Delay distribution of IEEE802.11 DCF: a comparative study under saturated conditions

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ABSTRACT

This work discusses the total transmission delay distribution in wireless networks where medium access follows the IEEE802.11 DCF medium access protocol. It does *not* propose a new model to derive the delay distribution of such protocol but focuses on providing a precise performance evaluation method to characterize the accuracy of analytical derivations of the literature. More specifically, this method separates the error introduced by the two main steps of the delay distribution characterization, naming the calculation of the delay probability generating function (PGF) and its numerical inversion. It is illustrated to assess the performance of the main Markov-based MAC model found in the literature together with two different types of queues (M/M/1 and M/G/1).

Categories and Subject Descriptors

C.2.1 [**Network Architecture and Design**]: Wireless Communication

General Terms

Theory, Performance

Keywords

Performance measure; wireless networks; delay distribution; IEEE 802.11 DCF; saturated traffic

1. INTRODUCTION

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Wireless networks based on the IEEE802.11 technology [5] are now deployed widely for non-critical applications. The flexibility of wireless connectivity is gaining momentum in the context of real-time networks (wireless industrial fieldbuses, wireless embedded networks, etc..)[7][8][12]. The main pitfall of wireless communications is of course the increased unreliability the medium suffers from due to interference and pathloss compared to shielded wires. Moreover, the main IEEE802.11 technology is based on is based on CSMA/CA (Carrier Sense Multiple Access with Collision Avoidance), which is non-deterministic but highly flexible.

Carrying soft real-time data over wireless is a feasible option[8], however, it is therefore necessary to characterize the response time of the network to forecast its behavior and dimension the real-time application using it. When a CSMA/CA type of medium access (MAC) is considered, it is thus necessary to have a precise and accurate knowledge of the delay distribution. Not this many works have targeted the analytical derivation of the delay distribution. This makes sense since for non-critical wireless networks, the knowledge of the average delay for a node to access the medium is enough. But for real-time applications, it is not sufficient and the total delay distribution needs to be derived.

The main works that discuss the delay distribution derivation use different analytical models for the MAC and queuing delay [4][9][10][13]. All these works calculate first the probability generating function (PGF) of the MAC and queuing delays. From these, they deduce the PGF of the total transmission delay which has to be inverted to obtain the total delay distribution. For instance, in [13] and [9], the MAC delay PGF is derived from the well-known Markov model of Bianchi [2]. An important step to get the delay distribution is to invert the PGF to obtain the corresponding probability mass function. This step can introduce errors. Similarly, different inversion methods have been proposed in these works.

Our aim in this paper is to propose a clear and precise performance evaluation method to *i*) assess the quality of the analytical model leading to the total delay PGF, *ii*) select the most accurate numerical inversion method. Therefore, we define two performance measures whose aim is to characterize the error originating from the analytical model on the one side and from the PGF inversion method on the other side. Our performance evaluation method is illustrated on the specific case of a IEEE802.11 DCF medium access where two different types of queues are assumed, naming M/M/1 and M/G/1.

This paper is organized as follows. In Section 2, the overall analytical derivation of the total delay distribution is presented. Detailed calculations for the individual MAC, queuing and total delays for IEEE802.11 DCF protocol and specific queues are given in Section 3. In Section 4, we introduce the method proposed to assess the performance of a delay distribution model. This method is leveraged in Section 5 to select the most accurate analytical model that derives the total delay distribution of an IEEE802.11 wireless network using DCF. Finally, Section 6 concludes this work.

2. TOTAL DELAY DISTRIBUTION

This section starts by introducing the wireless system of interest and then presents the overall analytical derivation of the total transmission delay distribution.

2.1 System model

In this paper, a source node is directly transmitting its packets to a destination node. These two nodes belong to a set of n stationary nodes sharing a common wireless medium. Each emitted packet experiences a *total transmission delay* d_t which is measured from its time of generation to the time its sender gets an acknowledgement (ACK) from the destination node or a maximum number of transmission trials has been reached.

At the time of generation, the emitted packet enters the transmission queue. Once it has reached the head of its queue, it will compete for channel access with the other stations. If the packet has been emitted, the sender waits for a positive ACK from the destination. Thus, the total delay is the sum of:

- a *queueing delay*, which is the time for the packet to reach the head of the transmission queue, and
- a *medium access delay (MAC delay)*, which is the time needed by the medium access protocol to either successfully deliver the packet or drop it in case of repeated failures.

The rest of the paper discusses the analytical derivation of the distribution of the total delay experienced by packets for an IEEE802.1 DCF medium access (with or without RTS/CTS mechanism). We consider a saturated traffic where all nodes of the network always have a packet ready for transmission. Two types of queues are investigated as well, naming M/M/1 and M/G/1. Ideal channel conditions are assumed as well (no channel errors, no hidden terminals).

2.2 Overall analytical derivation

MAC and queuing delays are independent discrete random variables. Indeed, the MAC delay experienced by a head of line packet is completely independent from the time it has spent in the queue [13]. It is just a function of the number of nodes contenting for medium access with him.

In the rest of the paper, the following notation is adopted: $d_t(k)$, $d_q(k)$ and $d_m(k)$ represent the probability mass functions (PMF) of the total transmission, queuing and MAC delays, respectively. $D_t(Z)$, $D_q(Z)$ and $D_m(Z)$ are the probability generating functions (PGF) of total, queuing and MAC delay, respectively. We recall that the probability-generating function of a discrete random variable X is the Z-transform of its PMF. It is calculated following $D(Z) = \sum_{k=0}^{\infty} d(k)Z^k$, with $Z \in \mathbb{C}$ and $d(k)$ the PMF of X.

