throughput or capacity, end-to-end transmission delay, overall energy consumption or transmission robustness. The purpose of the multiobjective framework presented in this work is to determine, given a network and a communication pattern, what kind of trade-offs arise between chosen performance metrics when varying the routing strategies. It relies on the cross-layer probabilistic network model presented in Section II.

A. Variables of the Multiobjective (MO) Framework

The routing strategies are the variables of our multiobjective optimization problem and a solution is defined by:

Definition 1 A solution $\mathcal{S}$ of the MO framework is defined by the set of transmission rates $\tau_i(r) \in [0, 1]$ used by each node i on each resource r:

$$\mathcal{S} = \{\tau_i(r)\}_{i \in [1,..,N], r \in [1..R]} \tag{8}$$

The set of node probabilities $x_{i, i \in [1..N]}$ is derived according to Eq.(7) and represents the routing strategy of the network. Each variable $\tau_i(r)$ takes its values in a discrete set Γ of size $T = |\Gamma|$. As a consequence, the solution space is derived as:

$$|\mathcal{S}| = \sum_{m=0}^{N-2} \binom{N-2}{m} T^{R.m} \tag{9}$$

In order to reduce the size of this very big search space, we only consider solutions where at least one cumulative time slot per node is available in the frame, i.e. $s.t.$ $\forall i \in [1, N]$, $\sum_{t=1}^{R} \tau_i(r) \leq R - 1$. The solutions that do not meet this constraint are usually very bad solutions since at least one of the nodes of the solution is transmitting in all its time slots preventing a failure free packet reception.

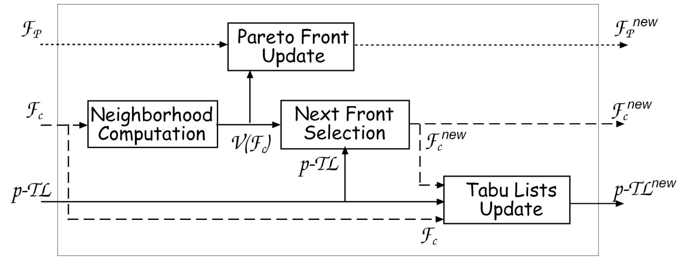

Fig. 2. PMOTS: Description of one search iteration.

Using this definition of a routing strategy, a solution may reflect various features: it can be single-hop or multi-hop, single path or multi-path, probabilistic or deterministic.

B. MO-Tabu: a multiobjective optimization heuristic

The aim of our MO framework is to obtain the set of Pareto-optimal routing strategies of the MO problem. A Pareto-optimal set is composed of all the non-dominated solutions of the MO problem with respect to the performance metrics considered. The definition of dominance is:

Definition 2 A solution A dominates a solution B for a $n-$objective MO problem if A is at least as good as B for all the objectives and A is strictly better than B for at least one objective. Mathematically, we have for a minimization problem:

$$\forall i \in [1, n] : f_i(A) \leq f_i(B), \exists j \in [1, n] : f_j(A) < f_j(B) \tag{10}$$

The considered optimization problem is solved using a multiobjective metaheuristic called PMOTS (*Parallel Multi-Objective Tabu Search*) described in [11]. It is based on the Tabu metaheuristic [12], a local search using a list of Tabu solutions to reduce the occurrence of loops in the search. PMOTS is a multiobjective extension of Tabu search where K Tabu searches are performed in parallel. Its macro-algorithm is given in Algorithm 1 and a graphical description is shown in Fig. 2. The goal of this algorithm is to obtain the best possible approximation of the Pareto-optimal set of solutions F_P.

In a search iteration, the K parallel search paths are represented as a search set or search front $F_c(i)$ of K solutions. The first set of K solutions is randomly created. A set of neighbor solutions $V(S_k)$ for each solution S_k of the search front is computed according to a set of rules. Further details on these rules can be found in [11]. The pool of neighbor solutions is added to the current Pareto-optimal front F_P, and the new front F_P is extracted from it using a dominance criterion.

