

Argumentation Using Temporal Knowledge

Nicholas Mann and Anthony Hunter

Arguments

Let Δ be set of formulas in classical propositional logic

An **argument** is a pair $\langle \Phi, \alpha \rangle$ such that

1. $\Phi \not\vdash \perp$
2. $\Phi \vdash \alpha$
3. Φ is a minimal subset of Δ satisfying 2

Φ is the **support** and α is the **consequent** of the argument

Using Time in Arguments

House prices have fallen throughout 2008.

- $\forall i. In2008(i) \rightarrow \neg Holds(house_price_rise, i)$

During periods of falling house prices, many mortgages are foreclosed.

- $\forall i. \neg Holds(house_price_rise, i) \rightarrow Holds(many_mortgages_foreclosed, i)$

In March, house prices bucked the trend, rising slightly, but April prices were down again. Despite the house price fall, however, few mortgages were foreclosed in April

- $Holds(house_price_rise, march) \wedge \neg Holds(house_price_rise, april) \wedge \neg Holds(many_mortgages_foreclosed, april)$

What to do?

- Use first order logic?
- Adopt an existing temporal logic?

What we want:

- Simple; as close to propositional logic as possible
- Able to include variable intervals or timepoints.
- Able to handle relationships between intervals or timepoints.

Talk overview

- \mathcal{T} : A calculus which is suitable for the expression of temporal data.
- Define a consequence relationship $\vdash_{\mathcal{T}}$ in terms of \vdash .
- Arguments in \mathcal{T} ; Support and database ground arguments.
- Show how these behave, and compare and contrast the two types of argument.

Timeline and Intervals

A **timeline** is a tuple $\langle T, \preceq \rangle$, where T is a finite set of **timepoints**, and \preceq is a total linear ordering over these timepoints. The symbol \prec is used such that $t_1 \prec t_2$ iff $t_1 \preceq t_2 \wedge t_1 \neq t_2$.

An **interval** (on a timeline $\langle T, \preceq \rangle$) is a pair (t_1, t_2) where t_1 and t_2 are both timepoints in T , and $t_1 \prec t_2$.

- Given a subset $T \subset \mathbb{Z}$, then $\langle T, \leq \rangle$ is suitable for a timeline.

Interval relations

Relationships between intervals are defined as follows:

(a, b) **precedes** (c, d) iff $b < c$

(a, b) **meets** (c, d) iff $b = c$

(a, b) **overlaps** (c, d) iff $a < c < b < d$

(a, b) **during** (c, d) iff $c < a < b < d$

(a, b) **starts** (c, d) iff $a = c$ and $b < d$

(a, b) **ends** (c, d) iff $c < a$ and $b = d$

(a, b) **equals** (c, d) iff $a = c$ and $b = d$

- $(0,2)$ **meets** $(2,8)$
- $(3,6)$ **starts** $(3, 12)$
- $(5,7)$ **during** $(3, 12)$

Syntax of \mathcal{T}

The syntax for the letters of \mathcal{T} is defined as below.

<i>symbol</i>	$::=$	$\alpha \mid \beta \mid \gamma \mid \delta \dots$
<i>timepoint</i>	$::=$	$0 \mid 1 \mid 2 \mid 3 \dots$
<i>varinterval</i>	$::=$	$i \mid j \mid k \dots$
<i>interval</i>	$::=$	$(\textit{timepoint}, \textit{timepoint})$ $\mid \textit{varinterval}$
<i>relation</i>	$::=$	precedes \mid meets \mid overlaps \mid during \mid starts \mid ends \mid equals
<i>letter</i>	$::=$	$\textit{Holds}(\textit{symbol}, \textit{interval})$ $\mid (\textit{interval} \textit{relation} \textit{interval})$

Examples of formulae in \mathcal{T}

$Holds(\alpha, i)$

$((Holds(\alpha, i) \wedge (i \text{ **meets** } j)) \rightarrow Holds(\beta, j))$