Since MAC and queueing delay random variables are independent, the PGF of the total delay $D_t(Z)$ is equal to the product of $D_m(Z)$ and $D_q(Z)$:

$$D_t(Z) = D_m(Z)D_q(Z) \tag{1}$$

and the complimentary cumulative distribution (CCDF) of the total delay is given by

$$\tilde{D}_t(Z) = \frac{1-D_m(Z)D_q(Z)}{1-Z} \tag{2}$$

The mean of the total delay $E[D_t]$ is obtained by summing the mean MAC and queuing delays:

$$E[D_t] = E[D_m] + E[D_q] \tag{3}$$

with $E[D] = D'(Z)|_{Z=1}$.

In this work we are interested in extracting the probability mass function $d_t(k)$ of the total delay. Therefore, we'll need first to derive $D_m(Z)$ and $D_q(Z)$, the PGF of MAC and queuing delays. Previous works have tackled these problems with different perspectives and models. Our aim in this paper is to present a performance evaluation analysis of such derivations in order to select the ones which provide the best trade-off between accuracy and complexity. Therefore, the analytical delay distributions have to be compared to their empirical counterparts using extensive simulations.

Having $D_t(Z) = \sum_{k=0}^{\infty} d_t(k)Z^k$, $d_t(k)$ is obtained by the Z-transform inversion of the PGF. This last inversion step is critical and can introduce errors. This inversion error adds to the error an imperfect analytical model creates as well. We argue in this paper that to have a clear view of the performance of a given analytical derivation of a delay distribution, its validation has to be done in two steps. First, the model used to derive the individual PGFs has to be validated before numerical inversion. Second, the numerical inversion has to be tailored to reduce the inversion error.

3. INDIVIDUAL PGF DERIVATIONS

This section recalls briefly the derivation of the individual PGFs for the MAC and queueing delays we have selected from the literature.

3.1 PGF of MAC delay

Two different types of models have been proposed in the literature to characterize the medium access delay distribution. Zhai et al. [13] and Vardakas et al. [9] rely on a Markov chain model originating from the work of Bianchi [2]. Vu and Sakurai [10] proposed a different probabilistic derivation for the PGF of MAC delay, but their paper presents a very limited performance evaluation. Thus, we have decided to look first at the better known Markov model of Vardakas et al [9] to derive the PGF of the MAC delay. In the following, we briefly review the Markov chain model and the related PGF derivation of the MAC delay of [9]. Then, the PGF of MAC delay is presented and we introduce a performance evaluation measure to validate its accuracy against simulation results.

3.1.1 Markov chain model for IEEE 802.11 DCF

Due to space limitation, we refer the reader to [5] for a detailed description of the IEEE802.11 DCF MAC protocol. Following the modeling and analysis proposed in [13] and [9], the state transition diagram of the discrete-time Markov chain has the following one-step transmission probabilities. Let $\{s(t), b(t)\}$ be a bi-dimensional, discrete-time Markov chain. Here $s(t)$ is the stochastic process which represents the backoff stage $i \in [0, \dots, m]$ at time slot t and $b(t)$ the stochastic process which represents the backoff counter (with different contention window sizes) for a given station at time slot t. There are m transmission attempts and we thus have m backoff stages. The aim of this model is to derive τ, the probability that the node gains access to the channel in any time slot.

COLLISION PROBABILITY p: The collision probability p is the probability of a collision seen by a packet transmitted on the channel, which is constant and independent of the number of the collisions that the packet has suffered from in the past. A fixed point formulation for p was introduced by Bianchi [2], who proposed the relationship:

$$p = 1 - (1 - \tau)^{n-1} \tag{4}$$

CHANNEL BUSY PROBABILITY p_b: As in [14], we assume that p_b stands for the probability that the channel is busy. This probability is independent of the backoff procedure, that is, independent not only from the backoff stage (number of retransmissions), but also from the value of the backoff counter. The channel is detected busy when at least one of the $n-1$ nodes transmits during a system time slot. Note also that a station remains with probability p_b at state $(i,k), k \leq 1$, when at least one of the $n-1$ remaining nodes transmit. Therefore the values of p_b and p coincide.

$$\begin{aligned} p_b &= \textstyle\sum_{i=1}^{n-1}[\binom{n-1}{i}.\tau^i.(1-\tau)^{n-1-i}] \\ &= 1-(1-\tau)^{n-1} \end{aligned} \tag{5}$$

A brief summary of the non null one-step transmission probabilities are provided here.

$$\begin{aligned} &P\{i,k|i,k+1\} = 1 - p_b, k \in [0, W_i - 2], i \in [0,m] \\ &P\{i,k|i,k\} = p_b, k \in [0, W_i - 1], i \in [0,m] \\ &P\{i,k|i,0\} = (1-p)/W_0, k \in [0, W_0 - 1], i \in [0, m-1] \\ &P\{i,k|i-1,0\} = p/W_i, k \in [0, W_i - 1], i \in [1,m] \\ &P\{0,k|m,0\} = 1/W_0, k \in [0, W_0 - 1] \end{aligned}$$