A new search front $F_c(i+1)$ is selected by choosing promising non-Tabu solutions that are not always non-dominated to avoid a premature convergence of the algorithm. Therefore for a path k, each new solution is selected randomly in the set of neighbor solutions of S_k which is limited to the solutions having a Pareto rank $R = R_{max}$. The rank of a solution x

is defined by $R(x) = 1 + d(x)$, where $d(x)$ is the number of solutions by which x is dominated in the set of feasible solutions S. The solutions of the Pareto-optimal set have a rank $R(x) = 1$. In this algorithm, the Pareto ranking is local to the set of neighbor solutions and does not include the current estimated Pareto set F_P. By not including F_P and selecting fairly good solutions with the Pareto rank constraint, diversity is introduced within the search strategy. Once $F_c(i+1)$ is chosen, the solutions of $F_c(i)$ are stored in the corresponding Tabu lists.

There is also a restart strategy that creates a new random search front if no solutions have been added to or suppressed from F_P for a given number of search iterations. The algorithm stops after a fixed number of iterations and provides an estimate of the Pareto front F_P.

Algorithm 1 Macro-Algorithm for PMOTS

Init K Tabu lists $TL_k = \emptyset, k \in [1, .., K]$; $F_P = \emptyset$;
Randomly create K solutions and include them into the search front $\mathcal{F}_c(0)$;
for $i \in [0, .., I_{max}]$ **do**
    $\mathcal{F}_c(i+1) = \emptyset$;
    for all $S_k \in \mathcal{F}_c(i)$ **do**
        Compute and evaluate the neighborhood set $V(S_k)$;
        Select from $V(S_k)$ the solutions with Pareto rank $R(S) = R_{max}$ and add them in $P_R(S_k)$;
        Select randomly a solution of $P_R(S_k)$ and add it into the new search front $F_c(i+1)$;
        Concatenate $P_R(S_k)$ with the Pareto front F_P;
        Update the Tabu list TL_k;
    end for
    Remove the solutions having a Pareto rank $R(S) > 1$ from F_P;
end for
Return F_P;

IV. A FIRST APPLICATION TO SENSOR NETWORKS

We propose in the following to assess the performance of a wireless sensor network (WSN) by capturing the trade-offs that arise between end-to-end robustness, overall energy consumption and end-to-end delay. These criteria are most relevant since providing a maximal network throughput is usually not the main task of a WSN. The criteria are defined for a single source-destination pair (S, D).

A. Robustness criterion

Robustness is defined as the probability that a message emitted at S successfully arrives at D in at most H_M hops. The robustness criterion is given by:

$$f_R = \mathcal{P}(T_{SD}^{H_M}) \tag{11}$$

For any two nodes i and j of the network, T_{ij}^{H} represents the event that a message transmitted by i successfully arrives at j in at most H hops. Our aim is to maximize $\mathcal{P}(T_{SD}^{H_M})$.

Definition 2: Global link probability.

For a link (i, j), the global link probability p_{ij} is the probability that a message arrives with success at node j. It is given by:

$$p_{ij} = \sum_{r=1;\tau_i(r)\neq 0}^{R} p_{ij}(r)\ \frac{\tau_i(r)}{\sum_r \tau_i(r)} \tag{12}$$

where $p_{ij}(r)$ is the link probability between i and j for resource r (*cf.* Eq. (4)), and $\tau_i(r)/\sum_r \tau_i(r)$ is the probability for the packet to be sent using r.

Definition 3: Robustness probability.

$\mathcal{P}(T_{SD}^{H_M})$ is the probability that the message arrives successfully at D in at most H_M hops and is given by:

$$\mathcal{P}(T_{SD}^{H_M}) = 1 - \prod_{h=1}^{H_M} (1 - \mathcal{P}(T_{SD}|H = h)) \tag{13}$$

where $\mathcal{P}(T_{SD}|H = h)$ is the probability for a packet to arrive in h hops at D. For $h = 1$, $\mathcal{P}(T_{SD}|H = 1) = p_{SD}$, the successful transmission probability on the link (S, D) following Eq. (12). For $h > 1$, we have:

$$\mathcal{P}(T_{SD}|H = h) = 1 - \prod_{j=1}^{N_S} [1 - p_{Sj}\ x_j\ \mathcal{P}(T_{jD}|H = h-1)] \tag{14}$$

with N_S the number of possible first hop relays of S; p_{Sj} the link probability between S and its neighbor j; $\mathcal{P}(T_{jD}|H = h-1)$ the probability to reach D in $(h-1)$ hops and x_j the forwarding probability of j. The set of N_S relays is given by all the nodes different from S that are active in at least one of the time slots in the current solution (i.e. having $\sum_{t=1}^{R}(x_i^t) > 0,\ i \neq \{j, S\}$).