$(((0, 1) \text{ **starts** } (4, 7)) \wedge Holds(\alpha, (3, 4)))$

$\neg((0, 1) \text{ **starts** } (4, 7))$

$((0, 1) \text{ **starts** } (0, 7))$

Grounding

A **grounding** is pair $[\alpha, \Theta]$, where α is a formula in \mathcal{T} and Θ is a set of substitutions.

$$[Holds(\alpha, i), \{i/(2, 3)\}]$$

$$[Holds(\alpha, i), \emptyset]$$

$$[Holds(\alpha, i) \wedge (i \text{ meets } j) \rightarrow Holds(\beta, j), \{i/(1, 2), j/(2, 5)\}]$$

If Θ contains a substitution for all variables in α , then we have a **complete grounding**, otherwise it is a **partial grounding**.

A **zero grounding** is when the set is empty.

Grounding Functions

The **complete grounding function** $G_C(\Phi)$ results in the set containing all complete groundings $[\phi, \Theta]$ where $\phi \in \Phi$.

$G_C(\text{Holds}(\alpha, i))$ with timepoints (0,1,2) is

$\{[\text{Holds}(\alpha, i), \{i/(0, 1)\}], [\text{Holds}(\alpha, i), \{i/(0, 2)\}], [\text{Holds}(\alpha, i), \{i/(1, 2)\}]\}$

$G_C(\text{Holds}(\alpha, (2, 3)))$ is $\{[\text{Holds}(\alpha, (2, 3)), \emptyset]\}$

$G_C([\text{Holds}(\alpha, i) \wedge \text{Holds}(\beta, j), \{i/(0, 2)\}])$ contains

$[\text{Holds}(\alpha, i) \wedge \text{Holds}(\beta, j), \{i/(0, 2), j/(0, 1)\}],$

$[\text{Holds}(\alpha, i) \wedge \text{Holds}(\beta, j), \{i/(0, 2), j/(0, 2)\}]$ etc

The **zero grounding function** $G_0(\Phi)$ results in the set of all zero groundings of each formula in Φ .

Intervals Revisited

The set I of interval relations is $I^\top \cup I^\perp$, where:

- I^\top is the set of all formulae $((a, b) \text{ **relation** } (c, d))$ where the conditions on a, b, c and d for that relation are met.

$((1, 2) \text{ **meets** } (2, 3)) \in I^\top$

- I^\perp is the set of all formulae $\neg((a, b) \text{ **relation** } (c, d))$ the conditions on a, b, c and d are for that relation not met.

$\neg((6, 7) \text{ **starts** } (1, 3)) \in I^\perp$

The $\vdash_{\mathcal{T}}$ relation

Let Φ be a set of formulae in \mathcal{T} and α a formula in \mathcal{T} . Then $\Phi \vdash_{\mathcal{T}} \alpha$ iff

$$G_C(\Phi) \cup I \vdash \bigwedge G_C(\alpha)$$

where I is the interval lookup set

$Holds(\alpha, i) \vdash_{\mathcal{T}} Holds(\alpha, (0, 1))$

◦ $\alpha_{(0,1)}$ is present in $G_C(Holds(\alpha, i))$

$Holds(\alpha, i) \vdash_{\mathcal{T}} Holds(\alpha, i)$

◦ Every $\alpha_{(a,b)}$ is present in $G_C(Holds(\alpha, i))$

$\vdash_{\mathcal{T}} (0, 1) \mathbf{meets} (1, 2)$

◦ $(0, 1) \mathbf{meets} (1, 2) \in I$

Example of an Invalid Argument

Given the following database

$$(1) \neg \text{Holds}(\text{house_price_rise}, i) \rightarrow \\ \text{Holds}(\text{many_mortgages_forclosed}, i)$$

$$(2) \neg \text{Holds}(\text{house_price_rise}, (0, 1)) \wedge \\ \neg \text{Holds}(\text{house_price_rise}, (9, 10)) \wedge \\ \neg \text{Holds}(\text{many_mortgages_forclosed}, (9, 10))$$

$\langle \{(1), (2)\}, \text{Holds}(\text{many_mortgages_forclosed}, (0, 1)) \rangle$ is not an argument.

