Strong and Weak Forms of Abstract Argument Defense

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Attackers with different strength

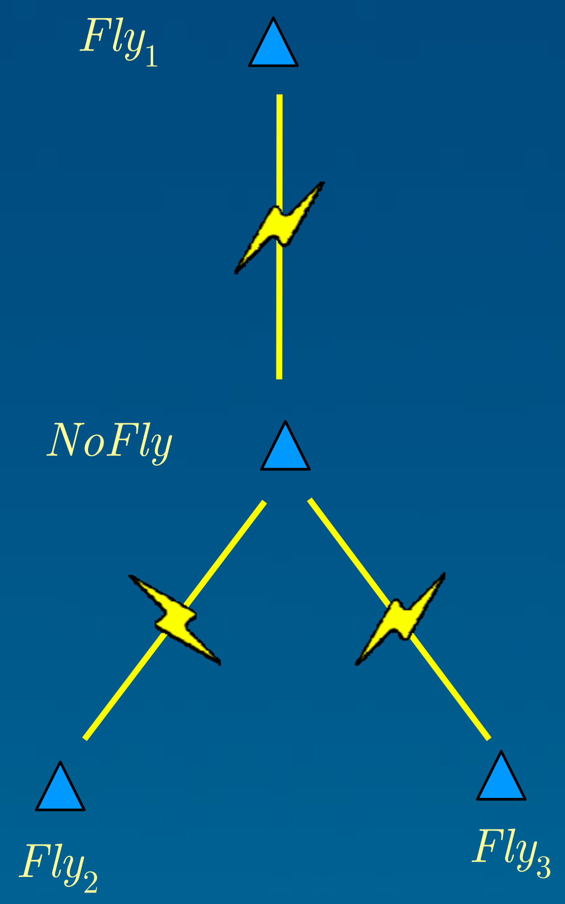
Fly_1: Fly *Oceanic Airlines* because it has the cheapest tickets.

stronger

$NoFly$: Do not fly *Oceanic Airlines* because the accident rate is high and the onboard service is not good.

Fly_2: Fly *Oceanic Airlines* because the accident rate is normal and the onboard service is improving.

Fly_3: Fly *Oceanic Airlines* because you can see some islands in the flight route.



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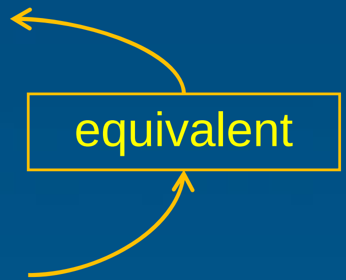

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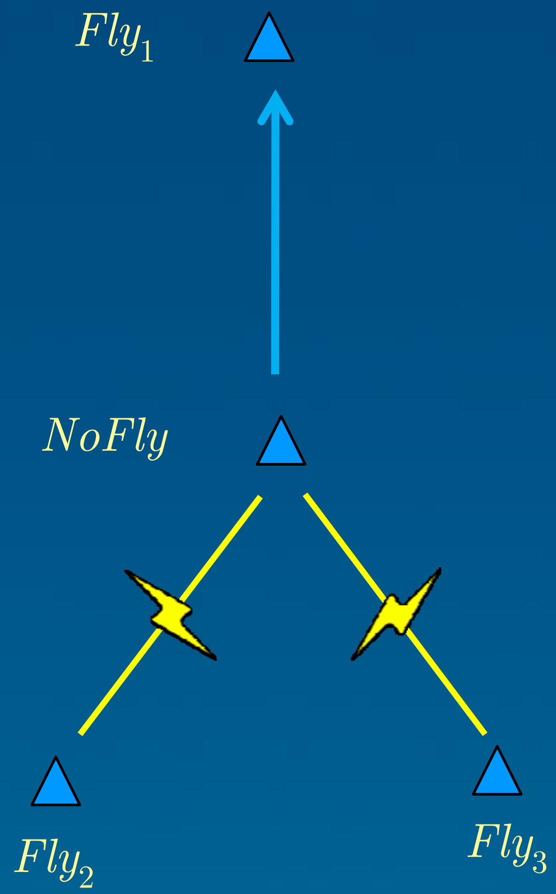

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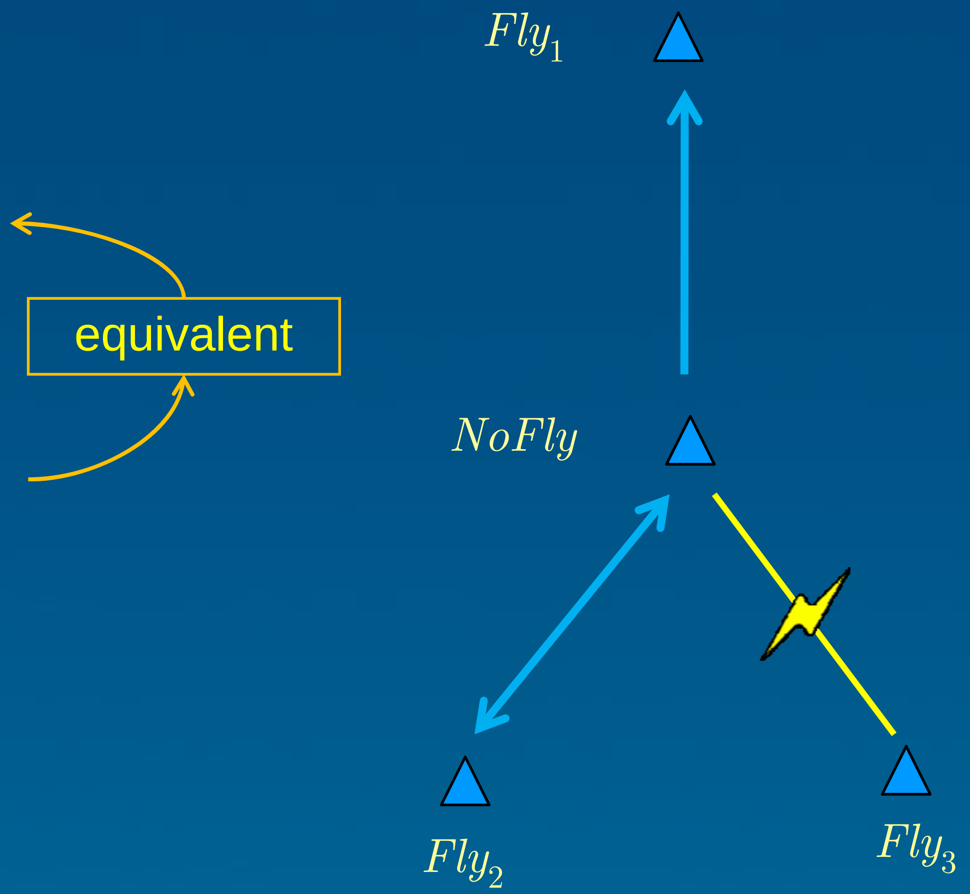