To reduce the computational complexity of the robustness probability, a restricted set N_S of first hop relays may be considered but the loss in terms of accuracy is hard to quantify. Therefore, we rather introduce a *link threshold value* $\mathcal{P}_{th}$ computed for each path made of h hops. While recursively calculating $\mathcal{P}(T_{SD}|H = h)$, if the probability of a path gets lower than $\mathcal{P}_{th}$, the recursion is stopped for that particular path and its contribution to $\mathcal{P}(T_{SD}|H = h)$ is set to zero.

B. Delay criterion

The end-to-end delay is the sum of the times spent at each relay on a multi-hop path where each relay introduces a delay of 1. The criterion f_D is defined by:

$$f_D = R \cdot \sqrt{\sum_{h=1}^{H_M} (h-1)^2 . R_h} \tag{15}$$

The quantity $(h-1)$ is the delay needed by a packet to arrive in h hops using $(h-1)$ relay nodes. The scaling factor R represents the delay induced by the R resources. R_h is the probability that the packet arrived in exactly h hops and did

Transmission Power	151mW	N_0	-154dBm/Hz
Bandwidth	1Mbps	f	$2.4GHz$
Pathloss exponent α	3	Channel Model	AWGN
Antenna gains	G_T=G_R=1	Modulation	BPSK

Fig. 3. Propagation and physical layer parameter values.

not arrive in 1, or 2 or $(h-1)$ hops. For $h=1$, we have $R_h = P(T_{SD}|h=1)$ and for $h>1$:

$$R_h = \mathcal{P}(T_{SD}|H=h).\prod_{i=1}^{h-1}(1-\mathcal{P}(T_{SD}|H=i)) \qquad (16)$$

If no route exists between S and D then $f_D = +\infty$.

C. Energy criterion

The energy criterion f_E is given by the total *forwarding energy* needed for a packet sent by S to reach D. We do not account for the energy spent by the initial transmission in S. The reception (respectively transmission) of a packet at node j in resource r consumes $e_j^R(r)$ (resp. $e_j^T(r)$). Hence, the energy criterion is defined as:

$$f_E = \sum_{h=1}^{H_M} \mathcal{E}(T_{SD}|H=h) \qquad (17)$$

where $\mathcal{E}(T_{SD}|H=h)$ is the total energy needed by the h-hop communications between S and D defined by:

$$\mathcal{E}(T_{SD}|H=h) = \sum_{j=1}^{N_S}\left(p_{Sj}.e_j^R + p_{Sj}.x_j.\left[e_j^T + \mathcal{E}(T_{jD}|H=h-1)\right]\right) \qquad (18)$$

In Eq. (18), $p_{Sj}.e_j^R$ is the energy consumed for a packet reception by the neighbor j of S; $p_{Sj}.x_j.e_j^T$ is the energy consumed for the packet transmitted by neighbor j and $p_{Sj}.x_j.\mathcal{E}(T_{jD}|H=h-1)$ is the total energy consumed by the following possible paths made of $(h-1)$ hops between neighbor j and the destination. For $h=1$, $\mathcal{E}(T_{SD}|H=1)=0$ since the energy in S is not accounted for.

V. First Results

A. M-Relay problem

The results presented in this section are obtained for a small problem instance for two reasons. First, we are able to determine the whole Pareto-optimal set of solutions using an exhaustive search. Secondly, such a problem can be easily analyzed and provides a first illustration of our multiobjective framework. Thirdly, it is used to assess the efficiency of the multiobjective optimization metaheuristic we developed to tackle bigger problem instances. [11].