- (1) and (2) are inconsistent in time period (9,10)

Database Ground Arguments

A **database ground argument** is a pair $\langle \Phi, \alpha \rangle$, assuming a database Δ , such that:

- (1) $\Phi \not\vdash_{\mathcal{T}} \perp$
- (2) $\Phi \vdash_{\mathcal{T}} \alpha$
- (3) Φ is a minimal subset of $G_C(\Delta)$ satisfying (1) and (2)

With the database:

- (1) $Holds(high_sales, i) \wedge (j \text{ during } i) \rightarrow Holds(high_sales, j)$
- (2) $Holds(high_sales, (0, 8)) \wedge \neg Holds(high_sales, (6, 7))$

We have a database ground argument:

$$\langle \{(2), (1)_{i=(0,8), j=(3,4)}\}, Holds(high_sales, (3, 4)) \rangle$$

Support Ground Arguments

A **support ground argument** is a pair $\langle \Phi, \alpha \rangle$, assuming a database Δ , such that:

- (1) $\Phi \not\vdash_{\mathcal{T}} \perp$
- (2) $\Phi \vdash_{\mathcal{T}} \alpha$
- (3) Φ is a minimal subset of $G_0(\Delta)$ satisfying (1) and (2)

With the database:

- (1) $Holds(high_sales, i) \wedge (j \text{ during } i) \rightarrow Holds(high_sales, j)$
- (3) $Holds(high_sales, (0, 5))$

We have a support ground argument:

$\langle \{(3), (1)\}, Holds(high_sales, (3, 4)) \rangle$

Examples

With the database:

(1) $Holds(high_sales, i) \wedge (j \text{ during } i) \rightarrow Holds(high_sales, j)$

(2) $Holds(high_sales, (0, 8)) \wedge \neg Holds(high_sales, (6, 7))$

(3) $Holds(high_sales, (0, 5))$

We have three arguments for $Holds(high_sales, (3, 4))$

- S1: $\langle \{(3), (1)\}, Holds(high_sales, (3, 4)) \rangle$
- D1: $\langle \{(3), (1)_{i=(0,5), j=(3,4)}\}, Holds(high_sales, (3, 4)) \rangle$
- D2: $\langle \{(2), (1)_{i=(0,8), j=(3,4)}\}, Holds(high_sales, (3, 4)) \rangle$

Relationships

Theorem

Given a finite database Δ , for every support ground argument $\langle \Phi, \alpha \rangle$, there is at least one database ground argument $\langle \Psi, \alpha \rangle$, such that $\Psi \subseteq G_C(\Phi)$

- We have already shown the reverse does not hold; there are database ground arguments which have no comparable support ground argument.

Undercuts

Returning to the database

(1) $Holds(high_sales, i) \wedge (j \text{ during } i) \rightarrow Holds(high_sales, j)$

(2) $Holds(high_sales, (0, 8)) \wedge \neg Holds(high_sales, (6, 7))$

(3) $Holds(high_sales, (0, 5))$

And the arguments:

- S1: $\langle \{(3), (1)\}, Holds(high_sales, (3, 4)) \rangle$
- D1: $\langle \{(3), (1)_{i=(0,5), j=(3,4)}\}, Holds(high_sales, (3, 4)) \rangle$

$\langle \{(2)\}, \diamond \rangle$ is a undercut for S1, but not for D1.