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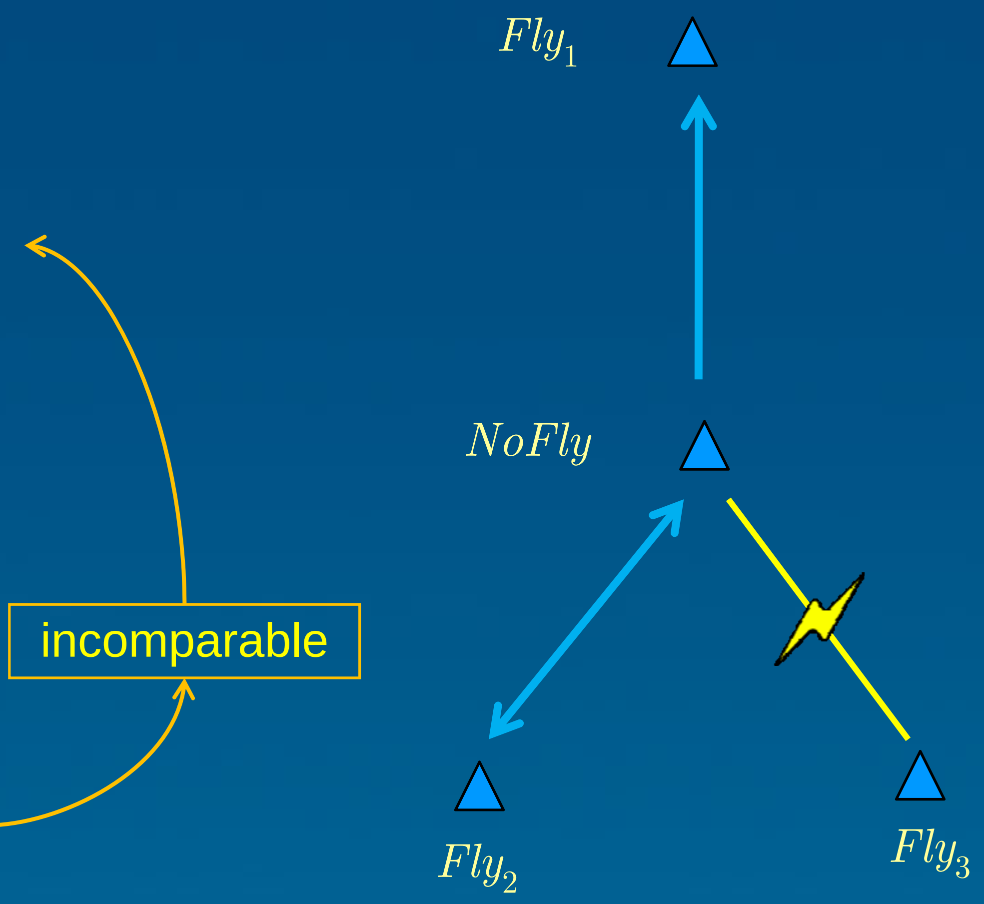

