Interferometric Phase Image Estimation via Sparse Coding in the Complex Domain

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Outline

1. The phase estimation problem
2. Examples in InSAR and in MRI
3. Phase unwrapping
4. Interferometric phase estimation via sparse coding
5. Non-Gaussian and non-additive noise
6. Concluding remarks

Absolute Phase Estimation problem

Given a set of observations $e^{j\phi_p} \equiv (\cos\phi_p, \sin\phi_p)$,
for $p \in \mathcal{V} \equiv \{1, \ldots, n\}$, determine ϕ_p (up to a constant)

$e^{j\phi_p}$ is 2π-periodic $\Rightarrow$ nonlinear and ill-posed inverse problem

Continuous/discrete flavor: $\phi = \mathcal{W}[\phi] + 2k\pi \qquad \mathcal{W} : \mathbb{R} \to [\pi, \pi[$

Phase Unwrapping (PU)

$\Downarrow$

Estimation of $k \in \mathbb{Z}$

Phase Denoising (PD)

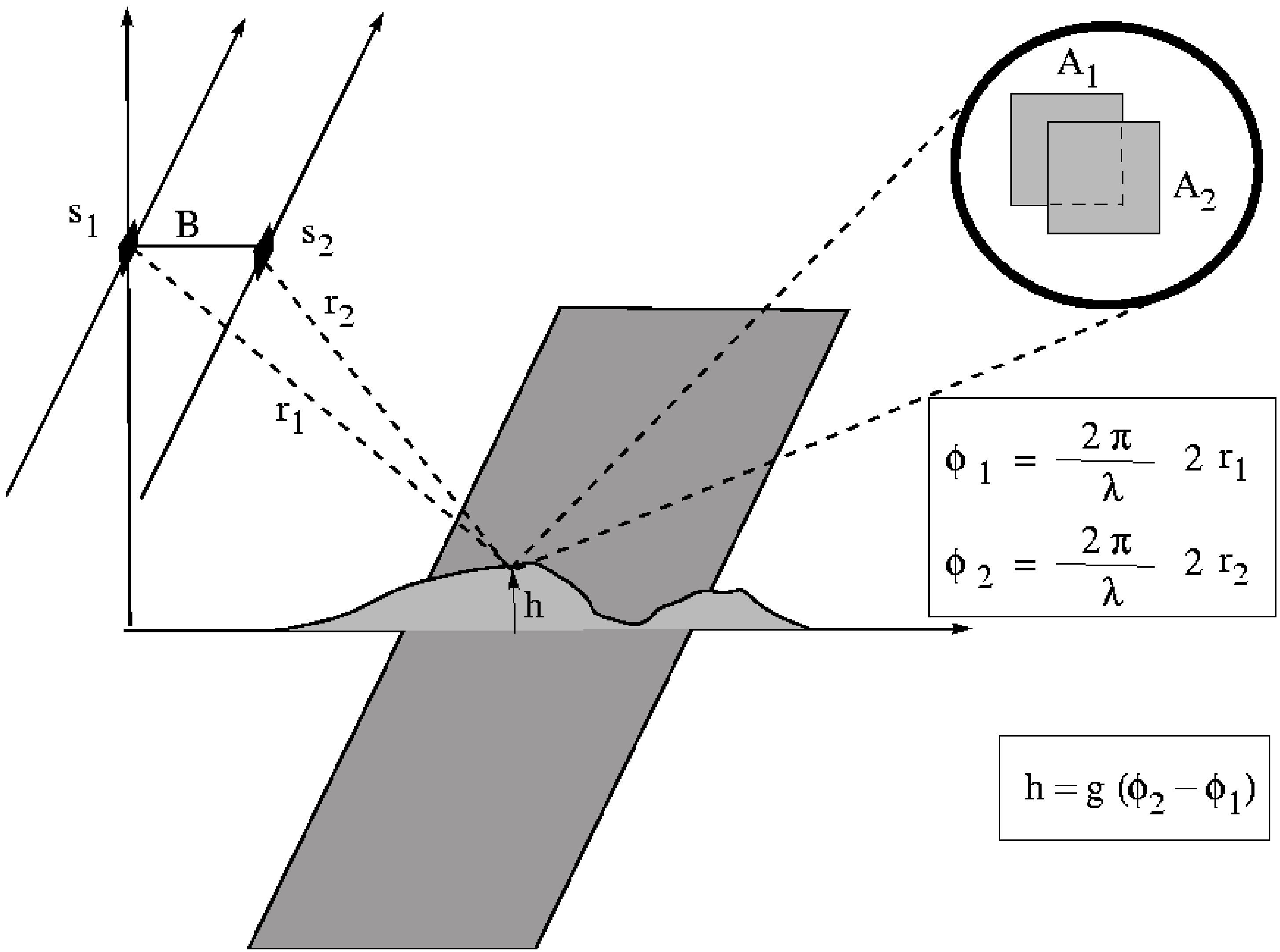
$\Downarrow$

Estimation of $\mathcal{W}(\phi) \in [\pi, \pi[$
(wrapped phase)

Applications

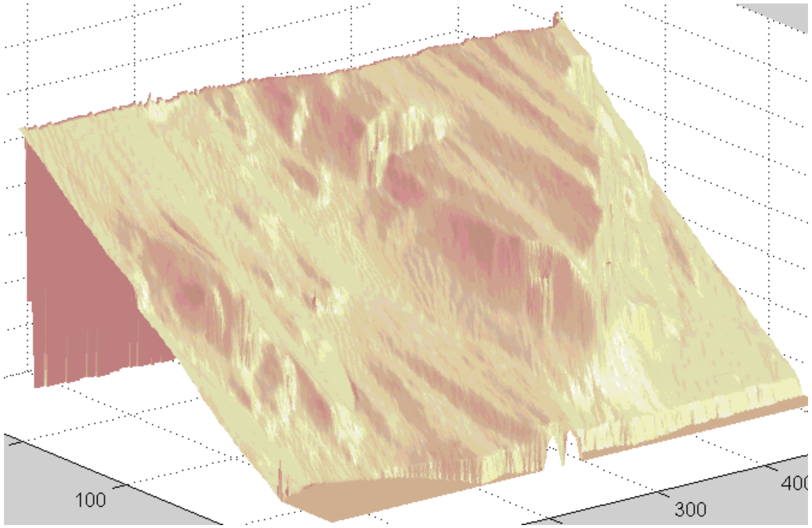
- Synthetic aperture radar/sonar
- Magnetic resonance imaging
- Doppler weather radar
- Doppler echocardiography
- Optical interferometry
- Diffraction tomography

Absolute Phase Estimation in InSAR (Interferometric SAR)



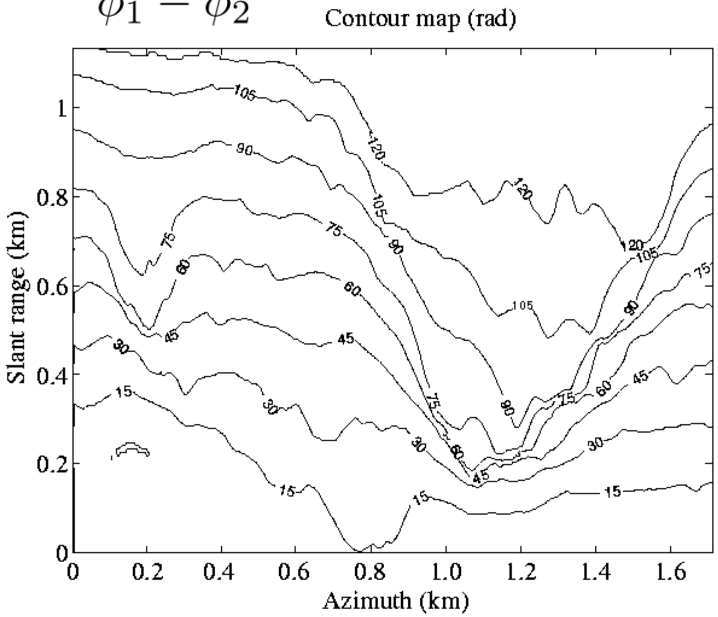
InSAR Problem: Estimate ϕ_2-ϕ_1 from signals read by s_1 and s_2

