A Systematic Classification of Argumentation Frameworks where Semantics Agree

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Many semantics

A variety of argumentation semantics have been proposed in the context of Dung's framework

Traditional semantics (Dung 95):

- Grounded
- Complete
- Stable
- Preferred

Recent semantics include:

- CF2 (Baroni, Giacomin, Guida 05)
- Semi-stable (Caminada 06)
- Ideal (Dung, Mancarella, Toni 06)

France or Italy?

W la différence !

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- alternative intuitions and viewpoints

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- alternative intuitions and viewpoints
- suitability for different application domains

France or Italy?

W la différence !

- alternative intuitions and viewpoints
- suitability for different application domains
- fruitful debates opening new research directions

France or Italy?

Don’t look to what divides you

Don’t look to what divides you ...

France or Italy?

Don’t look to what divides you, look to what unites you! (Pope John XXIII)

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- common behavior in some (many, most?) cases

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- common behavior in some (many, most?) cases
- shared principles behind (partial) differences

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- common behavior in some (many, most?) cases
- shared principles behind (partial) differences
- basic reference behavior

France or Italy?

Don’t look to what divides you, look to what unites you (Pope John XXIII)

- common behavior in some (many, most?) cases
- shared principles behind (partial) differences
- basic reference behavior
- (ir)relevance of choosing a specific semantics

Aim of the work

Creating a systematic basis for the study of agreement between argumentation semantics by

providing a classification of argumentation frameworks with respect to the issue of semantics agreement

considering the seven semantics mentioned before:

- ***GR***ounded
- ***CO***mplete
- ***ST***able
- ***PR***eferred
- ***CF2***
- ***S***emi***ST***able
- ***ID***eal

Presentation plan

Basic concepts and review of existing results

Presentation plan

Basic concepts and review of existing results

Description of the analysis carried out

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Basic concepts and review of existing results

Description of the analysis carried out

Unique-status agreement

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Multiple-status agreement

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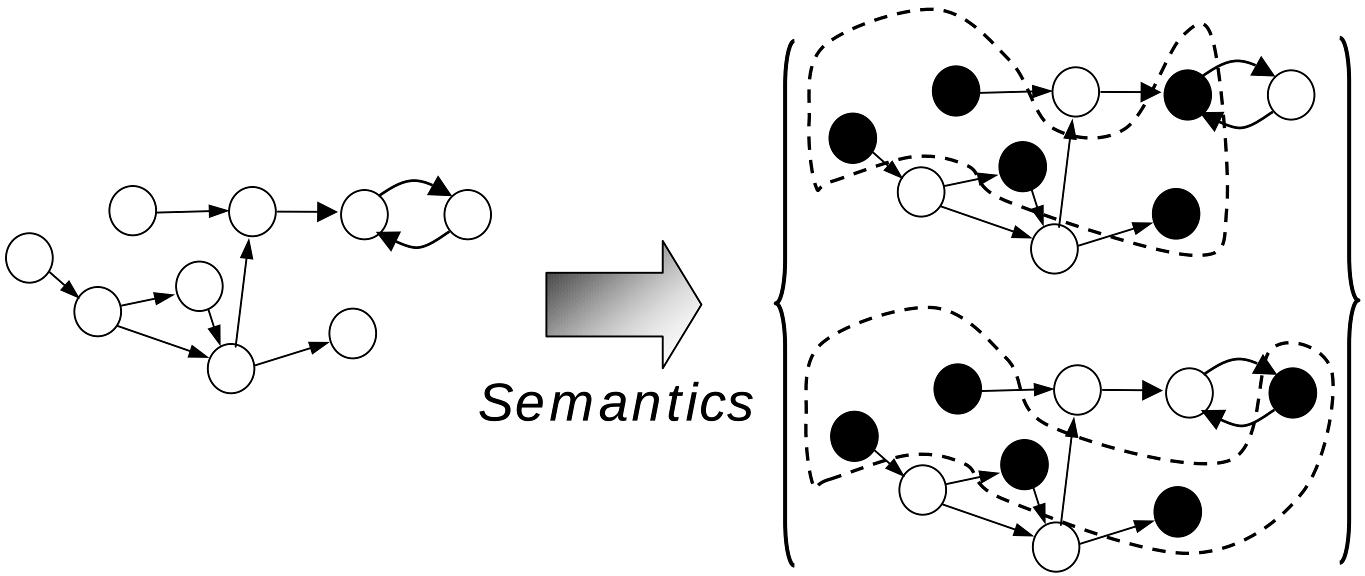
Conclusions

Dung's Argumentation Framework

$AF = \langle A, \rightarrow \rangle$

- A: arguments
- $\rightarrow$: attack relation



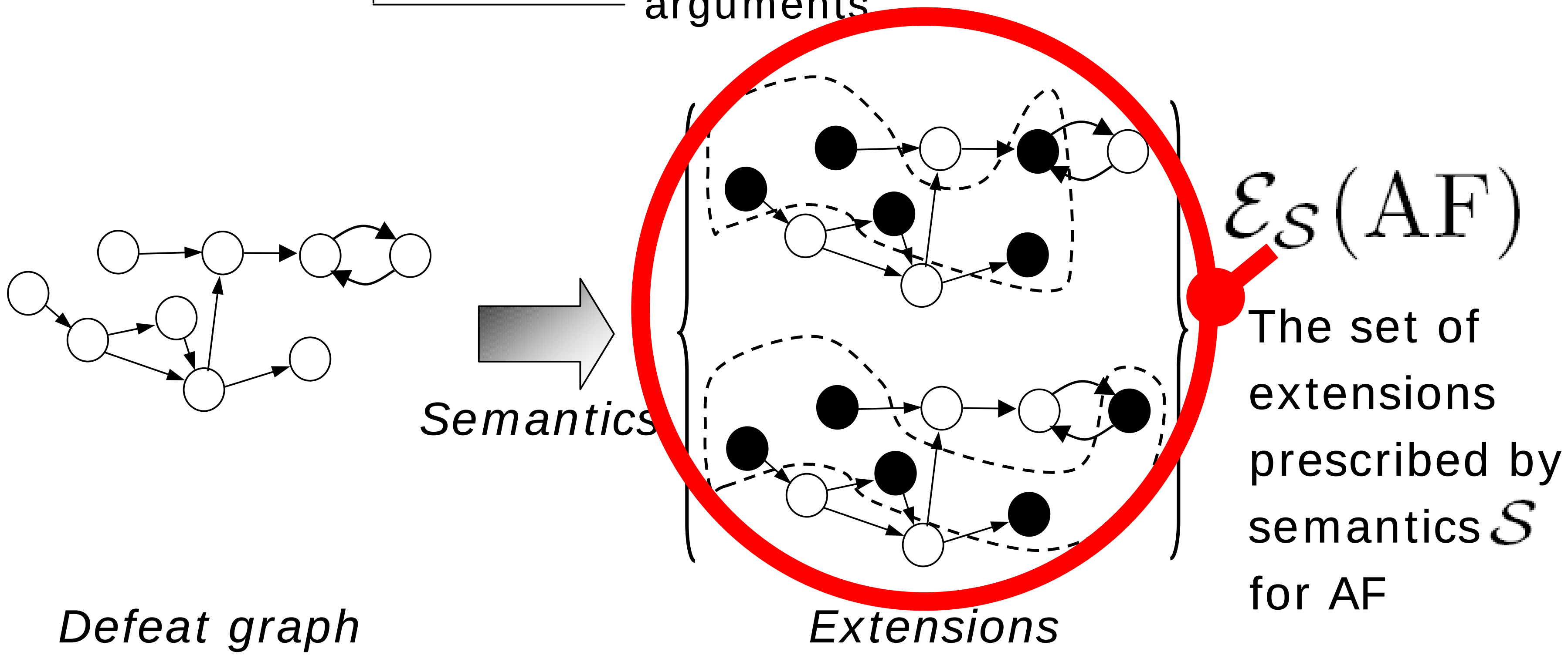
Defeat graph *Extensions*

Dung's Argumentation Framework

AF = <*A*, $\rightarrow$>

attack relation

arguments



Definition of semantics agreement

Two semantics $\mathcal{S}_1$ and $\mathcal{S}_2$ are in agreement about an argumentation framework AF if $\mathcal{E}_{\mathcal{S}_1}(\mathrm{AF}) = \mathcal{E}_{\mathcal{S}_2}(\mathrm{AF})$

We require that both $\mathcal{S}_1$ and $\mathcal{S}_2$ admit extensions for AF, namely $\mathcal{E}_{\mathcal{S}_1}(\mathrm{AF}) \neq \emptyset$ and $\mathcal{E}_{\mathcal{S}_2}(\mathrm{AF}) \neq \emptyset$

In other words, for a semantics $\mathcal{S}$, agreement is evaluated only about argumentation frameworks where $\mathcal{S}$ is defined

Only ***ST*** may be undefined for some AF

Existing results on agreement

Grounded, stable and preferred semantics are in agreement about AF if AF is well-founded [Dung, AIJ 95]
(when AF is finite, well-founded is equivalent to acyclic)

Stable and preferred semantics are in agreement about AF if AF is limited controversial [Dung, AIJ 95]
(when AF is finite, limited controversial is equivalent to free of odd-length cycles)

Stable, preferred and naïve semantics are in agreement about AF if AF is symmetric (all attacks are mutual) [Coste-Marquis et al., ECSQARU 05]

Agreement in three topological classes of argumentation frameworks related with the notion of strongly connected-components investigated in [Baroni&Giacomin, ARGNMR 07]

Existing results on agreement vs. ...

Relatively limited attention to the issue of agreement after Dung’s paper, with some recent revival

A few agreement classes identified considering mainly **topological properties** of argumentation frameworks

... a complementary perspective

Relatively limited attention to the issue of agreement after Dung's paper, with some recent revival

A few agreement classes identified considering mainly **topological properties** of argumentation frameworks

Systematic identification of **all possible agreement classes** (given the considered set of semantics)

Based on general **set-theoretical properties of semantics extensions** rather than on topological properties of argumentation frameworks

Agreement classes: notation and basic properties

Given a set $\mathbb{S}$ of argumentation semantics, the set of argumentation frameworks where all semantics in $\mathbb{S}$ agree will be denoted as $\mathcal{AGR}(\mathbb{S})$

E.g. $\mathcal{AGR}(\{\mathcal{PR}, \mathcal{ST}, \mathcal{SST}\})$
denotes the set of argumentation frameworks where preferred, stable and semi-stable semantics agree

Clearly $\mathbb{S}_1 \subseteq \mathbb{S}_2 \Rightarrow \mathcal{AGR}(\mathbb{S}_2) \subseteq \mathcal{AGR}(\mathbb{S}_1)$
E.g.

$$\mathcal{AGR}(\{\mathcal{ID}, \mathcal{ST}, \mathcal{PR}, \mathcal{SST}\}) \subseteq \mathcal{AGR}(\{\mathcal{PR}, \mathcal{ST}, \mathcal{SST}\})$$

It may be (and it is) the case that for some different sets of semantics $\mathbb{S}_1$ and $\mathbb{S}_2$ it holds that $\mathcal{AGR}(\mathbb{S}_1) = \mathcal{AGR}(\mathbb{S}_2)$

Agreement classes: how many?

Given that we consider a set of 7 argumentation semantics,
$\Omega = \{\mathcal{GR}, \mathcal{ID}, \mathcal{CO}, \mathcal{PR}, \mathcal{ST}, CF2, \mathcal{SST}\}$
any subset $\mathbb{S}$ of Ω such that $|\mathbb{S}| \geq 2$ gives rise, in principle, to an agreement class (**120** classes in total)

We have proved that most of these 120 classes are not actually different: only **14** distinct classes exist

Agreement classes: which kind of analysis?

Agreement classes are denoted as $\Sigma_1 \ldots \Sigma_{14}$

We proceed by partial order of inclusion: if $\Sigma_i \subset \Sigma_j$ then $j > i$

For each class Σ_i three main steps have been carried out:

1. identifying which classes Σ_k, with $k<i$, are included in Σ_i
2. for each of these classes Σ_k, showing that $\Sigma_i \setminus \Sigma_k \neq \emptyset$
3. for any Σ_h with $h<i$ and $\Sigma_h \not\subset \Sigma_i$, examining $\Sigma_i \cap \Sigma_h$

For any set $\mathbb{S}$ of semantics not directly corresponding to any of $\Sigma_1 \ldots \Sigma_{14}$ it is shown that $\mathcal{AGR}(\mathbb{S})$ coincides with one of them

Agreement classes: which (known) properties we use?

Several kinds of inclusion relations between (sets of) extensions: for any argumentation framework AF

- $\mathcal{E}_{\mathcal{PR}}(\mathrm{AF}) \subseteq \mathcal{E}_{\mathcal{CO}}(\mathrm{AF})$
- $\mathcal{E}_{\mathcal{SST}}(\mathrm{AF}) \subseteq \mathcal{E}_{\mathcal{PR}}(\mathrm{AF})$
- $\mathrm{GE}(\mathrm{AF}) \in \mathcal{E}_{\mathcal{CO}}(\mathrm{AF})$
- $\forall E \in \mathcal{E}_{\mathcal{S}}(\mathrm{AF})$
  $\mathrm{GE}(\mathrm{AF}) \subseteq E$ with $\mathcal{S} \in \{\mathcal{CO}, \mathcal{PR}, \mathcal{ST}, \mathcal{SST}, CF2\}$
- $\forall E \in \mathcal{E}_{\mathcal{CO}}(\mathrm{AF})\ \exists E' \in \mathcal{E}_{\mathcal{PR}}(\mathrm{AF}) : E \subseteq E'$
- $\forall E \in \mathcal{E}_{\mathcal{PR}}(\mathrm{AF})\ \exists E' \in \mathcal{E}_{CF2}(\mathrm{AF}) : E \subseteq E'$

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- $\mathcal{E}_{\mathcal{SST}}(\mathrm{AF}) \subseteq \mathcal{E}_{\mathcal{PR}}(\mathrm{AF})$
- $\mathrm{GE}(\mathrm{AF}) \in \mathcal{E}_{\mathcal{CO}}(\mathrm{AF})$

Inclusion of the whole set of extensions

- $\forall E \in \mathcal{E}_{\mathcal{S}}(\mathrm{AF})$
  $\mathrm{GE}(\mathrm{AF}) \subseteq E$ with $\mathcal{S} \in \{\mathcal{CO}, \mathcal{PR}, \mathcal{ST}, \mathcal{SST}, CF2\}$
- $\forall E \in \mathcal{E}_{\mathcal{CO}}(\mathrm{AF})\ \exists E' \in \mathcal{E}_{\mathcal{PR}}(\mathrm{AF}) : E \subseteq E'$
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The grounded extension is included in many kinds of extensions

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- $\forall E \in \mathcal{E}_{\mathcal{S}}(\mathrm{AF})$
  $\mathrm{GE}(\mathrm{AF}) \subseteq E$ with $\mathcal{S} \in \{\mathcal{CO}, \mathcal{PR}, \mathcal{ST}, \mathcal{SST}, \mathcal{C}F2\}$
- $\forall E \in \mathcal{E}_{\mathcal{CO}}(\mathrm{AF})\ \exists E' \in \mathcal{E}_{\mathcal{PR}}(\mathrm{AF}) : E \subseteq E'$
- $\forall E \in \mathcal{E}_{\mathcal{PR}}(\mathrm{AF})\ \exists E' \in \mathcal{E}_{CF2}(\mathrm{AF}) : E \subseteq E'$

Any extension of one kind is included in an extension of another kind

Agreement classes: which kind of properties we prove?