In the following, the network is composed of $N = 333$ nodes uniformly distributed with density $\rho = 0.004$ over a disk $\mathcal{D}$ of radius $R_\mathcal{D}$. The distance between S and D is of about 215 meters. To reduce border effects, S and D are selected within a radius $R_\mathcal{C} << R_\mathcal{D}$ which ensures that the power of a node at distance $R_\mathcal{C}$ is below the noise power for the nodes located at distance $R_\mathcal{D}$. We consider $R=2$ time slots and use a probabilistic discrete variable space. A link robustness threshold of $\mathcal{P}_{th} = 10^{-10}$ is set. Propagation and physical layer parameters are summarized in Fig. 3.

The dimension of the search space can be modified by setting a maximum number of forwarding nodes M in a solution $\mathcal{S}$. This sub-problem is addressed in the following as the M-relay problem instance.

B. Pareto-optimal set for the 1-relay problem

In this problem instance, we set $M=1$ and $H_M=2$. $\tau_i(r)$ takes its values in the set $\Gamma = \{0, 0.05, 0.1, \ldots 0.9, 0.95, 1.0\}$ of $|\Gamma| = 21$ elements. In that particular case, the search space has a dimension of 76131 solutions and the Pareto-optimal set is obtained with an exhaustive search.

For this instance, the direct link (S, D) is very weak. A robustness of only $\mathcal{P}(T_{SD}^{H_M}) = 0.0003$ is achieved with a delay of $f_D = 0$ and an energy of $f_E = 0$. Only 24820 solutions fulfill the constraint $x_i \leq 1$ that forbids a node to duplicate packets. Among these solutions, 3855 solutions are Pareto-optimal, representing respectively about 5% and 15% of the whole and the constrained solution space. For all the Pareto-optimal solutions the relay never transmits in the first time slot concurrently with the source. The performance of the Pareto-optimal set of solutions is represented in Fig. 4 in the space defined by the three evaluation functions. For clarity purposes, the projections of the Pareto set on the robustness-delay, robustness-energy and the delay-energy planes are displayed. The plots of Fig. 4 show that an improved robustness is obtained at the price of an increase in delay and energy. The trade-off between robustness and delay can be easily understood since higher robustness is achieved when the relay contributes with a higher forwarding probability x_i, inducing an increase in delay. Similarly, an increase of x_i triggers an accrued average energy consumption since the relay is forwarding packets more often.

The energy consumption for all the Pareto-optimal solutions belongs to a discrete set of 21 energy levels which is a direct consequence of the 21 values of $\tau_i(r)$ defined in this problem instance. Hence, the definition of a continuous transmission rate variable $\tau_i(r)$ would provide the most precise description of the Pareto set. However, tackling the continuous formulation of our problem is much more challenging and for our study, we will stick to the simpler discrete formulation which still provides a fair representation of the Pareto set.

The Pareto set is composed of solutions where relays belong to a set of 226 nodes, which represents about two thirds of the number of nodes of the network. The location in the network of these 226 nodes is presented in Fig. 5. We also highlighted on this figure the relays that provide a near perfect transmission. We can conclude that the relays located in an ellipse near the middle of the (S, D) distance provide the best robustness at the price of the highest delay and energy. The other relays present in the Pareto set provide various trade-offs depending on their values of $\tau_i(r)$.