InSAR Example

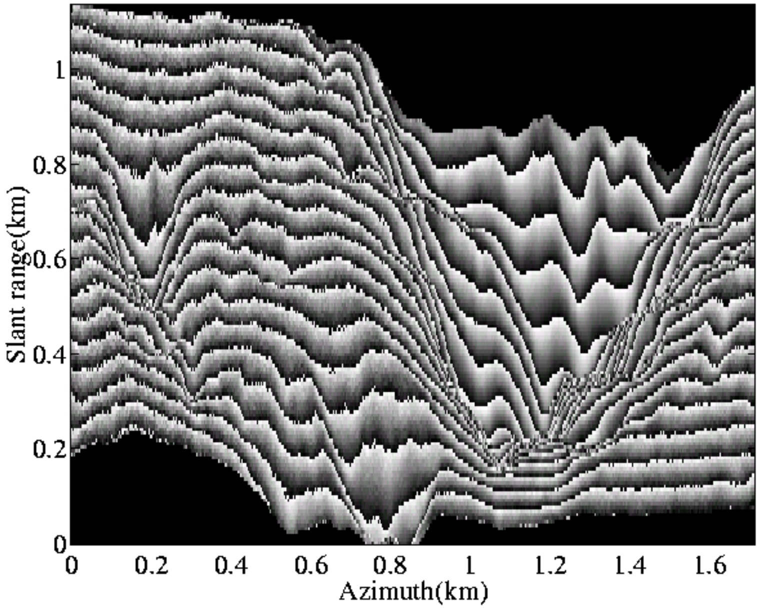


Mountainous terrain around Long's Peak, Colorado

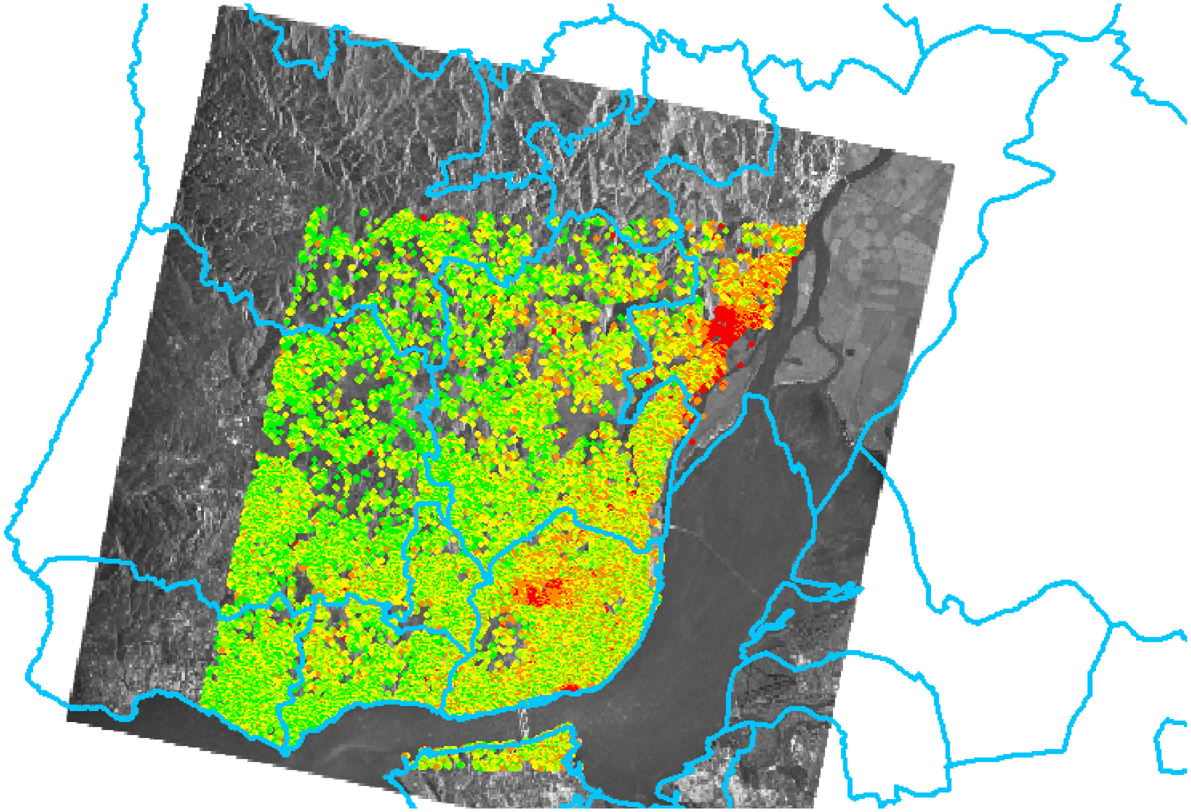
$\phi_1 - \phi_2$



Interferogram $\mathcal{W}(\phi_1 - \phi_2)$



Differential Interferometry

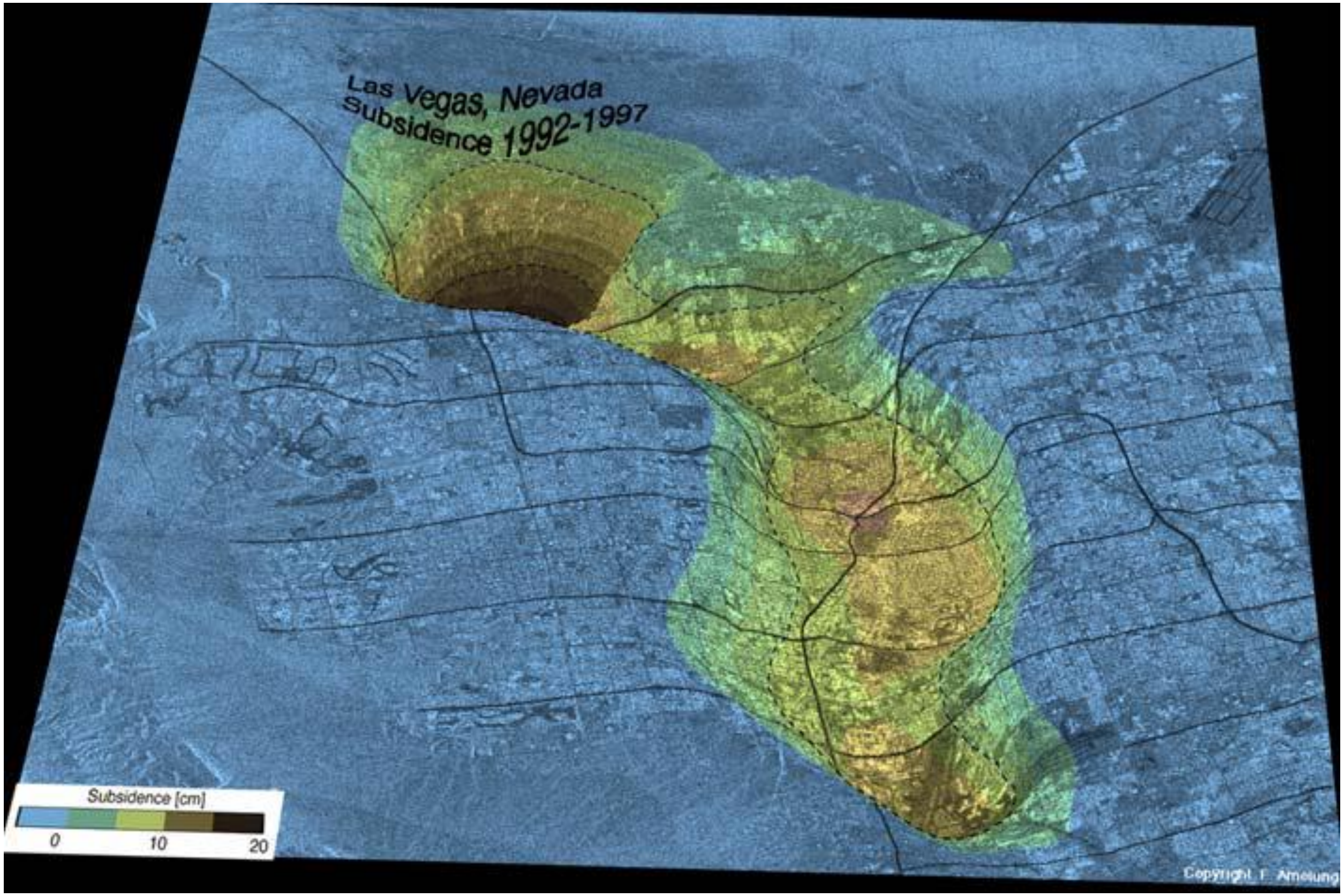


Height variation

7 mm/year

-17 mm/year

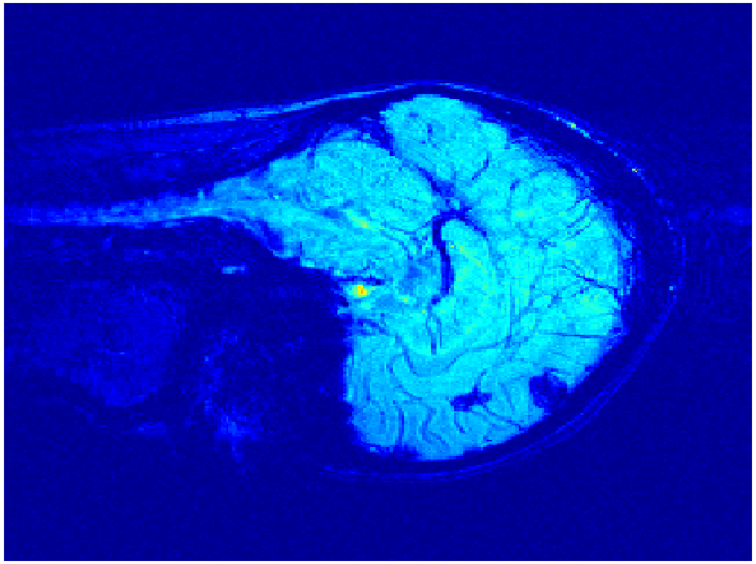
Differential Interferometry



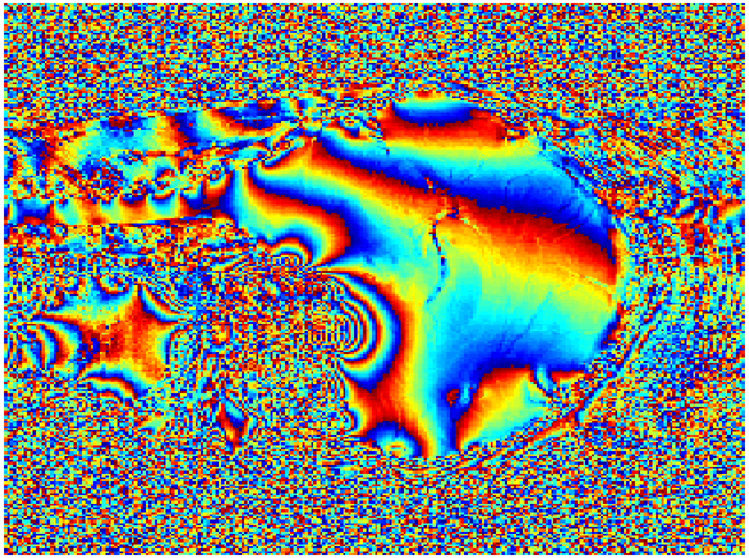
Differential InSAR derived subsidence in Las Vegas between 1992 and 1997 (from [Amelung et al., 1999]).

Magnetic Resonance Imaging - MRI

Intensity



Interferometric phase



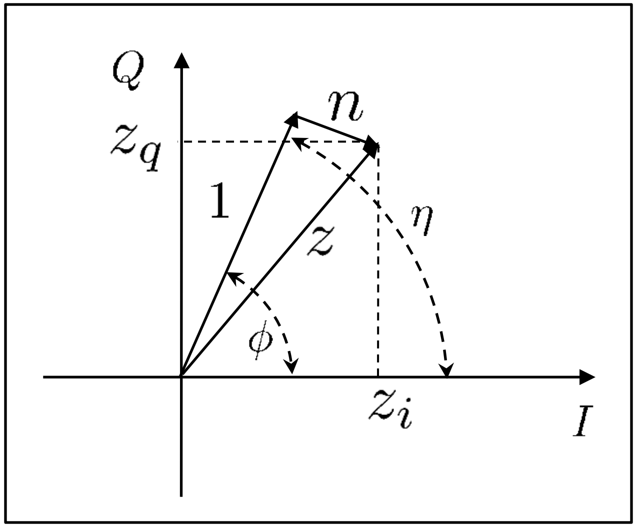
Interferomeric phase

- measure temperature
- visualize veins in tissues
- water-fat separation
- map the principal magnetic field

Forward Problem: Sensor Model

$$z_i = \cos\phi + n_i \qquad n = (n_i, n_q)$$

$$z_q = \sin\phi + n_q \qquad z = (z_i, z_q)$$



$$(n_i, n_q) \sim \mathcal{N}(0, \sigma^2 \mathbf{I}_2)$$

$$\boxed{p(z|\phi) \propto c\, e^{\lambda \cos(\phi - \eta)}} \qquad \widehat{\phi}_{ML} = \eta + 2k\pi$$

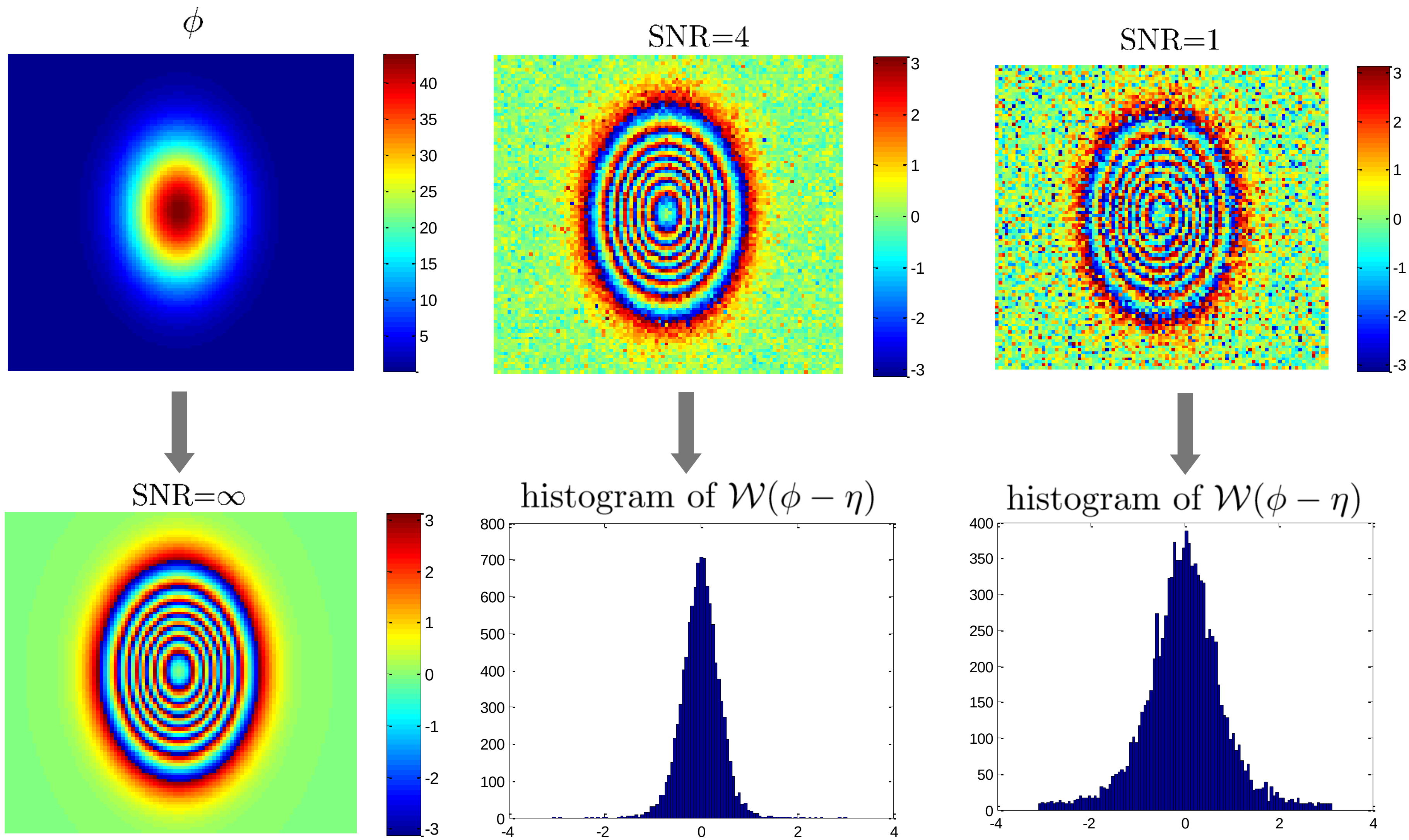
$$\eta = \arg(z)$$

$$\lambda = \frac{2|z|}{\sigma^2}$$

Simulated Interferograms

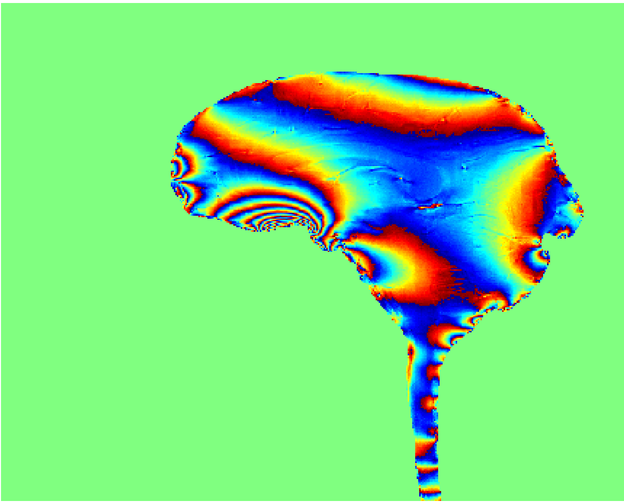
Images of $\eta = \arg(e^{j\phi} + n)$

$$\mathrm{SNR} \equiv \frac{1}{2\sigma^2}$$

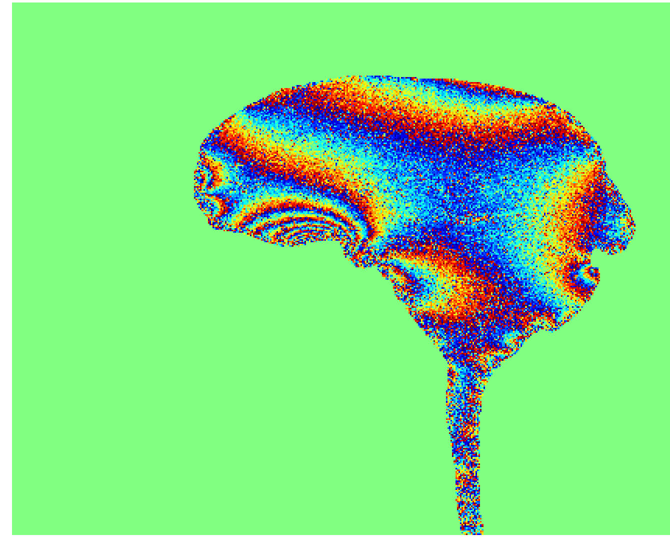


Real Interferograms

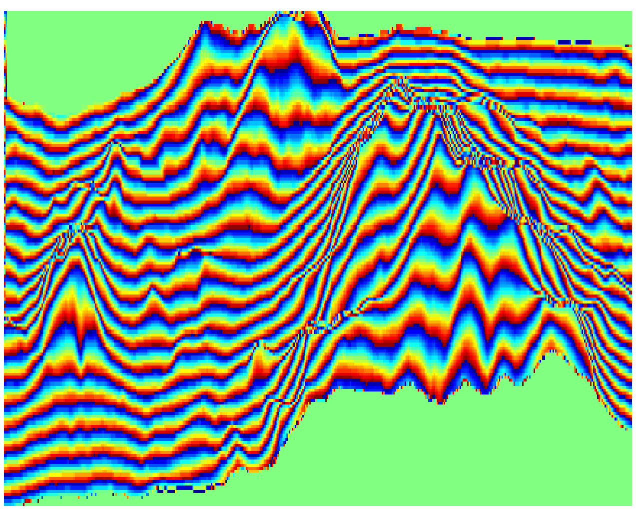
MRI



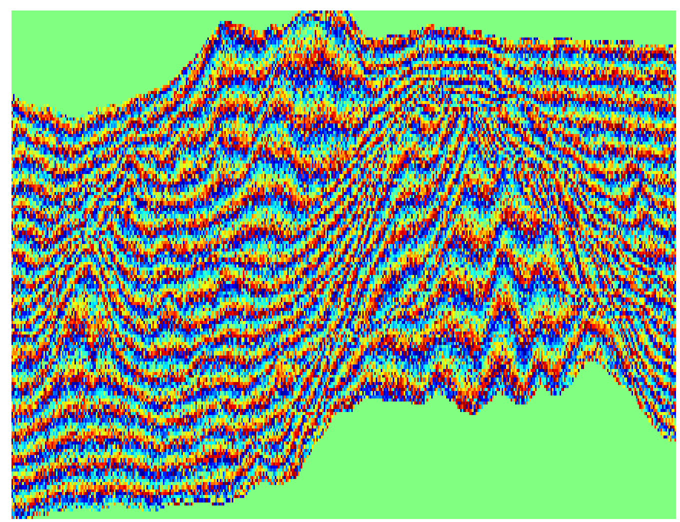
MRI



InSAR



InSAR



Bayesian Approach

data density: $$p(\mathbf{z}|\boldsymbol{\phi}) = \prod_{p \in \mathcal{V}} p(z_p|\phi_p)$$

prior (MRF): $$p(\boldsymbol{\phi}) = \frac{1}{Z} e^{-\sum_{\{p,q\} \in \mathcal{E}} E_{pq}(\phi_p - \phi_q)}$$

- $\mathcal{E} = \{\{p, q\} \, : \, p \sim q\}$ clique set
- $E_{pq}(\cdot)$ clique potential (pairwise interaction)

$E_{pq}(\cdot)$ convex	$E_{pq}(\cdot)$ non-convex
↓	↓
Enforces smoothness	Enforces piecewise smoothness (discontinuity preserving)

Maximum a Posteriori Estimation Criterion

- $\widehat{\phi} \in \arg\max_{\boldsymbol{\phi}\in\mathbb{R}^n} \; p(\boldsymbol{\phi}|\mathbf{z})$ $\qquad p(\boldsymbol{\phi}|\mathbf{z}) \propto p(\mathbf{z}|\boldsymbol{\phi})\, p(\boldsymbol{\phi})$ posterior density

$$= \arg\min_{\boldsymbol{\phi}} E(\boldsymbol{\phi}) \qquad E(\boldsymbol{\phi}) = -\log p(\boldsymbol{\phi}|\mathbf{z}) + c^{te}$$

$$\boxed{E(\boldsymbol{\phi}) = \sum_{p\in\mathcal{V}} -\lambda_p \cos(\phi_p - \eta_p) + \sum_{\{p,q\}\in\mathcal{E}} E_{pq}(\phi_p - \phi_q)}$$