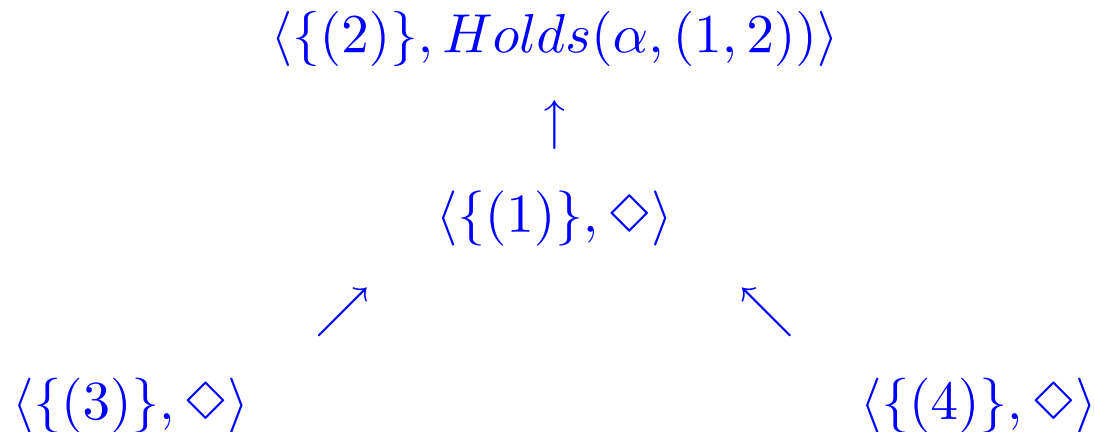
Argument Trees

Considering the following database:

- (1) $Holds(\beta, i) \rightarrow Holds(\alpha, i)$
- (2) $Holds(\alpha, (1, 2)) \wedge Holds(\beta, (2, 3)) \wedge \neg Holds(\alpha, (2, 3))$
- (3) $Holds(\beta, (1, 2)) \wedge Holds(\beta, (3, 4)) \wedge \neg Holds(\alpha, (3, 4))$
- (4) $Holds(\beta, (5, 6)) \wedge \neg Holds(\alpha, (5, 6))$

We wish to argue for $Holds(\alpha, (1, 2))$.

We have the following support ground argument tree:



Example Trees, continued

We have the following pair of database ground argument trees,

- One related to the support ground argument tree

$$\langle \{(2)\}, Holds(\alpha, (1, 2)) \rangle$$

↑

$$\langle \{(1)_{i=(2,3)}\}, \diamond \rangle$$

- One which does not have a corresponding support ground argument tree.

$$\langle \{(3), (1)_{i=(1,2)}\}, Holds(\alpha, (1, 2)) \rangle$$

↑

$$\langle \{(1)_{i=(3,4)}\}, \diamond \rangle$$

Problem trees

- (1) $Holds(\alpha, i)$
- (2) $Holds(\beta, i)$
- (3) $Holds(\beta, i) \rightarrow \neg Holds(\alpha, i)$

The support ground argument tree for $Holds(\alpha, i)$.

$$\begin{array}{c} \langle \{(1)\}, Holds(\alpha, i) \rangle \\ \uparrow \\ \langle \{(2), (3)\}, \diamond \rangle \end{array}$$

The database ground argument tree for $Holds(\alpha, i)$.

$$\begin{array}{c} \langle G_C((1)), Holds(\alpha, i) \rangle \\ \nearrow \qquad \qquad \qquad \uparrow \\ \langle \{(2)_{i=(1,2)}, (3)_{i=(1,2)}\}, \diamond \rangle \quad \langle \{(2)_{i=(2,3)}, (3)_{i=(2,3)}\}, \diamond \rangle \quad \dots \end{array}$$

- Leaves are $\langle \{(2)_{i=(a,b)}, (3)_{i=(a,b)}\}, \diamond \rangle$ where $a \in T, b \in T, a \prec b$

Conclusions

- There is a need for temporal knowledge in argumentation
- A calculus \mathcal{T} can be defined in terms of propositional logic
- There is the possibility of support ground and database ground arguments
- Each type of argument has its advantages and disadvantages.