Scenario Update
Applied to Causal Reasoning

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Workshop Micrac Toulouse
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Scenario Update applied to Causal Reasoning

What would have happened if something had been different in a scenario?
⇒ Scenario Update

Overview:
• Reminder on Belief Change
• A Toy Example
• Extrapolation
• Scenario Update
• Causality
Reminder on Belief Change

- I know that a given door can only be one of those:
Reminder on Belief Change

- I know that a given door can only be one of those:

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« the door is red »
Reminder on Belief Change

- I know that a given door can only be one of those:

« the door is red »
(I learn that it was red)
Reminder on Belief Change

- I know that a given door can only be one of those:

\[\text{« the door is red »} \quad \text{(I learn that it was red)}\]

\[\text{« the door is red »} \quad \text{(I learn that the door has been painted in red)}\]
Reminder on Belief Change

- I know that a given door can only be one of those:

```
« the door is red »
(I learn that it was red)
```

```
« the door is red »
(I learn that the door has been painted in red)
```
Reminder on Belief Change

- I know that a given door can only be one of those:

```
REVISION
piece of info: « the door is red»
(I learn that it was a red one)

UPDATE
piece of info: « the door is red»
(I learn that the door has been painted in red)
```
Reminder on Belief Change

- I know that a given door can only be one of those:

  
  ```
  REVISION
  piece of info: « the door is orange »
  ```
Reminder on Belief Change

- I know that a given door can only be one of those:

  \[\text{REVISION piece of info: « the door is orange»}\]
Reminder on Belief Change

- I know that a given door can only be one of those:

```
REVISION
piece of info: « the door is orange»
```

```
UPDATE
piece of info: « the door is orange»
```
Overview

What would have happened if something had been different in a scenario?

⇒ Scenario Update

✓ Reminder on Belief Change
✓ A Toy Example
  • Extrapolation
  • Scenario Update
  • Causality
A Spy Story

- I am in contact with two agents both of whom live alternatively in Toulouse and London. I received a postcard from London but I couldn’t read the signature. It was from one of them but I don’t know which.
A Spy Story

- I am in contact with two agents both of whom live alternatively in Toulouse and London. I received a postcard from London but I couldn’t read the signature. It was from one of them but I don’t know which.
  - a or b was in London at time point 1

\[
\begin{align*}
& m1 = a \quad b \\
& m2 = a \neg b \\
& m3 = \neg a \quad b \\
& m4 = \neg a \neg b \\
\end{align*}
\]
A Spy Story

- I am in contact with two agents both of whom live alternatively in Toulouse and London. I received a postcard from London but I couldn’t read the signature. It was from one of them but I don’t know which.

- I learned that, the day after, they had met each other (since they have exchanged secret documents).
  - at 2 either they both were in London or they both were in Toulouse.

\[
\begin{align*}
\Sigma : & \quad a \vee b \quad a \leftrightarrow b \\
\sum_{t=1}^{2} & \\
\text{m1} & = a \quad b \\
\text{m2} & = a \neg b \\
\text{m3} & = \neg a \quad b \\
\text{m4} & = \neg a \neg b
\end{align*}
\]
A Spy Story

• I am in contact with two agents both of whom live alternatively in Toulouse and London. I received a postcard from London but I couldn’t read the signature. It was from one of them but I don’t know which.
• I learned that, the day after, they had met each other.
• I know that one of the agents was seen in Toulouse two days later, but I don’t know which.
Spy Story Extrapolation

A possible trajectory:

\[
\begin{align*}
\text{t:} & \quad 1 & 2 & 3 \\
\Sigma: & \quad a \lor b & a \iff b & \neg a \lor \neg b \\
m1 &= a \quad b \\
m2 &= a \neg b \\
m3 &= \neg a \quad b \\
m4 &= \neg a \neg b
\end{align*}
\]

b flies to Toulouse
Spy Story Extrapolation

They prefer not to travel together.