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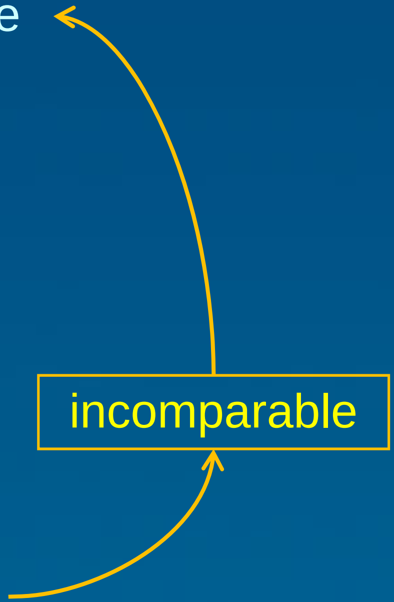
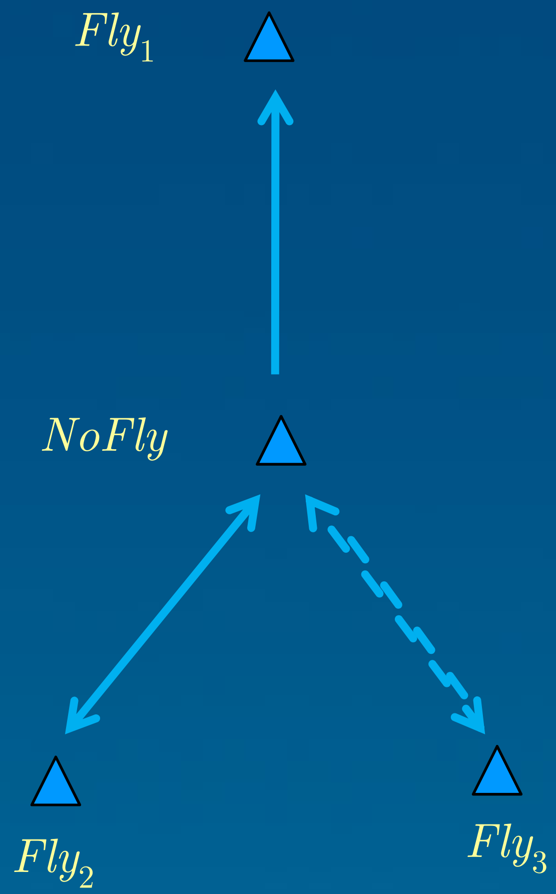
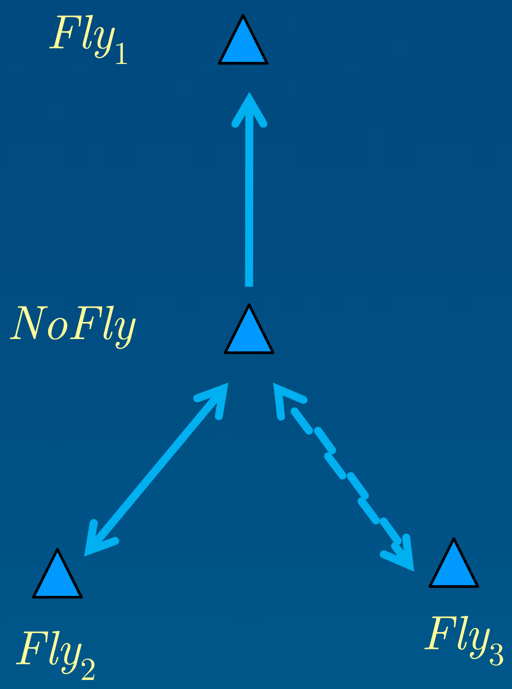




The strenght of defenses



Argument Fly_1 is defended by Fly_2 and Fly_3

The defense provided by Fly_2 may be considered stronger than the defense provided by Fly_3

This is the main motivation of this work. We explore this idea in the context of extended argumentation frameworks…

Extended Abstract Frameworks - definition

An extended abstract argumentation framework (EAF) is a triplet

$$< AR, \sqsubseteq, C, R >$$

- AR: Finite set of arguments
- $\sqsubseteq$: Subargument relation
- C: Binary conflict relation between arguments $C \subseteq AR \times AR$
- R: Preference relation for conflictive arguments

- Arguments are abstract entities: $\mathcal{A}$,$\mathcal{B}$,$\mathcal{C}$,….
- The symbol $\sqsubseteq$ denotes *subargument relation*: $\mathcal{A} \sqsubseteq \mathcal{B}$ means $\mathcal{A}$ is a *subargument* of $\mathcal{B}$.
- In this work, the subargument relation is not relevant for the topic addressed and therefore we will assume $\sqsubseteq = \emptyset$.

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- The conflict relation states the incompatibility of acceptance between arguments.
- It is a symmetric relation.
- It is devoided of any form of argument evaluation.

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- C: Binary conflict relation between arguments $C \subseteq AR \times AR$
- R: Preference relation for conflictive arguments

- The preference relation is used to compare conflicting arguments.
- It captures *any* form of evaluation. For example, an argument may be preferred to other if,
  - it exposes more specific information., or
  - it was constructed recently, or
  - it is proposed by a more reliable agent, or
  - it is undercutting the other argument, or
  - it simply satisfies a particular bias.

Extended Abstract Frameworks - defeaters

Relation R represents an order on the set of arguments.

- If $\mathcal{A}R\mathcal{B}$ but not $\mathcal{B}R\mathcal{A}$ then $\mathcal{A}$ is preferred to $\mathcal{B}$, denoted $\mathcal{A}\succ\mathcal{B}$
- If $\mathcal{A}R\mathcal{B}$ and $\mathcal{B}R\mathcal{A}$ then $\mathcal{A}$ and $\mathcal{B}$ are arguments with equal relative preference, denoted $\mathcal{A}\equiv\mathcal{B}$
- If neither $\mathcal{A}R\mathcal{B}$ and $\mathcal{B}R\mathcal{A}$ then $\mathcal{A}$ and $\mathcal{B}$ are incomparable arguments, denoted $\mathcal{A}\bowtie\mathcal{B}$