As intermediate steps, we have proved several lemmata based on the inclusion relationships like the following

Lemma 3 *If* $|\mathcal{E}_{\mathcal{CO}}(\mathrm{AF})| = 1$, *then* $\mathcal{E}_{\mathcal{CO}}(\mathrm{AF}) = \mathcal{E}_{\mathcal{GR}}(\mathrm{AF}) = \mathcal{E}_{\mathcal{PR}}(\mathrm{AF})$

Lemma 4 *Let* $\mathcal{S} \in \{\mathcal{PR}, \mathcal{ST}, \mathcal{SST}, CF2\}$, *if* $\mathrm{GE}(\mathrm{AF}) \in \mathcal{E}_{\mathcal{S}}(\mathrm{AF})$ *then* $\mathcal{E}_{\mathcal{S}}(\mathrm{AF}) = \{\mathrm{GE}(\mathrm{AF})\}$

Lemma 6 *If* $\mathcal{E}_{\mathcal{ST}}(\mathrm{AF}) = \{\mathrm{GE}(\mathrm{AF})\}$ *then* $\mathcal{E}_{CF2}(\mathrm{AF}) = \{\mathrm{GE}(\mathrm{AF})\}$

Lemma 8 *If* $\mathcal{E}_{CF2}(\mathrm{AF}) = \mathcal{E}_{\mathcal{ID}}(\mathrm{AF}) = \{ID(\mathrm{AF})\}$ *then* $\mathcal{E}_{CF2}(\mathrm{AF}) = \mathcal{E}_{\mathcal{PR}}(\mathrm{AF})$

Lemma 10 *If* $\mathcal{E}_{CF2}(\mathrm{AF}) \subseteq \mathcal{AS}(\mathrm{AF})$ *then* $\mathcal{E}_{CF2}(\mathrm{AF}) = \mathcal{E}_{\mathcal{PR}}(\mathrm{AF})$

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Implications of cardinality

Lemma 4 *Let* $\mathcal{S} \in \{\mathcal{PR}, \mathcal{ST}, \mathcal{SST}, CF2\}$, *if* $\mathrm{GE}(\mathrm{AF}) \in \mathcal{E}_{\mathcal{S}}(\mathrm{AF})$ *then* $\mathcal{E}_{\mathcal{S}}(\mathrm{AF}) = \{\mathrm{GE}(\mathrm{AF})\}$

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Lemma 6 *If* $\mathcal{E}_{\mathcal{ST}}(\mathrm{AF}) = \{\mathrm{GE}(\mathrm{AF})\}$ *then* $\mathcal{E}_{\mathcal{CF}2}(\mathrm{AF}) = \{\mathrm{GE}(\mathrm{AF})\}$

Implications of inclusion in the set of extensions

Lemma 8 *If* $\mathcal{E}_{\mathcal{CF}2}(\mathrm{AF}) = \mathcal{E}_{\mathcal{ID}}(\mathrm{AF}) = \{$ *then* $\mathcal{E}_{\mathcal{CF}2}(\mathrm{AF}) = \mathcal{E}_{\mathcal{PR}}(\mathrm{AF})$

Lemma 10 *If* $\mathcal{E}_{\mathcal{CF}2}(\mathrm{AF}) \subseteq \mathcal{AS}(\mathrm{AF})$ *then* $\mathcal{E}_{\mathcal{CF}2}(\mathrm{AF}) = \mathcal{E}_{\mathcal{PR}}(\mathrm{AF})$

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Some agreements imply others

Lemma 6 *If* $\mathcal{E}_{\mathcal{ST}}(\mathrm{AF}) = \{\mathrm{GE}(\mathrm{AF})\}$ *then* $\mathcal{E}_{CF2}(\mathrm{AF}) = \{\mathrm{GE}(\mathrm{AF})\}$

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Implications of extension properties

Lemma 10 *If* $\mathcal{E}_{\mathcal{CF}2}(\mathrm{AF}) \subseteq \mathcal{AS}(\mathrm{AF})$ *then* $\mathcal{E}_{\mathcal{CF}2}(\mathrm{AF}) = \mathcal{E}_{\mathcal{PR}}(\mathrm{AF})$

Agreement classes: coincidence

Coincidence of agreement classes follows (almost directly) from the lemmata. Examples are:

If $\{\mathcal{GR}, \mathcal{ST}\} \subseteq \mathbb{S}$ then $\mathcal{AGR}(\mathbb{S}) = \Sigma_1$

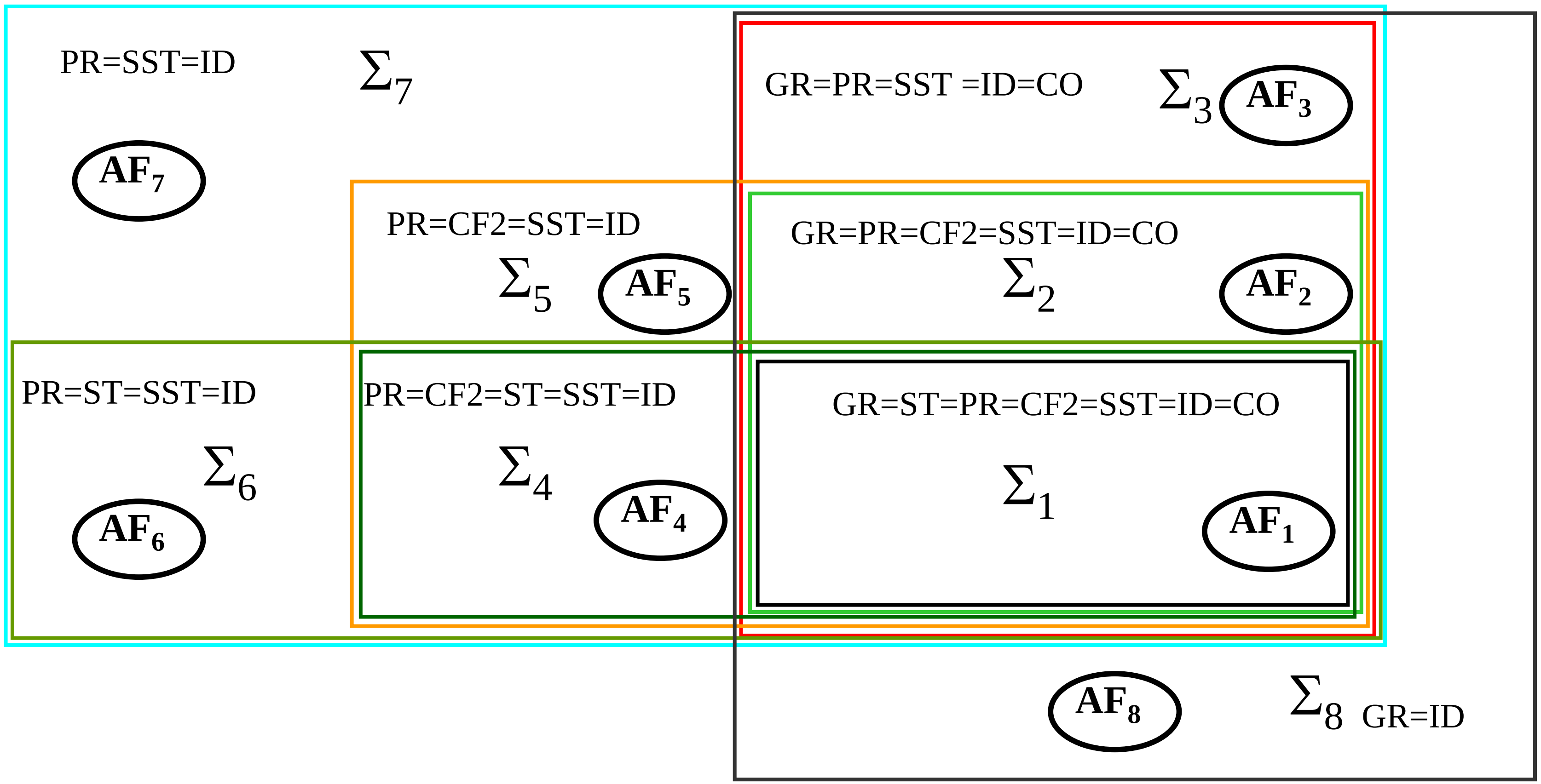
If $\{\mathcal{ID}, CF2\} \subseteq \mathbb{S}$ then $\mathcal{AGR}(\mathbb{S}) \in \{\Sigma_4, \Sigma_5\}$

If $\{\mathcal{GR}, \mathcal{SST}\} \subseteq \mathbb{S} \wedge (\{\mathcal{ST}, CF2\} \cap \mathbb{S}) = \emptyset$
then $\mathcal{AGR}(\mathbb{S}) = \Sigma_3$

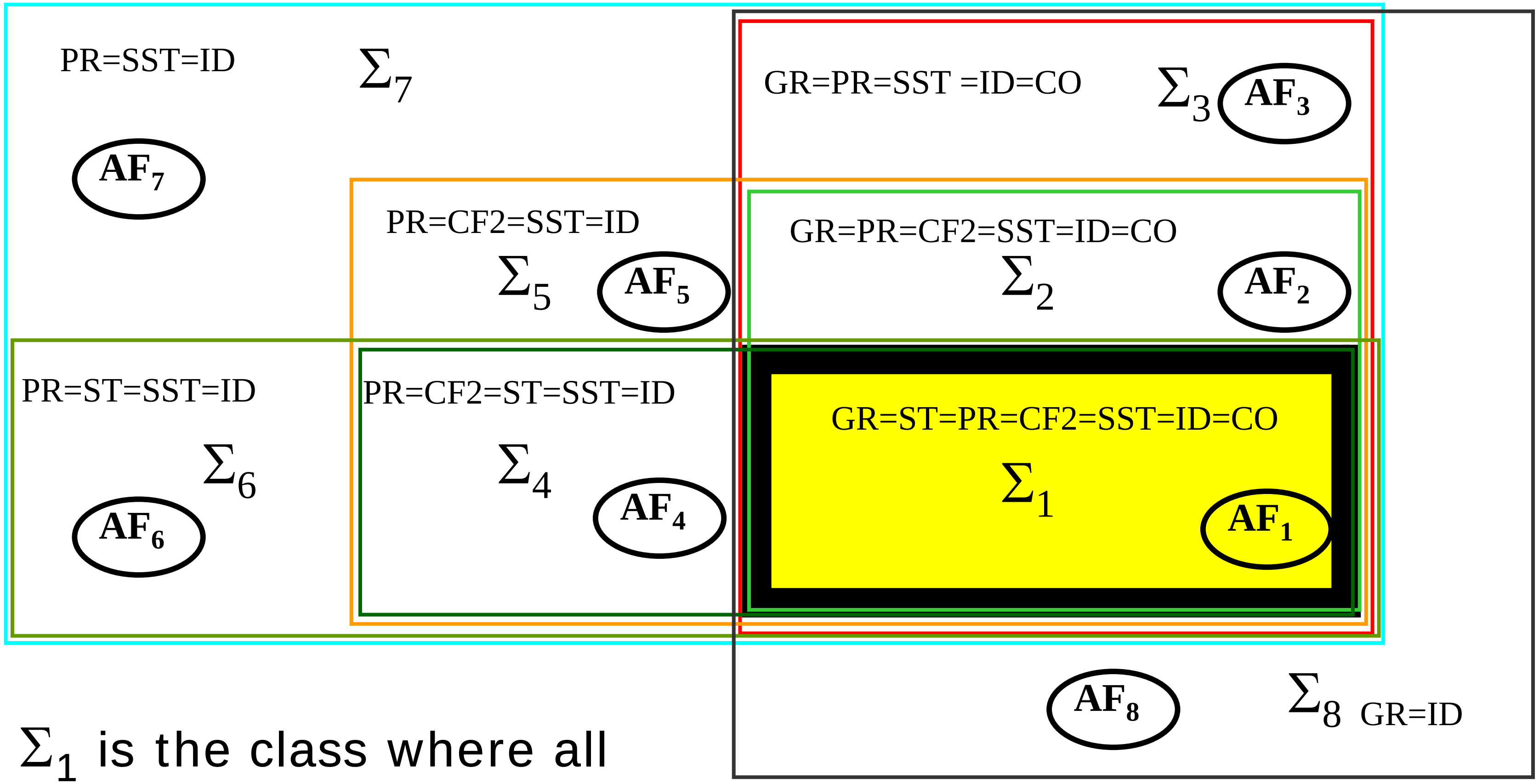
$$\{CF2, \mathcal{ST}\} \subseteq \mathbb{S} \subseteq \{CF2, \mathcal{ST}, \mathcal{PR}, \mathcal{SST}\} \Rightarrow \mathcal{AGR}(\mathbb{S}) = \Sigma_9$$

$$\{CF2, \mathcal{SST}\} \subseteq \mathbb{S} \subseteq \{CF2, \mathcal{PR}, \mathcal{SST}\} \Rightarrow \mathcal{AGR}(\mathbb{S}) = \Sigma_{10}$$

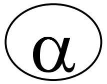
Agreement classes: unique-status behavior