- A surprising trajectory:

\[ t : 1 \quad 2 \quad 3 \]
\[ \Sigma : a \lor b \quad a \leftrightarrow b \quad \neg a \lor \neg b \]

- \[ m_1 = a \quad b \]
- \[ m_2 = a \quad \neg b \]
- \[ m_3 = \neg a \quad b \]
- \[ m_4 = \neg a \quad \neg b \]
The four less surprising trajectories when they are not frequent flyers:
Two questions

• I learn that Gatwick airport in London was closed (air traffic controller strike) at time point 1:
  ➢ "Gatwick departure closed"$^{(1)}$

• I wonder what would have happened if Gatwick Airport had been closed at time point 1 …
  ➢ "Gatwick departure closed"$^{(1)}$
Question 1: Gatwick was closed at time 1

I shall correct (revise) my initial beliefs.

\[
\begin{align*}
   \Sigma & : \ a \lor b \quad a \leftrightarrow b \quad \neg a \lor \neg b \\
   m1 & = \ a \quad b \\
   m2 & = \ a \neg b \\
   m3 & = \neg a \quad b \\
   m4 & = \neg a \neg b
\end{align*}
\]
Question 2:
What would have happened if...

I am confident in my previous beliefs, but I want to make a hypothesis.

- I shall consider every initial possibility and make it evolve.
Question 2: What would have happened if...

I am confident in my previous beliefs, but I want to make a hypothesis.

- I shall consider every initial possibility and make it evolve.
- Trajectory 1:

\[
\begin{align*}
m1 & = a \ b \\
m2 & = a \neg b \\
m3 & = \neg a \ b \\
m4 & = \neg a \neg b
\end{align*}
\]
Question 2:
What would have happened if...

I am confident in my previous beliefs, but I want to make a hypothesis.

- I shall consider every initial possibility and make it evolve.
- Trajectory 1: no change

```
t : 1  2  3
m1 = a  b  
m2 = a ¬b
m3 = ¬a  b
m4 = ¬a ¬b
```

Gatwick closed

b flies to Toulouse
Question 2: What would have happened if…

I am confident in my previous beliefs, but I want to make a hypothesis.

- I shall consider every initial possibility and make it evolve.
- Trajectory 2: no change
Question 2: What would have happened if...

I am confident in my previous beliefs, but I want to make a hypothesis.

- I shall consider every initial possibility and make it evolve.
- Trajectory 3 must change!

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<tr>
<td>m1</td>
<td>a b</td>
<td></td>
<td></td>
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<tr>
<td>m2</td>
<td>a\neg b</td>
<td></td>
<td></td>
</tr>
<tr>
<td>m3</td>
<td>\neg a b</td>
<td>a flies to</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Toulouse</td>
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<tr>
<td>m4</td>
<td>\neg a \neg b</td>
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Question 2: What would have happened if…

I am confident in my previous beliefs, but I want to make a hypothesis.

- I shall consider every initial possibility and make it evolve.
- Trajectory 3 must change!

\[
\begin{align*}
m1 &= a \ b \\
m2 &= a \neg b \\
m3 &= \neg a \ b \\
m4 &= \neg a \neg b
\end{align*}
\]
Question 2: What would have happened if…

I am confident in my previous beliefs, but I want to make a hypothesis.

- I shall consider every initial possibility and make it evolve.
- Trajectory 3 must change!

\[
\begin{align*}
\text{t:} & \quad 1 & \quad 2 & \quad 3 \\
\text{m1} = & \quad a \quad b \\
\text{m2} = & \quad a \neg b \\
\text{m3} = & \quad \neg a \quad b \\
\text{m4} = & \quad \neg a \neg b
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\]
Question 2: What would have happened if…

I am confident in my previous beliefs, but I want to make a hypothesis.

- I shall consider every initial possibility and make it evolve.
- Trajectory 3 must change!

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<td>m1</td>
<td>a</td>
<td>b</td>
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<tr>
<td>m2</td>
<td>a</td>
<td>¬b</td>
</tr>
<tr>
<td>m3</td>
<td>¬a</td>
<td>b</td>
</tr>
<tr>
<td>m4</td>
<td>¬a</td>
<td>¬b</td>
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Question 2: What would have happened if…

I am confident in my previous beliefs, but I want to make a hypothesis.

- I shall consider every initial possibility and make it evolve.
- Trajectory 3 must change: 8 possibilities

\[
\begin{align*}
m1 &= a \ b \\
m2 &= a \neg b \\
m3 &= \neg a \ b \\
m4 &= \neg a \neg b
\end{align*}
\]
Question 2: What would have happened if...