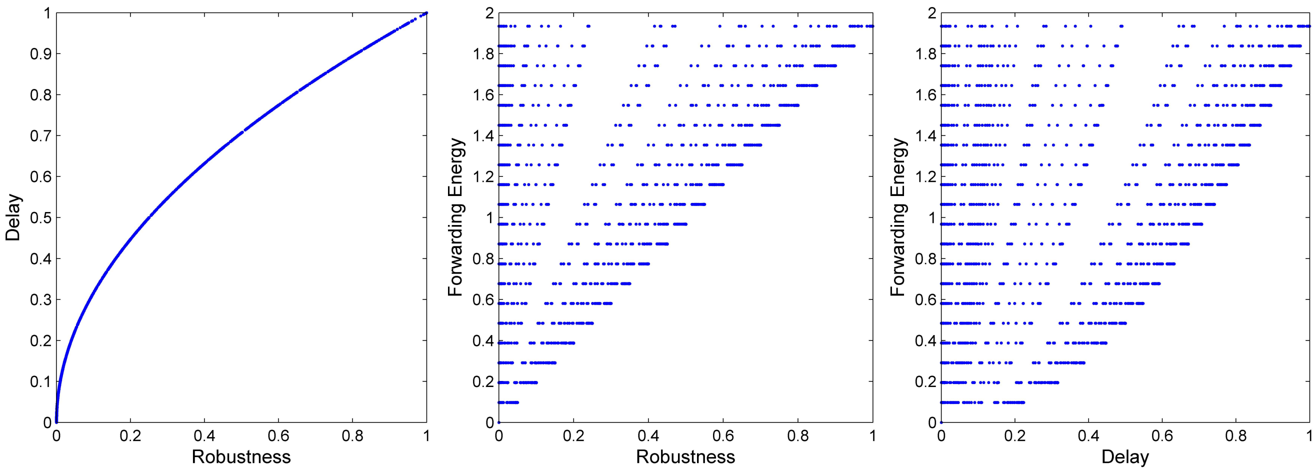

Fig. 4. Representation of the projections of the Pareto-optimal set for the 1-relay problem.

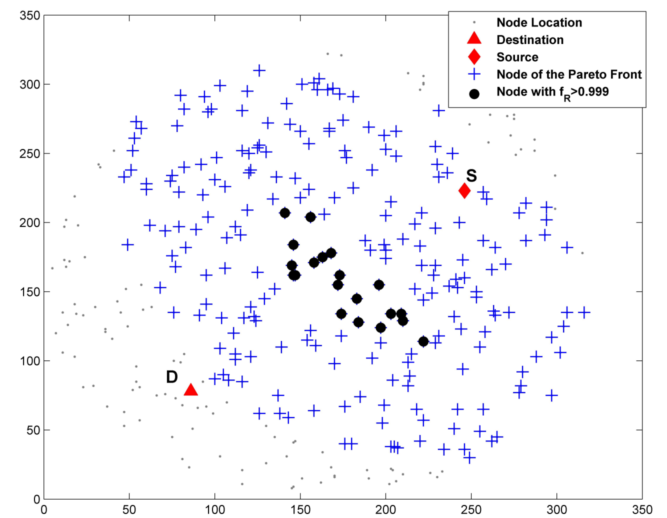

Fig. 5. Location of the nodes that provide Pareto-optimal solutions in the network (blue crosses) and of the nodes that provide a near quasi-perfect robustness (full black dots), i.e. $f_R > 0.999$.

C. Validation of PMOTS on the 1-Relay problem

This first simple study shows that the proposed multiobjective probabilistic network model provides a coherent and complete view of the trade-offs that arise between robustness, delay, and energy in our network. A more extensive analysis of the performance of the model has to be performed next by considering a solution space that considers all the possible relaying strategies (i.e. no M-relay search space reduction) and various network topologies. For such instances, our problem is solved using the multiobjective optimization algorithm PMOTS as presented in Section III-B. Although this paper does not concentrate on the description and the performance analysis of PMOTS, Fig. 6 highlights the convergence properties of the algorithm for the 1-relay problem.

Three performance metrics measure the convergence of PMOTS towards the Pareto-optimal set F_P^* obtained through exhaustive search. The approximated Pareto sets F_P obtained by PMOTS are compared to F_P^* with respect to the number of iterations the search has performed using the following metrics [13]:

- **The error ratio** that measures the non-convergence of a search method to F_P^*. It is given by:

$$ER = \frac{\sum_{i=1}^{n} e_i}{n} \tag{19}$$

where $e_i = 0$ if solution i of F_P belongs to F_P^* and $e_i = 1$ otherwise, and n is the number of solutions in the approximated Pareto front F_P.

- **The generational distance** that measures the distance between a set of n solutions and the theoretical Pareto front F_P^*. It is defined by:

$$GD = \frac{\left(\sum_{i=1}^{n} d_i^p\right)^{1/p}}{n} \tag{20}$$

where d_i is the smallest distance between a solution of F_P and F_P^*. Here, we use $p = 2$ and n the number of solutions of the approximated Pareto front F_P.

- **The similarity ratio** that measures the proportion of solutions of F_P^* present in the approximated Pareto set F_P. It is given by:

$$SR = \frac{\sum_{i=1}^{n} f_i}{n*} \tag{21}$$

where $f_i = 1$ if solution i of F_P^* belongs to F_P and $f_i = 0$ otherwise, and $n*$ the number of solutions of the Pareto optimal front F_P^*.