- Phase unwrapping ($\lambda_p \to \infty$):

$$\phi_p = \eta_p + 2k\pi \;\; \text{for} \;\; k_p \in \{0, 1, \ldots, K-1\}$$

$$\widehat{\phi} \in \arg\min_{\mathbf{k}\in\mathbb{Z}^n} \; E(\mathbf{k}) \qquad E(\mathbf{k}) = \sum_{\{p,q\}\in\mathcal{E}} V_{pq}(k_p - k_q)$$

$$V_{pq}(k_p - k_q) = E_{pq}(\eta_p - \eta_q + 2\pi(k_p - k_q))$$

Phase Unwrapping Algorithms

$$E(\mathbf{k}) = \sum_{\{p,q\}\in\mathcal{E}} V_{pq}(k_p - k_q)$$

- $E_{pq}(\cdot) = |\cdot|_{2\pi-\text{quantized}}$

  [Flynn, 97] (exact) $\rightarrow$ sequence of positive cycles on a graph

  [Costantini, 98] (exact) $\rightarrow$ min-cost flow on a graph $(|\mathcal{V}| = n, |\mathcal{E}| = 4n)$

- $E_{pq}(\cdot) = (\cdot)^2$

  [B & Leitao, 01] (exact) $\rightarrow$ sequence of positive cycles on a graph $(|\mathcal{V}| = n, |\mathcal{E}| = 4n)$
  [Frey et *al*., 01] (approx) $\rightarrow$ belief propagation on a 1st order MRF

- $E_{pq}(\cdot)$ convex

  [B & Valadao, 05,07,09] (exact) $\rightarrow$ Sequence of *K* min cuts ($KT(n, 6n)$)

- $E_{pq}(\cdot)$ non-convex

  [Ghiglia, 96] $\rightarrow$ LPN0 (continuous relaxation)

  [B & G. Valadao, 05, 07,09] $\rightarrow$ Sequence of min cuts ($KT(n, 6n)$)

PUMA (Phase Unwrapping MAx-flow)

[B & Valadao, 05,07,09]

$$\phi^{(0)} = \boldsymbol{\eta}$$

$$\textbf{while} \quad \text{success} == \text{false}$$

$$\boxed{\boldsymbol{\delta}' := \arg \min_{\boldsymbol{\delta} \in \{0,1\}^{|\mathcal{V}|}} E(\boldsymbol{\phi} + 2\boldsymbol{\delta}\pi)}$$

$$\textbf{if } E(\boldsymbol{\phi} + 2\boldsymbol{\delta}'\pi) < E(\boldsymbol{\phi}) \textbf{ then } \boldsymbol{\phi} := \boldsymbol{\phi} + 2\boldsymbol{\delta}'\pi$$

$$\textbf{else} \quad \text{success} = \text{true}$$

$$\textbf{end}$$

PUMA finds a sequence of steepest descent binary images

PUMA: Convex Priors

- A local minimum is a global minimum
- Takes at most *K* iterations
- E is submodular: $2V_{pq}(0) \leq V_{pq}(1) + V_{pq}(-1)$

  $\Rightarrow$ each binary optimization has the complexity of a min cut $T(n, 6n)$

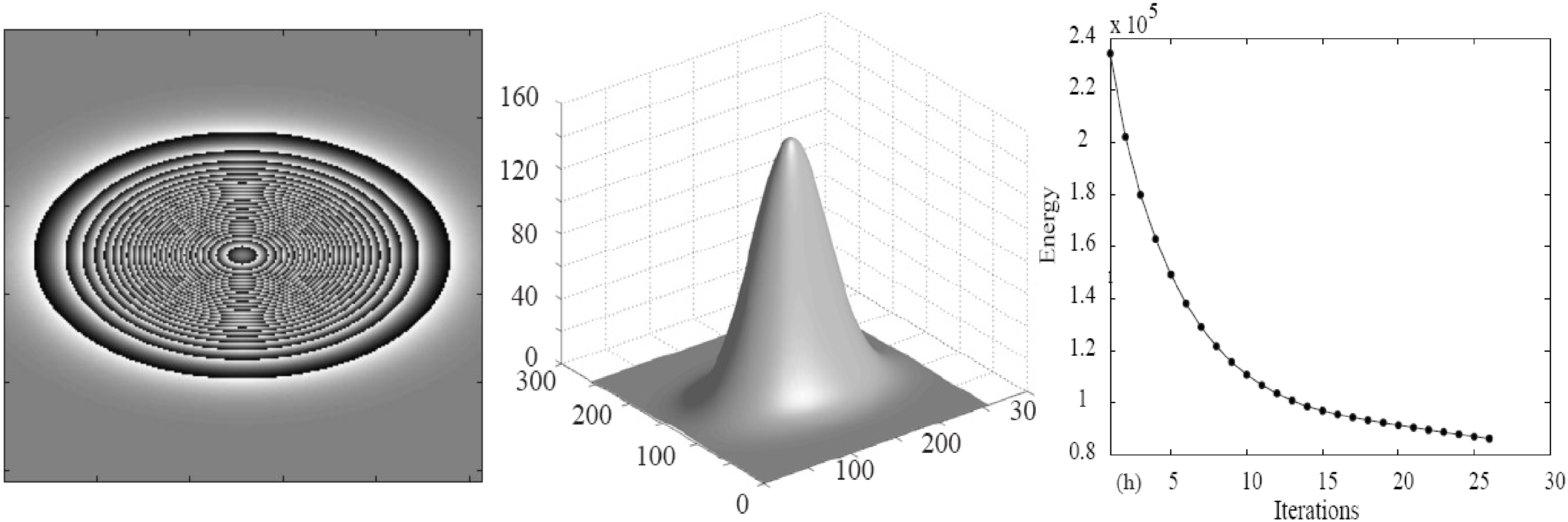
$$E(\mathbf{k}) = \sum_{p \in \mathcal{V}} U_p(k_p) + \sum_{\{p,q\} \in \mathcal{E}} V_{pq}(k_p - k_q)$$

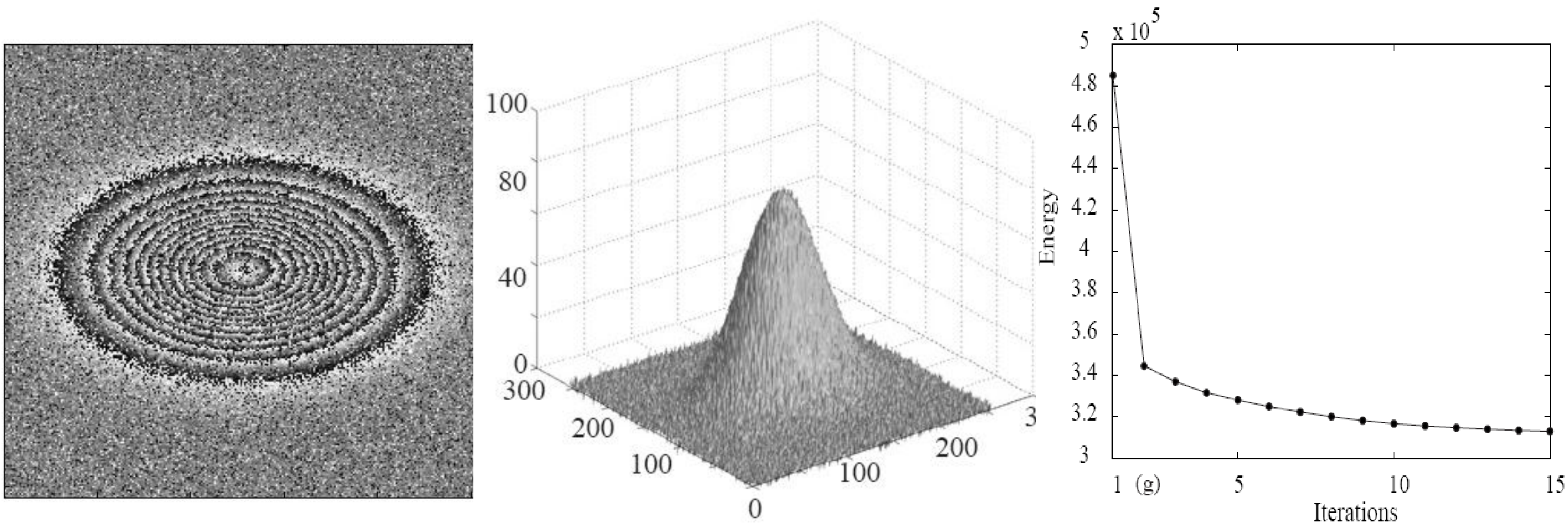
- Related algorithms

  [Veksler, 99] (1-jump moves)
  [Murota, 03] (steepest descent algorithm for L-convex functions)
  [Ishikawa, 03] (MRFs with convex priors)
  [Kolmogorov & Shioura, 05,07], [Darbon, 05] (Include unary terms)
  [Ahuja, Hochbaum, Orlin, 03] (convex dual network flow problem)

Results

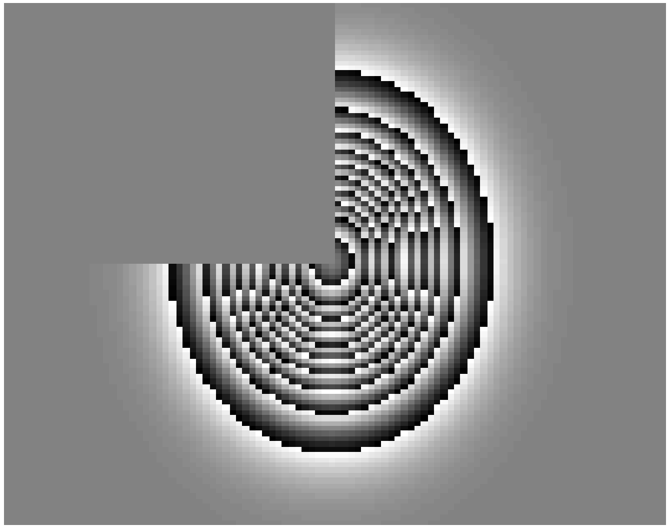
$$E_{pq}(\cdot) = (\cdot)^2$$



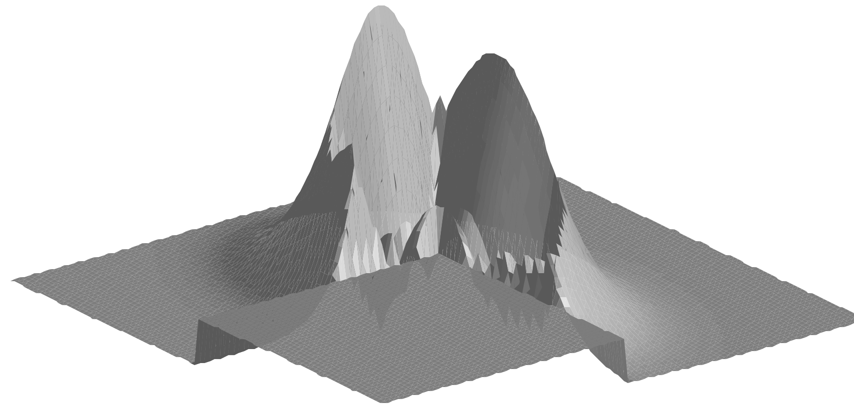


Results

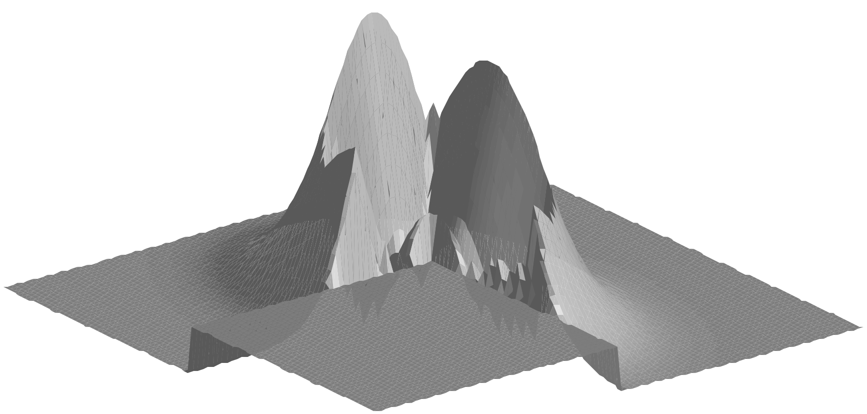
Convex priors do not preserve discontinuities



$$E_{pq}(x) = x^2$$

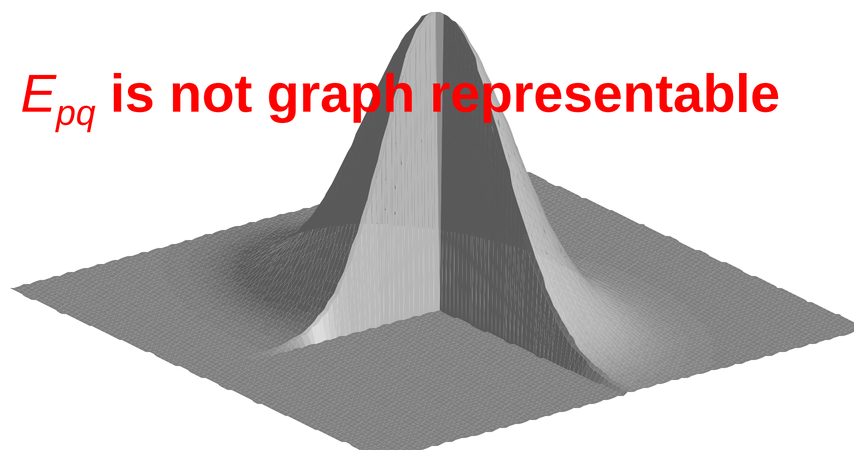


$$E_{pq}(x) = |x|$$



$$E_{pq}(x) = \begin{cases} x^2 & |x| \leq \pi \\ \pi^2 |x/\pi|^{0.5} & |x| > \pi \end{cases}$$

***E**$_{pq}$* **is not graph representable**

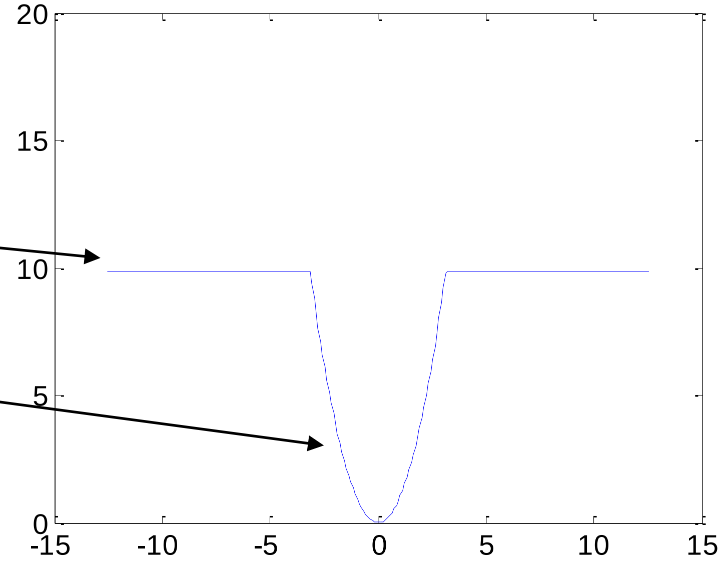


PUMA: Non-convex priors

Ex: $E(x) = \min(x^2, \pi^2)$

Models discontinuities

Models Gaussian noise



Shortcomings

- ❑ Local minima are no more global minima
- ❑ Energy contains nonsubmodular terms (NP-hard)