I am confident in my previous beliefs, but I want to make a hypothesis.

- I shall consider every initial possibility and make it evolve.
- Trajectory 3 must change: (agents are not frequent flyers)
Question 2: What would have happened if...

I am confident in my previous beliefs, but I want to make a hypothesis.

- I shall consider every initial possibility and make it evolve.
- Trajectory 3 must change! (a: frequent flyer, b: no)
Question 2: What would have happened if...

I am confident in my previous beliefs, but I want to make a hypothesis.

- I shall consider every initial possibility and make it evolve.
- Trajectory 4 must change! (inert system)

\[
\begin{align*}
\text{m1} &= a \quad b \\
\text{m2} &= a \neg b \\
\text{m3} &= \neg a \quad b \\
\text{m4} &= \neg a \neg b
\end{align*}
\]
Question 2:
What would have happened if…

I am confident in my previous beliefs, but I want to make a hypothesis.

- I shall consider every initial possibility and make it evolve.
- Trajectory 4 must change! (a: frequent flyer, b: no)
Question 2: What would have happened if...

I am confident in my previous beliefs, but I want to make a hypothesis.

- I shall consider every initial possibility and make it evolve.
- A possible result (with an inert system):

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<tr>
<td>m2</td>
<td>a ¬b</td>
<td></td>
<td></td>
</tr>
<tr>
<td>m3</td>
<td>¬a b</td>
<td></td>
<td></td>
</tr>
<tr>
<td>m4</td>
<td>¬a ¬b</td>
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F. Dupin de Saint-Cyr Scenario Update Applied to Causal Reasoning
Question 2: What would have happened if...

I am confident in my previous beliefs, but I want to make a hypothesis.

- I shall consider every initial possibility and make it evolve.
- Another possible result (with $a$: frequent flyer, $b$: no):

$$
\begin{align*}
  m1 &= a \quad b \\
  m2 &= a \quad \neg b \\
  m3 &= \neg a \quad b \\
  m4 &= \neg a \quad \neg b
\end{align*}
$$
The question “what would happen if…”

- Given a story = sequence of observations
  - some facts are true at some time points
  - some events have occurred
- What would happen if a given formula had had another value at time t?
  - would the end of the story be the same?
  - what would have changed?
- Aim of this question:
  - assign responsibilities
  - distinguish influential/non-influential variables
  - ...
- Application to causal ascription
Our proposal

• Compute the best world evolutions satisfying the initial scenario
  (scenario extrapolation)
  ➢ less surprising trajectories

• Compute the closest trajectories satisfying the new observation
  (scenario update)
  ➢ closest less surprising trajectories
Overview

✓ Reminder on belief change
✓ A toy example
✓ Extrapolation

• Scenario Update
• Causality
Event-Based Extrapolation

- Event:
  - Operation that induces a change in the normal course of the evolution
  - Represented by 2 functions:
    - $k_e$: surprise degree associated to an event occurrence
      - Example: $a$ and $b$ prefer not to travel together, they are not frequent flyers
        - $k_e(\text{``a & b fly to Tlse''}) > k_e(\text{``a flies to Tlse''}) = k_e(\text{``b to Tlse''}) = k_e(\text{``a to London''}) = k_e(\text{``b to Ln''}) > k_e(\text{``no event''})=0$
Event-Based Extrapolation

- **Event:**
  - Operation that induces a change in the normal course of the evolution
  - Represented by 2 functions:
    - $k_e$: surprise degree associated to an event occurrence

- **Example:**
  - a and b prefer not to travel together, they are not frequent flyers
    - $k_e(\text{"a & b fly to Tlse"}) >$
    - $k_e(\text{"a flies to Tlse"}) = k_e(\text{"b to Tlse"}) = k_e(\text{"a to London"}) = k_e(\text{"b to Ln"}) >$
    - $k_e(\text{"no event"})=0$
  - a and b prefer not to travel together, a: frequent flyer, b: no
    - $k_e(\text{"a & b fly to Tlse"}) >$
    - $k_e(\text{"b to Tlse"}) = k_e(\text{"b to Ln"}) >$
    - $k_e(\text{"a to Tlse"}) = k_e(\text{"a to Ln"}) = k_e(\text{"no event"})=0$
Event-Based Extrapolation