Let $b_{i,k} = \lim_{t\to+\infty} P\{s(t) = i, b(t) = k\}$, with i, k integers, $i \in [0,m]$ and $k \in [0, W_i - 1]$, be the stationary distribution of the Markov chain. The probability of being in state $b_{0,0}$ can be computed as:

$$b_{0,0} = \begin{cases} \frac{2(1-p)(1-p_b)(1-2p)}{W_0(1-p)(1-(2p)^{m+1})+(1-2p)(1-p^{m+1})}, m \leq m' \\ \frac{2(1-p)(-p_b)(1-2p)}{P_1+P_2}, m > m'. \end{cases} \tag{6}$$

where

$$\begin{aligned} &P_1 = W_0(1-p)(1-(2p)^{m'+1}) \\ &P_2 = (1-2p)[1 - p^{m+1} + pW_0(2p)^{m'}(1 - p^{m-m'})] \end{aligned}$$

Using (4), (5) and (6), the probability τ is finally derived as:

$$\tau = \textstyle\sum_{i=0}^{m} b_{i,0} = \frac{1-p^{m+1}}{1-p}.b_{0,0} \tag{7}$$

3.1.2 PGF of the MAC delay derivation

The interruption of the backoff period is a result of two different events: the collision of two or more nodes with probability p and the transmission of only one node other than the tagged one, with probability

$$p' = \binom{n-1}{1}.\tau.(1-\tau)^{n-2} \tag{8}$$

Following [9], the binary exponential backoff algorithm can be envisioned as a function of two coordinates (x,y), where $(x \in [0,m])$ is the backoff stage and $(y \in [0, W_x - 1])$ is the value of the backoff counter at the backoff stage x. The authors of [9] deduce that the PGF of the duration that a packet stays in stage x with backoff counter y is given by:

$$B_{x,y}(Z) = \frac{(1-p).Z^\sigma}{1-(p'S(Z)+(p-p')C(Z))} \tag{9}$$

where Z^σ is the PGF of the propagation time σ, $S(Z) = Z^{T_s}$ and $C(Z) = Z^{T_c}$ are the PGFs of the duration of a successful transmission period T_s and of a collision period T_c, respectively. They depend on the type of service (basic or RTS/CTS) and their derivation can be found in [9]. Main DCF timing values are given in Table 1.

PHY	slot σ	SIFS	DIFS	CW_{min}	CW_{max}	m'
DSSS	20 μs	10 μs	50 μs	31	1023	5

Table 1: DCF parameters for DSSS-PHY.

The PGF of the duration the packet stays in the backoff stage x follows:

$$B_x(Z) = \begin{cases} \sum_{y=0}^{W_x-1} \frac{B_{x,y}(Z)}{W_x}, 0 \leq x \leq m' \\ B_{m'}(Z), m' < x \leq m. \end{cases} \tag{10}$$

From this, the PGF of the MAC delay is derived as:

$$\begin{aligned} D_m(Z) &= (1-p).S(Z).\textstyle\sum_{x=0}^{m}[(p.C(Z))^x \prod_{i=0}^{x} B_i(Z)] + \\ &(p.C(Z))^{m+1}.\textstyle\prod_{i=0}^{m} B_i(Z) \end{aligned} \tag{11}$$

It represents the duration for the packet to reach the end state (i.e. being transmitted successfully or discarded after maximum m retransmission failures) from the start state (i.e. beginning to be served). The first term relates to the delay of a successfully transmission including the delay spent in the previous x and y backoff stages, while the second term calculates the delay for dropping the packet after m trials.

Then, the mean MAC delay $E[D_m]$ can be derived by evaluating the first derivative of $D_m(Z)$ at $Z = 1$:

$$E[D_m] = D'_m(Z)|_{Z=1} \tag{12}$$

3.2 PGF of queueing delay

This section presents the derivation of the queuing delay PGF, $D_q(Z), Z \in \mathbb{C}$ for both M/M/1 and M/G/1 queues. Packets enter the queue according to a Poisson distribution of rate λ. The packet transmission process introduced by the DCF medium access can be modeled as a general single server whose service time distribution is known from Eq. (11).

Engelstad and Osterbo[4], Zhai et al.[13] and Vardakas et al.[9] have introduced models for the queuing delay in their total delay derivation. Zhai et al. study M/M/1/K and M/G/1/K queues while Vardakas et al. assume an M/G/1 queue. Zhai et al. show that MAC service times can be reasonably approximated by an exponential distribution. Thus, using an M/M/1/K model seems reasonable with respect to the complex derivations entailed by an M/G/1/K model.

3.2.1 M/M/1 queue

For the M/M/1 queue, the service time is exponentially distributed with parameter μ. Thus, the cumulative distribution function (CDF) and probability density function (PDF) of the service delay are $F(t) = 1 - e^{\mu t}$ and $f(t) = \mu e^{-\mu t}$, respectively. The service times have an average value of μ equal to the mean MAC delay: $\mu^{-1} = E[D_m]$. The Laplace transform of F is the function $L_f(s) = \frac{\mu}{s+\mu}$ [11]. According to the Pollaczek-Khintchine (P-K) transform equation, the Laplace transform $L_{D_q}(s)$ of the queueing delay can be expressed as:

$$L_{D_q}(s) = \frac{s(1-\rho)}{s-\lambda+\lambda L_f(s)} \tag{13}$$

with $\rho = \lambda/\mu$ the server utilization. According to the relationship between Laplace and Z-transform [11] (*cf.* Appendix A), it is possible to deduce the Z-transform $D_q(Z)$ from $L_{D_q}(s)$ by substituting $s = -\ln Z$ into (13):

$$D_q(Z) = \frac{-ln(Z)(1-\rho)}{-ln(Z)-\lambda+\lambda L_f(-ln(Z))} \tag{14}$$

$D_q(Z)$ is the queuing delay PGF. We can derive the PMF $d_q(k)$ by inverting $D_q(Z)$.