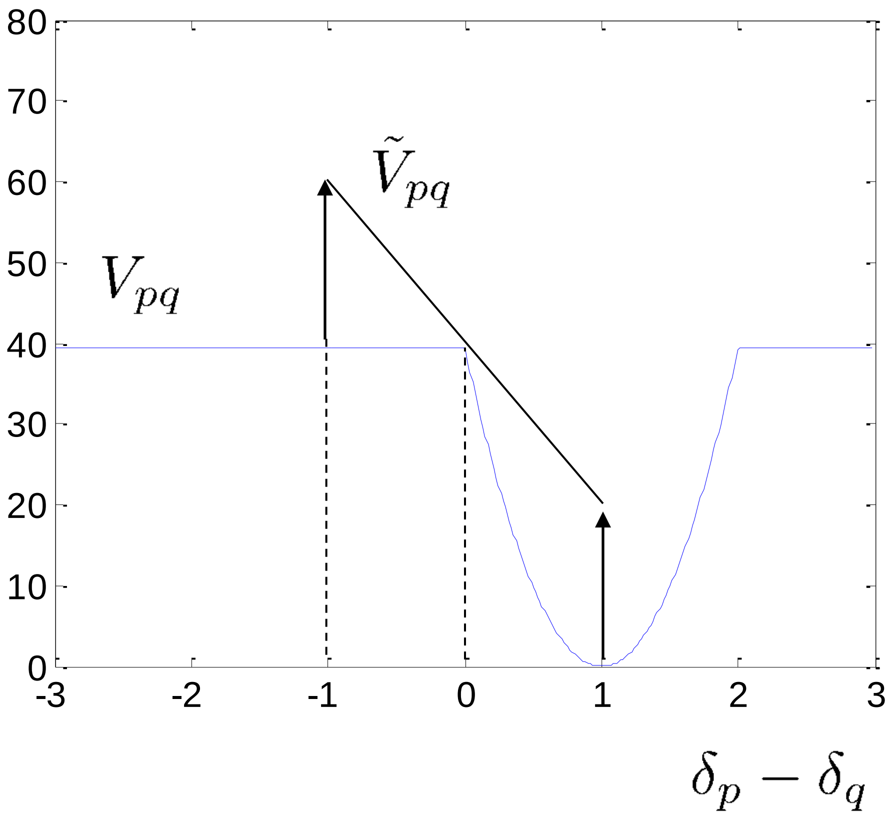
Proposed suboptimal solution: majorization minimization applied
To PUMA binary problem

Other suboptimal approaches

- ❑ Quadratic Pseudo Boolean Optimization (Probing [Boros et *al*., 2006], Improving [Rother et *al*., 2007])
- ❑ Sequencial Tree-Reweighted Message Passing (TRW-S) [Kolmogorov, 2006]
- ❑ Dual decomposition (DD) [Komodakis et al., 2011]

Majorizing Nonsubmodular Terms

Majorization Minimization (MM) [Lange & Fessler, 95]



$$\begin{cases} \tilde{V}(\mathbf{k}) = V(\mathbf{k}) \\ \tilde{V}(\mathbf{k}+\boldsymbol{\delta}) \geq V(\mathbf{k}+\boldsymbol{\delta}) \end{cases}$$

$$\boldsymbol{\delta}' = \arg\min_{\boldsymbol{\delta}} \tilde{V}(\mathbf{k}+\boldsymbol{\delta})$$

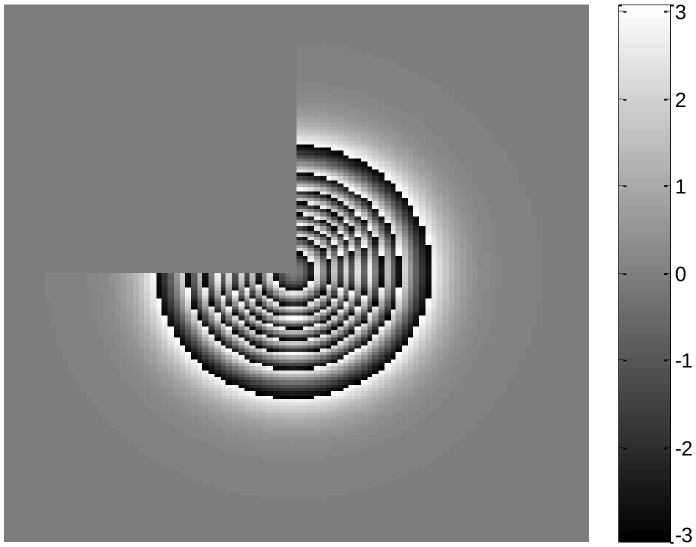
Non-increasing property

$$\boxed{V(\mathbf{k}+\boldsymbol{\delta}') \leq V(\mathbf{k})}$$

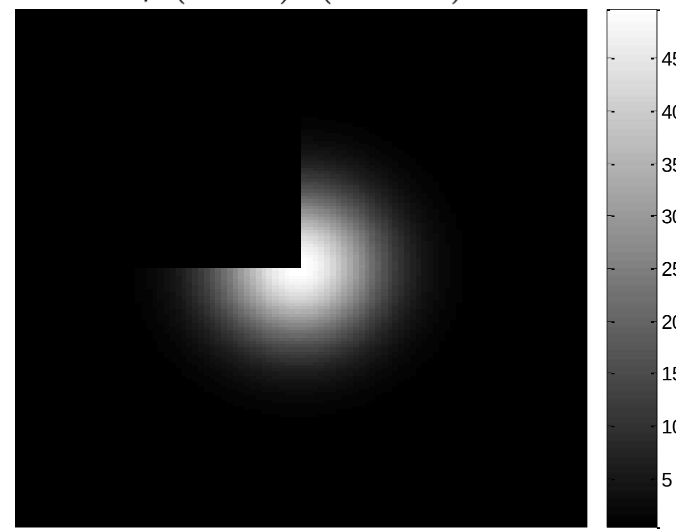
[Rother *et al.*, 05] $\rightarrow$ similar approach for alpha expansion moves

Results with PUMA (MM)

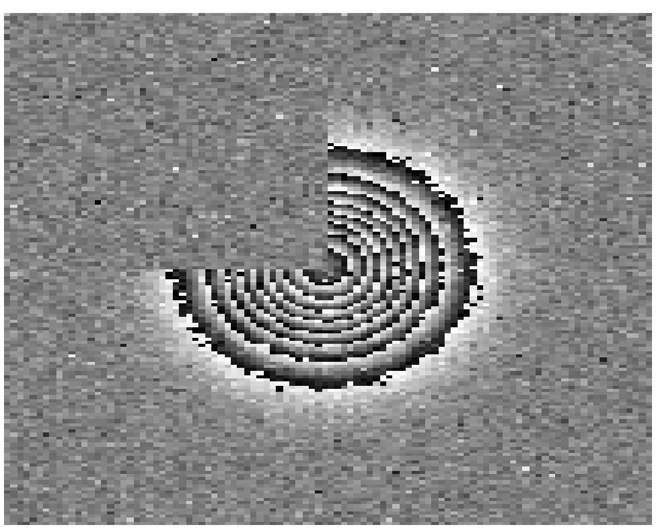
Interferogram η



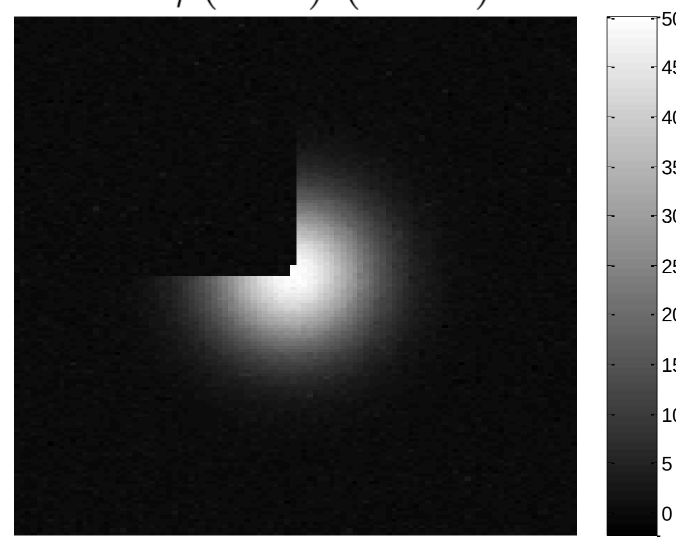
$\widehat{\phi}$(MM) (8 iter)

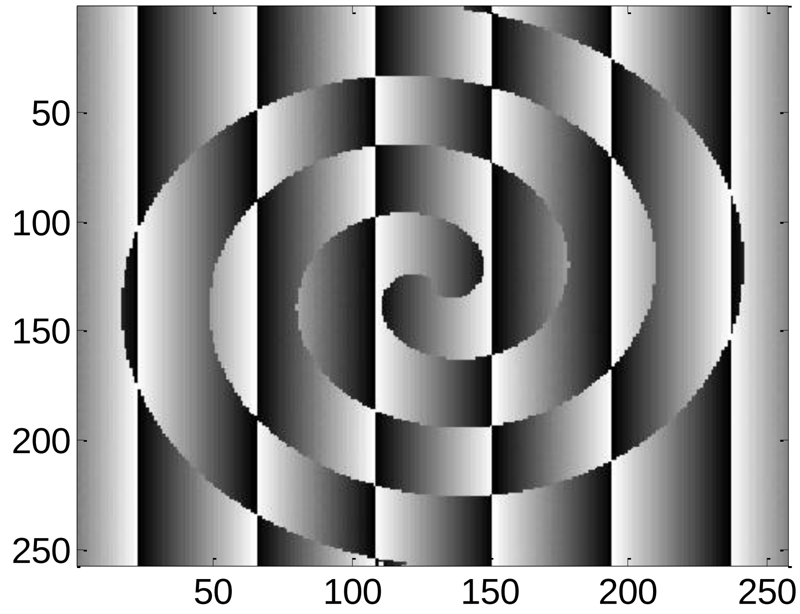


Interferogram η



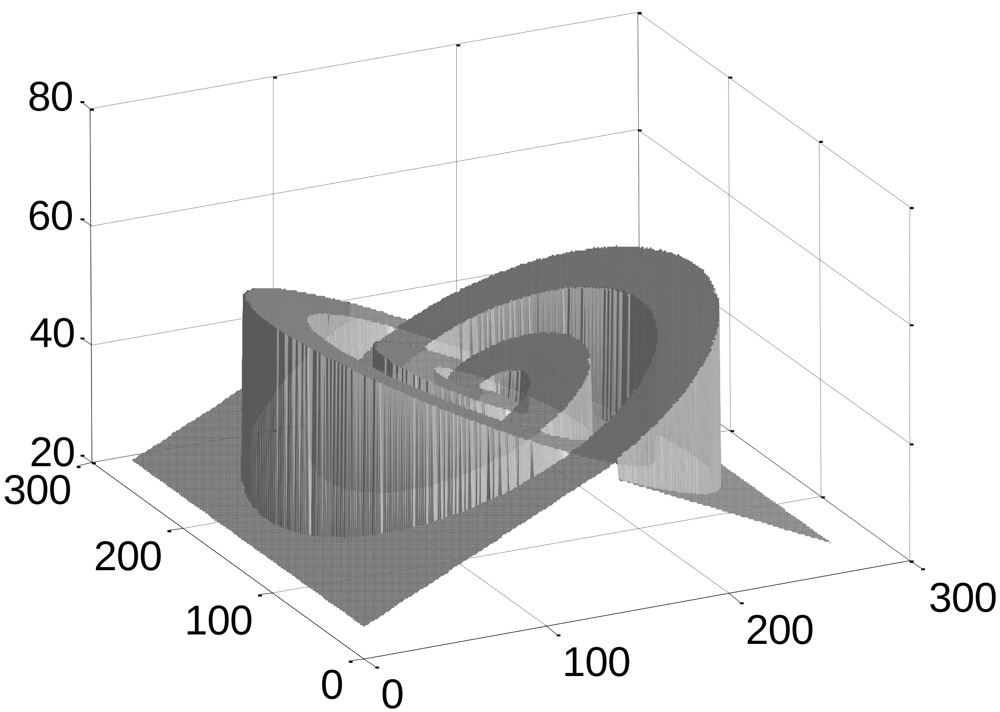
$\widehat{\phi}$(MM) (8 iter)

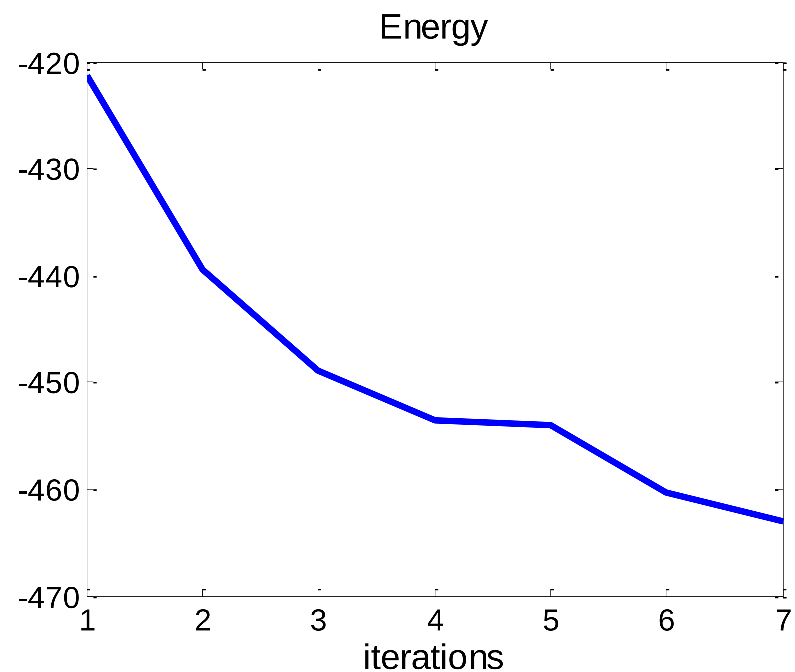




Multi-jump version of PUMA (MM)
jumps d $\in$ [1 2 3 4]

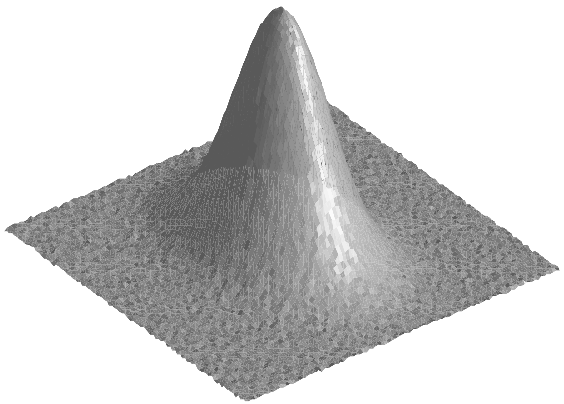
$$\boldsymbol{\delta}' := \arg \min_{\boldsymbol{\delta} \in \{0,d\}^{|\mathcal{V}|}} E(\boldsymbol{\phi} + 2\boldsymbol{\delta}\pi)$$