- **Event:**
  - Operation that induces a change in the normal course of the evolution
  - Represented by 2 functions:
    - $k_e$: surprise degree associated to an event occurrence
    - $k_m$: surprise degree associated to a transition w.r.t. a given event

Example:
- It is impossible to take a flight to Toulouse if you are already in Toulouse:
  - $k_m$ (m4="a in Tlse and b in Tse", event="a flies to Tlse", m) = $\infty$
- If a takes a plane alone then b's location is not affected:
  - $k_m$ (m1="a and b in London", event="a flies to Tlse", m4) = $\infty$
- If a takes a plane then normally a arrives to its destination:
  - $k_m$ (m2="a and b in Tse", event="a flies to Tlse", m4) < $k_m$ (m2="a and b in Tse", event="a flies to Tlse", m2)
Event-Based Extrapolation

• Event:
  - Operation that induces a change in the normal course of the evolution
  - Represented by 2 functions:
    - $k_e$: surprise degree associated to an event occurrence
    - $k_m$: surprise degree associated to a transition w.r.t. a given event

• Example:
  - it is impossible to take a flight to Toulouse if you are already in Tse
    $k_m(m4=\text{"a in Tlse and b in Tse"}, \text{event=\"a flies to Tlse"}, \text{m1}) = \infty$
  - if a takes a plane alone then b's location is not affected
    $k_m(m1=\text{"a and b in London"}, \text{event=\"a flies to Tlse"}, \text{m4}) = \infty$
  - if a takes a plane then normally a arrives to its destination:
    $k_m(m2=\text{"a in Ln and b in Tlse"}, \text{event=\"a flies to Tlse"}, \text{m4}) < k_m(m2, \text{event=\"a flies to Tlse"}, \text{m3=\"a in Tlse b in Ln")}$
Extrapolation and Inertia

- **Event:**
  - Operation that induces a change in the normal course of the evolution
  - Represented by 2 functions:
    - $k_e$: surprise degree associated to an event occurrence
    - $k_m$: surprise degree associated to a transition w.r.t. a given event
- **The system is inert $\iff k_e$ and $k_m$ are such that**
  - if no event occurs then every change is surprising
  - each event occurrence is surprising
Extrapolation and Inertia

• **Event:**
  - Operation that induces a change in the normal course of the evolution
  - Represented by 2 functions:
    - $k_e$: surprise degree associated to an event occurrence
    - $k_m$: surprise degree associated to a transition w.r.t. a given event

• The system is **inert** $\iff$ $k_e$ and $k_m$ are such that
  - if no event occurs then every change is surprising
  - each event occurrence is surprising
  - if $a$ and $b$ are not frequent flyers we can consider that the system is inert
Extrapolation and Surprise degrees

- **Event:**
  - Operation that induces a change in the normal course of the evolution
  - Represented by 2 functions:
    - $k_e$: surprise degree associated to an event occurrence
    - $k_m$: surprise degree associated to a transition w.r.t. a given event
- The system is **inert** $\iff k_e$ and $k_m$ are such that
  - if no event occurs then every change is surprising
  - each event occurrence is surprising
- **Surprise of a trajectory** = for each time point
  - events surprise degree
  - + surprise degree of the transition given the events
Surprise of a trajectory

If the system is inert:

- $k(\tau) = 0$
- $k(\tau) = k_e("b flies to Tlse"(2))$
- $k(\tau) = k_e("a,b fly to Tlse"(1)) + k_e("a flies to London"(2))$
- $k(\tau) = k_m(m3, "a flies to Tlse",m3) + k_e("a flies to Tlse"(2)) = \infty$
- $\tau < \tau < \tau < \tau$
Extrapolation of a temporal formula

=> new temporal formula characterizing the less surprising trajectories

Σ = (a \(1\) \lor b \(1\)) \land (a \(2\) \iff b \(2\)) \land (\neg a \(3\) \lor \neg b \(3\))

Extrapolation(Σ) =

(a \(1\) \land b \(1\) \land a \(2\) \land b \(2\) \land ((bfT \(2\) \land a \(3\) \land \neg b \(3\)) \lor (afT \(2\) \land \neg a \(3\) \land b \(3\))))
\lor (\neg a \(2\) \land \neg b \(2\) \land \neg a \(3\) \land \neg b \(3\) \land ((a \(1\) \land \neg b \(1\) \land afT \(2\)) \lor (\neg a \(1\) \land b \(1\) \land bfT \(2\))))
Overview