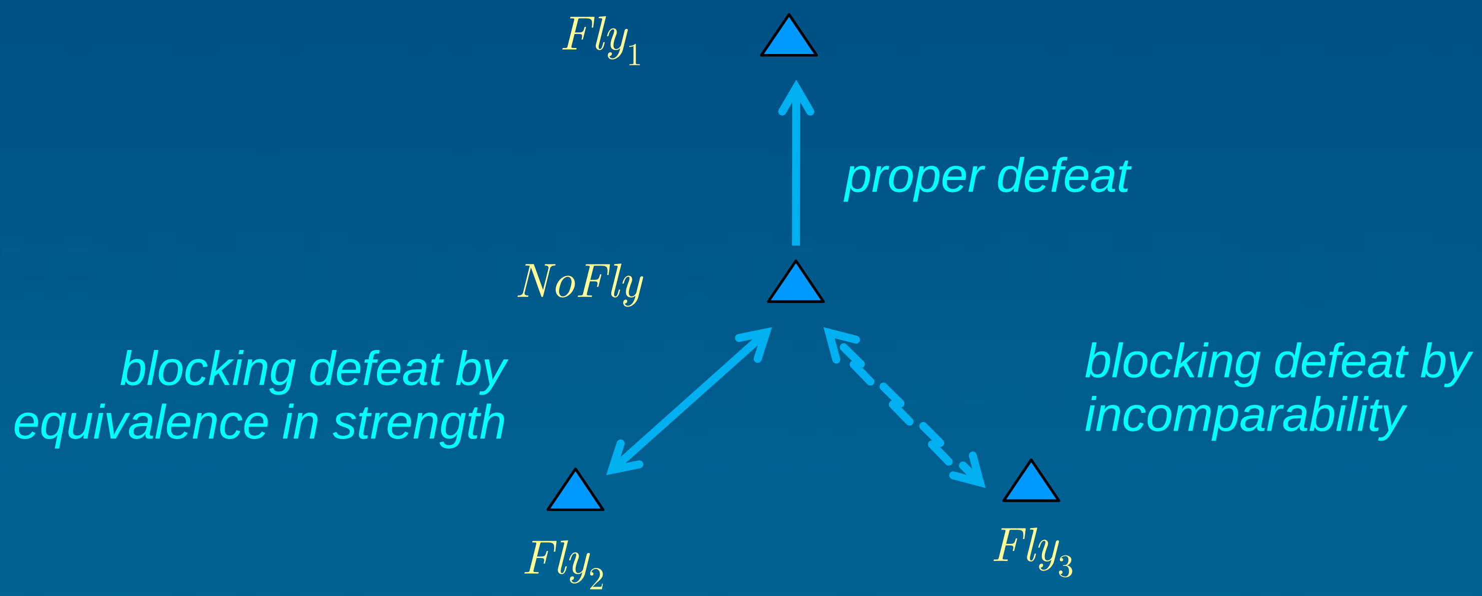
Let $\mathcal{A}$ and $\mathcal{B}$ be two arguments in AR such that $\{\mathcal{A},\mathcal{B}\} \in$ C.

- If $\mathcal{A}$ *is preferred to* $\mathcal{B}$, then it is said that $\mathcal{A}$ is a proper defeater of $\mathcal{B}$.
- If $\mathcal{A}$ and $\mathcal{B}$ have the *equal relative strength*, or are incomparable then no proper defeat relation can be established, and it is said that $\mathcal{A}$ and $\mathcal{B}$ are blocking defeaters.

EAF - example

$< AR, \sqsubseteq, C, R >$

$$
\begin{cases}
AR = \{\ Fly_1,\ NoFly,\ Fly_2,\ Fly_3\ \} \\
\sqsubseteq\ = \varnothing \\
C = \{\{Fly_1, NoFly\},\ \{Fly_2, NoFly\},\ \{Fly_3, NoFly\}\ \} \\
NoFly \succ Fly_1\ ,\quad Fly_2 \equiv NoFly,\quad Fly_3 \bowtie NoFly
\end{cases}
$$



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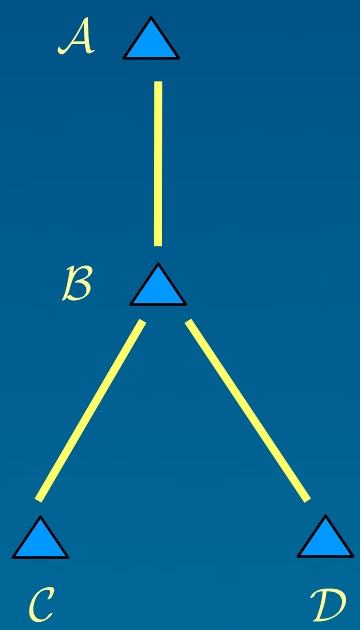
Comparing individual defenses

Let AF=<Args, $\sqsubseteq$,C,R> be an EAF.
Let $\mathcal{A}$ and $\mathcal{B}$ be two arguments in Args.
The function pref: Args$\times$Args $\longrightarrow$ {0,1,2} is defined as follows

$$\text{pref}(\mathcal{A}, \mathcal{B}) = \begin{cases} 0 \text{ if } \mathcal{A} \bowtie \mathcal{B} \\ 1 \text{ if } \mathcal{A} \equiv \mathcal{B} \\ 2 \text{ if } \mathcal{A} \succ \mathcal{B} \end{cases}$$

Let AF=<Args, $\sqsubseteq$,C,R> be an EAF.
Let $\mathcal{A} \in$ Args be an argument with defeater $\mathcal{B}$, which is defeated, in turn, by arguments $\mathcal{C}$ and $\mathcal{D}$. Then

- $\mathcal{C}$ and $\mathcal{D}$ are equivalent in force defenders of $\mathcal{A}$ if
  pref($\mathcal{C}$, $\mathcal{B}$)=pref($\mathcal{D}$, $\mathcal{B}$)
- C is a stronger defender than $\mathcal{D}$ if
  pref($\mathcal{C}$, $\mathcal{B}$)>pref($\mathcal{D}$, $\mathcal{B}$)
  It is also said that $\mathcal{D}$ is a weaker defender than $\mathcal{C}$



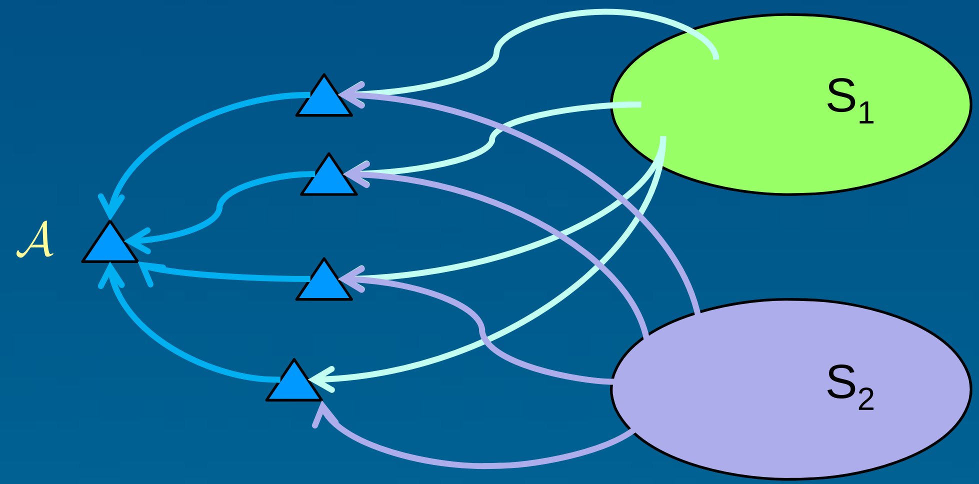
Stronger Defense

Let Φ =<Args, $\sqsubseteq$,C,R> be an EAF.