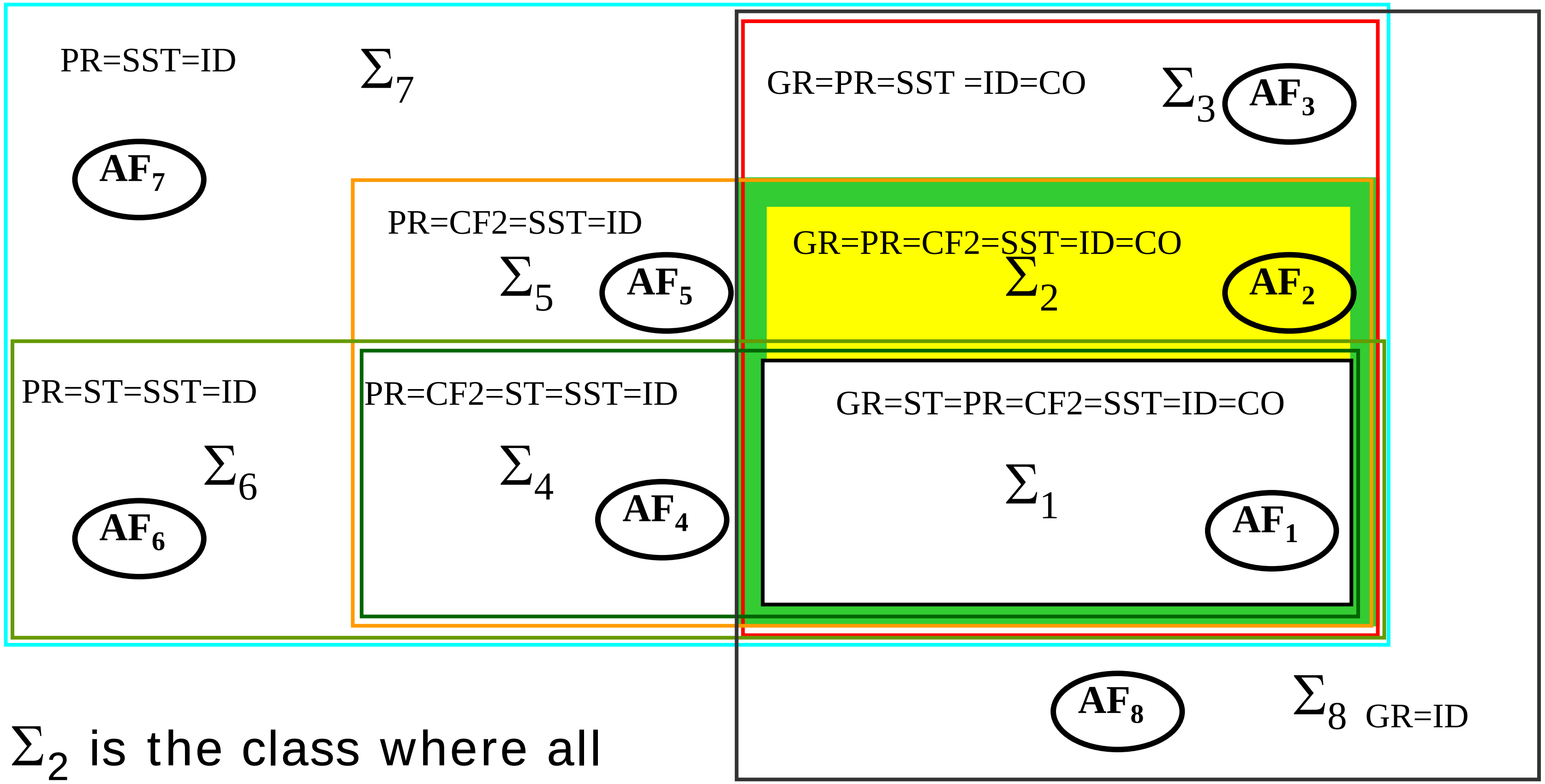
Agreement classes: ***GR*** unique-status behavior



Σ_1 is the class where all the considered semantics agree (in particular with ***GR***)

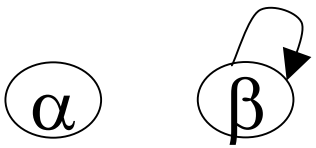
It includes (for instance) acyclic argumentation frameworks like AF_1 = 

Agreement classes: ***GR*** unique-status behavior

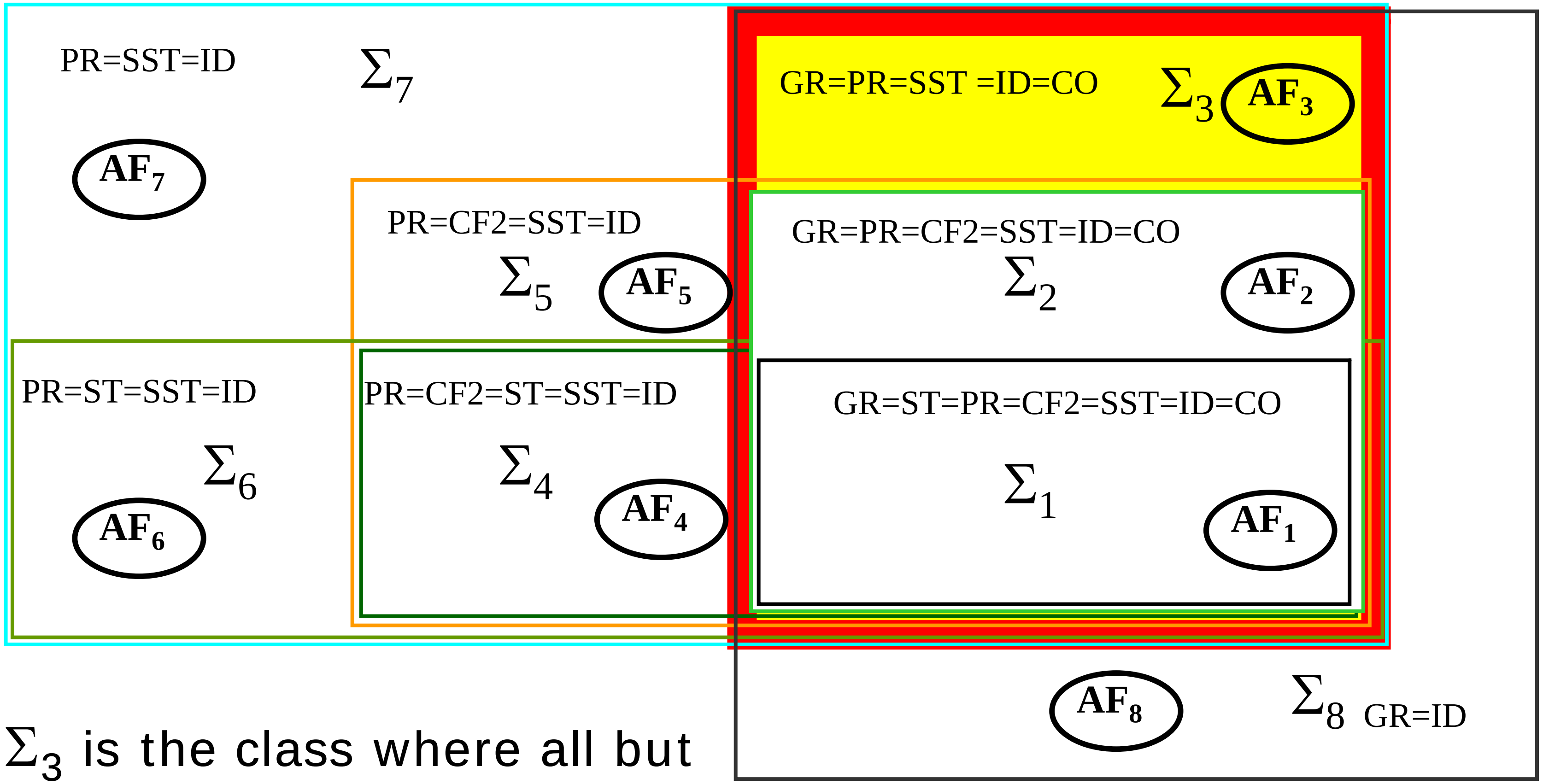


Σ_2 is the class where all the considered semantics agree (but ***ST*** may be undefined)

$\Sigma_2 \setminus \Sigma_1 \neq \emptyset$ as it includes AF_2=

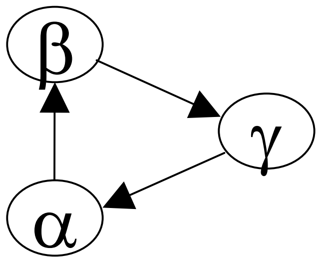


Agreement classes: ***GR*** unique-status behavior



Σ_3 is the class where all but ***CF2*** semantics agree (while ***ST*** may be undefined)

$\Sigma_3 \setminus \Sigma_2 \neq \emptyset$ as it includes AF_3=



A note on complete semantics

The grounded extension belongs to the set of complete extensions (is the least complete extension)

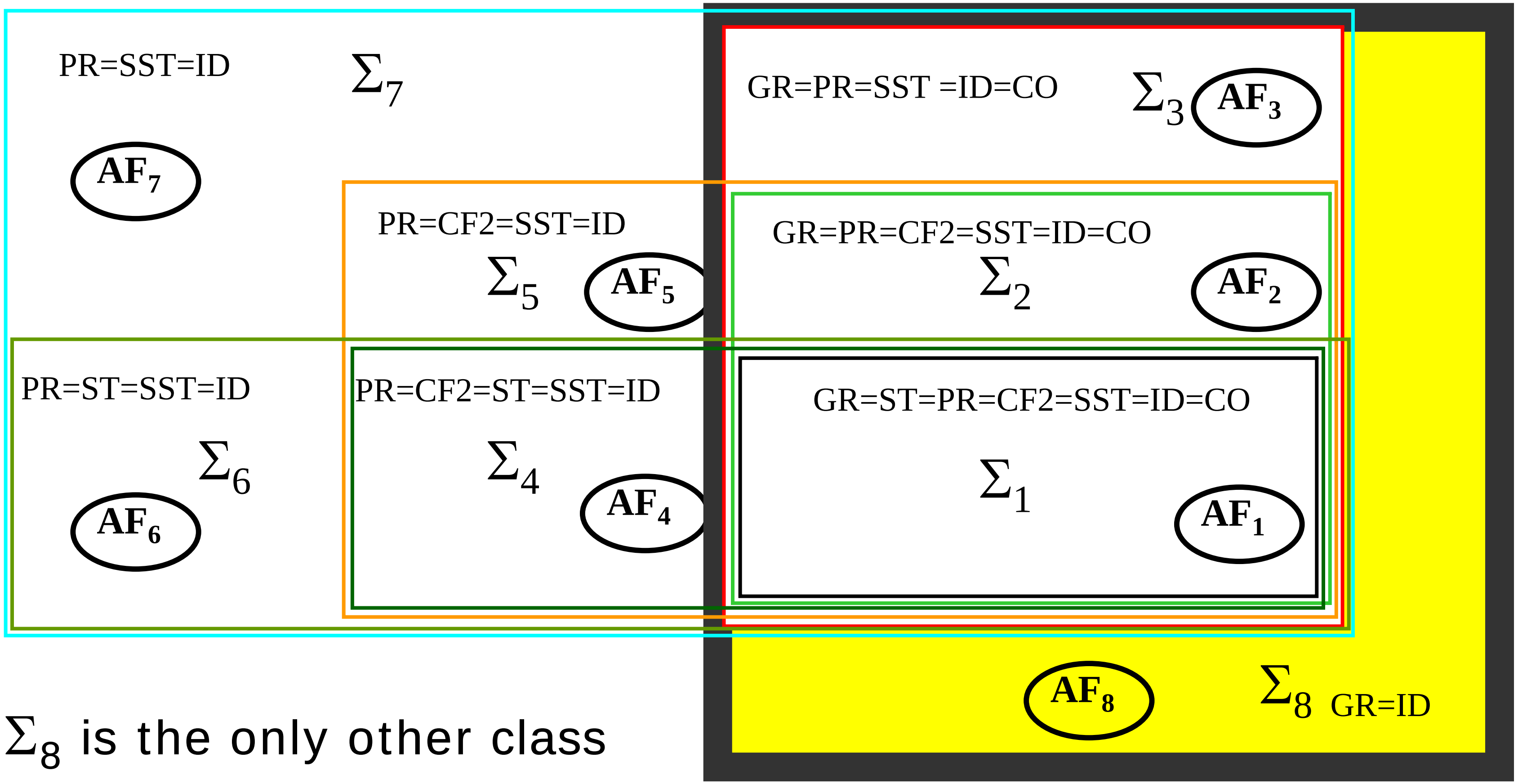
For all semantics considered in this paper, except ***CO***, it holds that no extension can be a proper subset of another extension

For all semantics considered in this paper, any extension is a superset of the grounded extension

It follows that agreement with ***CO*** is possible for a multiple-status semantics only if also agreement with ***GR*** holds

As a consequence ***CO*** only appears in agreement classes $\Sigma_1, \Sigma_2, \Sigma_3$

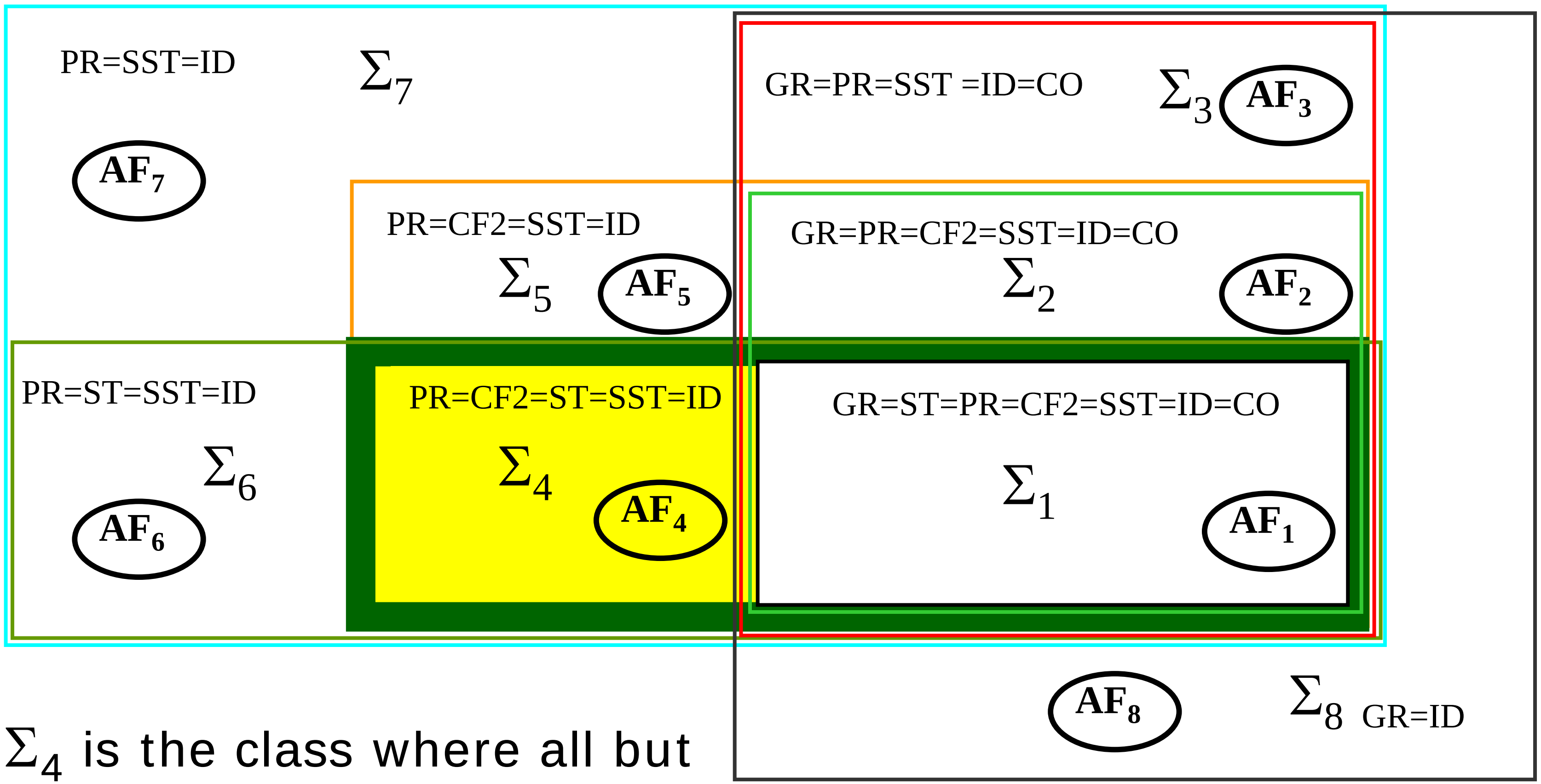
Agreement classes: ***GR*** unique-status behavior