✓ Reminder on Belief Change
✓ A toy example
✓ Extrapolation
✓ Scenario Update
  • Causality
Scenario update

• Update requires (Katsuno-Mendelzon 91)
  ➢ a family of preference relations
  ➢ \( \tau^1 \leq_\tau \tau^2 \) compares trajectories \( \tau^1 \) and \( \tau^2 \) w.r.t. a given trajectory \( \tau \)

• Several proposals
  ➢ compare the number of fact values differing between \( \tau \) and \( \tau^1 \) to that of \( \tau \) and \( \tau^2 \)
  ➢ compare the surprise degree of the events differing between \( \tau \) and \( \tau^1 \) to that of \( \tau \) and \( \tau^2 \)
  ➢ minimize chronologically the differences between events and then minimize fact distances
Chronological preference relation

\[ \begin{align*}
\text{in } \tau_2: & \quad \text{no event occurs at time 1 while in } \tau \text{ "a flies to Tlse",} \\
& \quad \text{at time 2: no event occurs in the two trajectories} \\
\text{in } \tau_1: & \quad \text{at time 1 same event than in } \tau, \\
& \quad \text{at time 2: no event occurs in the two trajectories} \\
\text{in } \tau_3: & \quad \text{at time 1 same event than in } \tau, \\
& \quad \text{at time 2: event "b flies to London" while no event occurs in } \tau \\
\text{Hence } \tau_3 \text{ is closer from } \tau \text{ than } \tau_2 \\
\tau_3 \text{ is closer from } \tau \text{ than } \tau_2 \text{ since a difference occurs at time 1 for } \tau_2 \text{ while only at 2 for } \tau_3 \\
\end{align*} \]

\[ \tau \preceq \tau_1 \preceq \tau_3 \preceq \tau_2 \]

F. Dupin de Saint-Cyr  
Scenario Update Applied to Causal Reasoning
Scenario update (2)

Update of a temporal formula $f$ by a formula $\varphi(t)$:
- compute for each trajectory $\tau$ in Extrapolation ($f$)
- the closest trajectories (w.r.t. $\leq_\tau$) satisfying $\varphi(t)$

• Example: $\Sigma \Diamond$ ("Gatwick departure closed"$_{(1)}$)

\[ t : \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \]

- m1 = a b
- m2 = a $\neg$ b
- m3 = $\neg$a b
- m4 = $\neg$a $\neg$b
Overview

✓ Reminder on Belief Change
✓ A toy example
✓ Extrapolation
✓ Scenario Update
✓ Causality
Causality, update and extrapolation

- Event causation: find a concrete cause in a given story

- New definition of a cause based on counterfactual:
  - if the cause had not been there the conclusion would not have been obtained

Given a formula $f$ describing a scenario

\[ A(t) \text{ causes } B_{(t+k)} \text{ iff } \]

\[ \text{Extrapolation (f) satisfies } A(t) \land B_{(t+k)} \]

\[ f \not\diamond \neg A(t) \text{ do not satisfy } B_{(t+k)} \]
"Gatwick departure not closed at t=1" is a cause of the meeting of the two agents at t=2?

- In Extrapolation(Σ): the event "Gatwick departure closed" did not happen at t=1 and the agents met at t=2,
"Gatwick departure open at t=1" is a cause of the meeting of the two agents at t=2?

- In Extrapolation(Σ): the event "Gatwick departure closed" did not happen at t=1 and the agents met at t=2,
- If Gatwick had been closed at t=1 then they would have had two possibilities to meet each other,
- Hence, this is not a cause of their meeting.
Conclusion

• Our contribution:
  - development of event-based extrapolation
  - claim: “what would have happened if ….”
    amounts to computing a scenario update
  - causality definition in terms of scenario update

• Further research:
  - perceived causality
  - DNA sequences alignment => distance between trajectories
Distance between trajectories and DNA sequences alignment

m1 = a b
m2 = a¬ b
m3 = ¬a b
m4 = ¬a¬ b