The smaller the error ratio and the generational distance metrics, the smaller in number and amplitude are the errors between F_P and F_P^*. The higher the value of the similarity ratio, the more solutions of F_P^* are present in F_P. These three metrics have been calculated for the Pareto fronts obtained with PMOTS every 20 iterations. Average and standard deviation values are computed over 10 runs of PMOTS using the same test environment.

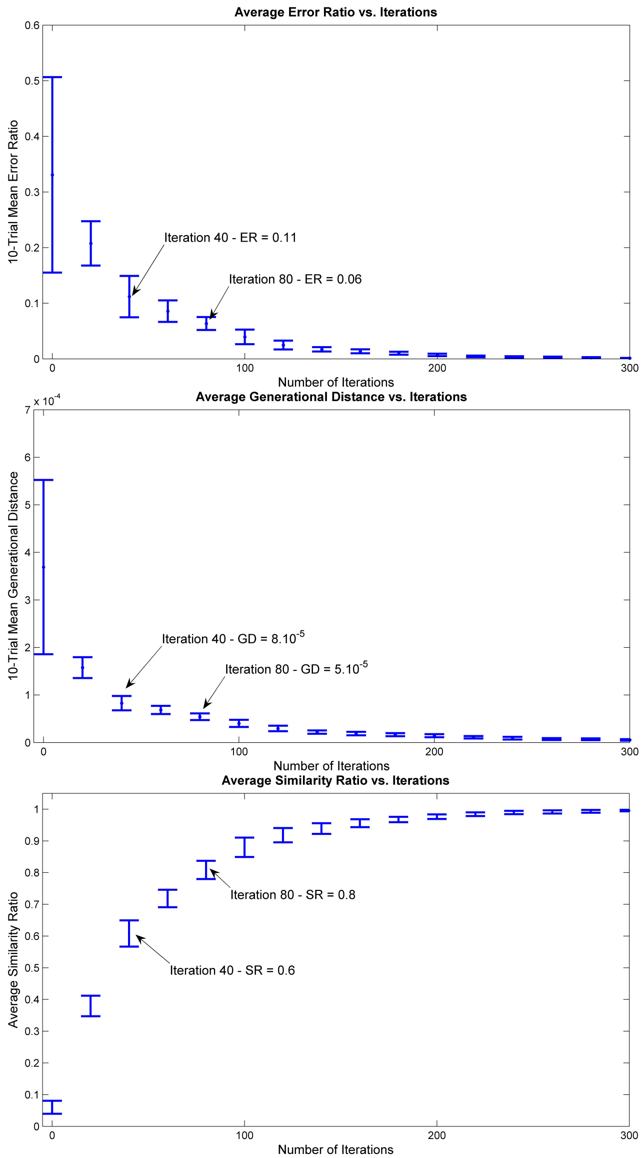

Fig. 6. PMOTS performance: Error Ratio, Generational Distance and Similarity Ratio statistics for the 1-Relay problem.

On Fig. 6, it can be seen that both ER and GD quickly decrease with time while SR increases as more solutions are added to the Pareto-optimal set. Iterations 40 and 80 have been highlighted on Fig. 6 because they represent the times at which PMOTS has evaluated the number of solutions equal to half the search space and the entire search space, respectively. At iteration 80, only about 6% of the solutions of F_P do not belong to F_P^*, and these solutions are really close to the Pareto optimal front as shown by the GD measure of 5.10^{-5}. In iteration 40, we already have a good first picture of the Pareto-optimal set since we have 60% of the solutions of F_P^* and the erroneous solutions of F_P are very close to F_P^* having a generational distance value of $GD = 8.10^{-5}$ value. PMOTS performs well on this case and we will use it on higher order problem instances. However, we are still working on improving its performance in terms of convergence speed.

D. *Pareto-optimal set for the 2-relay problem*

In this problem instance, we set $M = 2$ such as each solution is made of either one or two active relays. We set $H_M = 3$, meaning that we account for all the paths having $h \leq 3$ hops in the criteria computation. The precision of the $\tau_i(r)$ variable is reduced and it takes its values in the set $\Gamma = \{0, 0.1, \ldots 0.9, 1.0\}$ of $|\Gamma| = 11$ elements. The search space has a cardinality of $230,769,891$ solutions and hence, the Pareto-optimal set presented in Fig. 7 has been obtained with PMOTS, after 10 days, 4 hours of computing and having evaluated $8,446,029$ solutions. This estimated Pareto-optimal set presented here is composed of 58799 solutions.