Let $\mathcal{A} \in$ Args be an argument acceptable with respect to $S_1 \subseteq$ Args.

A set of arguments $S_2 \subseteq$ Args is said to be a stronger collective defense of $\mathcal{A}$ if $\mathcal{A}$ is acceptable with respect to S_2, and

- There are no two arguments $\mathcal{X} \in S_1$ and $\mathcal{Y} \in S_2$ such that $\mathcal{X}$ constitutes a stronger defense than $\mathcal{Y}$.
- For at least one defender $\mathcal{X} \in S_1$ of $\mathcal{A}$, there exists an argument $\mathcal{Y} \in S_2 - S_1$ which constitutes a stronger defense of $\mathcal{A}$.



No argument in S_1 is a stronger defender than an argument in S_2 .
S_2 provides at least one stronger defender than an argument in S_1.

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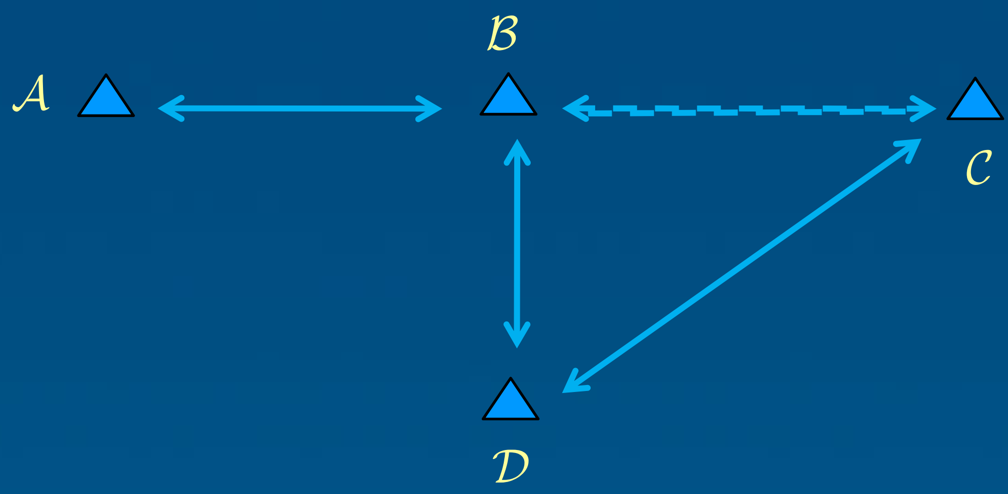
Stronger defenses

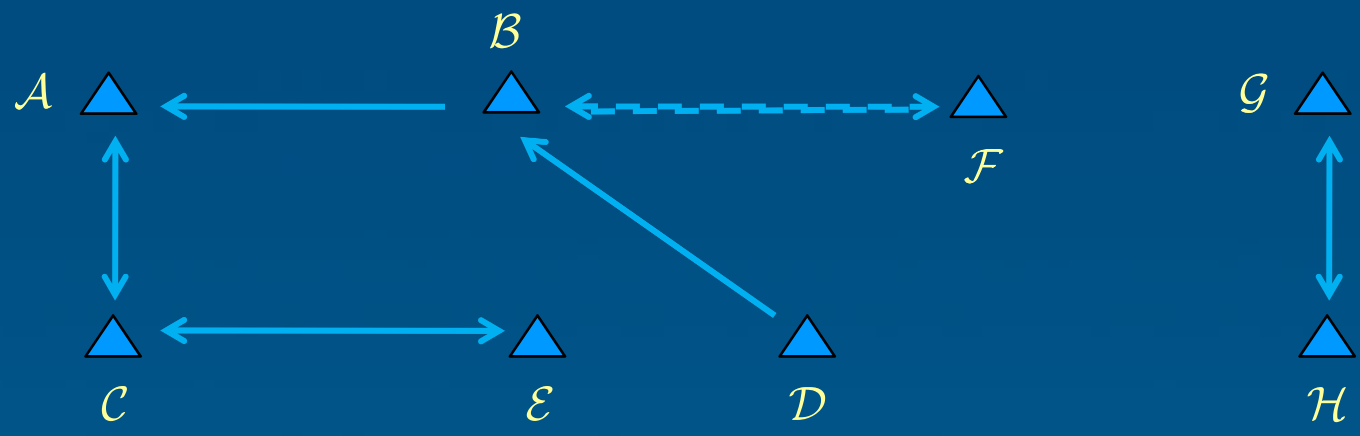


$\{\mathcal{D}\}$ is a stronger collective defense for $\mathcal{A}$ than $\{\mathcal{C}\}$

Admissible Sets:
- empty set
- every singleton set
- $\{\mathcal{A}, \mathcal{D}\}$ → Admissible set with stronger inner defenses
- $\{\mathcal{A}, \mathcal{C}\}$

Top-admissible sets

An admissible set of arguments S is said to be top-admissible if, for any argument $\mathcal{A} \in$ S, no other admissible set S' includes a stronger defense of $\mathcal{A}$ than S.