Σ_8 is the only other class of agreement involving ***GR***

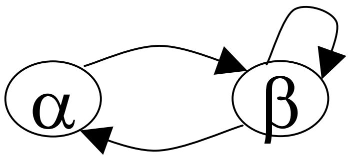
Examples of argumentation frameworks in $\Sigma_8 \setminus \Sigma_7$ will be given when examining its intersections with multiple-status agreement classes

Agreement classes: ***ID*** unique-status behavior

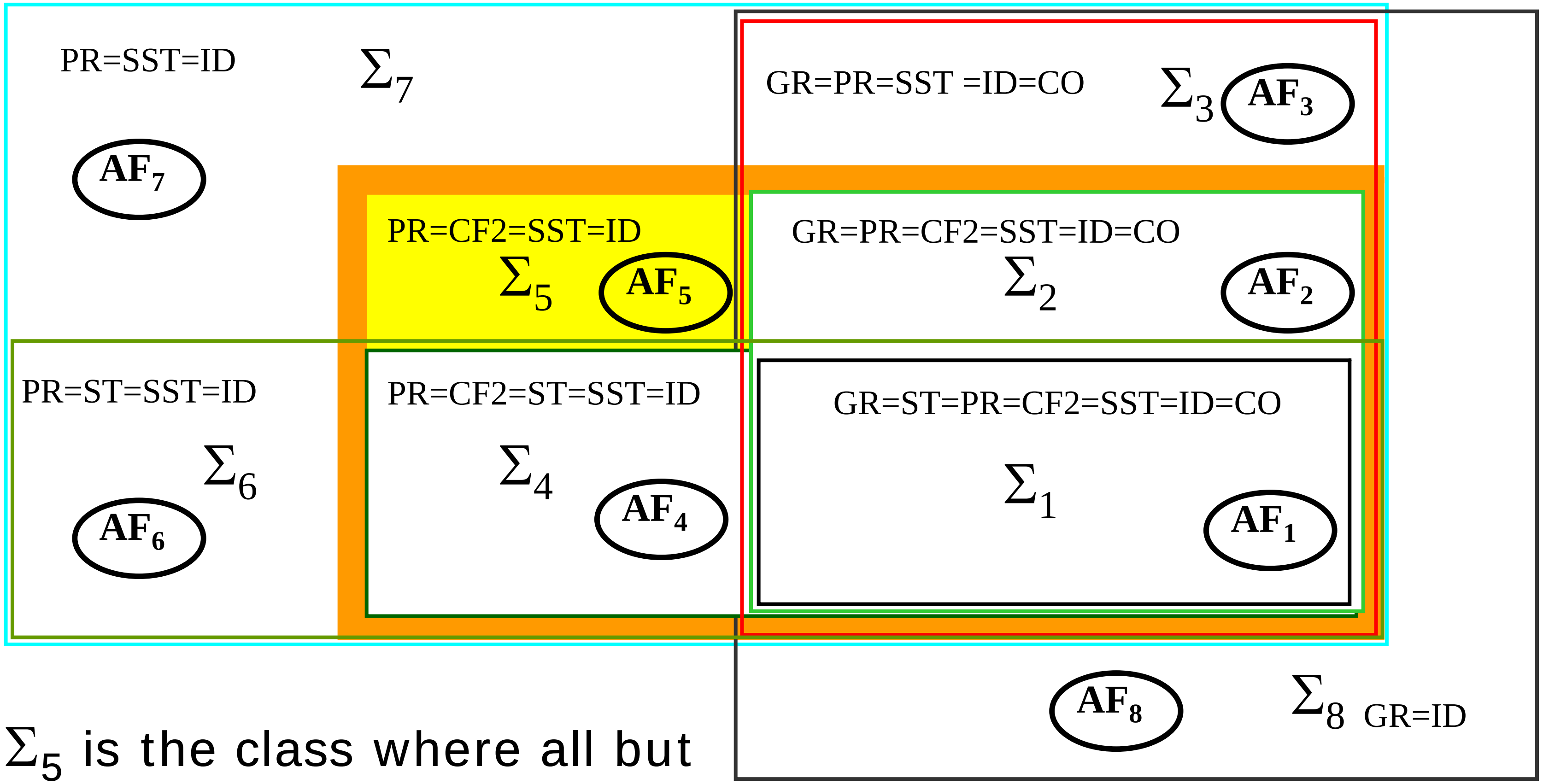


Σ_4 is the class where all but ***CO*** and ***GR*** semantics agree

$\Sigma_4 \setminus \Sigma_1 \neq \emptyset$ as it includes AF_4 =

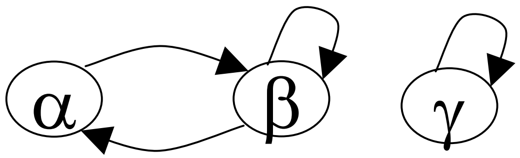


Agreement classes: *ID* unique-status behavior

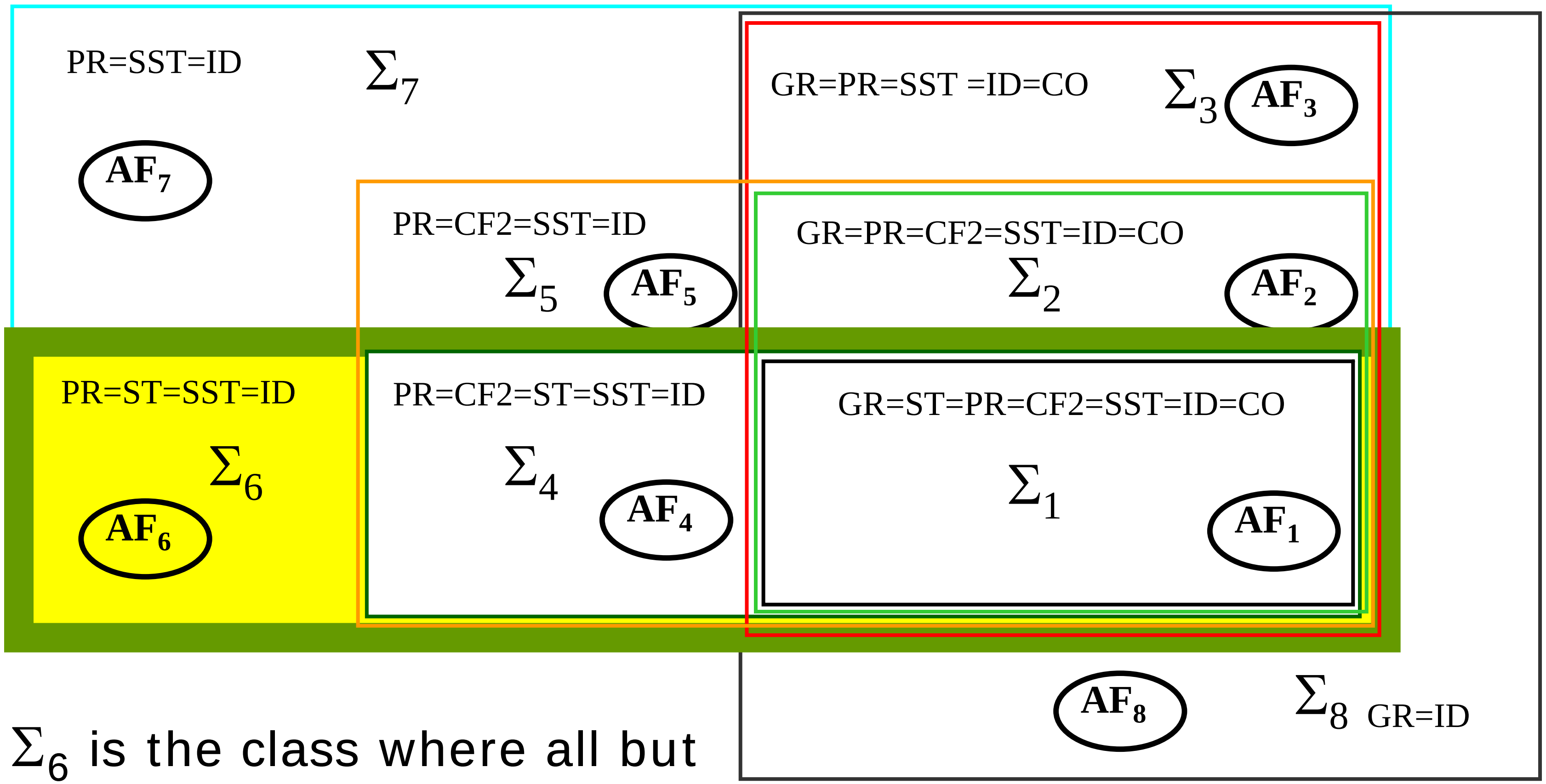


Σ_5 is the class where all but ***CO*** and ***GR*** semantics agree, while ***ST*** may be undefined

$\Sigma_5 \setminus (\Sigma_4 \cup \Sigma_2) \neq \emptyset$ as it includes AF_5 =

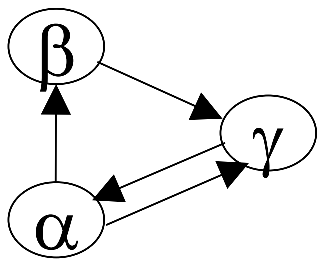


Agreement classes: ***ID*** unique-status behavior

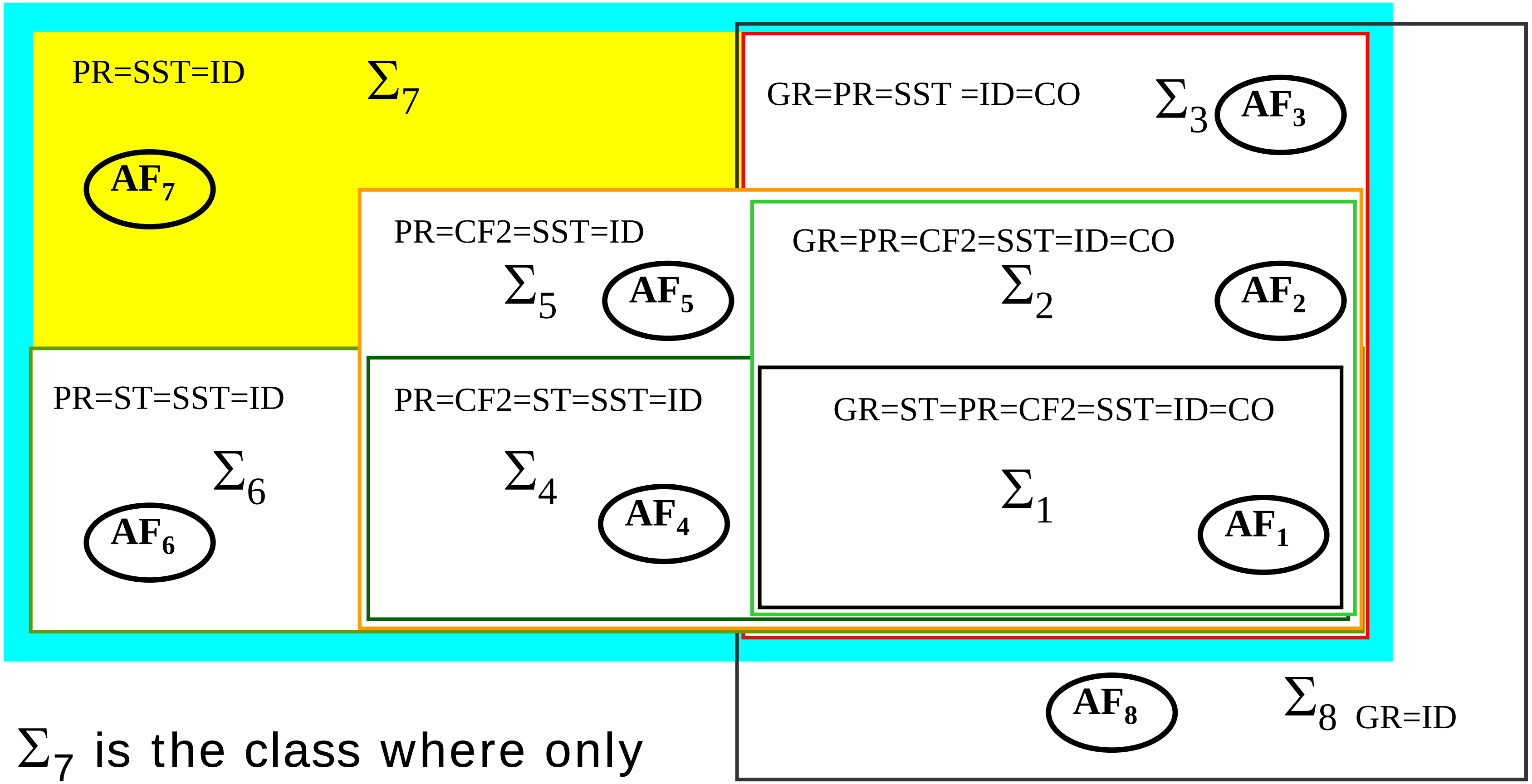


Σ_6 is the class where all but ***CO***, ***GR***, and ***CF2*** semantics agree

$\Sigma_6 \setminus (\Sigma_4 \cup \Sigma_5) \neq \emptyset$ as it includes AF_6 =