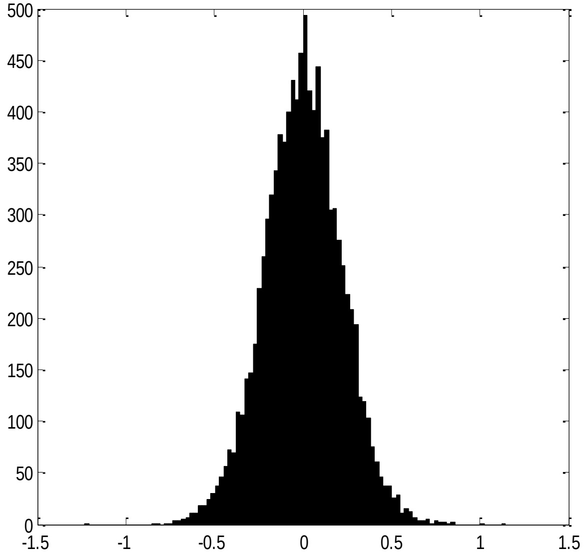




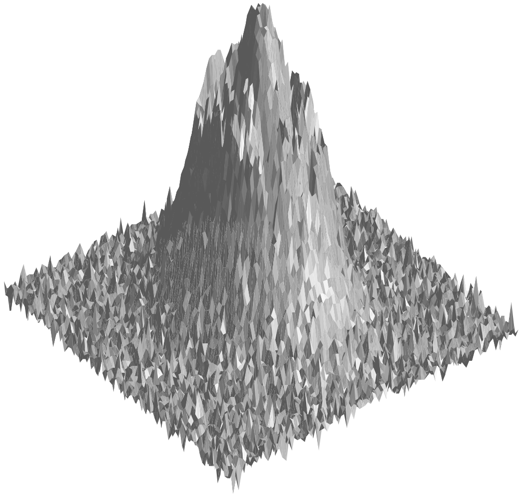
Interferometric Phase Denoising

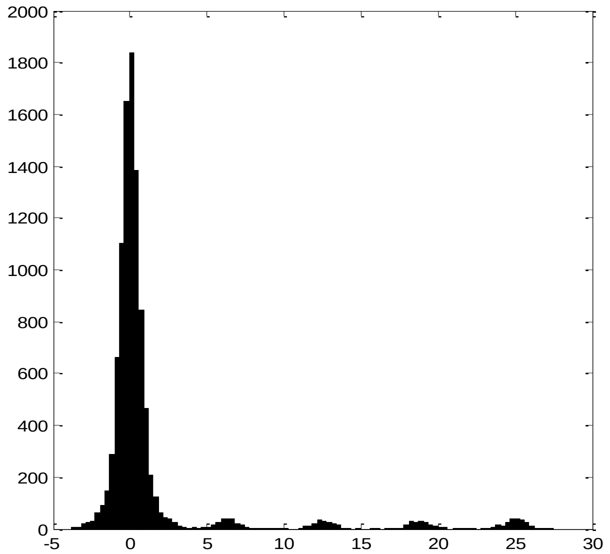
$\sigma = 0.3$ PU works





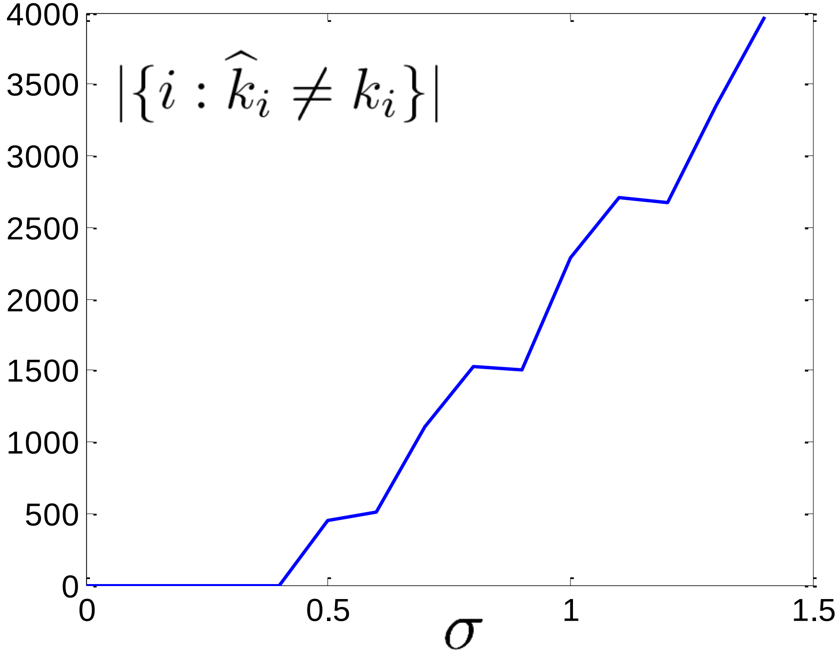
$\sigma = 0.9$ PU does not work



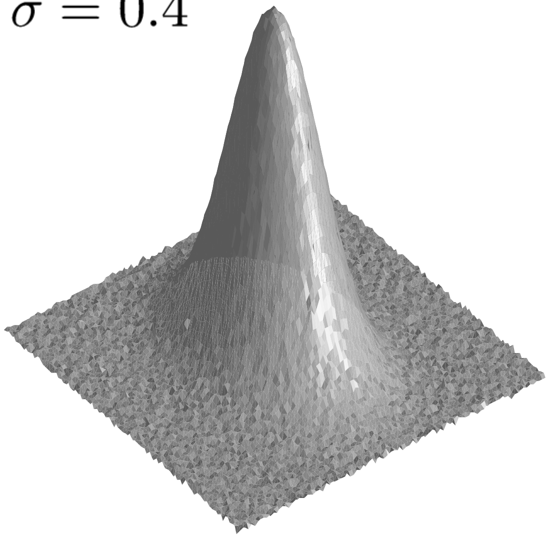


PU Errors

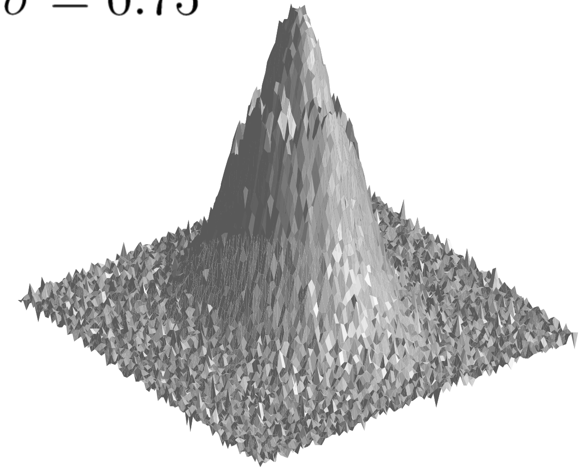
Gaussian shaped image (100×100, max ϕ = 20π)



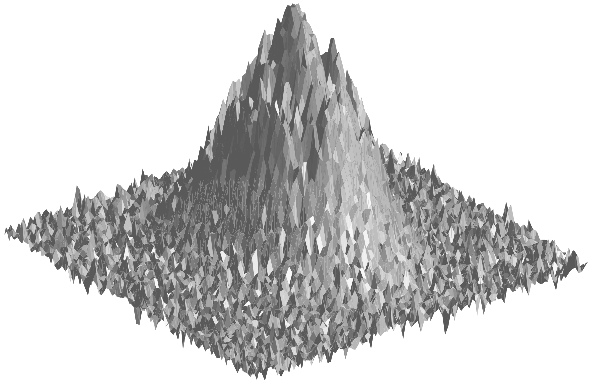
$\sigma = 0.4$



$\sigma = 0.75$



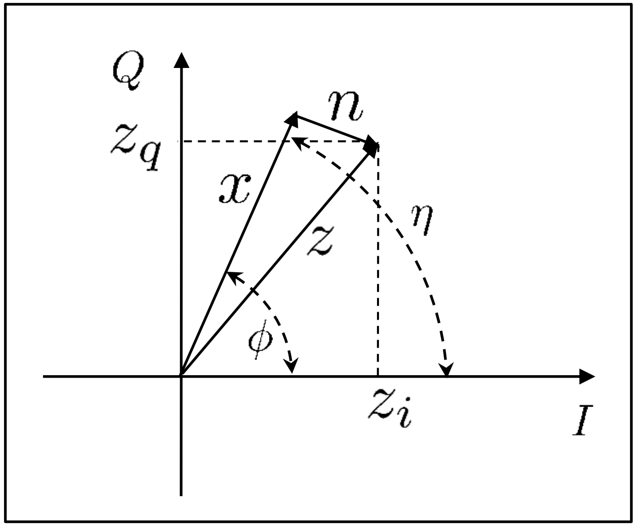
$\sigma = 1.0$



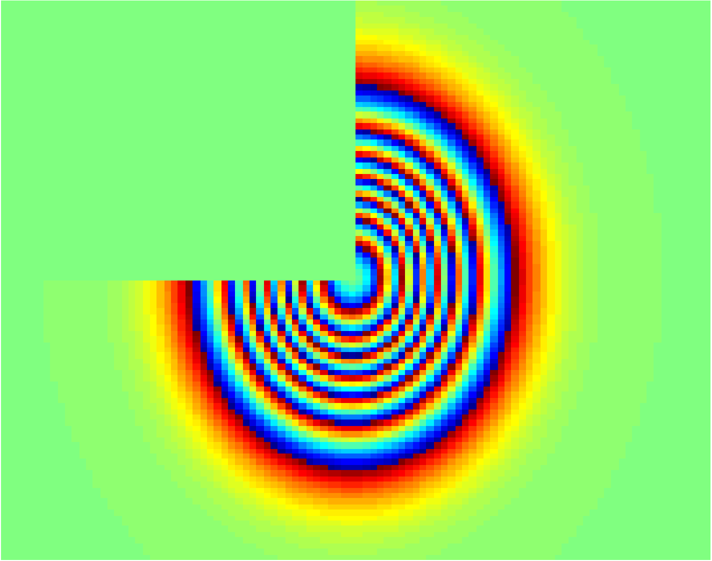
Interferometric Phase Denoising

objective: estimate $\mathcal{W}[\phi]$ from η

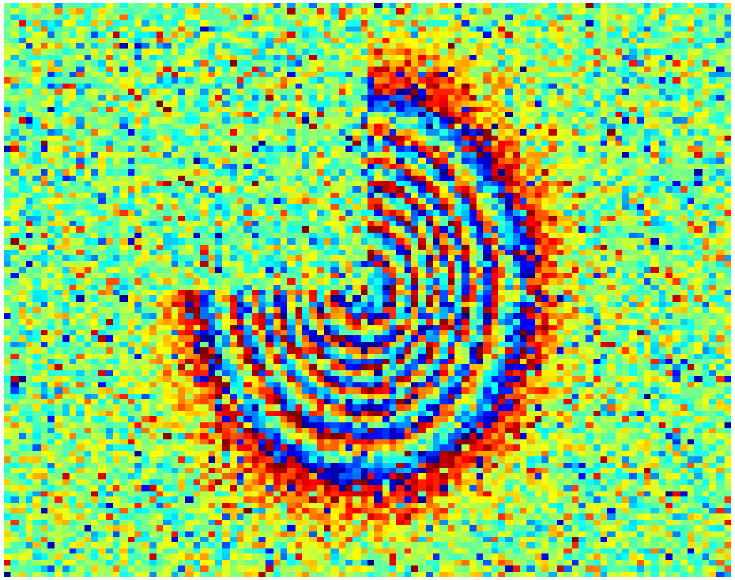
phase modulo 2π



original interf. image $\phi_{2\pi} \equiv \mathcal{W}[\phi]$



observed interf. image $\boldsymbol{\eta}$



State-of-the-art Interferometric Phase Estimation

$$z = e^{j\phi} + n$$

- parametric model for ϕ

  PEARLS [B et al., 2008]: local first order approximation for phase and adaptive window selection (ICI [Katkovnik et al., 2006])

- denoise $\mathbf{z}$

  WFT [Kemao, 2007]: windowed Fourier thresholding

- non-local means filtering

  NL-InSAR [Deledalle, et al., 2011]: patch similarity criterion suitable to SAR images and a weighted maximum likelihood estimation interferogram with weights derived in a data-driven way

Dictionary Based Interferometric Phase Estimation

Motivation

1) sparse and redundant representations are at the heart of many state-of-the-art applications namely in image restoration

2) phase images exhibit a high level of self-similarity. So they admit sparse representations on suitable dictionaries.

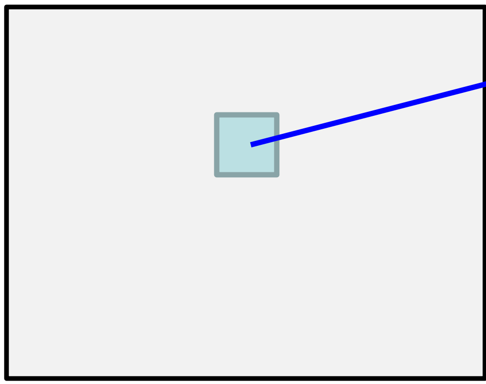
Challenge: the observation mechanism linking the observed phase η with the interferometric phase $\phi_{2\pi}$ is nonlinear.

Observation: the fact that the phase image $\boldsymbol{\phi}$ is self-similar implies that $e^{j\boldsymbol{\phi}}$ is self-similar

Our approach: learn sparse representations for $e^{j\boldsymbol{\phi}}$ and from them infer $\boldsymbol{\phi}$

Interferometric Phase Estimation via Sparse Regression

Complex valued image



patch of size $\sqrt{m} \times \sqrt{m}$ at pixel *i*

$$\mathbf{z}_i = \mathbf{x}_i + \mathbf{n}_i \in \mathbb{C}^m$$

noise vector

original vector

observed vector

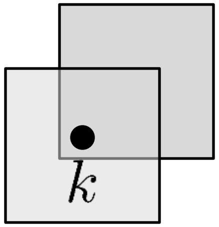
$\mathbf{D} \equiv [\mathbf{d}_1, \ldots, \mathbf{d}_k] \in \mathbb{C}^{m \times k}$ dictionary with respect to which $\mathbf{x}_i$ admits a sparse representation

$$\widehat{\mathbf{x}}_i = \mathbf{D}\widehat{\boldsymbol{\alpha}}_i \qquad \min_{\boldsymbol{\alpha}} \|\boldsymbol{\alpha}\|_0, \quad \text{s.t.:} \quad \|\mathbf{D}\boldsymbol{\alpha} - \mathbf{z}_i\|_2^2 \leq \delta$$

estimation error $\boldsymbol{\varepsilon}_i = \widehat{\mathbf{x}}_i - \mathbf{x}_i$

iid noise $\Rightarrow$ $\boxed{\dfrac{\|\boldsymbol{\varepsilon}_i\|_2^2}{\|\mathbf{n}_i\|_2^2} \simeq \dfrac{p}{m}}$ $\qquad p = \|\widehat{\boldsymbol{\alpha}}\|_0$

Interferometric Phase Estimation



$\mathcal{P}_k \rightarrow$ the set of patches containing the pixel k

$\widehat{x}_i = x_i + \varepsilon_i, \quad i \in \mathcal{P}_k$ the set of estimates of x_k obtained from patches $i \in \mathcal{P}_k$

Maximum likelihood estimate of $x_i = ae^{j\phi}$

(assume that $\boldsymbol{\varepsilon}_i = [\varepsilon_1, \ldots, \varepsilon_p]$ is $\mathcal{N}(\mathbf{0}, \mathbf{C})$)

$$\boxed{\widehat{\phi}_{2\pi} = \arg\left(\sum_{j=1}^{q} \widehat{x}_j \gamma_j\right)} \qquad \widehat{a} = \frac{\left|\sum_{j=1}^{q} \widehat{x}_j \gamma_j\right|}{\sum_{j=1}^{q} \gamma_j}$$

where $\gamma_j := \sum_{k=1}^{q} [\mathbf{C}^{-1}]_{jk}$.

in practice γ_j is very hard to compute and we take $\gamma_j = c^{te}$

Dictionary Learning

find a dictionary representing accurately the image patches with the smallest possible number of atoms.

formalization under the regularization framework

$$\min_{\mathbf{D}\in\mathcal{C},\mathbf{A}} L(\mathbf{D},\mathbf{A}) \qquad\qquad L(\mathbf{D},\mathbf{A}) = (1/2)\big\|\mathbf{Z}-\mathbf{D}\mathbf{A}\big\|_F^2 + \lambda\|\mathbf{A}\|_1,$$

where $\mathcal{C} := \left\{\mathbf{D}\in\mathbb{C}^{m\times k}\ :\ \left|\mathbf{d}_j^H\mathbf{d}_j\right| \le 1,\ j=1,\ldots,k\right\}$

and $\mathbf{Z} = [\mathbf{z}_1,\ldots,\mathbf{z}_{N_p}]$ and $\mathbf{A} = [\boldsymbol{\alpha}_1,\ldots,\boldsymbol{\alpha}_{N_p}]$