$S_1 = \{ \mathcal{A}, \mathcal{D}, \mathcal{E} \}$ is top-admissible
$S_2 = \{ \mathcal{A}, \mathcal{E}, \mathcal{F} \}$ is not top-admissible, as S_1 provides a stronger defense for $\mathcal{A}$
$S_3 = \{ \mathcal{G} \}$ is top-admissible and so is $\{ \mathcal{H} \}$

Adjusted defense

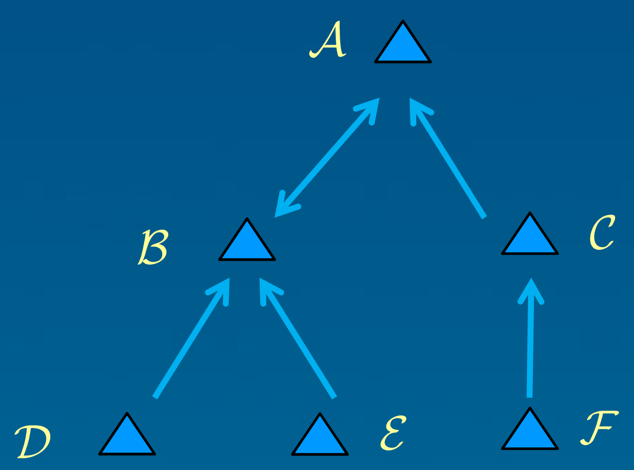
Let Φ =<Args, $\sqsubseteq$,C,R> be an EAF.

Let S be a set of arguments.

Let $\mathcal{A}\in$Args be an argument acceptable with respect to S.

An argument $\mathcal{B}\in$S is a superfluous defender of $\mathcal{A}$ in S, if $\mathcal{A}$ is acceptable with respect to S $- \{\mathcal{B}\}$.

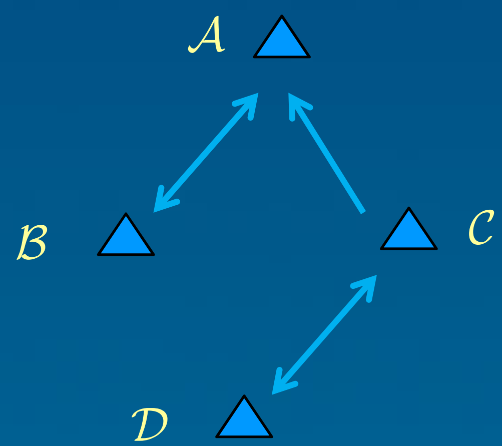
If no argument in S is a superfluous-defender, then the set S is said to be an adjusted defense of $\mathcal{A}$.



Adjusted defenses of $\mathcal{A}$

{ $\mathcal{D}$, $\mathcal{F}$ }

{ $\mathcal{E}$, $\mathcal{F}$ }



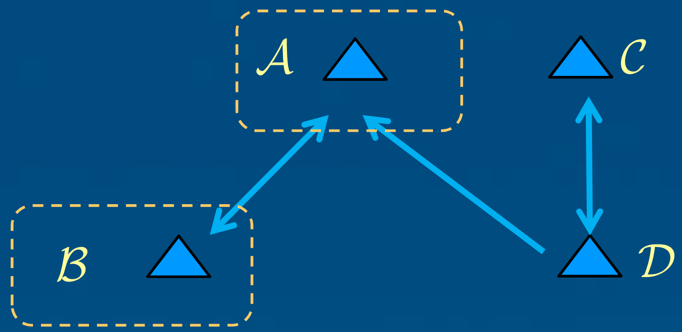
Adjusted defenses of $\mathcal{A}$

{ $\mathcal{A}$, $\mathcal{D}$ }

Adjusted defenses of $\mathcal{D}$

{ $\mathcal{D}$ }

Dead-end defeaters and weak acceptability



An argument $\mathcal{B}$ is said to be a dead-end defeater of an argument $\mathcal{A}$ if the only defense of $\mathcal{A}$ against $\mathcal{B}$ is $\mathcal{A}$ itself.

An argument $\mathcal{A}$ is said to be a self-defender if for every adjusted defense S of $\mathcal{A}$, then $\mathcal{A} \in$ S.

In that case, $\mathcal{A}$ is said to be weak-acceptable with respect to S if

1. $|S| > 1$, and
2. $\mathcal{A}$ is defended by S $- \{\mathcal{A}\}$ against every non dead-end defeater.

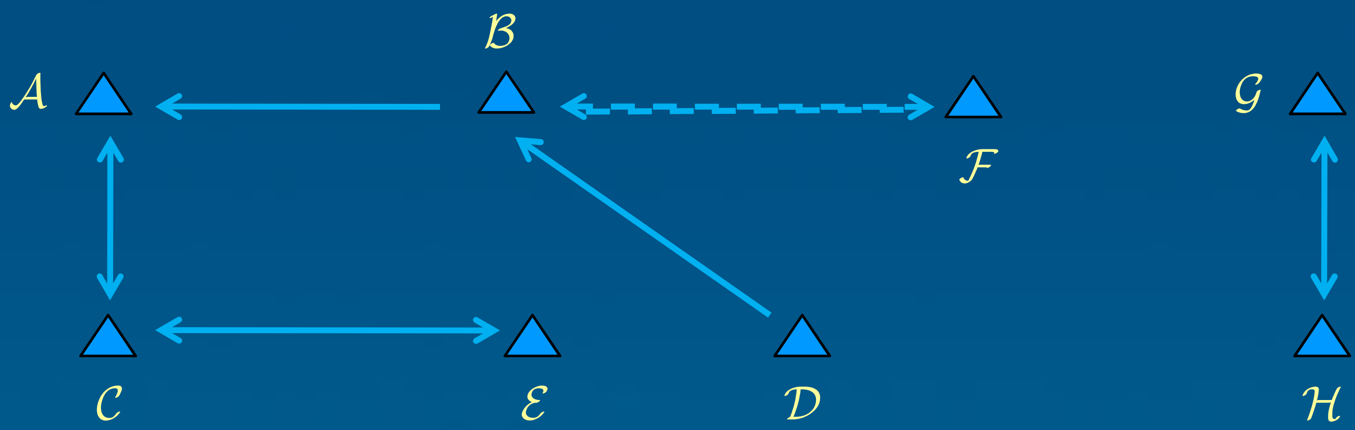
A self-defender argument $\mathcal{A}$ is weak acceptable wrt S if its self-defense is necessary only on dead-end defeaters. For the rest, it is defended by S.