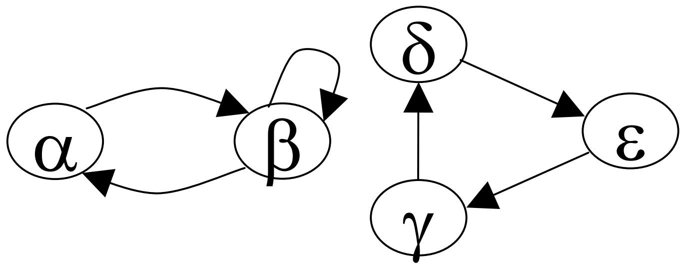


Agreement classes: ***ID*** unique-status behavior

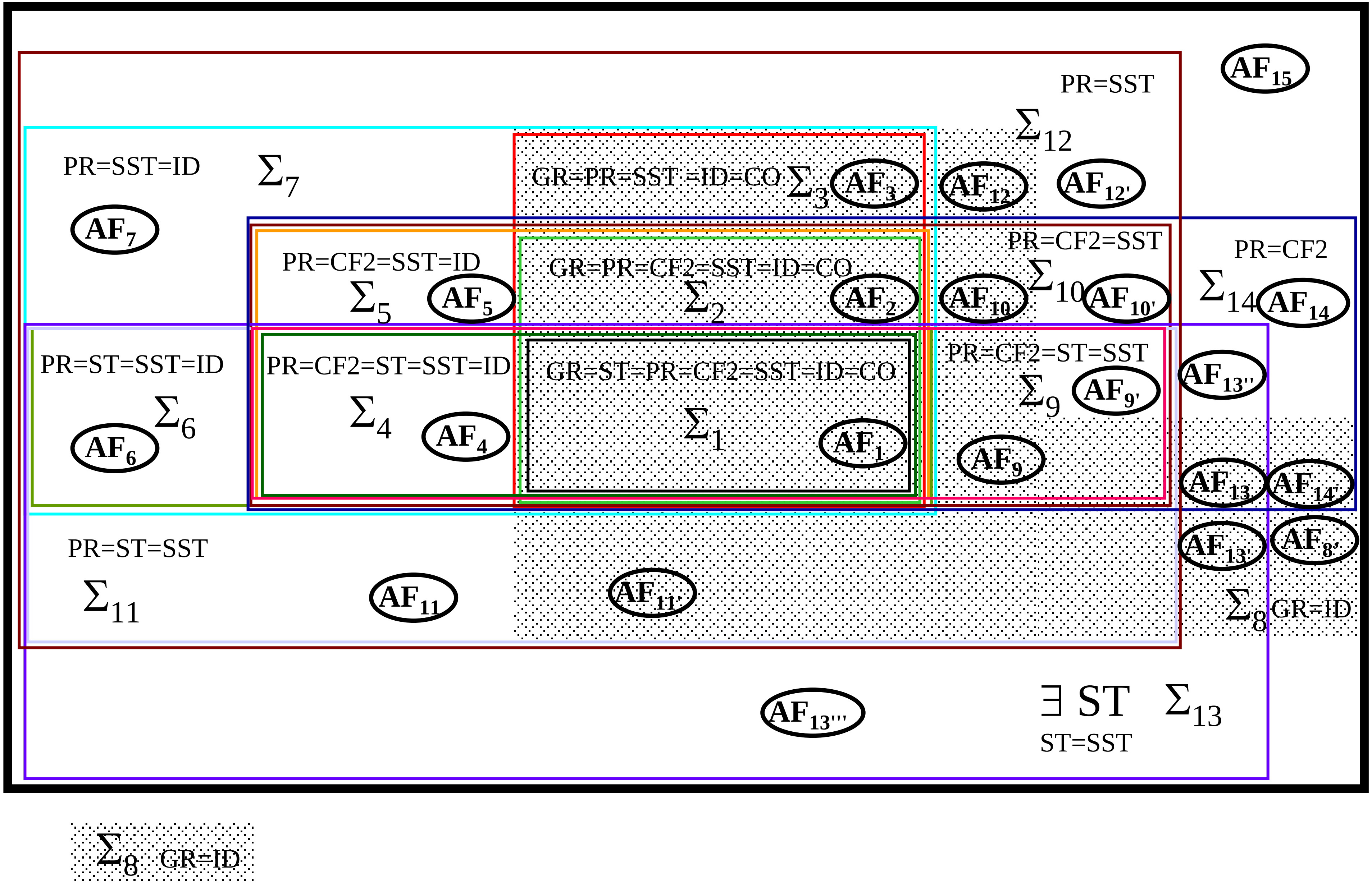


Σ_7 is the class where only ***PR***, ***SST***, and ***ID*** semantics agree

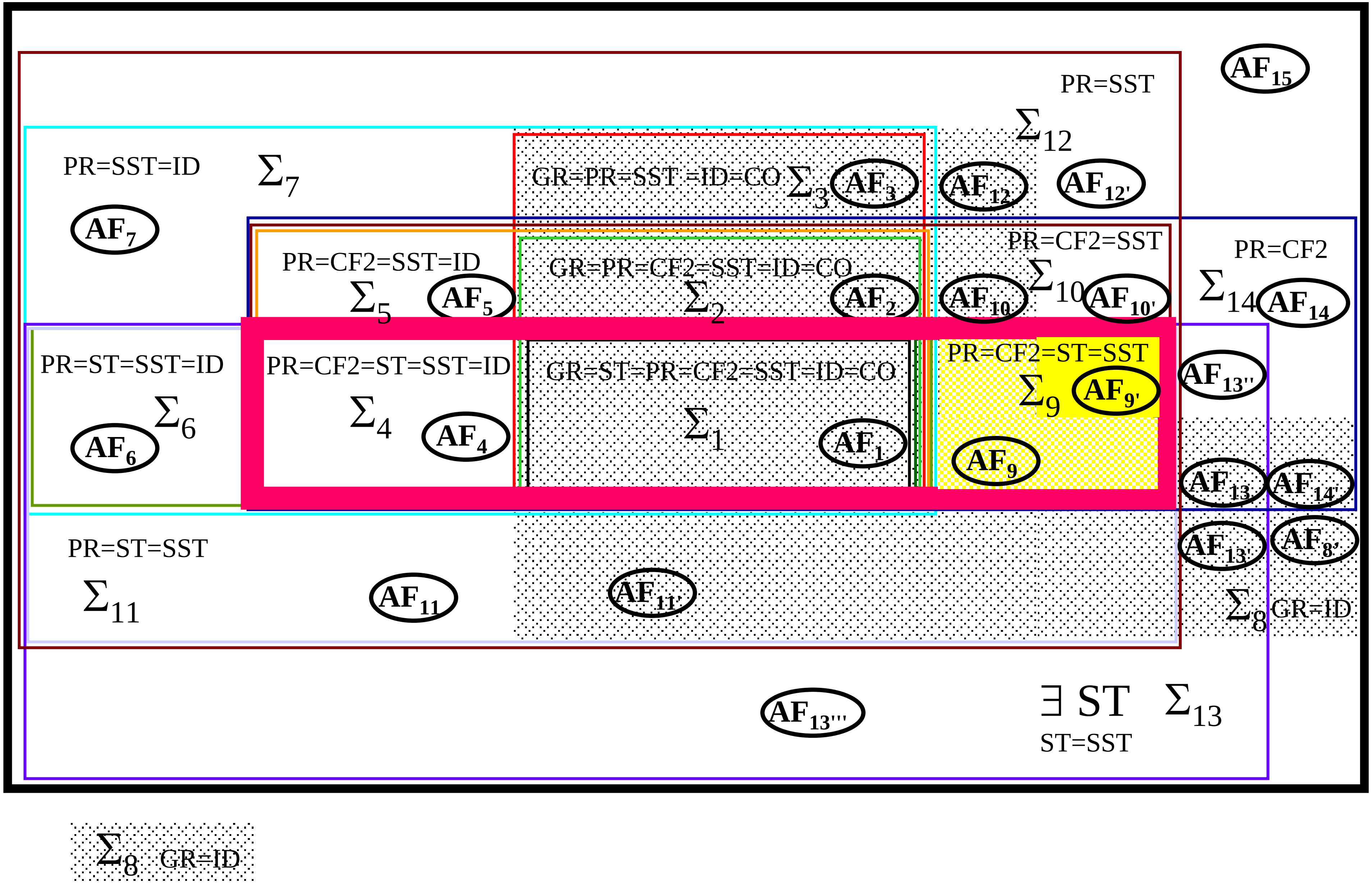
$\Sigma_7 \setminus (\Sigma_6 \cup \Sigma_5 \cup \Sigma_3) \neq \emptyset$ as it includes AF_7 =



Agreement classes: multiple-status behavior



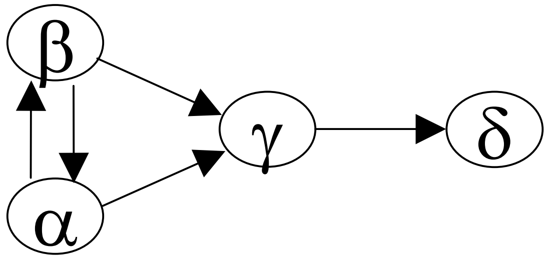
Agreement classes: multiple-status behavior



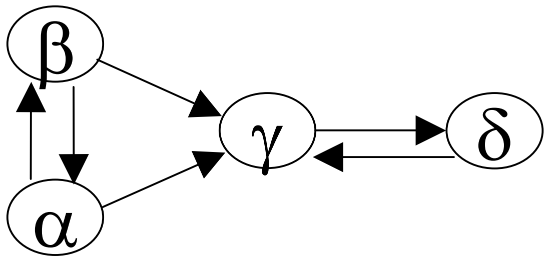
The Σ_9 class

Σ_9 is the class where all multiple-status semantics (except ***CO***) agree

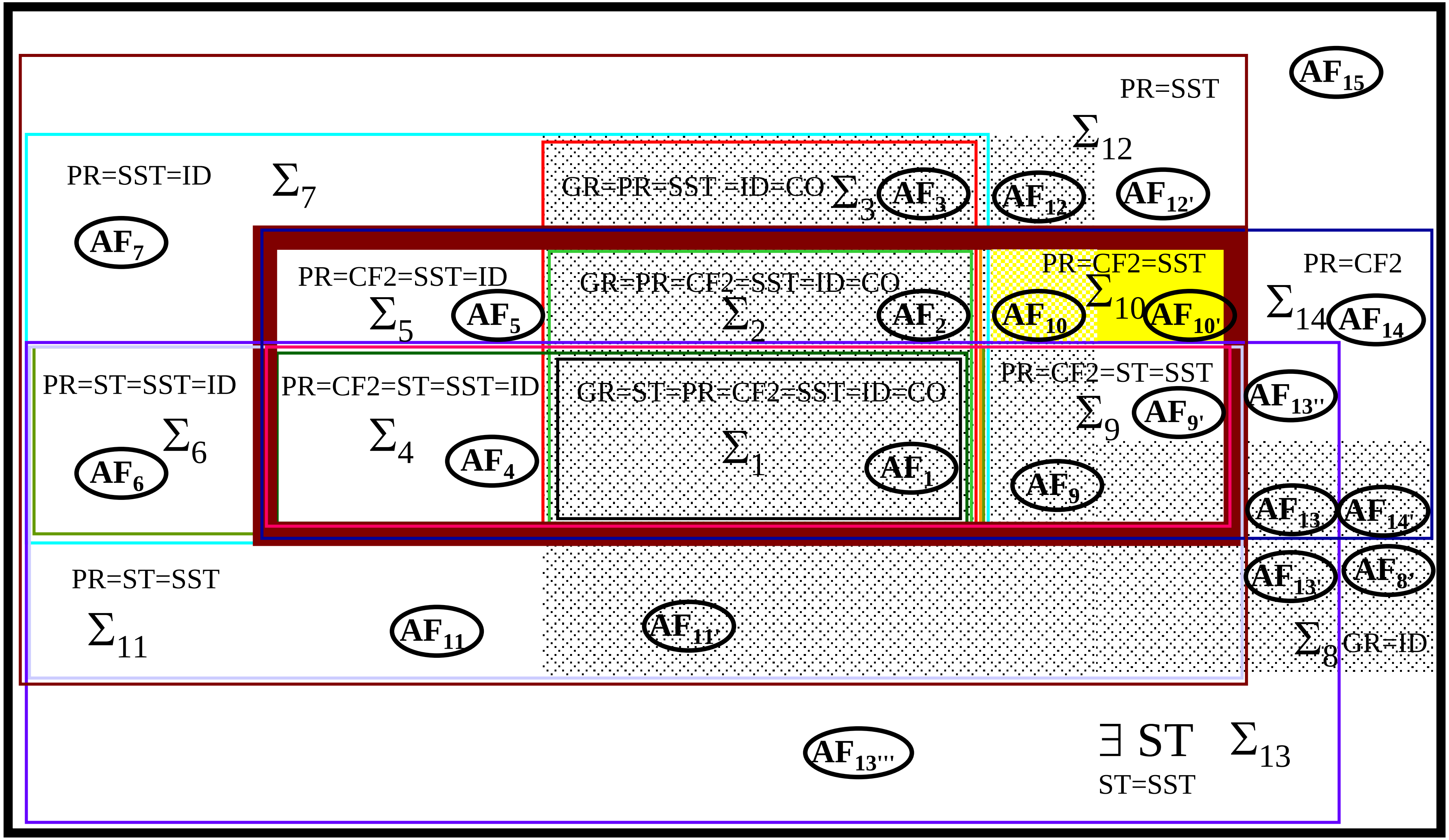
$(\Sigma_9 \setminus \Sigma_4) \cap \Sigma_8 \neq \emptyset$ as it includes AF_9



$\Sigma_9 \setminus (\Sigma_4 \cup \Sigma_8) \neq \emptyset$ as it includes $AF_{9'}$



Agreement classes: multiple-status behavior

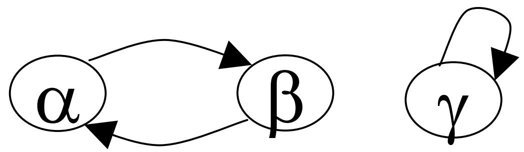


Σ_8 GR=ID

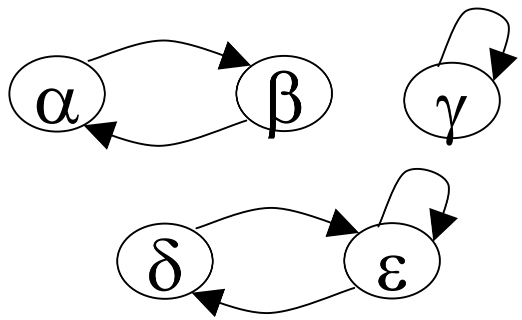
The Σ_{10} class

Σ_{10} is the class where all multiple-status semantics (except ***CO***) agree on a multiple-status behavior but ***ST*** may be undefined

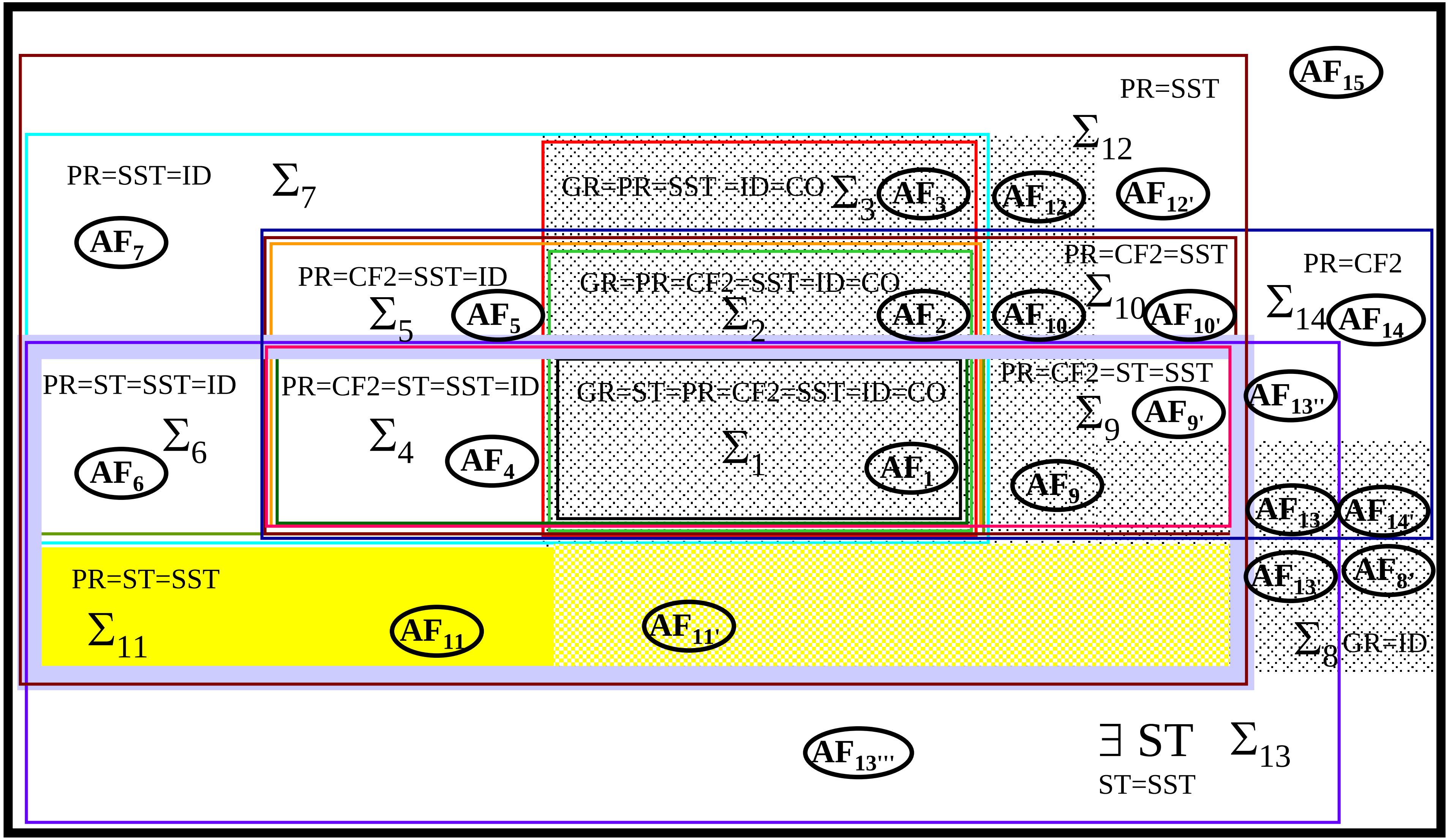
$(\Sigma_{10} \cap \Sigma_8) \setminus (\Sigma_9 \cup \Sigma_5) \neq \emptyset$ as it includes AF_{10}