3.2.2 M/G/1 queue

The M/G/1 queue is a single-server system with Poisson arrivals and arbitrary service-time distribution. Similarly, Laplace transform of the queueing delay gives $L_{D_q}(s) = \frac{s(1-\rho)}{s-\lambda(1-L_f(s))}$, where $L_f(s)$ is the Laplace transform of the service time distribution function. The Z-transform of the queueing delay $D_q(Z)$ is derived as in [4]:

$$D_q(Z) = \frac{(1-Z)(1-\rho)}{1-Z-\lambda(1-D_m(Z))} \tag{15}$$

Similarly to the M/M/1 case, the PMF $d_q(k)$ can be derived by inverting $D_q(Z)$.

3.3 PGF of the total delay

The PGF of the total delay is computed by multiplying the PGF of queuing and MAC delay as given in (1). We investigate two different models, a very simple and a more accurate one. The first one assumes an M/M/1 queue and a Markovian MAC delay distribution as well. The second one assumes an M/G/1 queue and a MAC delay that follows the $D_m(Z)$ PGF of (11). This last model is very heavy to compute compared to the simpler M/M/1 one.

3.3.1 With the M/M/1 queue

In this total delay derivation, we assume that the service times are exponentially distributed with an average μ^{-1} equal to the mean MAC delay of (12). We do not use the MAC delay distribution of (11), but assume that the packets are served by the MAC with an exponential distribution of PDF $f(t) = \mu e^{-\mu t}$ of mean MAC delay $\mu^{-1} = E[D_m]$. The corresponding PGF of the exponential MAC delay is given by:

$$D_m(Z) = \frac{\mu}{\mu - \ln(Z)} \tag{16}$$

This assumption may of course introduce errors but its derivation is much simpler. The point of this paper is to state whether the loss due to this approximation is reasonable or not compared to a precise (and complex) M/G/1 formulation and complete MAC delay derivation.

Similarly to (1), the Laplace transform of the total delay $L_{D_t}(s)$ is computed as the produce of the Laplace transforms of the queuing and MAC delay PDFs.

$$L_{D_t}(s) = L_f(s)L_{D_q}(s) = L_f(s)\frac{s(1-\rho)}{s-\lambda+\lambda L_f(s)} \tag{17}$$

From $L_f(s) = \mu/(s+\mu)$ and (17), $L_{D_t}(s)$ is given by

$$L_{D_t}(s) = \frac{\mu(1-\rho)}{s+\mu-\lambda} = \frac{\mu-\lambda}{s+\mu-\lambda} \tag{18}$$

The Z-transform of the total transmission delay $D_t(Z)$ can be expressed as

$$D_t(Z) = L_{D_t}(-\ln Z) = \frac{\mu-\lambda}{-\ln Z+\mu-\lambda} \tag{19}$$

The mean queueing delay $E[D_q]$ for M/M/1 queue is computed using Little's law as $\rho/(\mu-\lambda)$ and the corresponding mean total delay $E[D_t]$ as $1/(\mu-\lambda)$.

3.3.2 With the M/G/1 queue

In this case, the service times of the queue are distributed according to the MAC delay distribution given by the PGF $D_t(Z)$ in Eq. (11). Following (1), the Z-transform of the total delay $D_t(Z)$ follows:

$$D_t(Z) = D_m(Z)D_q(Z) = \frac{D_m(Z)(1-Z)(1-\rho)}{1-Z-\lambda(1-D_m(Z))} \tag{20}$$

The mean queueing delay $E[D_q]$ for M/G/1 is derived by the Pollaczek-Khinchin mean value formula [6] [Kleinrock 1975(sec 5.6)], given though the second moment of D_m:

$$E[D_q] = \frac{\lambda E[D_m^2]}{2(1-\rho)} \tag{21}$$

where $E[D_m^2]$ is derived as $E[D_m^2] = var(D_m) + (E[D_m])^2$ and $var(D_m)$ is the variance of $D_m(Z)$:

$$var(D_m) = D''_m(Z)|_{Z=1} + D'_m(Z)|_{Z=1} - (D'_m(Z)|_{Z=1})^2$$

4. EVALUATING THE PERFORMANCE OF A DELAY DISTRIBUTION

This section proposes a performance evaluation measure to characterize the accuracy of a given delay distribution model. As presented in Section 2, it is very convenient to express the delay distribution as a PGF. Thus, to obtain the PMF values $p(k)$, the corresponding PGF $D(Z)$ has to be inverted. This numerical inversion introduces errors. Thus, we argue that directly comparing the final $p(k)$ with the PMF obtained by simulations $p^s(k)$ is not meaningful enough. In other words, it is not possible with such a comparison, as done in [13] to know whether the errors between the simulated and analytical PMF come from an inaccurate model or originate from the inversion of $D(Z)$. We argue in this paper that to have a clear view of the performance of a given analytical derivation of a delay distribution, its validation has to be done in two steps. First, the model used to derive the individual PGFs has to be validated before numerical inversion. Second, the numerical inversion has to be tailored to reduce the inversion error.

Indeed, even though the models proposed in previous works [9][10][13] are very interesting, they suffer from a limited or not convincing performance evaluation of the delay distribution. More specifically, the MAC delay distribution models of [9] and [10] show little results in their papers. Vardakas et al.[9] mostly validate the *average* MAC delay against simulations but don't give results for the total distribution. Vu and

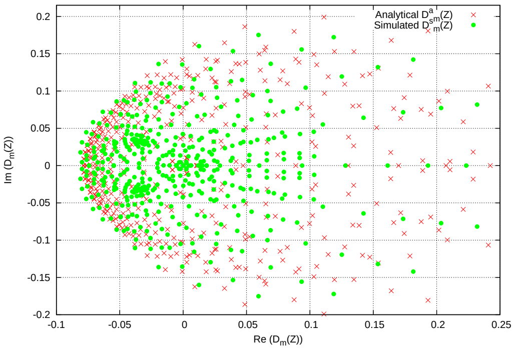


Figure 1: Comparison of $D^a_m(Z)$ and $D^s_m(Z)$ for $n = 5$.