Forceful defense

Let Φ =<Args, $\sqsubseteq$,C,R> be an EAF.

Let S be a set of arguments and let $\mathcal{A} \in$ Args.

The set S is a forceful-defense of $\mathcal{A}$ if S is an adjusted defense of $\mathcal{A}$ and no other adjusted defense is a stronger defense than S.



Adjusted defenses of $\mathcal{A}$

- $\{\mathcal{A}, \mathcal{D}\}$
- $\{\mathcal{A}, \mathcal{F}\}$
- $\{\mathcal{E}, \mathcal{D}\}$
- $\{\mathcal{E}, \mathcal{F}\}$

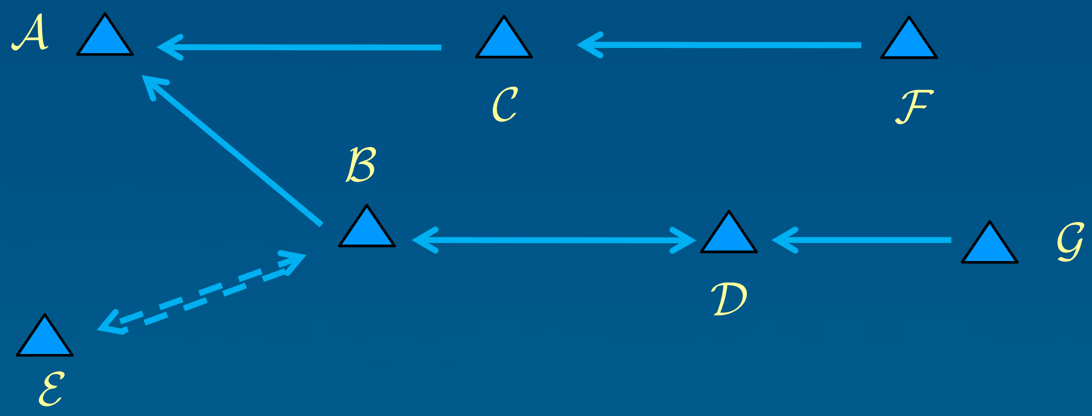
Forceful defenses of $\mathcal{A}$: $\{\mathcal{A}, \mathcal{D}\}$, $\{\mathcal{E}, \mathcal{D}\}$

Arguments forcefully included in a set

Let Φ =<Args, $\sqsubseteq$,C,R> be an EAF.

Let S be a set of arguments and let $\mathcal{A}\in$Args.

The argument $\mathcal{A}$ is said to be forcefully-included in S if at least one forceful-defense of $\mathcal{A}$ is included in S.



S = { $\mathcal{A}$, $\mathcal{E}$, $\mathcal{F}$, $\mathcal{G}$ } is admissible
$\mathcal{A}$ is not forcefully included in S as { $\mathcal{E}$, $\mathcal{F}$ } is its adjusted defense, not the strongest one.

The forceful defense of $\mathcal{A}$ is { $\mathcal{D}$, $\mathcal{F}$ }, but $\mathcal{D}$ cannot be included in an admissible set.
This may be considered a sign of weakness of $\mathcal{A}$, as it is not forcefully included in any admissible set.

Forceful inclusion and extensions

Let Φ =<Args, $\sqsubseteq$,C,R> be an EAF.

Let S be an admissible set of arguments.

If every argument in S is forcefully included in S, then S is top-admissible

Dung's grounded extension is a strong admissible set, in the sense it only includes forcefully included arguments…

Let Φ =<Args, $\sqsubseteq$,C,R> be an EAF.

Let GE be the grounded extension of AF.

Then every argument in GE is forcefully included in GE.

General idea:

A blocked argument is not acceptable wrt to the empty set.

Thus, $F_{AF}(\emptyset)$ includes arguments with no defeaters. They may be defeaters of other arguments, of course, by proper defeat.

Thus, $F_{AF}(F_{AF}(\emptyset))$ includes arguments defended by proper defeat, which is the strongest form of defense in EAF.

It can be proved by induction that every argument is defended by at least one proper defeater of a defeater.

Weak grounded extension

It is possible to adopt a more credulous approach, expanding the acceptance by considering self-defender arguments…

Let Φ=<Args, $\sqsubseteq$,C,R> be an EAF.

