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Natural Calculation

Gentzen introduces in his famous paper "Untersuchungen über das logische Schließen" two types of calculi for formal syntactical reasoning. Before defining the sequent calculus, which is a technically convenient calculus for proving his Hauptsatz, Gentzen discusses first the so called calculus of natural deduction. The main advantage of natural deduction is its close relationship to informal mathematical reasoning. In Gentzen's own words:[1]

> We wish to set up a formalism that reflects as accurately as possible the actual logical reasoning involved in mathematical proofs. (p. 74)

> The calculus lends itself in particular to the formalization of mathematical proofs. (p. 80)

Besides logical argumentation, mathematicians also do some calculations within proofs. The usual way to handle this is to extend the calculus by some identity rules and axioms. This allows the treatment of equality statements in a technically perfect way. But due to the involved axioms the resulting proofs seem to be somehow artificial.

In our talk, we introduce an alternative approach, which allows the formalisation of such calculations in a more natural way: we add some term inference rules, which allow to calculate directly with terms before establishing some equality statements.

As a next step, we will generalise the introduced method to allow the treatment of other binary relations as, for example, $(\cdot \leq \cdot)$ in arithmetics, $(\cdot \subseteq \cdot)$ in set theory or even to some philosophically motivated relations describing the real world.

If time permits, we will consider, how to handle relations with other arities, and also have a look at other strategies for syntactical subatomic reasoning.

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[1] Quotes taken from: *The Collected Papers of Gerhard Gentzen*, M. E. Szabo (Editor), North Holland Publishing Company, 1969