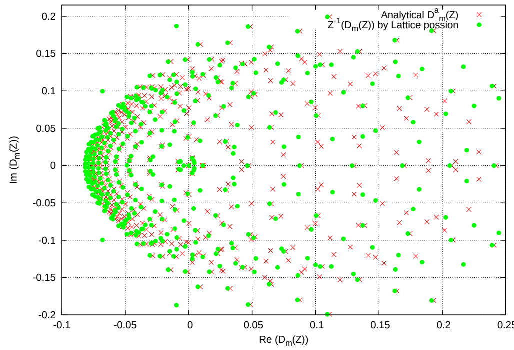


Figure 2: Inversion error for $D_m(Z)$ for $n = 5$.

Sakurai [10] present a single figure to validate their model against simulations and [13]'s results. Zhai et al. [13] provide PMF results for several cases, but they directly compare the PMF to the simulated distribution, completely ignoring the fact that errors can originate from the numerical inversion of the PGF. There is clearly a need for clear performance evaluation measures to assess the quality of a delay distribution model.

In the following, we describe first a performance measure to assess the analytical model's accuracy. Then, we give a performance measure to calculate the error introduced by the PGF numerical inversion.

4.1 Analytical performance measure

From now on, we will denote the PMF (resp. PGF) values obtained by simulation using $d^s(k)$ (resp. $D^s(Z)$) and the ones obtained analytically using $d^a(k)$ (resp. $D^a(Z)$).

It is straightforward to calculate the PMF values $d^s(k)$ from the statistics of the delay obtained by simulation. Thus, to avoid the inversion of the analytical PGF for its performance evaluation, we propose to compare directly the analytical PGF values $D^a(Z), Z \in \mathbb{C}$ to the PGF $D^s(Z)$ derived from the simulated PMF. The value of the PGF $D^s(Z)$ for any complex $Z \in \mathbb{C}$ is given by the Z-transform of the $d^s(k)$:

$$D^s(Z) = \textstyle\sum_{k=0}^{\infty} d^s(k) Z^k. \tag{22}$$

This calculation doesn't introduce any errors. Thus for a same set of complex values $\mathcal{C} \subseteq \mathbb{C}$, it is possible to calculate the analytical PGF $D^a(Z), Z \in \mathcal{C}$ and its simulated counterpart $D^s(Z), Z \in \mathcal{C}$. Figure 1 illustrates both analytical and simulated complex sets ($D^a_m(Z)$ and $D^s_m(Z)$) in a real and imaginary plot obtained for the MAC delay. Analytical PGF is derived following Eq. (11). The set of complex values used to calculate $D^a_m(Z)$ and $D^s_m(Z)$ is here defined as $\mathcal{C} = \{re^{-i\pi h/k}\}$ where $r = 10^{-4/k}$, k varies from 1 to 50 with step 5 and h varies from $-k$ to $+k$ with step 1.

For a perfect analytical model, the points calculated for $D^a(Z)$ would exactly match the ones obtained by simulation (providing that the simulation is extensive enough). From Figure 1, it is clear that there is an error between analytical and simulated values. Therefore we propose to quantify this error by defining a normalized root mean squared error (NRMSE) as:

$$f_{model} = \frac{1}{\mathrm{Card}(\mathcal{C})} \sum_{Z \in \mathcal{C}} \sqrt{\frac{|D^s(Z) - D^a(Z)|^2}{|D^s(Z)|^2}} \tag{23}$$

4.2 PGF inversion performance measure

By definition, a perfect PGF inversion is characterized by:

$$\mathcal{Z}\{\mathcal{Z}^{-1}\{D(Z), Z \in \mathbb{C}\}\} \equiv \{D(Z), Z \in \mathbb{C}\}$$

where $D(Z)$ is the PGF of a delay distribution, $\mathcal{Z} : Z \in \mathbb{C} \rightarrow \sum_{k=0}^{\infty} d^s(k) Z^k$ is the Z-transform function and $\mathcal{Z}^{-1} : D(Z), Z \in \mathbb{C} \rightarrow d(k)$ is the inverse Z-transform.

In the following, the PMF obtained after inversion is denoted $\{\hat{d}(k), k \in \mathbb{N}\}$. Thus, for a perfect inversion, there is a perfect match between the original PGF values $D(Z)$ and the Z-transform of the PMF $\hat{d}(k) = \mathcal{Z}^{-1}\{D(Z), Z \in \mathbb{C}\}, \forall k \in \mathbb{N}$. A non perfect inversion yields a difference between the two obtained complex sets. This is illustrated on Figure 2 for the MAC delay PGF calculated for $n = 5$. The same set $\mathcal{C}$ as for Figure 1 has been used to represent the PGF.