Even though the approximation of the Pareto-optimal set is not the most accurate one, it is already possible to understand the composition of the trade-offs between the three criteria. As a matter of fact, it is already clear that the same trade-off between robustness and delay exists as in the 1-relay subproblem. On the robustness-energy projection and on the delay-energy projection, we have highlighted the solutions composed of only one relay using a red cross marker. It can be seen that the solutions providing a quasi perfect robustness ($f_R > 0.999$)) are composed of single relay solutions. As for the 1-relay case, most of the solutions in the front are divided into various energy levels because of the discretization of the $\tau_i(r)$ space.

So far, PMOTS has not yet found a solution made of 2 relays that outperforms the 1-relay performance in terms of robustness. For this particular configuration and network, it makes sense since the use of a single relay is the best possible configuration to mitigate interference for a 2-time slot system. For the 2-relay case, our model clearly accounts for the interference created between the source node and one of the two relays and impacts accordingly the performance of a 2-relay solution. From this basic illustration of our framework for a simple case study, we can conclude (knowing the 1-relay and 2-relay Pareto-optimal sets) that the best possible robustness is achieved for a 1-relay configuration using nodes located between the source and the destination. As shown in Fig. 5, several central nodes provide similar performance and hence, could be used for instance as opportunistic relays in the transmission from S to D. From our first results, we can conjecture that for this specific example network, using a single relay provides better performance when robustness is considered as a primary objective. We have also highlighted the optimal trade-off between robustness and delay, and the impact of the third objective, the energy, on the optimal compromise surface.

VI. CONCLUSION

In this paper, we have proposed a novel multiobjective optimization framework for network routing in wireless ad hoc networks. Our proposed framework consists of a general probabilistic network model capable of capturing the impact

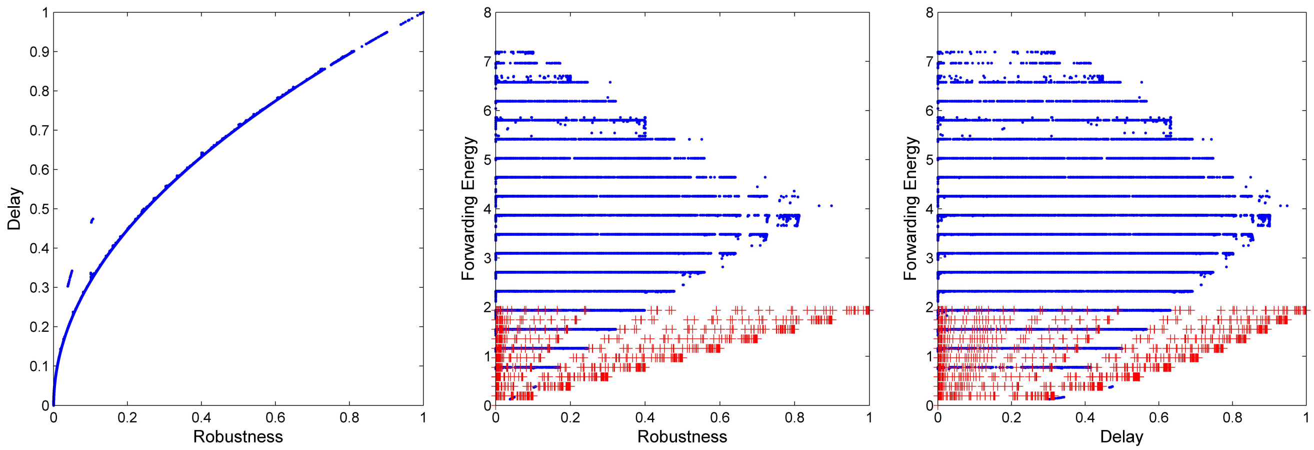

Fig. 7. Representation of the projections of the estimated Pareto-optimal set for the 2-relay problem. The solutions are divided into two sets: the set of 1894 solutions using only one relay (red cross marker) and the set of 56905 solutions using two relays (blue dot marker).