DL Algorithm: alternating proximal minimization (APM)

$$\begin{cases} \mathbf{D}^{k+1} \in \arg\min\limits_{\mathbf{D}\in\mathcal{C}} L(\mathbf{D},\mathbf{A}^k) + \lambda\|\mathbf{D}-\mathbf{D}^k\|_F^2 \\ \mathbf{A}^{k+1} \in \arg\min\limits_{\mathbf{A}} L(\mathbf{D}^{k+1},\mathbf{A}) + \lambda\|\mathbf{A}-\mathbf{A}^k\|_F^2 \end{cases}$$

Convergence (based on the Kurdyka- Lojasiewicz inequality)
[Attouch et al. 10], [Xu, Yin, 2012]

Dictionary Learning

drawback: alternating proximal minimization takes too long (order of 10^4 sec) in a typical image scenario (N_p = 100000, m = 100, and k = 200)

another DL algorithm: Direct Optimization (DM) method (non-convex proximal splitting results) [Rakotomamonjy, 2012]

projection on the unit sphere

$$\begin{cases} \mathbf{D}^{k+1} = \boxed{\mathrm{prox}_{\eta_k \iota_{\mathcal{D}}}}\left(\mathbf{D}^k - \eta_k(\mathbf{Z} - \mathbf{D}^k\mathbf{A}^k)(\mathbf{A})^H\right) \\ \mathbf{A}^{k+1} = \boxed{\mathrm{prox}_{\eta_k \Omega_A}}\left(\mathbf{D}^k - \eta_k(\mathbf{D^k})^H(\mathbf{Z} - \mathbf{D}^k\mathbf{A}^k)\right) \end{cases}$$

soft threshold

drawback: direct optimization yields local minima poorer than APM

another DL algorithm: Online Dictionary Learning (ODL): [Mairal et al. 2010]

Dictionary Learning

Online Dictionary Learning (ODL): [Mairal et al. 2010]

Select randomly $\mathbf{z}^t \equiv [\mathbf{z}_i^t \; i = 1, \ldots \eta]$ from $\mathbf{z}$

(*Sparse coding:* BPDN)

$$\boldsymbol{\alpha}^t := \arg \min_{\boldsymbol{\alpha} \in \mathbb{C}^{k \times \eta}} (1/2)\|\mathbf{z}^t - \mathbf{D}\boldsymbol{\alpha}\|_F^2 + \lambda\|\boldsymbol{\alpha}\|_1$$

$$\min_{\mathbf{D} \in \mathcal{C}} \frac{1}{S_t} \sum_{i=1}^{t} w_i \left\{ (1/2)\left\|\mathbf{z}^i - \mathbf{D}\boldsymbol{\alpha}^i\right\|_F^2 + \lambda\|\boldsymbol{\alpha}^i\|_1 \right\}$$

$\mathbf{D}^t$ converges to the stationary points of

$$(1/2)\left\|\mathbf{Z} - \mathbf{D}\mathbf{A}\right\|_F^2 + \lambda\|\mathbf{A}\|_1,$$

$$\mathbf{D} \in \mathcal{C}$$

fast sparse coding via ADMM [B & Figueiredo, 2010]

$\boldsymbol{\alpha}^t := \mathbf{D}^H \mathbf{z}^t$
$\mathbf{u}^t := \boldsymbol{\alpha}^t, \; \mathbf{v}^t := \mathbf{0}$
$\mathbf{F} = (\mathbf{D}^H \mathbf{D} + \mu \mathbf{I})^{-1}$
while *not converge* **do**
  $\mathbf{u}^t := \mathrm{soft}(\boldsymbol{\alpha}^t - \mathbf{v}^t, \lambda/\mu)$
  $\boldsymbol{\alpha}^t := \mathbf{F}(\mathbf{D}^H \mathbf{z}^t + \mu(\mathbf{u}^t + \mathbf{v}^t))$
  $\mathbf{v}^t := \mathbf{v}^t - (\boldsymbol{\alpha}^t - \mathbf{u}^t)$
end

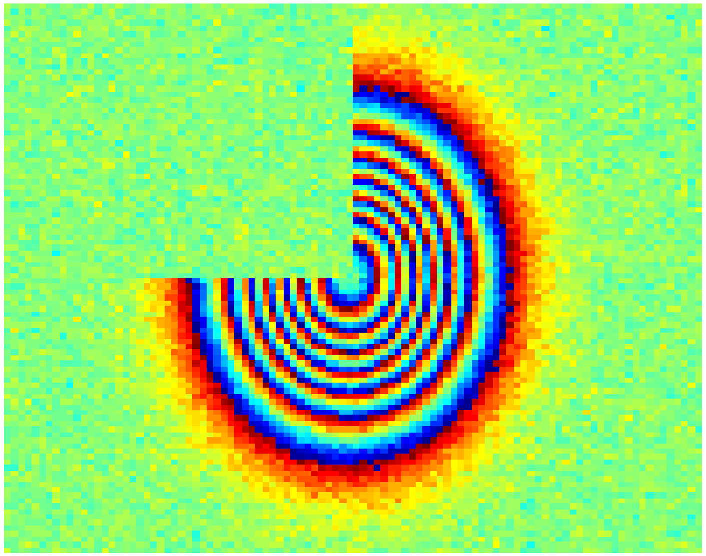
computational complexity: $O(km^2 + \eta k m)$

The Proposed Algorithm

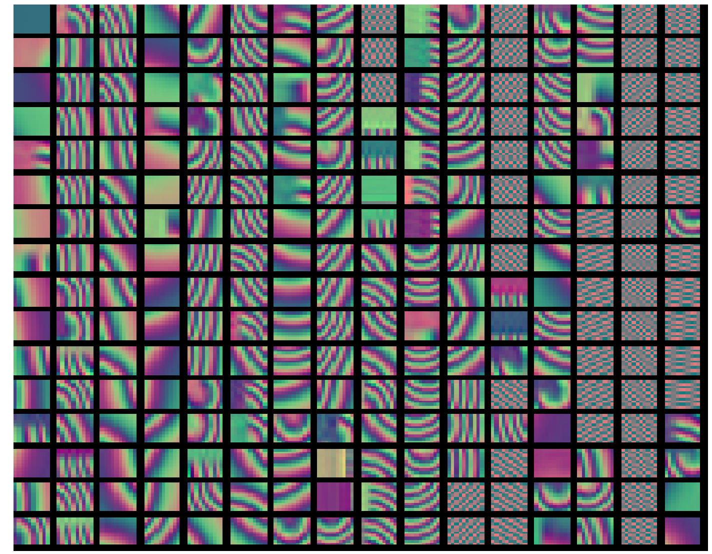
SpInPHASE [Hongxing, B, Katkovnik, 2013]

Input: $\mathbf{z} \in \mathbb{C}^{N_1 \times N_2}$	(complex valued image)
Ouput: $\widehat{\phi} \in \mathbb{R}^{N_1 \times N_2}$	(absolute phase estimate)
Begin	
$\mathbf{z}_i \leftarrow \mathbf{M}_i\mathbf{z}, \;\; i = \ldots, N_p$	(extract patches)
$\mathbf{D} \leftarrow \mathrm{DL}(\mathbf{z}_i, i = 1, \ldots, N_p)$	(learn the dictionary)
$\boldsymbol{\alpha}_i \leftarrow \mathrm{OMP}(\mathbf{D}, \mathbf{z}_i, i = 1, \ldots, N_p)$	(sparse coding)
$\mathbf{x}_i \leftarrow \mathbf{D}\boldsymbol{\alpha}_i, i = 1, \ldots, N_p)$	(path estimate)
$\mathbf{x} \leftarrow \mathrm{compose}(\mathbf{x}_i, i = 1, \ldots, N_p)$	(path compose)
$\widehat{\phi}_{2\pi} \leftarrow \arg(\mathbf{x})$	(interferometric phase estimate)
$\widehat{\phi} \leftarrow \mathrm{PUMA}(\widehat{\phi}_{2\pi})$	(phase unwrapping)
End	

DL (APM): Example (truncated Gaussian - $\sigma = 0.3$) $\sqrt{m} = 12,\ k = 256$



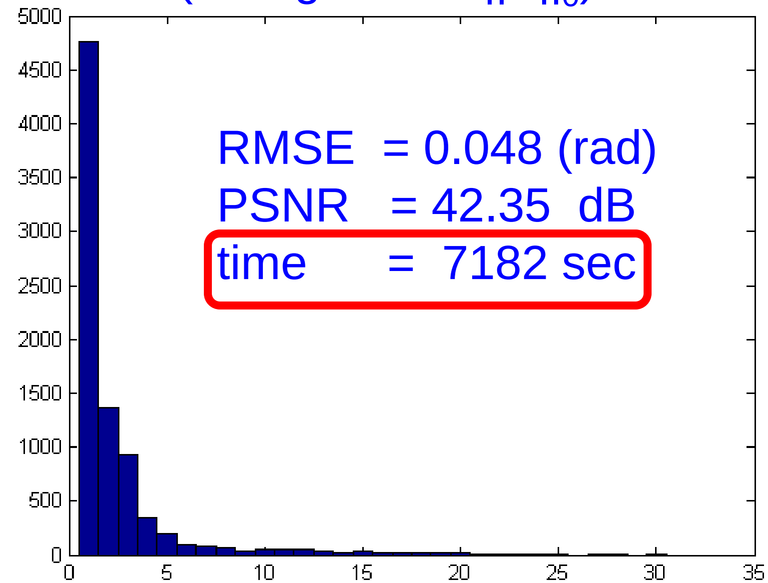
(learned dictionary – APM)



$$\mathrm{RMSE} := \frac{\|\mathcal{W}(\widehat{\phi}_{2\pi} - \phi_{2\pi})\|_F}{\sqrt{N}}$$

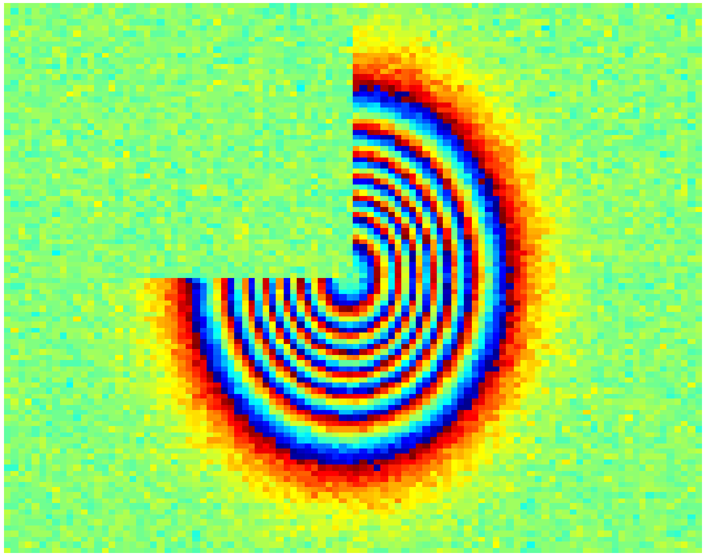
$$\mathrm{PSNR} := \frac{4N\pi^2}{\|\mathcal{W}(\widehat{\phi}_{2\pi} - \phi_{2\pi})\|_F^2}$$

(histogram of $\|\alpha\|_0$)

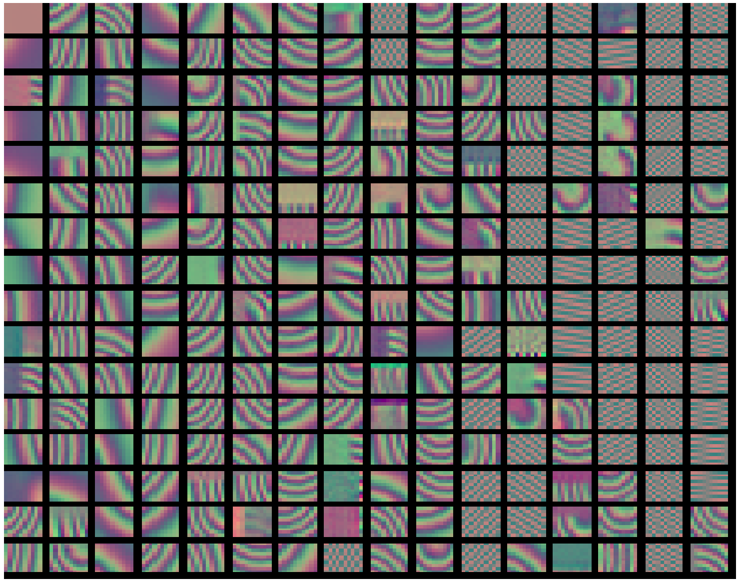


$$\frac{\|\mathcal{W}(\boldsymbol{\eta} - \phi_{2\pi})\|_F^2}{\|\mathcal{W}(\widehat{\phi}_{2\pi} - \phi_{2\pi})\|_F^2} = 20 \simeq \frac{1}{2}\frac{m}{\overline{p}}$$

DL (ODL): Example (truncated Gaussian - σ = 0.3) $\sqrt{m} = 12,\ k = 256$

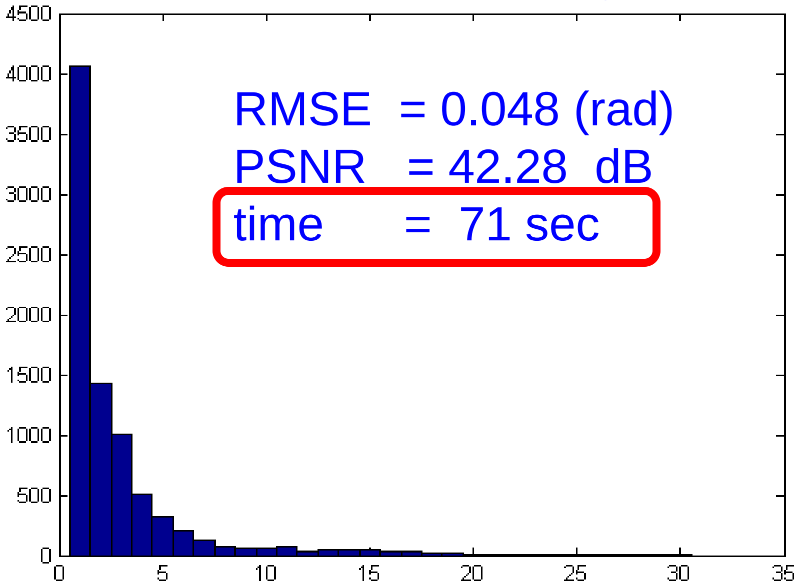


(learned dictionary – ODL)



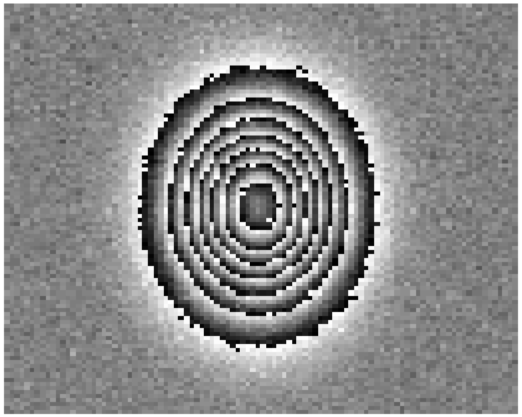
$$\frac{\|\mathcal{W}(\boldsymbol{\eta} - \phi_{2\pi})\|_F^2}{\|\mathcal{W}(\widehat{\phi}_{2\pi} - \phi_{2\pi})\|_F^2} = 20 \simeq \frac{1}{2}\frac{m}{\overline{p}}$$