To assess the quality of a PGF inversion method, we propose to simply calculate, for each Z in a complex set $\mathcal{C} \subseteq \mathbb{C}$, the NRMSE between the original PGF and the Z-transform of the delay PMF obtained by inversion, naming $\hat{d}(k) = \mathcal{Z}^{-1}\{D(Z), Z \in \mathbb{C}\}, \forall k \in \mathbb{N}$. Formally, our performance measure is:

$$f_{inv} = \frac{1}{\mathrm{Card}(\mathcal{C})} \sum_{Z \in \mathcal{C}} \sqrt{\frac{|D(Z) - \mathcal{Z}\{\mathcal{Z}^{-1}\{D(Z)\}|^2}{|D(Z)|^2}} \tag{24}$$

5. PERFORMANCE RESULTS

This section illustrates first our performance metric f_{model} to derive the error related to the analytical model and then, the performance metric f_{inv} to derive the error related to the numerical inversion step. For the model performance evaluation, we compare analytical results to simulations, thus simulation settings are presented next.

5.1 Simulation settings

The wireless network composed of n nodes and one sink is simulated using the discrete event-driven network simulator WSNet[1]. Presented results for the IEEE 802.11 DCF MAC delay are obtained for the RTS/CTS scheme. The DSSS-PHY layer is considered with rate 11Mbps using packets of size 1400 bytes. Propagation delay $\delta = 1\mu s$. Further investigations will be done for OFDM-PHY layers in the future. Main DCF timing parameters for analysis and simulation

[1] http://wsnet.gforge.inria.fr/

Table 2: Queue parameters to reach saturation

Number of nodes	$\lambda(packet/ms)$	$\mu(packet/ms)$
$n = 5$	0.07799	1/12.1808
$n = 15$	0.02665	1/36.4052
$n = 30$	0.01359	1/71.3596

can be found in Table 1. Simulations have been conducted for 7 hours, experimenting the transmission of ∼70 000 packets.

All nodes experience the same Poisson arrival rate λ. Since the MAC model assumes saturated conditions, λ should be chosen such as to satisfy that the transmission queue remains nonempty. Therefore, utilization of the queue ρ should be more than 95%. And to satisfy Pollaczek-Khintchine (P-K) transform equation condition, $\rho < 1$ (i.e. $\lambda < \mu$). Since we set $\mu^{-1} = E[D_m]$, values for λ are calculated for different network sizes in Table 2.

5.2 Model performance evaluation

This section illustrates our model performance measure f_{model} on the following two models for DCF MAC:

1- the Markov chain based MAC PGF of [9],[13] of Eq (11).

2- the simple exponential MAC PGF of Eq. (16).

The PGF of the first model is complex to evaluate while the second one is very light, since it simply necessitates the derivation of the mean MAC delay of Eq. (12). Our point is to use the f_{model} measure to quantify the error induced by both models and check whether the exponential distribution is a valid assumption or not.

The same type of evaluation will be applied in future works to test the accuracy of Vu and Sakurai's MAC model [10]. Indeed, their results show (on a single plot) that it performs much better than the Markov chain based model of [13].

5.2.1 MAC delay PGF evaluation

Most of the works on DCF modeling have been validated by comparing the mean values. Table 3 gives the analytical and simulated mean values obtained for different network sizes. Note that both Markov based and exponential MAC models provide the same average delay.

Table 3: $E[D_m^a]$, $E[D_m^s]$ and $\Delta = |E[D_m^a] - E[D_m^s]|$ (ms)

Number of nodes	$E[D_m^a]$	$E[D_m^s]$	Δ
$n = 5$	12.1808	12.1582	0.0226
$n = 15$	36.4052	36.0012	0.404
$n = 30$	71.3596	71.8879	0.5283

Simulated and analytical mean values are really close. As can be seen on Figure 1, the analytical and simulated values of $D_m(Z)$ do not coincide for the Markov model. Thus, to better discriminate the quality of the distribution, the value of f_{model} is derived for the Markov model and the simple exponential MAC service distribution. Results are summarized in Table 4. Clearly, the Markov model is much more precise.

5.3 PGF Inversion performance evaluation

Table 4: Comparison of MAC distributions

Number of nodes	f_{model} for Markov model	f_{model} for Exponential MAC
$n = 5$	0.0547	0.1736
$n = 15$	0.0789	0.1258
$n = 30$	0.0729	0.1040

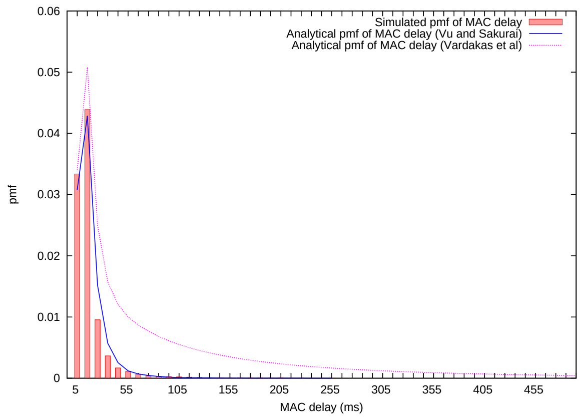


Figure 3: Comparison of MAC delay PMF for different inversion methods (accuracy 10^{-6}, $n = 5$)

Inverting the probability generating function can be done by repeatedly differentiating and evaluating it at $Z = 0$:

$$d(k) = \left.\frac{D^{(k)}(Z)}{k!}\right|_{Z=0}$$

This type of inversion has been done by Zhai et al. in [13] using numerical differentiation techniques and symbolic mathematical software. However, it is often difficult to achieve desired accuracy with numerical differentiation techniques, especially for large k_{max} (k_{max} being the number of PMF values obtained after inversion). It is also difficult to invoke symbolic mathematical software when the generating function is only expressed implicitly. Fortunately, in this setting numerical inversion is a viable alternative that has been chosen by Vardakas et al. [9] and Vu and Sakurai [10]. The numerical inversion of a PGF is based on the Lattice-Poisson (LP) algorithm [1]. Two different derivations of the LP algorithm have been proposed to numerically invert a delay PGF in [9] and [10].