The extended characteristic function of Φ is defined as

$F_{\Phi}^{\cup}(S) = F_{\Phi}(S) \cup \{ \mathcal{A} : \mathcal{A}$ is weak acceptable with respect to $S \cup \mathcal{A} \}$

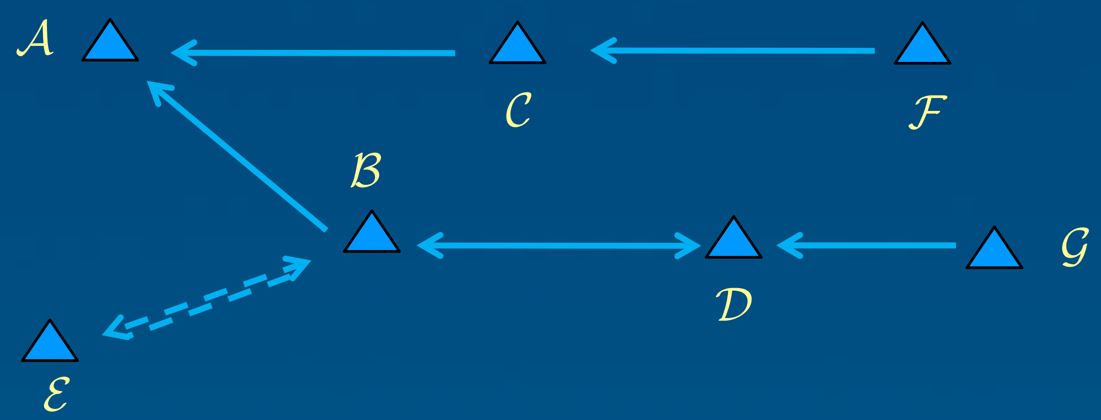
If S is an admissible set, then $F_{\Phi}^{\cup}(S)$ is admissible

This leads to the definition of an extension using the extended characteristic function:

Let Φ=<Args, $\sqsubseteq$,C,R> be an EAF.

The **weak grounded extension** of Φ is the least fixpoint of $F_{\Phi}^{\cup}(S)$

Weak grounded extension



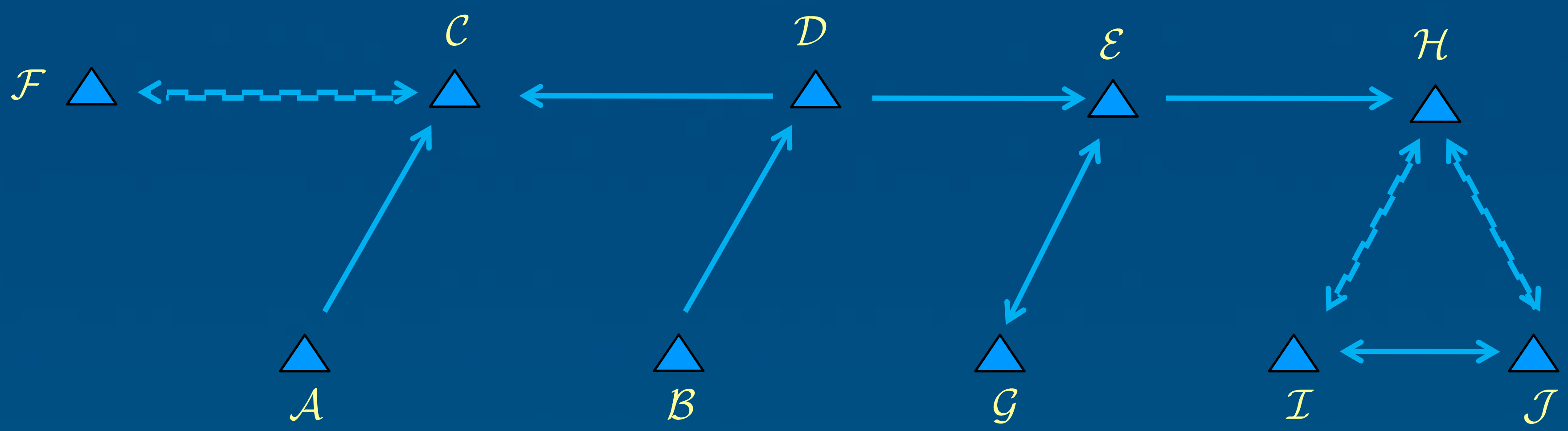
Arguments $\mathcal{G}$ and $\mathcal{F}$ are acceptable with respect to $\emptyset$

Argument $\mathcal{B}$ is
not acceptable with respect to $\{\mathcal{G}\}$
weak acceptable with respect to $\{\mathcal{G},\mathcal{B}\}$

Grounded Extension
$\{ \mathcal{F} , \mathcal{G} \}$

Weak Grounded Extension
$\{ \mathcal{F} , \mathcal{G} , \mathcal{B} \}$

Weak grounded extension



Arguments $\mathcal{A}$ and $\mathcal{B}$ are acceptable with respect to $\emptyset$

Argument $\mathcal{F}$ is acceptable with respect to $\{\mathcal{A}\}$

Argument $\mathcal{E}$ is defended from $\mathcal{D}$ by $\mathcal{B}$, and from $\mathcal{G}$ by itself, thus $\mathcal{E}$ is weak acceptable with respect to $\{\mathcal{B}, \mathcal{E}\}$

Arguments $\mathcal{I}$ and $\mathcal{J}$ are dead-end defeaters of each other

Grounded Extension
$\{\mathcal{A}, \mathcal{B}, \mathcal{F}\}$

Weak Grounded Extension
$\{\mathcal{A}, \mathcal{B}, \mathcal{F}, \mathcal{E}\}$

Conclusions

We analyzed the strength of defenses in extended argumentation frameworks, where the quality of a defense depends on the type of defeaters used (proper/blocking).

Proper-defeat defense is considered stronger than defense through blocking defeaters.

Blocking-defeat defense provided by equivalent in force arguments is considered stronger than the defense provided by incomparable arguments.

We defined forceful inclusion of arguments. An argument is forcefully included in an admissible set when the best defense is captured by that set.

We defined top-admissible sets in EAF. This admissible set includes, for every argument in the set, the strongest defense as it is possible to conform admissibility.

We introduced the notion of weak acceptability, allowing the definition of the weak grounded extension, where arguments can partially defend themselves.

We have shown that classic grounded extension GE in EAF is top-admissible and every argument in GE is forcefully included.