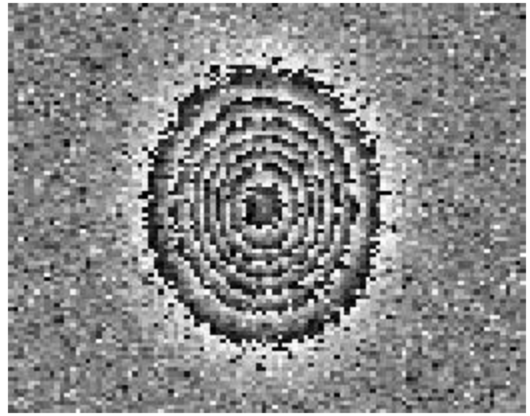
(histogram of $||\alpha||_0$)

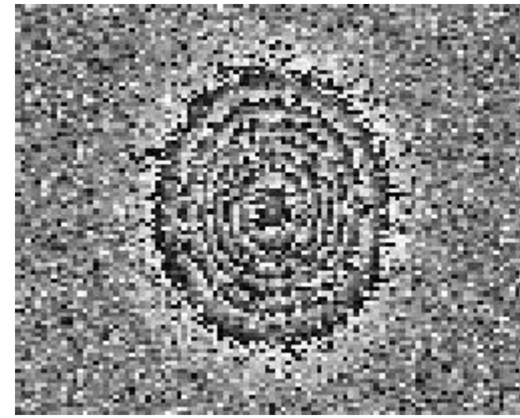


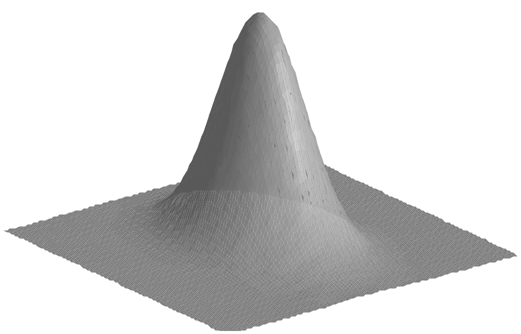
Restored Images

σ = 0.5 | σ = 1.0 | σ = 1.5

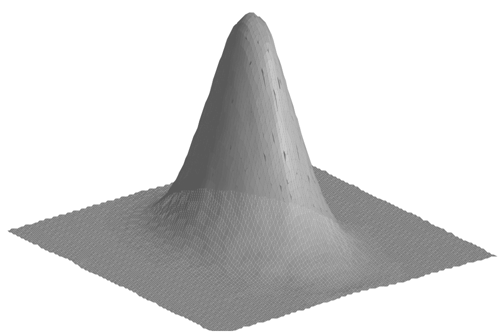




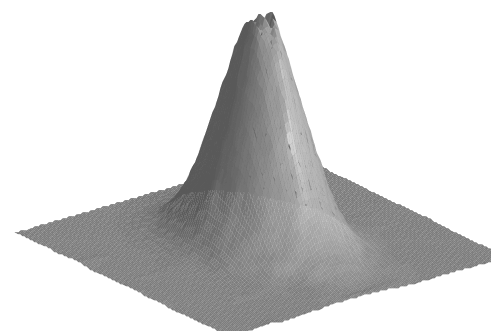




RMSE = 0.052

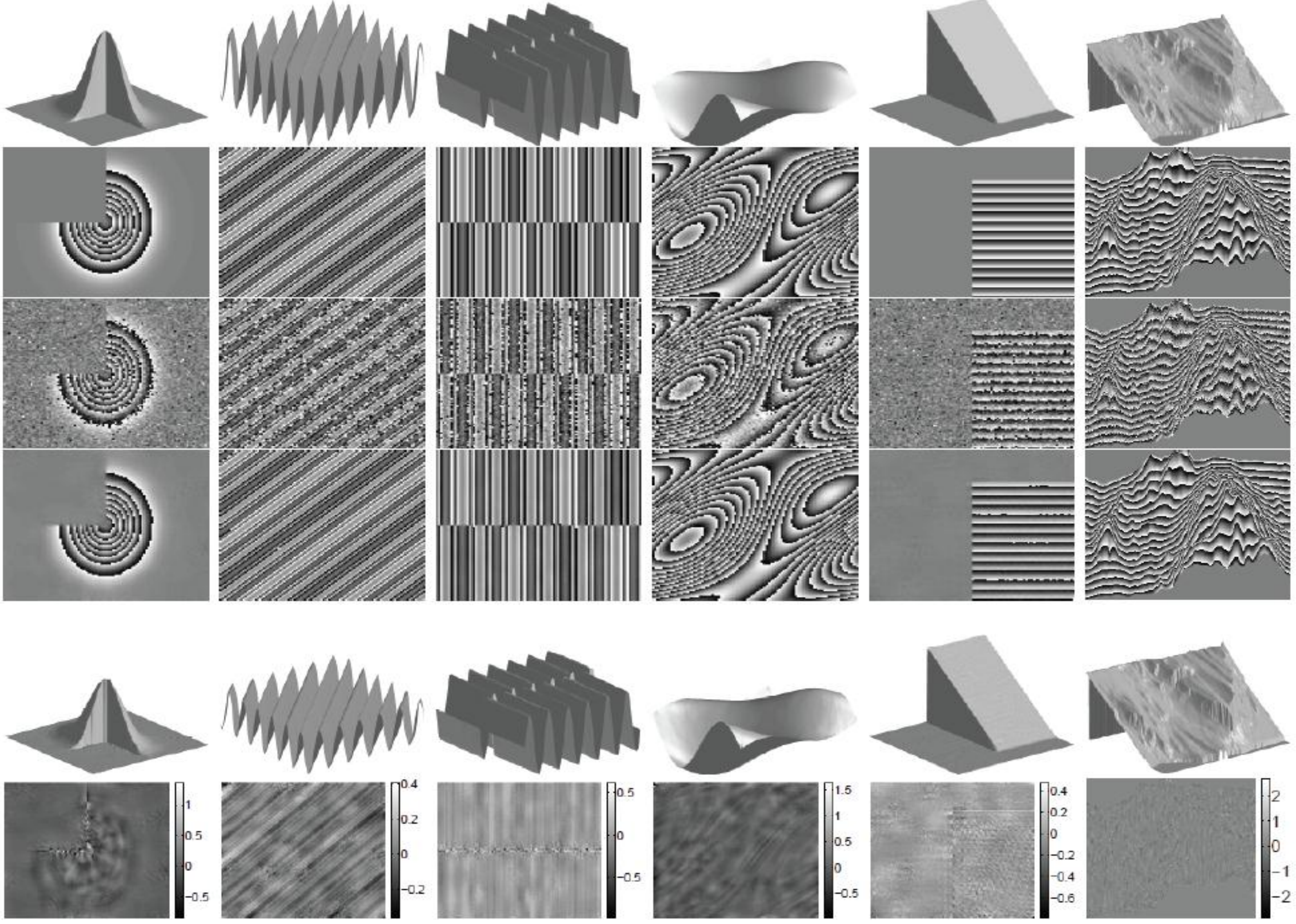


RMSE = 0.108



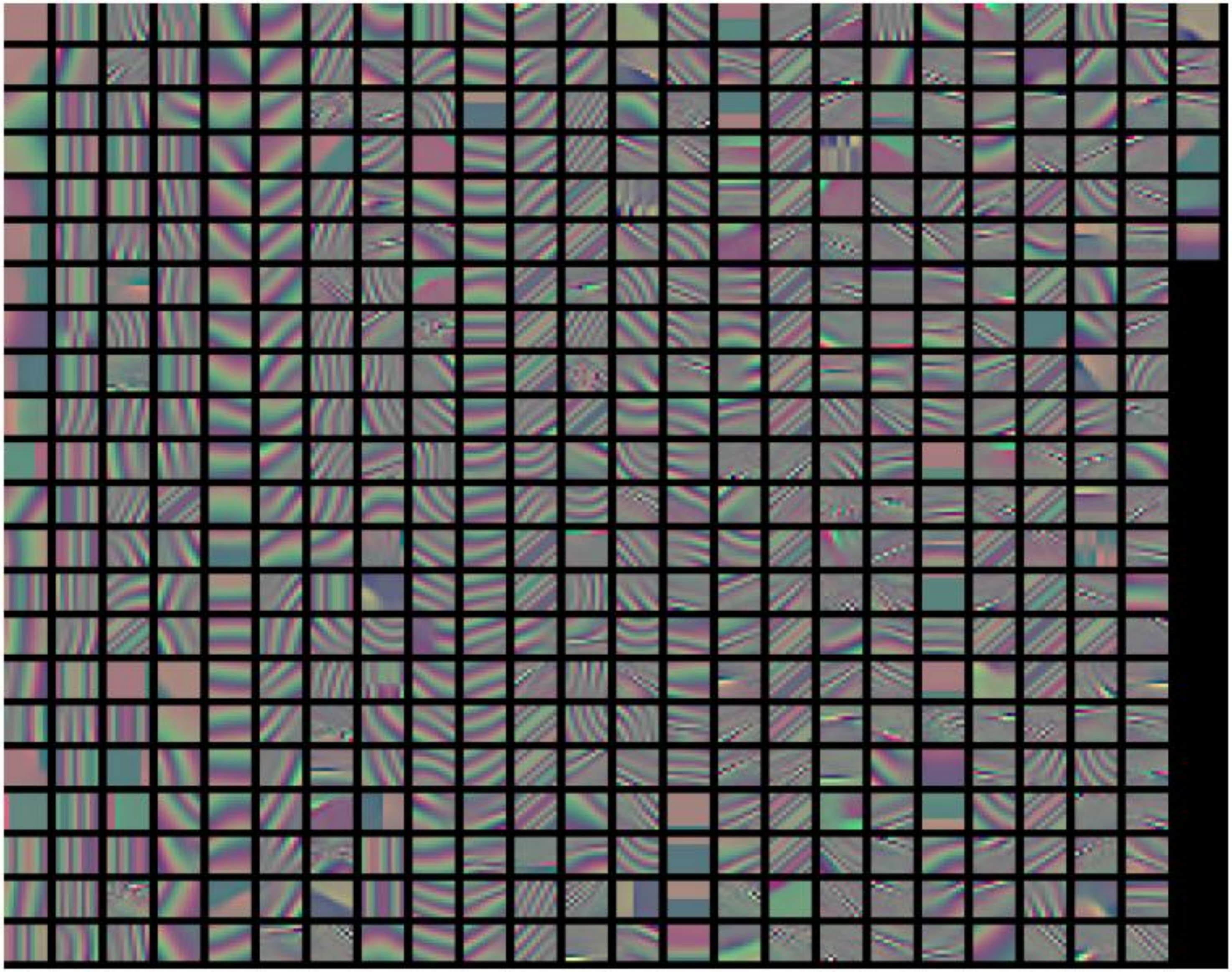
RMSE = 0.174

Results



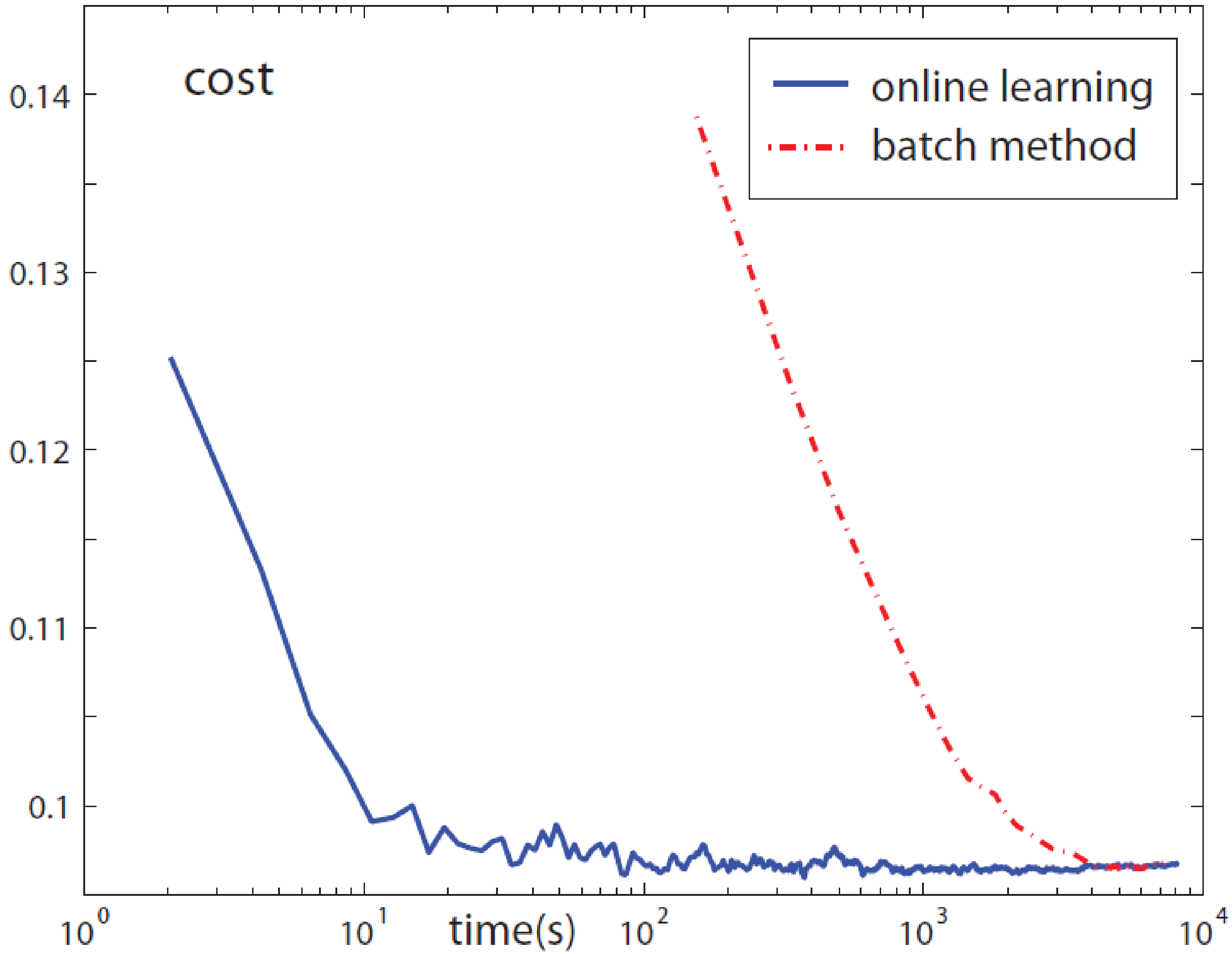
Dictionary Learned from 6 Images (shown before)

$\sqrt{m} = 12,\ k = 512$



DL: Online (ODL) Versus Batch (APM)