$\Sigma_9 \setminus (\Sigma_9 \cup \Sigma_5 \cup \Sigma_8) \neq \emptyset$ as it includes $AF_{10'}$



Agreement classes: multiple-status behavior

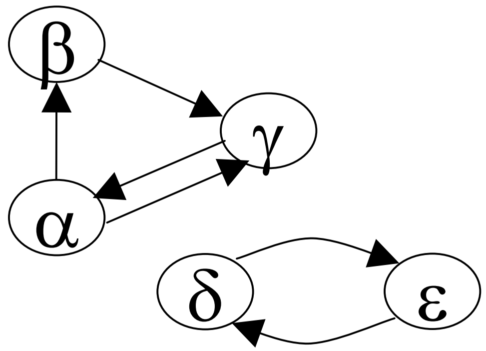


Σ8 GR=ID

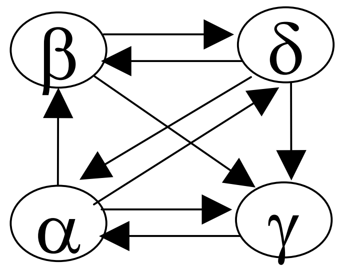
The Σ_{11} class

Σ_{11} is the class where only traditional ***PR*** and ***ST*** (and hence ***SST***) agree while ***CF2*** may differ

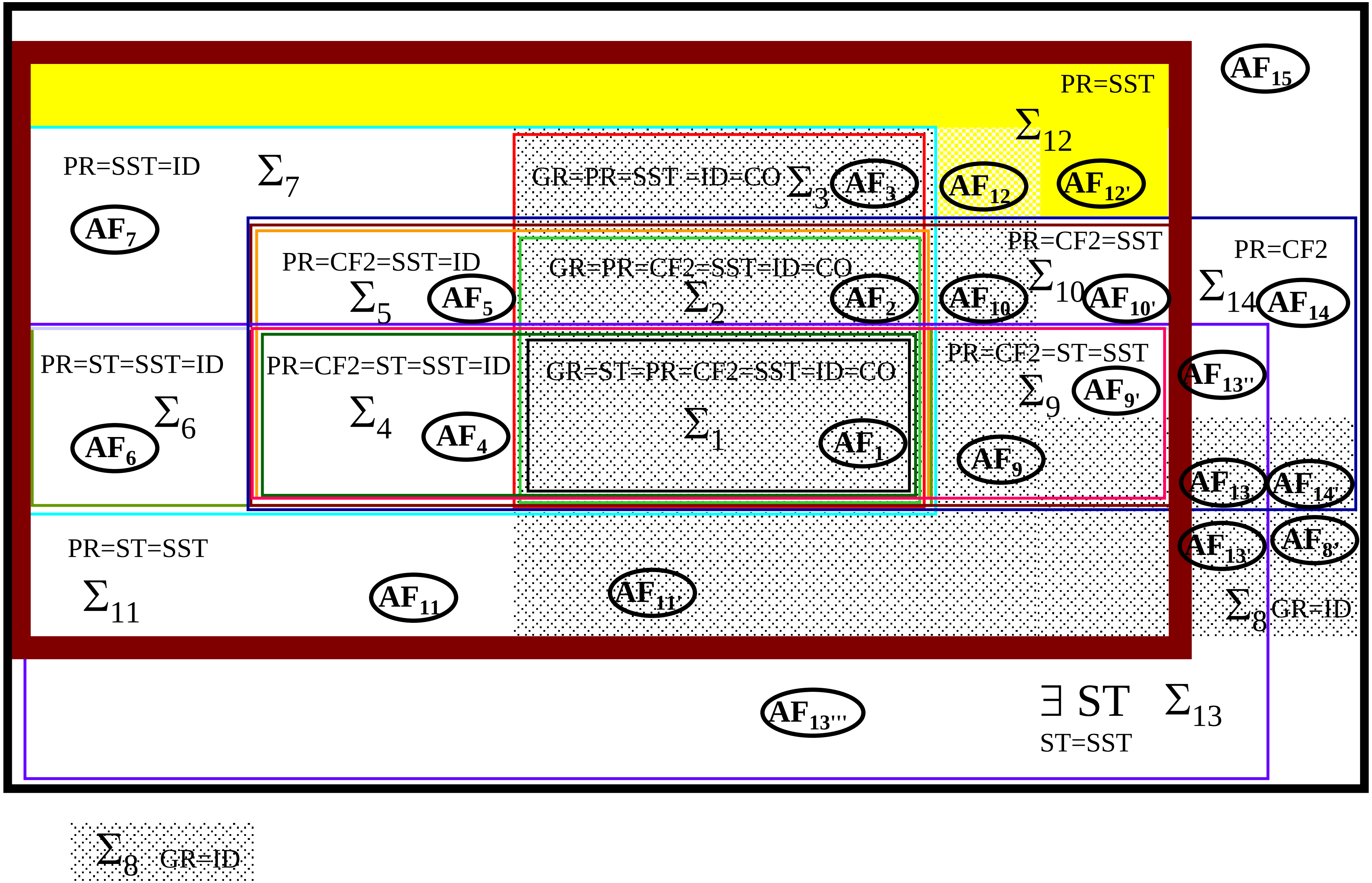
$\Sigma_{11} \setminus (\Sigma_6 \cup \Sigma_9 \cup \Sigma_8) \neq \emptyset$ as it includes AF_{11}



$(\Sigma_{11} \cap \Sigma_8) \setminus (\Sigma_6 \cup \Sigma_9) \neq \emptyset$ as it includes $AF_{11'}$



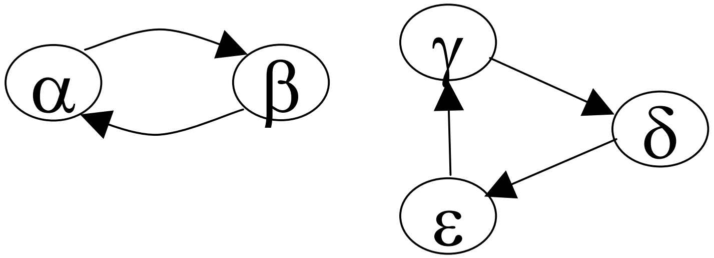
Agreement classes: multiple-status behavior



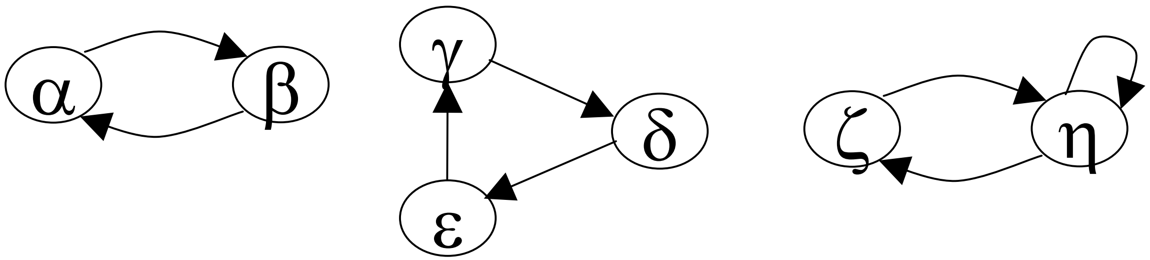
The Σ_{12} class

Σ_{12} is the class where ***PR*** and ***SST*** agree and may differ from ***CF2*** while ***ST*** is undefined

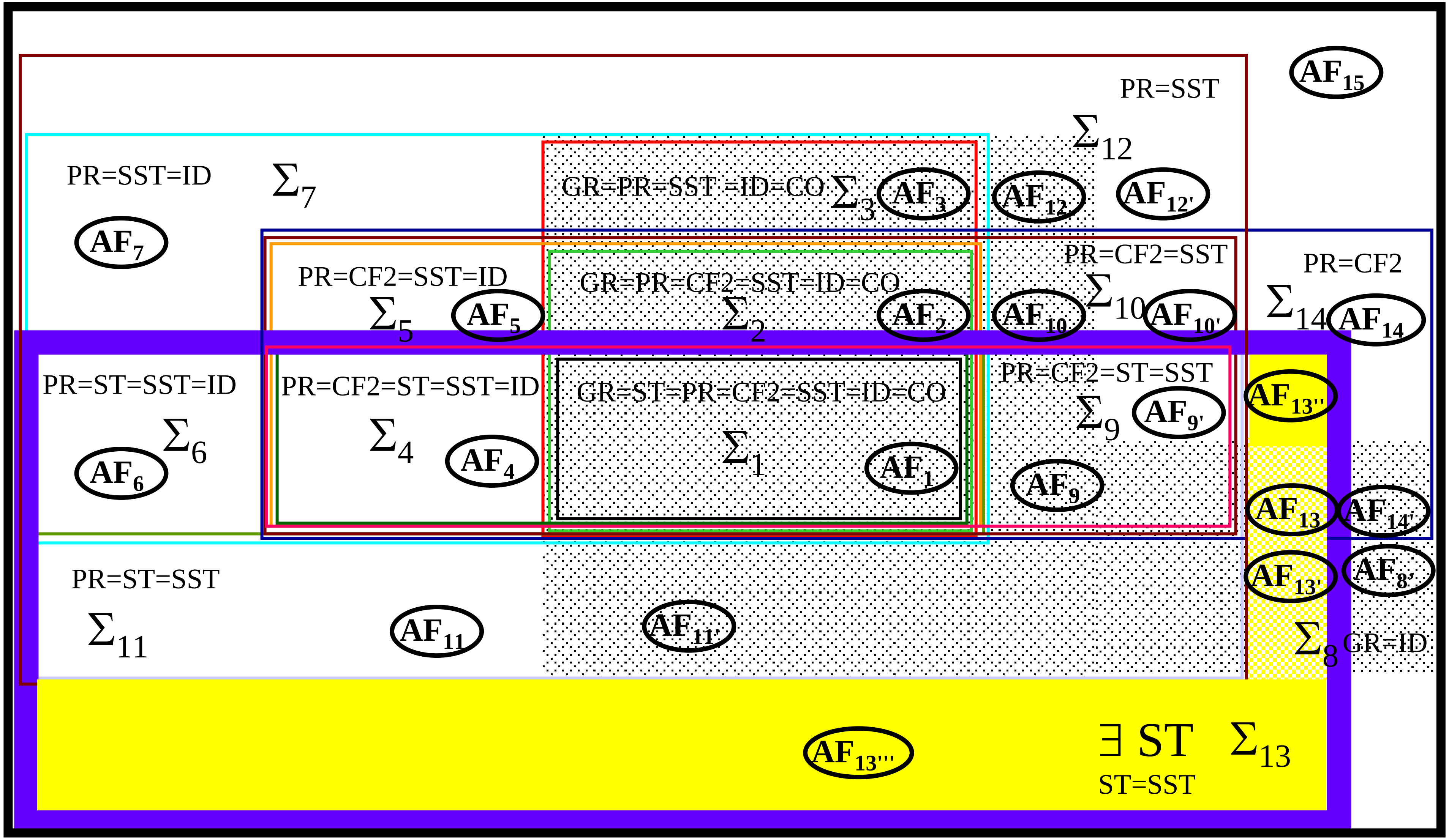
$(\Sigma_{12} \cap \Sigma_8) \setminus (\Sigma_7 \cup \Sigma_{10} \cup \Sigma_{11}) \neq \emptyset$ as it includes AF_{12}



$\Sigma_{12} \setminus (\Sigma_7 \cup \Sigma_{10} \cup \Sigma_{11} \cup \Sigma_8) \neq \emptyset$ as it includes $AF_{12'}$



