Reasoning about the opinions of groups of agents

Asma Belhadi[1] and Didier Dubois[2] and Faiza Khellaf-Haned[1] and Henri Prade[2]

1. LRIA, Univ. Sci. & Tech. Houari Boumediene, Alger

2. IRIT, Université de Toulouse

Introduction

- reasoning about pieces of (uncertain) information

  held by subgroups of agents

  (p, A) "all agents in A are certain that p is true"

- *not so much* to try to take the best of the information provided by sets of agents viewed as sources as in fusion

  rather to understand what claims a group of agents supports *with what other groups they are in conflict*, about what

- to distinguish the individual inconsistency of agents from the global inconsistency of a group of agents

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- multiple-agent logic
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- *multiple-agent* possibilistic logic

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Multiple-agent logic - Syntax

- pairs (p_i, A_i) $\quad p_i$ proposition $\quad A_i \neq \emptyset$ subset

  of agents $A_i \subseteq ALL$

- multiple-agent logic base = conjunction of such pairs

- $(\neg p \vee q, A), (p \vee r, B) \vdash (q \vee r, A \cap B))$

- inconsistency of K: $inc(K) = \cup\{A | K \vdash (\bot, A)\}$

- $inc(K)$ subset of the agents individually inconsistent

- one may have $inc(K) = \emptyset$ even if K^* is inconsistent

  $K^* = \{p_i | (p_i, A_i) \in K\}$

- Example $K = \{(p, B), (\neg p, \overline{B})\}$

Multiple-agent logic - Semantics

- (p_i, A_i) $\mathbf{N}(p_i) \supseteq A_i$

  set necessity $\mathbf{N}(p \wedge q) = \mathbf{N}(p) \cap \mathbf{N}(q)$

- $\mathbf{N}(p) = \overline{\mathbf{\Pi}(\neg p)}$ and $\mathbf{\Pi}(p) = \bigcup_{\omega:\ \omega \models p} \pi_K(\omega)$

- ***set-valued*** possibility distribution $\pi_K(\omega) =$

  $\pi_{\{(p_i, A_i) | i=1,m\}}(\omega) = \bigcap_{i=1,m}([p_i](\omega) \cup \overline{A_i}))$

  $[p_i](\omega) = ALL$ if $\omega \models p_i$; $[p_i](\omega) = \emptyset$ otherwise

- $K \models (p, A)$ iff $\forall \omega, \pi_K(\omega) \subseteq \pi_{\{(p,A)\}}(\omega)$

- $inc(K) = \cap_\omega \overline{\pi_K(\omega)}$ $inc(K) = \emptyset$ weaker than

$\exists \omega, \pi_K(\omega) = ALL$: the agents are collectively consistent

Standard possibilistic logic - Syntax

- pairs (p_i, α_i) $\quad p_i$ proposition $\quad \alpha_i$ certainty level
- standard possibilistic base = conjunction of such pairs
- $(\neg p \vee q, \alpha), (p \vee r, \beta) \vdash (q \vee r, \min(\alpha, \beta))$
- inconsistency level of a base K:

  $inc(K) = \max\{\alpha | K \vdash (\bot, \alpha)\}$
- $inc(K) = 0$ iff K^* is consistent $\quad K^* = \{p_i | (p_i, \alpha_i) \in K\}$
- $K \vdash (p, \alpha)$ iff $K^*_\alpha \vdash p$ and $\alpha > inc(K)$

$$K^*_\alpha = \{(p_i, \alpha_i) \in K, \alpha_i \geq \alpha\}$$

Standard possibilistic logic - Semantics

- (p_i, α_i) $\quad N(p_i) \geq \alpha_i$

  necessity $N(p \wedge q) = \min(N(p), N(q))$

- $N(p) = 1 - \Pi(\neg p)$ and $\Pi(p) = \max_{\omega:\ \omega \models p} \pi_K(\omega)$

- possibility distribution

  $\pi_K(\omega) = \pi_{\{(p_i,\alpha_i) | i=1,m\}}(\omega)$

  $= \min_{i=1,m} \max([p_i](\omega), 1 - \alpha_i)$

  $[p_i](\omega) = 1$ if $\omega \models p_i$; $[p_i](\omega) = 0$ otherwise

- $K \models (p, \alpha)$ iff $\forall \omega, \pi_K(\omega) \leq \pi_{\{(p,\alpha)\}}(\omega)$

- $inc(K) = 1 - \max_\omega \pi_K(\omega)$

Multiple-agent possibilistic logic. Syntax

- pairs $(p_i, \alpha_i/A_i)$ p_i prop., α_i certainty level, A_i subs. agents
- Multiple-agent possibilistic logic base: conjunction of such pairs
- $(\neg p \vee q, \alpha/A), (p \vee r, \beta/B) \vdash (q \vee r, \min(\alpha, \beta)/A \cap B)$
- inconsistency level of a base K:
  $inc(K) = \cup\{\alpha/A \mid K \vdash (\perp, \alpha/A)\}$
- $inc(K)$ fuzzy subset of agents individually inconsistent

Multiple-agent possibilistic logic - Semantics

- $(p_i, \alpha_i / A_i)$ $\qquad$ $\mathbf{N}(p_i) \supseteq \alpha_i / A_i$

  $\alpha_i / A_i(a) = \alpha_i$ if $a_i \in A_i$ et $\alpha_i / A_i(a) = 0$ si $a_i \notin A_i$

  more generally $(p_i, \bigcup_j \alpha_{i,j} / A_{ij})$

  ***fuzzy* set-valued** necessity $\mathbf{N}(p \wedge q) = \mathbf{N}(p) \cap \mathbf{N}(q)$

- $\mathbf{N}(p) = \overline{\mathbf{\Pi}(\neg p)}$ and $\mathbf{\Pi}(p) = \cup_{\omega:\ \omega \models p} \pi_K(\omega)$

- $inc(K)$ describes to what extent

  different subsets of agents are inconsistent

  to different degrees

Conclusion

- Multiple agent possibilistic logic

  (A. Belhadi, D. Dubois, F. Khellaf-Haned, H. Prade)

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- extensions

  at most the agents in A believe p

  at least one agent in A believes p

  generalized possibilistic logic

Propagating trust

- agent a trusts agent b at level θ: $(\mathbf{b}, \theta/\{a\})$

  $\mathbf{b}$ stands for "any proposition about which b is certain"

- agent b is certain at level α that p is true:

$$(p, \alpha/\{b\}) \qquad (= ((p, \alpha), 1/\{b\}))$$

$$(p, \alpha/\{b\}), (\mathbf{b}, \theta/\{a\}) \vdash (p, \min(\alpha, \theta)/\{a\})$$

- agent a is certain at level $\min(\alpha, \theta)$ that p is true

substituting (p, α) to $\mathbf{b}$ in $(\mathbf{b}, \theta/\{a\})$ yields $((p, \alpha), \theta/\{a\})$,

it reduces to $(p, \min(\alpha, \theta)/\{a\})$ in the possibilistic setting