and interaction of a wide range of resource/interference management techniques, various channel conditions, and network scenarios. Used in conjunction with metaheuristic optimization techniques, this framework provides an efficient tool to capture the trade-offs between different performance metrics and obtain bounds on the achievable performance of routing for a single source-destination transmission. Preliminary results were obtained in characterizing the delay, robustness, and energy tradeoffs for a two time slot sensor network model. Future work will extend the model to consider more complex network scenarios, such as to account for various network topologies, to consider multiple concurrent flows in the network, and to use more refined cross-layer interactions and interference models.

ACKNOWLEDGMENTS

This work was supported in part by the Marie Curie IOF Action of the European Community's Sixth Framework Program (DistMO4WNet project), by the ONR grant #N00014-06-1-0063, and by the NSF Alliances for Graduate Education and the Professoriate (AGEP MAGNET-STEM II project) at Stevens Institute of Technology. This article only reflects the authors' views and neither the European Community, the ONR, nor AGEP are liable for any use that may be made of the information contained herein.

REFERENCES

[1] P. Gupta and P.R. Kumar, "The Capacity of Wireless Networks" in *IEEE Trans. on Information Theory*, Vol. 46, No. 2, March 2000, pp. 388-404.
[2] Z. Wang, H.R. Sajadpour and J.J. Garcia-Luna-Aceves, "A Unifying Perspective on The Capacity of Wireless Ad Hoc Networks", in *Proc. of IEEE INFOCOM 2008*, April 2008, pp. 753-761.
[3] V. Mhatre, C. Rosenberg, and R. Mazumdar; On The Capacity of Ad-hoc Networks under Random Packet Losses, in *IEEE Transaction in Information Theory*, Vol 55, No. 6, June 2009, pp. 2494-2498
[4] A. Gamal, J. Mammen, B. Prabhakar and D. Shah, "Throughput-delay trade-off in wireless networks", in *Proceedings of INFOCOM 2004*, vol. 1, March 2004, pp. 464-475.
[5] C. Comaniciu and H.V. Poor, "On the Capacity of Mobile Ad Hoc Networks with Delay Constraints" in *IEEE Trans. on Wireless Communications*, Vol. 5, No. 8, August 2006, pp. 2061-2071.
[6] M. Brand and A.F. Molisch, "Delay-energy tradeoffs in wireless ad-hoc networks with partial channel state information", in *IEEE Global Telecommunications Conference (GLOBECOM 2008)*, Nov. 2008, pp. 1-6.
[7] F. Xue, A. Sanderson and R. Graves, "Multi-Objective Routing in Wireless Sensor Networks with a Differential Evolution Algorithm", in *IEEE International Conference on Networking, Sensing and Control*, 2006, pp. 880-885.
[8] M. Brand, P. Maymounkov and A.F. Molisch, "Routing with Probabilistic Delay Guarantees in Wireless Ad-Hoc Networks", in *IEEE Global Telecommunications Conference (GLOBECOM 2008)*, Nov. 2008, pp. 1-6.
[9] N. Vassileva and F. Barcelo-Arroyo, "A Survey of Routing Protocols for Energy Constrained Ad Hoc Wireless Networks", in *IEEE Future Generation Communication and Networking*, Dec. 2007, pp. 522-527.
[10] A. Eryilmaz and R. Srikant, "Joint congestion control, routing, and MAC for stability and fairness in wireless networks", in *IEEE Journal on Selected Areas in Communications*, Vol. 24, No. 8, Aug. 2006, pp. 1514-1524.
[11] K. Jaffrès-Runser, J.-M. Gorce and C. Comaniciu, "A multiobjective TABU Framework for the Optimization and Evaluation of Wireless Systems", book chapter in *Local Search Techniques: Focus on Tabu Search*, I-Tech Publishing, Sept. 2008, pp. 29-54.
[12] F. Glover and M. Laguna, "Tabu Search", Kluwer, 1997, 408p.
[13] Y. Collette and P. Siarry, "Multiobjective Optimization: Principles And Case Studies", Springer Berlin Heidelberg, August 2003, 320p.