Agreement classes: multiple-status behavior

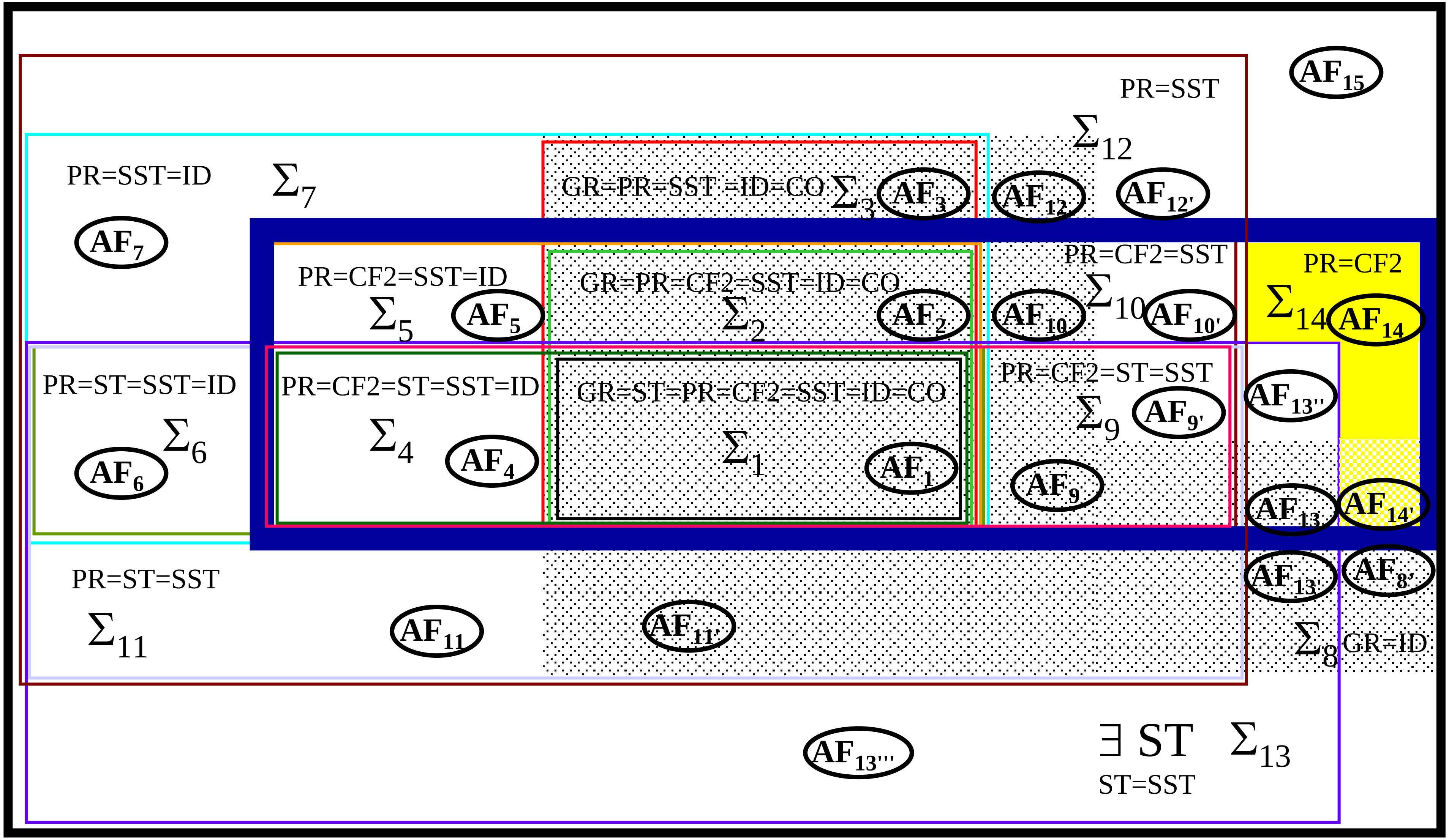


The Σ_{13} class

Σ_{13} is the class where ***ST*** is defined (and hence agrees with ***SST***) and may differ from any other

Σ_{13} has articulated intersections with Σ_{14} to be examined later

Agreement classes: multiple-status behavior



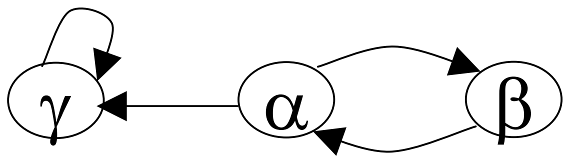
Σ_8 GR=ID

The Σ_{14} class – Regions of interest

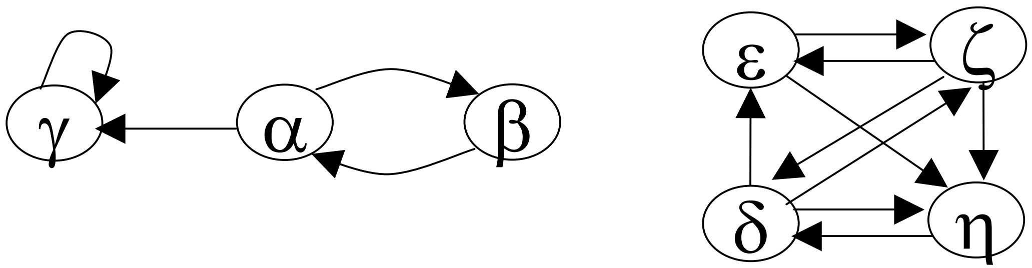
Σ_{14} is the class corresponding to agreement between the last pair to be considered: ***PR*** and ***CF2***

Let us now examine the distinct regions related to Σ_{13} and Σ_{14} in the Venn diagram

$(\Sigma_{13} \setminus \Sigma_{12}) \cap \Sigma_8 \cap \Sigma_{14} \neq \emptyset$ as it includes AF_{13}

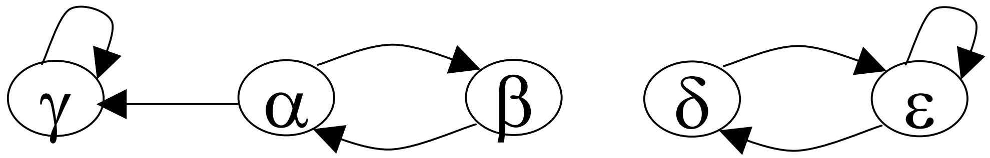


$(\Sigma_{13} \setminus \Sigma_{12}) \cap (\Sigma_8 \setminus \Sigma_{14}) \neq \emptyset$ as it includes $AF_{13'}$

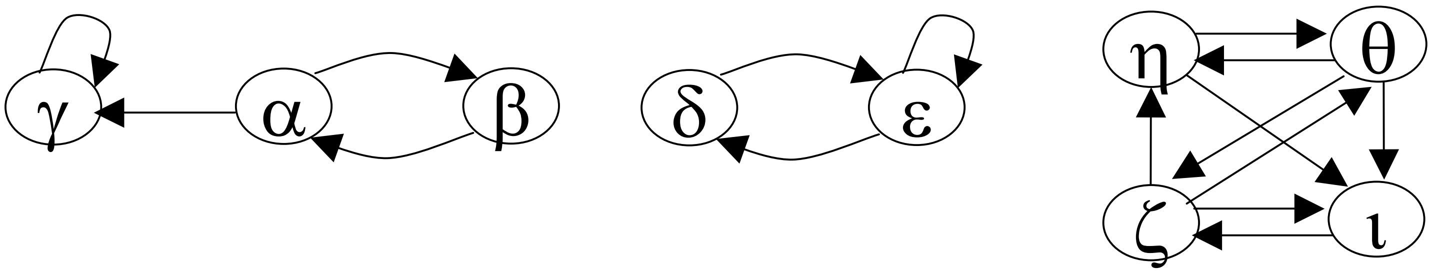


The Σ_{14} class – Regions of interest

$(\Sigma_{13} \setminus \Sigma_{12}) \cap (\Sigma_{14} \setminus \Sigma_8) \neq \emptyset$ as it includes $AF_{13''}$

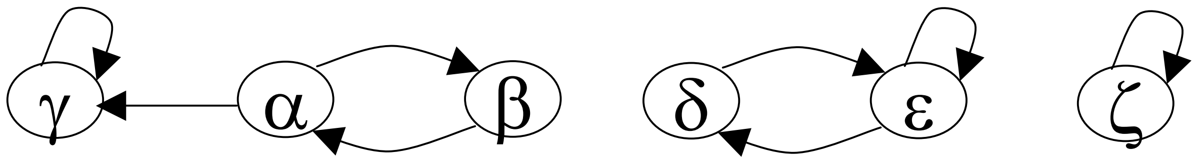


$\Sigma_{13} \setminus (\Sigma_8 \cup \Sigma_{12} \cup \Sigma_{14}) \neq \emptyset$ as it includes $AF_{13'''}$

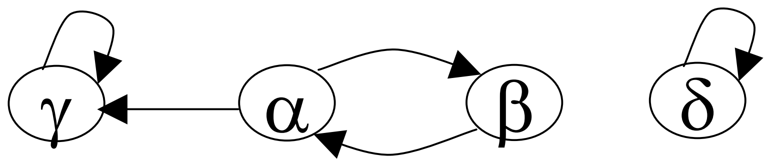


The Σ_{14} class – Regions of interest

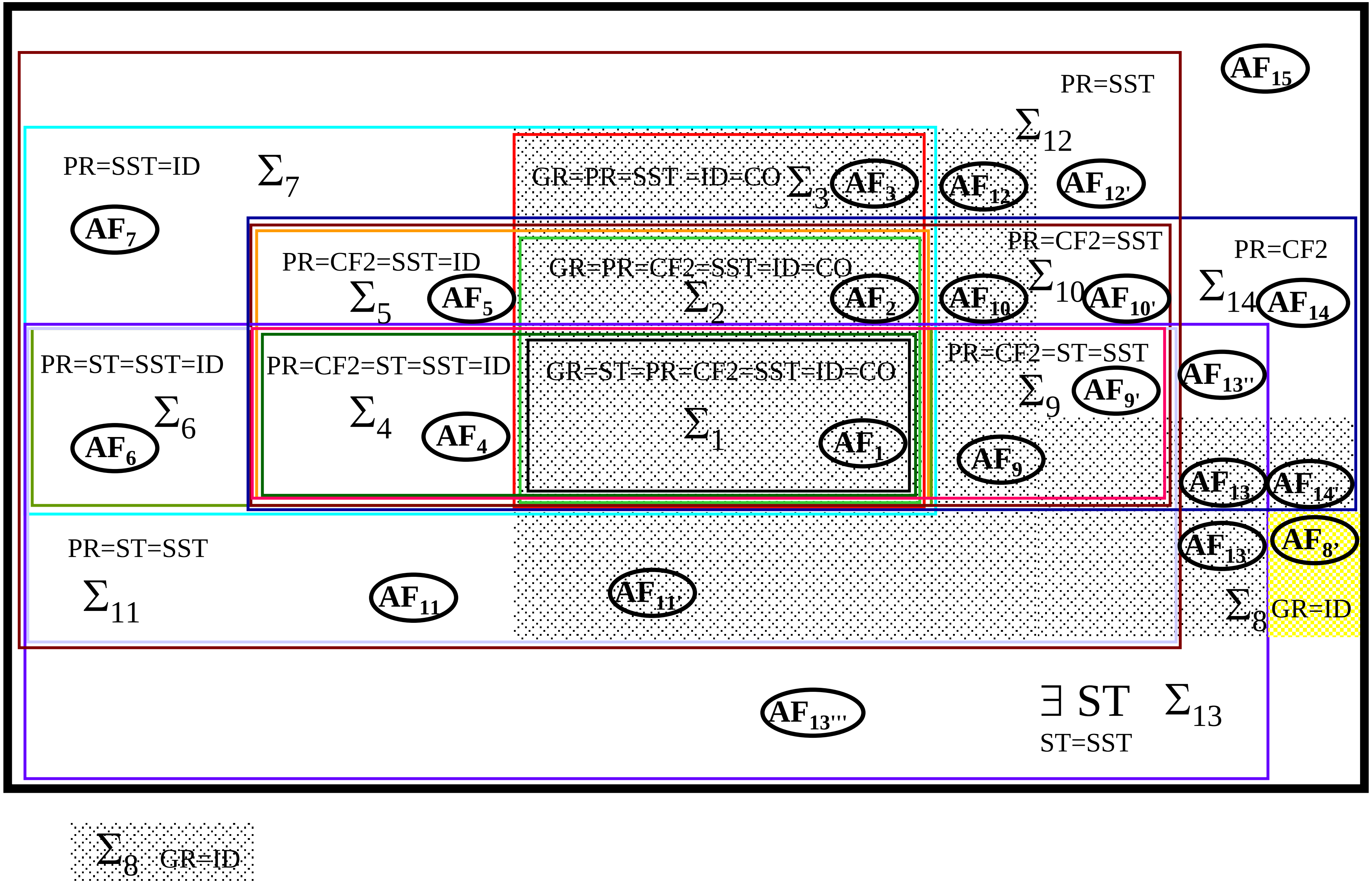
$\Sigma_{14} \setminus (\Sigma_8 \cup \Sigma_{12} \cup \Sigma_{13}) \neq \emptyset$ as it includes AF_{14}



$(\Sigma_{14} \cap \Sigma_8) \setminus (\Sigma_{12} \cup \Sigma_{13}) \neq \emptyset$ as it includes $AF_{14''}$



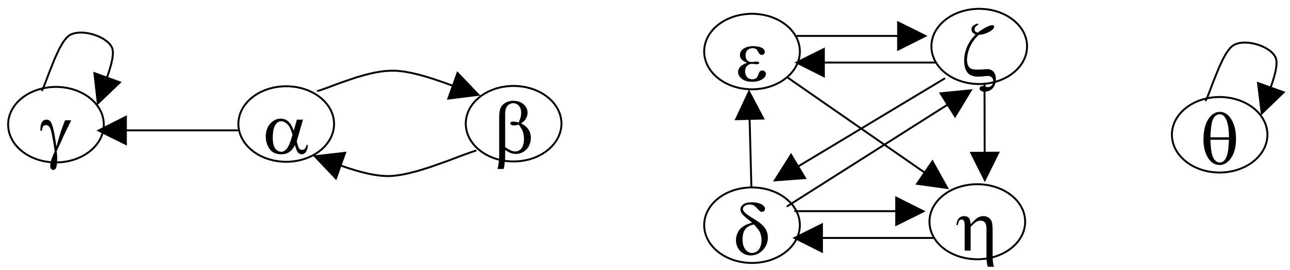
Agreement classes: multiple-status behavior



Σ_8 and nothing else

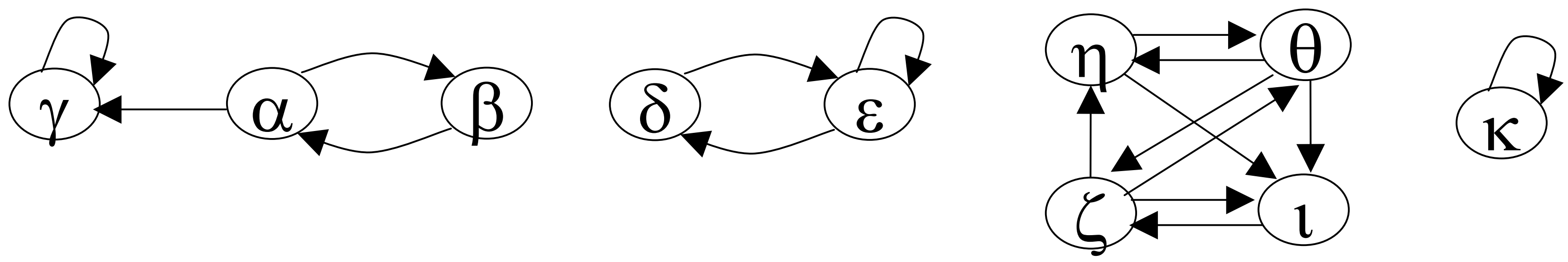
There are argumentation frameworks where ***GR*** and ***ID*** agree while all other semantics disagree (and ***ST*** is undefined)

$\Sigma_8 \setminus (\Sigma_{12} \cup \Sigma_{13} \cup \Sigma_{14}) \neq \emptyset$ as it includes AF_8