$\sqrt{m} = 12,\ k = 512$



Comparisons with Competitors

Surf.	σ	PSNR (dB) Sp(ld)	PSNR (dB) Sp(pd)	PSNR (dB) W	PSNR$_a$ (dB) Sp(ld)	PSNR$_a$ (dB) Sp(pd)	PSNR$_a$ (dB) W	NELP Sp(ld)	NELP Sp(pd)	NELP W	TIME (s) Sp(ld)	TIME (s) Sp(pd)	TIME (s) W
Trunc. Gauss.	0.3	42.51	**42.88**	40.29	42.51	**42.88**	40.29	**0**	**0**	**0**	69	**6**	10
	0.5	39.63	**39.95**	36.71	39.63	**39.95**	36.71	**0**	**0**	**0**	74	**4**	10
	0.7	35.69	**36.96**	34.26	35.85	**36.98**	34.37	8	**3**	10	72	**3**	10
	0.9	33.52	**36.04**	32.79	33.52	**36.23**	32.79	**0**	7	**0**	72	**3**	10
Sinu.	0.3	**48.94**	47.77	35.76	**48.94**	47.77	35.76	**0**	**0**	0	61	**2**	10
	0.5	41.91	**43.50**	31.48	41.91	**43.50**	31.48	**0**	**0**	**0**	65	**2**	10
	0.7	38.44	**41.20**	28.90	38.44	**41.20**	28.90	**0**	**0**	**0**	65	**2**	10
	0.9	36.42	**39.30**	26.36	36.42	**39.30**	26.36	**0**	**0**	**0**	63	**2**	10
Sinu. discon.	0.3	**44.45**	42.29	35.91	**44.45**	42.29	35.91	**0**	**0**	**0**	63	**6**	10
	0.5	**39.41**	38.61	31.86	**39.41**	38.61	31.86	**0**	**0**	**0**	72	**3**	10
	0.7	**37.09**	35.95	29.86	**37.09**	35.95	29.95	**0**	**0**	1	71	**2**	10
	0.9	**34.17**	34.00	27.64	**34.17**	34.00	27.71	**0**	**0**	6	66	**2**	10
Mount.	0.3	**40.66**	38.90	40.00	**40.66**	38.90	40.00	**0**	**0**	**0**	57	**10**	**10**
	0.5	**37.20**	35.66	36.55	**37.20**	35.66	36.55	**0**	**0**	**0**	60	**6**	10
	0.7	**34.35**	33.29	34.17	**34.35**	33.29	34.17	**0**	**0**	**0**	62	**5**	10
	0.9	**32.55**	31.66	32.31	**32.70**	31.79	32.31	1	1	**0**	60	**4**	10
Shear plane	0.3	**49.36**	47.01	40.67	**49.36**	47.01	40.67	0	**0**	**0**	57	23	**10**
	0.5	**42.95**	44.05	37.07	**42.95**	44.05	37.07	**0**	**0**	**0**	63	**2**	10
	0.7	38.39	**39.58**	34.13	38.39	**39.58**	34.13	**0**	**0**	**0**	68	**2**	10
	0.9	33.53	**38.72**	33.24	33.53	**38.72**	33.24	**0**	**0**	**0**	72	**2**	10
Long's Peak	0.3	35.49	**35.68**	35.40	35.51	**35.69**	35.41	**28**	**28**	**28**	515	179	**31**
	0.5	33.05	**33.19**	32.89	33.08	**33.24**	32.93	32	33	**31**	357	77	**30**
	0.7	31.32	**31.46**	31.19	31.46	**31.53**	31.28	**26**	48	32	326	42	**30**
	0.9	29.97	**30.17**	29.90	30.09	**30.26**	29.99	34	**32**	35	308	**27**	30

Non-Gaussian and Non-additive Noise

quite often, we are just given interferferograms $\boldsymbol{\eta} = \mathcal{W}(\boldsymbol{\phi} + \boldsymbol{\varepsilon})$

$$\Rightarrow \quad e^{j\boldsymbol{\eta}} = e^{j(\boldsymbol{\phi}+\boldsymbol{\varepsilon})}$$

if $|\boldsymbol{\varepsilon}| \ll \pi \quad \Rightarrow \quad e^{j\boldsymbol{\eta}} \simeq e^{j\boldsymbol{\phi}} + \boxed{je^{j\boldsymbol{\phi}}\boldsymbol{\varepsilon}}$ $\rightarrow$ additive noise

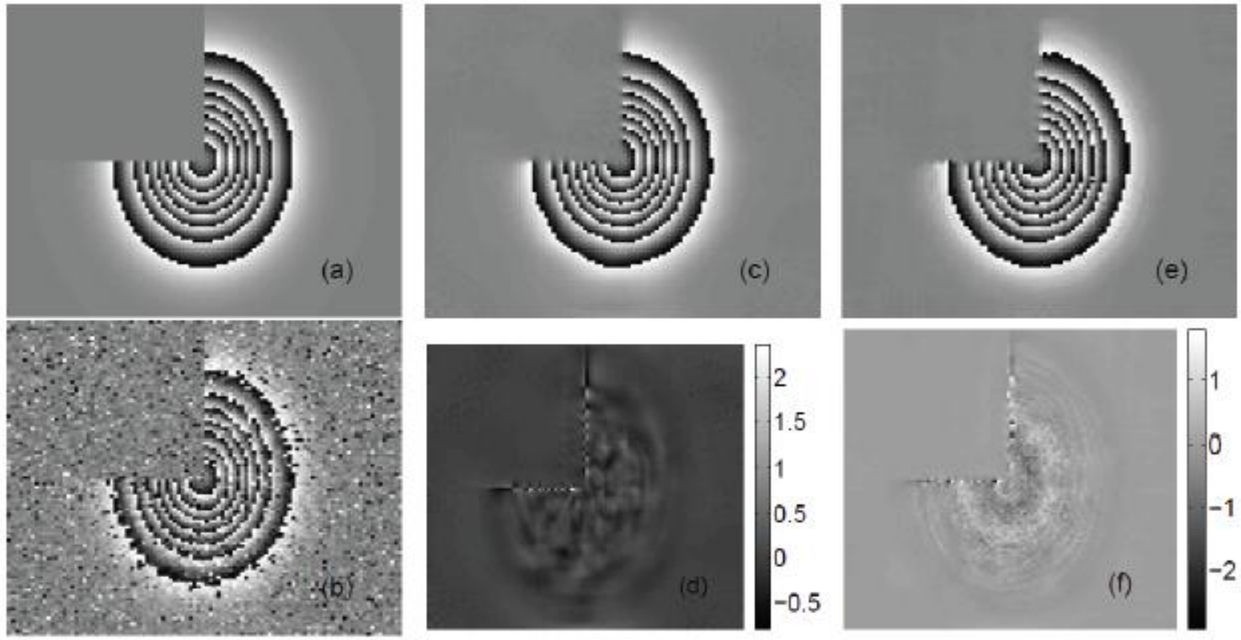
Define $\mathbf{z} = \dfrac{e^{j\boldsymbol{\eta}}}{\sigma_{\boldsymbol{\varepsilon}}}$

Example: Intereferometric SAR (InSAR)

$$\sigma_{\varepsilon}^{2}(\gamma) = \frac{\pi^{3}}{3} - \pi\arcsin(\gamma) + \arcsin^{2}(\gamma) + \frac{\mathrm{Li}_{2}(\gamma)}{2}$$

Intereferometric coherence

Application to InSAR



Indicator	Algorithm	coherence			
		0.95	0.9	0.85	0.8
PSNR (dB)	NL-InSAR	31.70	31.69	31.68	28.97
	SpInPHASE	**38.00**	**35.57**	**33.48**	**31.74**
$PSNR_a$ (dB)	NL-InSAR	32.09	32.49	32.82	31.52
	SpInPHASE	**38.00**	**36.05**	**33.67**	**32.99**
NELP	NL-InSAR	23	45	24	202
	SpInPHASE	**0**	**24**	**12**	**95**
TIME (s)	NL-InSAR	**34.07**	**33.43**	**32.60**	**32.07**
	SpInPHASE	368.63	362.56	380.97	365.11

Concluding Remarks

- New interferometric phase estimation via sparse coding in the complex domain

- Exploits the self similarity of the complex valued images $e^{j\phi}$

- SpInPHASE in short
  - Online dictionary learning
  - ADMM to solve the ODL BPDN step
  - Sparse coding via OMP
  - PUMA to solve the phase unwraping step

- State-of-the-art results, namely regarding the preservation of discontinuities coded in the interferometric phase

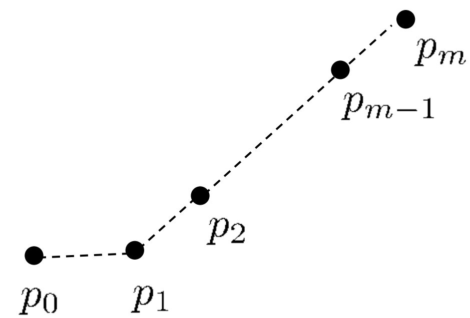
- Future research directions
  - Multisource phase estimation
  - Analysis dictionaries

Phase Unwrapping Path Following Methods

Assume that $|\phi_p - \phi_q| < \pi \qquad \phi_p = \eta_p + 2k_p\pi \qquad \phi_q = \eta_q + 2k_q\pi$

Then $\phi_p - \phi_q = \mathcal{W}(\phi_p - \phi_q) = \mathcal{W}(\eta_p - \eta_q)$

PU $\Rightarrow$ summing $\mathcal{W}(\eta_p - \eta_q)$ over walks



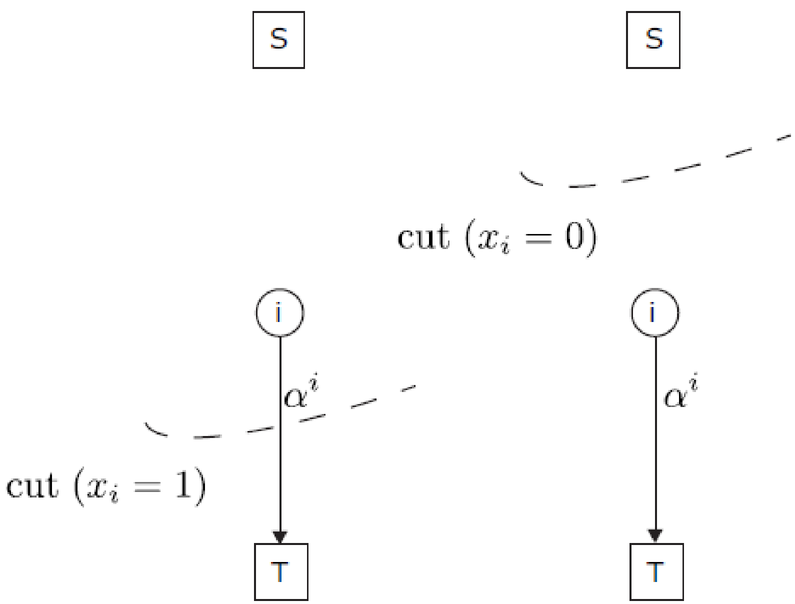
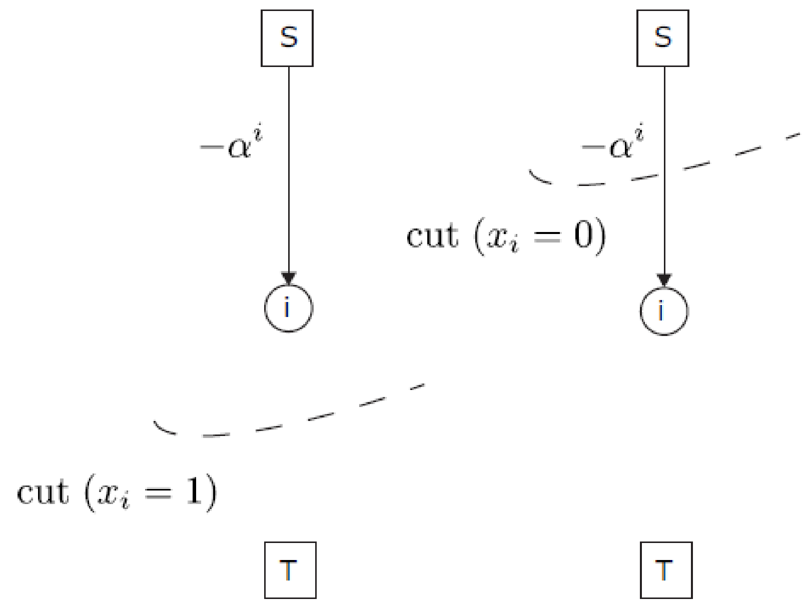
$$\phi_{p_m} = \phi_{p_0} + \sum_{i=1}^{m} W(\eta_{p_i} - \eta_{p_{i-1}})$$

Why isn't PU a trivial problem?

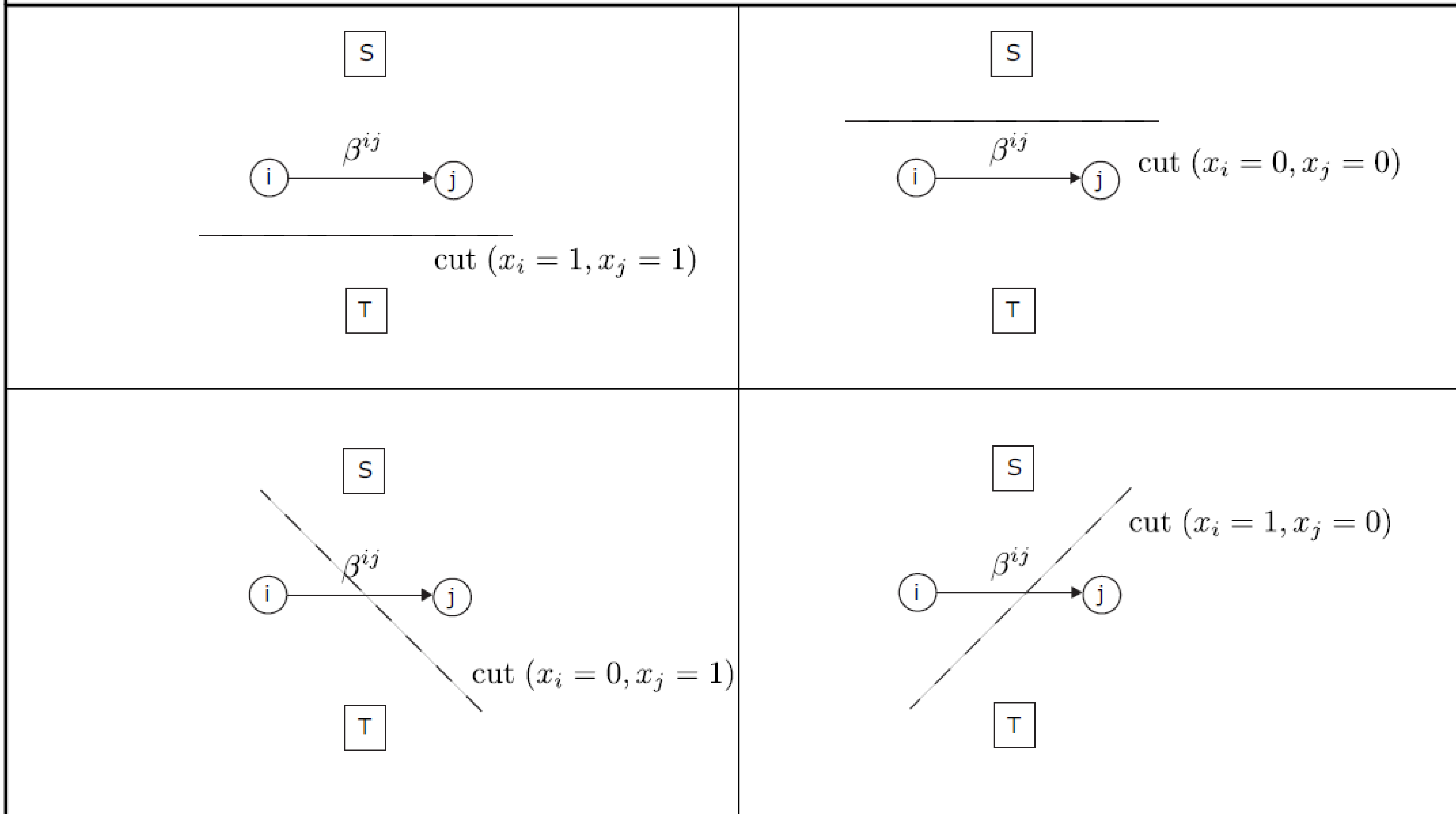
Discontinuities
High phase rate
Noise

$$|\phi_p - \phi_q| \geq \pi$$

Unary terms: $\alpha^i x_i$

$\alpha^i > 0$	$\alpha^i < 0$
	
$\text{cut}\ (x^i) = \alpha^i x_i$	$\text{cut}(x^i) = \alpha^i x_i - \alpha^i$

Pairwise terms: $\beta^{ij}(x_i - x_j)x_i, \ \beta^{ij} \geq 0$



$$\mathrm{cut}(x_i, x_j) = \beta^{ij}(x_i - x_j)x_i$$