The LP inversion formula of Vardakas et al. [9] is:

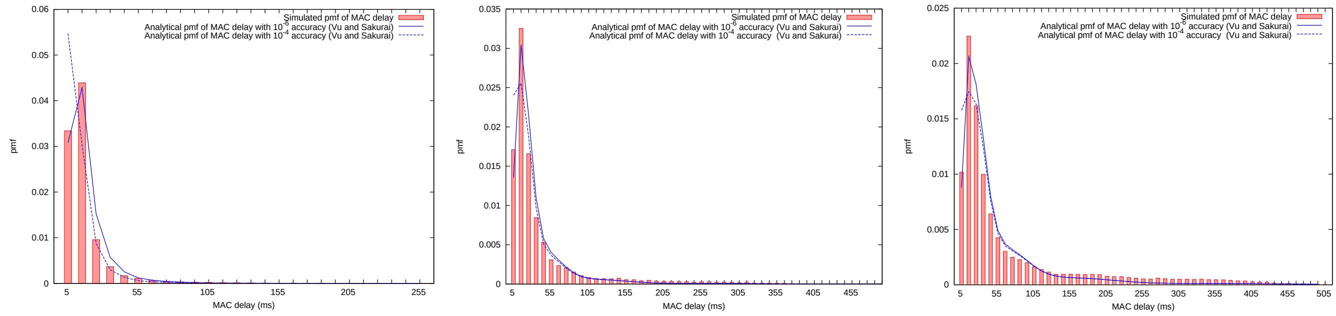
$$d(k) \approx \frac{1}{2kr^k} \sum_{j=1}^{2k} (-1)^j Re(D(re^{i\pi j/k})) \tag{25}$$

with $Re(D(Z))$ is real part of the complex $D(Z)$. For each k, $d(k)$ is derived by summing $Re(D_m(Z))$ over a circle of radius $r = 10^{-\gamma/(2k)}$ for an accuracy of $10^{-\gamma}$.

The LP inversion formula of Vu et Sakurai [10] is :

$$d(k) \approx \frac{1}{2klr^k} Re\left(\sum_{j=-kl}^{kl-1} D(re^{-i\pi j/(kl)})e^{i\pi j/l}\right) \tag{26}$$

where, $l = 1$ and $r = 10^{-\gamma/(2k)}$, which results in an accuracy of $10^{-\gamma}$ as well. Both formulas are almost equivalent. The only difference is how the real part is calculated. In Eq. (25), only the real part of $D(Z)$ is considered, while in Eq. (26), the real part of the whole sum is returned.



(a) $n = 5$ (b) $n = 15$ (c) $n = 30$

Figure 4: Comparison of MAC delay PMF for different accuracy using Vu and Sakurai's LP formula.

Table 5: f_{inv} for Vu's and Vardakas' LP algorithm, $n = 5$

Error bound	Inversion method	f_{inv}
$\gamma = 10^{-6}$	Vu and Sakurai	0.0195
	Vardakas et al.	0.0688
$\gamma = 10^{-4}$	Vu and Sakurai	0.0232
	Vardakas et al.	0.1814

Using f_{inv}, it is possible to decide which inversion formula is correct. It is the one with the smallest value of f_{inv}. The values are given in Table 5 for an accuracy of 10^{-6} and 10^{-4}. It is the LP formula of Vu and Sakurai which introduces the less error. It can be observed as well on the plot of Figure 3. It makes sense that Eq. (26) is correct since to inverse the PGF mathematically, a contour integral has to be computed of the complex values $D(Z).Z^{n-1}$. The real $d(k)$ values of this integral are obtained by taking the real part of the integration result and not by integrating $Re(D(Z)).Z^{n-1}$ over the contour.

The impact of the precision can be measured as well with f_{inv} as shown in Table 5 and presented in Figure 4. As expected, the best results are obtained for a 10^{-6} accuracy.

5.4 Queuing and total delay PMF

The inversion method of Vu and Sakurai with a precision of 10^{-6} will be used for following derivations as previous analysis showed its reduced inversion error. Queueing and total delay distributions for M/M/1 and M/G/1 are presented in Figure 5 for $n = 5$ nodes.

The total delay is clearly dominated by the queueing delay. This is not surprising since we are working at a very high utilization $\rho > 95\%$ to reach saturated conditions. For the queuing delay, the NRMSE calculated using the k_{max} PMF values is of 0.1036 for the M/M/1 model and of 0.0553 for the more precise M/G/1 model. It can be concluded that the M/G/1 model clearly better matches the simulated queuing delay, but the error with M/M/1 stays limited.

For the total delay, the NRMSE calculated using the k_{max} PMF values is of 0.1031 for the M/M/1 model and of 0.0549 for the more precise M/G/1 model. However, looking at the total delay, M/M/1 seems to be a better (and simpler) match for low delay values. In Table 4, it was clear using f_{model} that the MAC model of M/M/1 was less efficient than the Markov based model. As such, we can conclude that the Markov MAC model with the M/G/1 queue is the most efficient one and it is possible with Eq. (26) to keep a limited inversion error.

6. CONCLUSION

This paper proposes a performance evaluation method to characterize the accuracy of a delay distribution derivation. This method is capable of decoupling the error originating from the analytical model from the error induced by the probability generating function inversion. The method has been illustrated on MAC, queuing and total delay distribution models for an IEEE DCF medium access protocol under saturated conditions. Future work will leverage the proposed performance evaluation method to exhibit the most accurate delay distribution model, for saturated and unsaturated conditions and various IEEE 802.11 PHY layers.