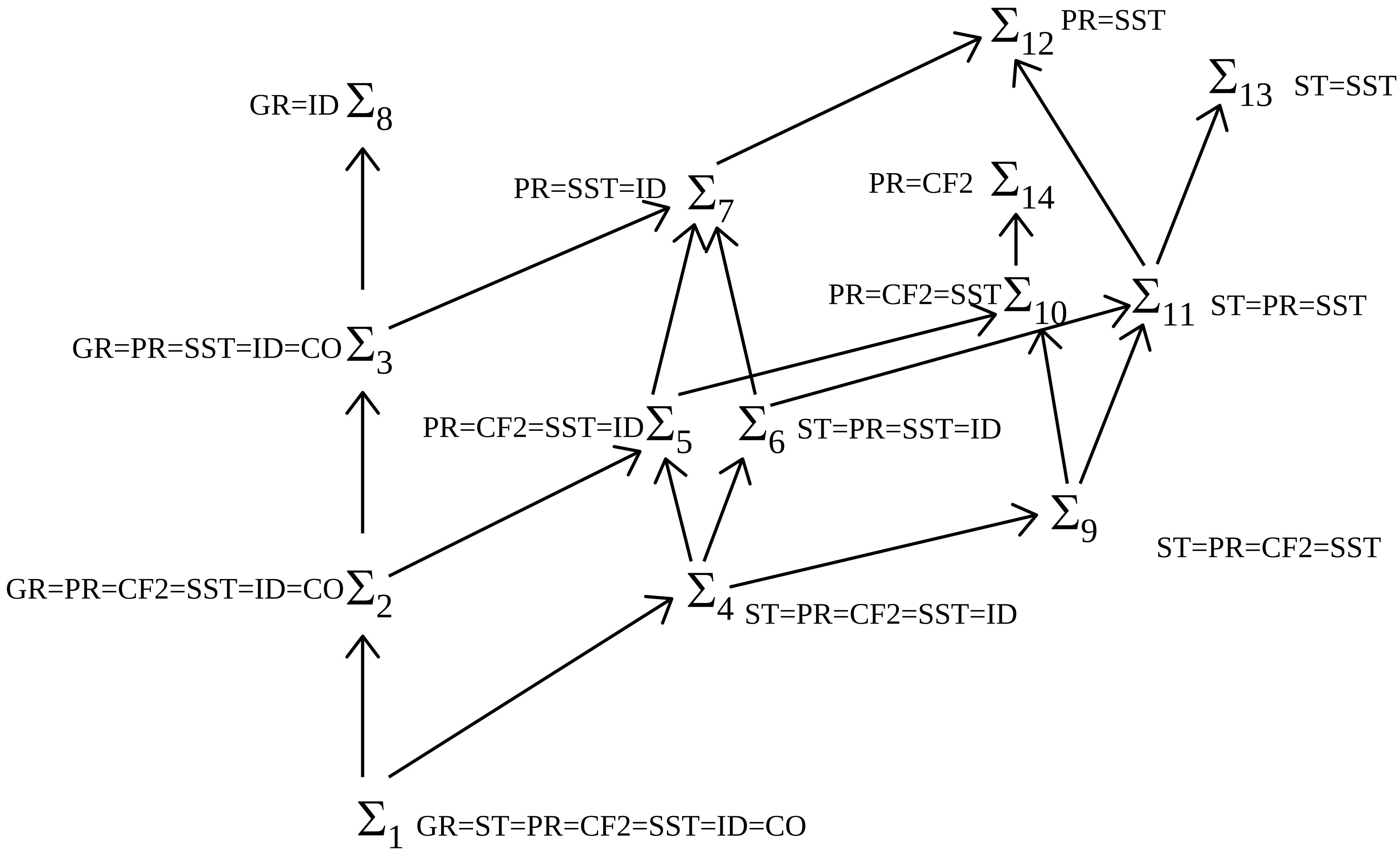


Universal disagreement is possible

There are argumentation frameworks where no two semantics agree (while ***ST*** is undefined) like AF_{15}

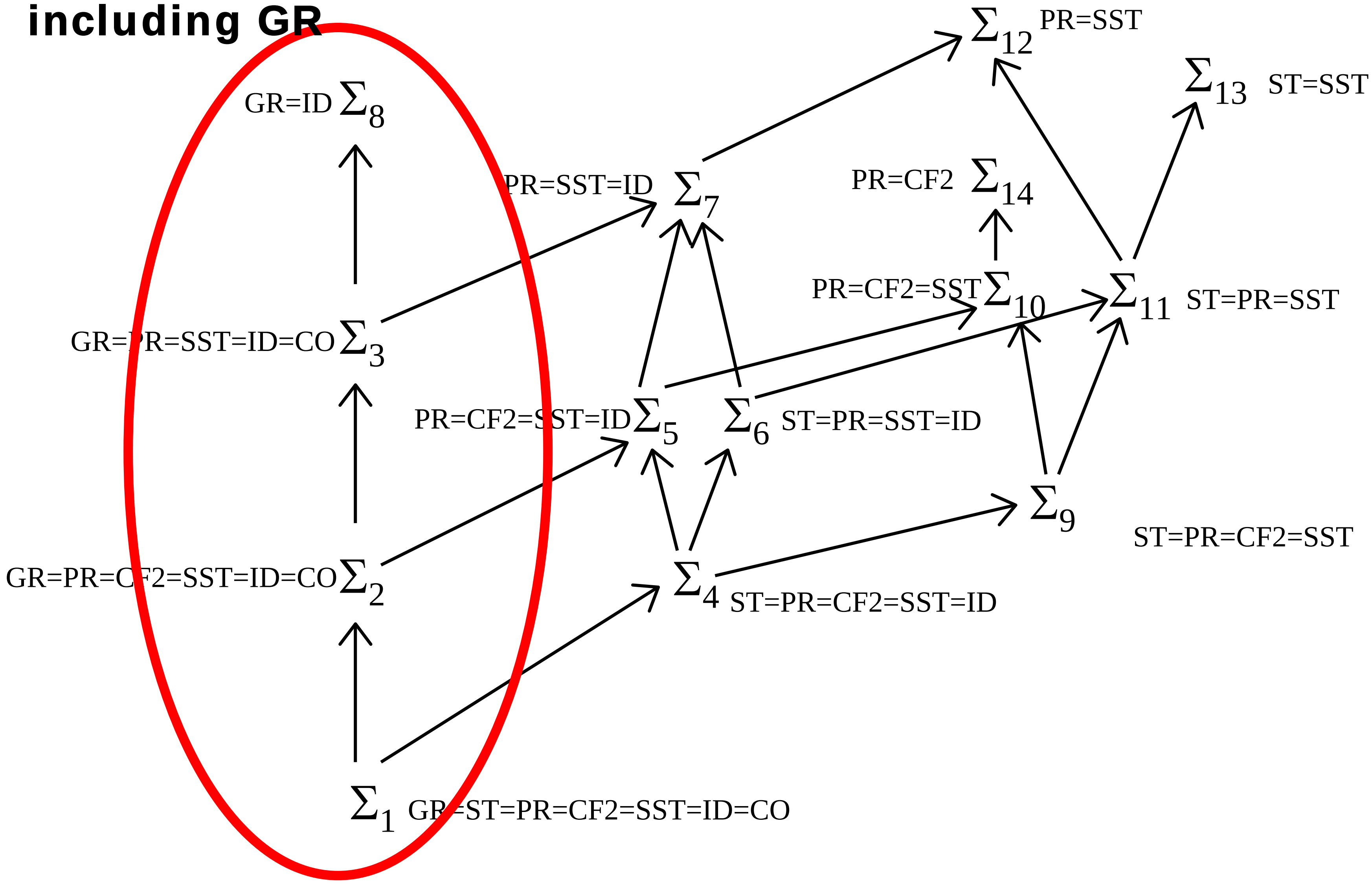


A synthetic view

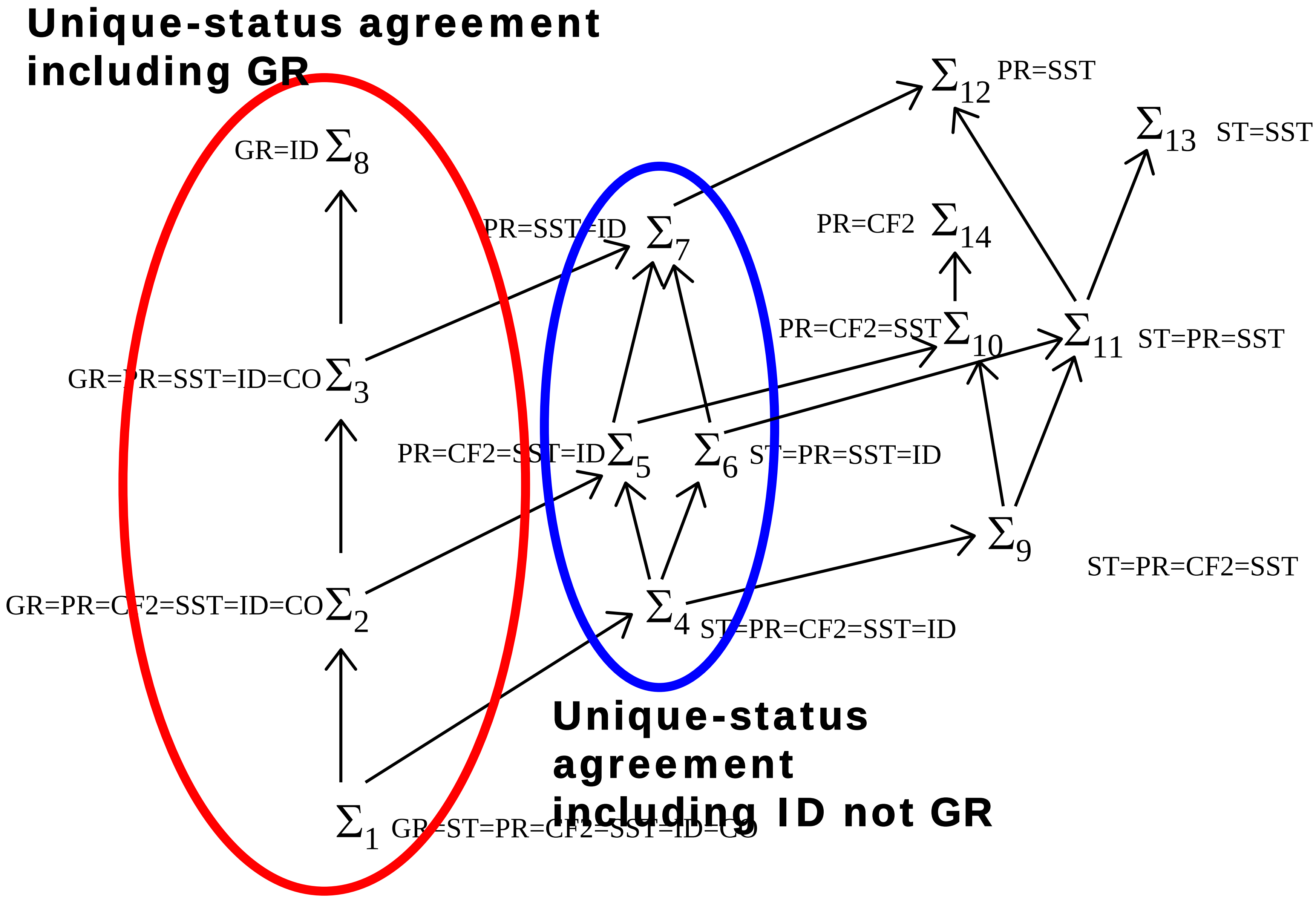


A synthetic view

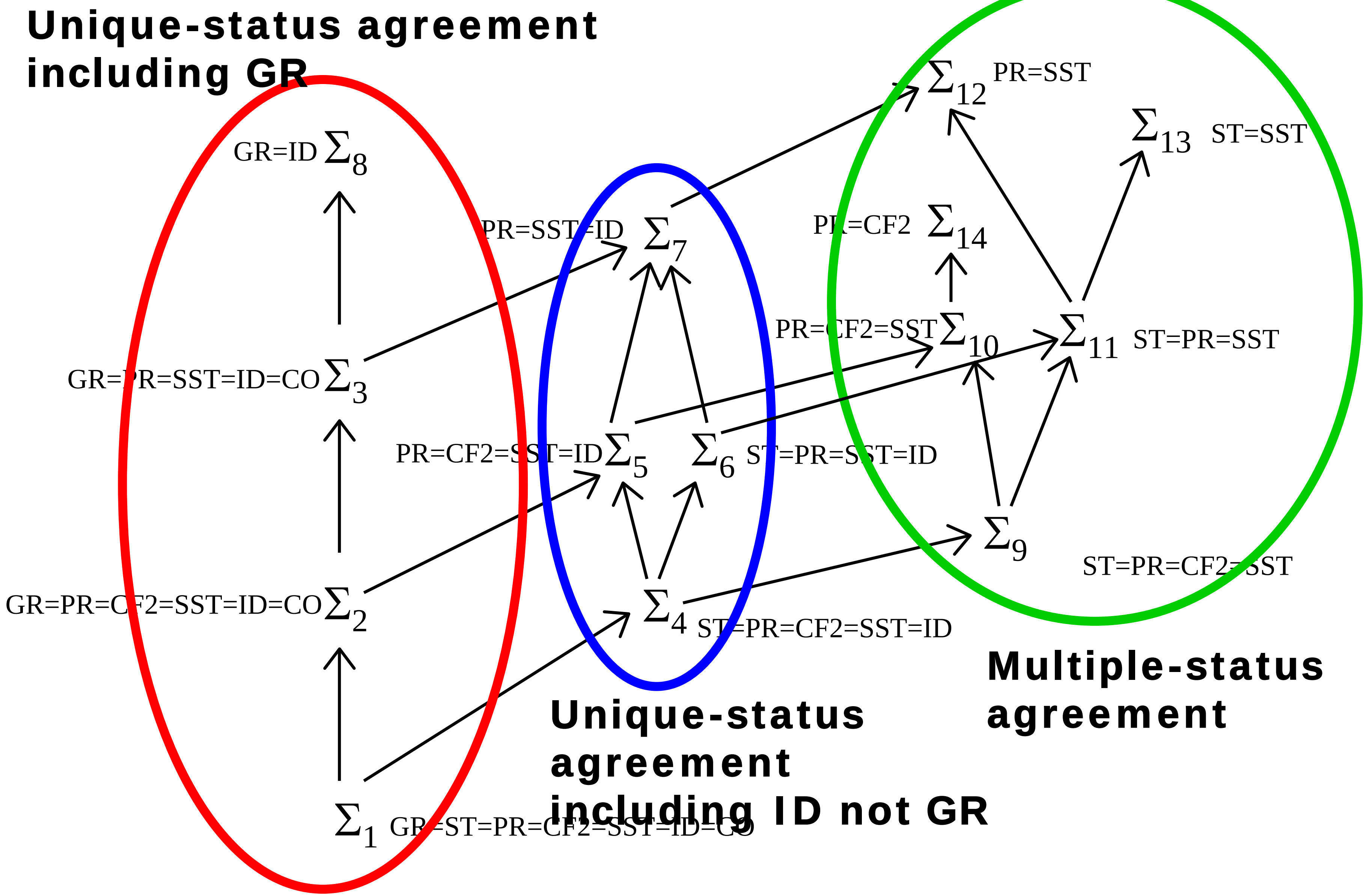
Unique-status agreement including GR



A synthetic view



A synthetic view



Conclusions

A systematic analysis of agreement providing a reference framework independent of any topological characterization

Next steps:

analyzing relationships between agreement classes and topological families of argumentation frameworks

considering other argumentation semantics