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8. REFERENCES

[1] J. Abate, G. L. Choudhury, and W. Whitt. *Computational Probability*, chapter .An Introduction to Numerical Transform Inversion and its Application to Probability Models. Kluwer, Boston, 2000.

[2] G. Bianchi. Performance analysis of the ieee 802.11 distributed coordination function. *Selected Areas in Communications, IEEE Journal on*, 18(3):535–547, 2000.

[3] R. B. Cooper. *Introduction to queueing theory*, volume 2. North Holland New York, 1981.

[4] P. Engelstad and O. Osterbo. The delay distribution of IEEE 802.11e EDCA and 802.11 DCF. In *IPCCC 2006*. IEEE, 2006.

[5] 802.11-2012 - IEEE Standard for Information technology–Telecommunications and information exchange between systems Local and metropolitan area networks
Specific requirements Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications, 2012.

[6] L. Kleinrock. *Theory, volume 1, Queueing systems.* Wiley-interscience, 1975.

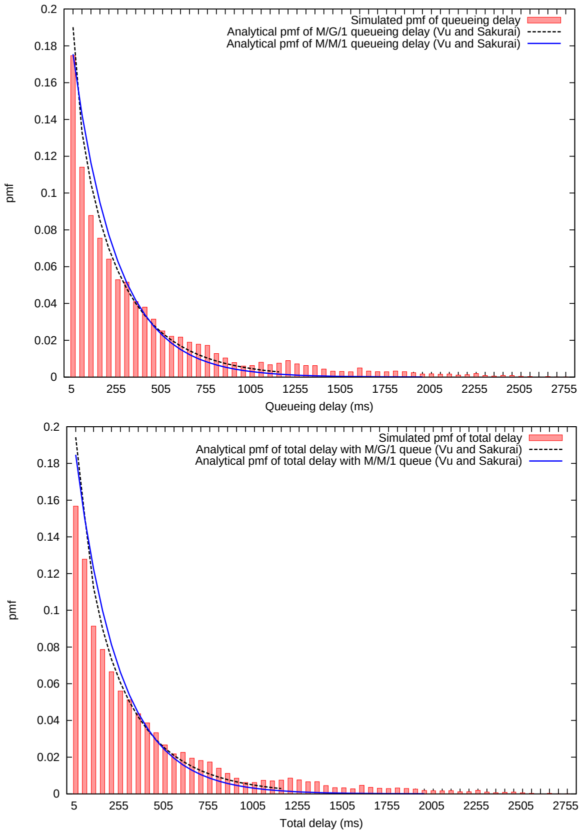

Figure 5: Analytical queueing (top) and total (bottom) delay PMFs for M/M/1 or M/G/1 queues vs. simulatins ($n = 5$).

[7] S. Petersen and S. Carlsen. WirelessHART versus ISA100.11a: The format war hits the factory floor. *Industrial Electronics Magazine, IEEE*, 5:23–34, Dec. 2011.

[8] T. Pulgar, J.-L. Scharbarg, K. Jaffrès-Runser, and C. Fraboul. Extending CAN over the air: an interconnection study through IEEE802.11. In *IEEE ETFA 2013*, pages 10–15, Cagliari, Italy, September 2013.

[9] J. Vardakas, M. K. Sidiropoulos, and M. Logothetis. Performance behaviour of IEEE 802.11 distributed coordination function. *Circuits, Devices Systems, IET*, 2(1):50–59, 2008.

[10] H. Vu and T. Sakurai. Accurate delay distribution for IEEE 802.11 DCF. *Communications Letters, IEEE*, 10(4):317–319, 2006.

[11] A. Willig. A short introduction to queueing theory. Technical report, TU Berlin, Telecommunication Networks Group, 1999.

[12] A. Willig, K. Matheus, and A. Wolisz. Wireless technology in industrial networks. *Proceedings of the IEEE*, 93(6):1130–1151, 2005.

[13] H. Zhai, Y. Kwon, and Y. Fang. Performance analysis of IEEE 802.11 MAC protocols in wireless LANs. *Wireless Communications and Mobile Computing*, 4(8):917–931, 2004.

[14] E. Ziouva and T. Antonakopoulos. CSMA/CA performance under high traffic conditions: throughput and delay analysis. *Comput. Commun.*, 25(3):313–321, Feb. 2002.

APPENDIX

A. LAPLACE TRANSFORM

The Laplace transform of a cumulative distribution function (CDF) $F(t)$, defined for all real numbers $t \geq 0$, is the function $L_f(s)$, defined by:

$$L_f(s) = \int_0^\infty e^{-st} dF(t), s \in \mathbb{C} \tag{27}$$

Similar with the above property of probability generating function, the Laplace transform of the sum of independent random variables equals to the product of the Laplace transform of each variable.

The relationship between the probability generating function and Laplace transform is the following[11]: be f the probability density function (PDF) corresponding to the CDF $F(t)$ of a random variable ; be $\{f_i : i \in \mathbb{N}, f_i := f(i)\}$ the sequence of "samples" of $f(t)$ taken at the discrete times $i = 0, 1, 2, \ldots$; be $G_f(z)$ the probability generating function of that sequence and $L_f(s)$ the Laplace transform of f. Then it is straightforward to show

$$L_f(s) = G_f(e^{-s}). \tag{28}$$

That is $L_f(s)$ differs from $G_f(z)$ only by the change of variable $z = e^{-s}$ [3].