Second-hand information and imperfect information sources

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My neighbour tells me that his friend, who lives in the mountains, told him that the temperature has risen there. (second-hand information)

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My friend, who lives in the mountains, tells me that the temperature has risen there ?

My neighbour tells me that his friend, who lives in the mountains, told him that the temperature has risen there. (second-hand information)

My neighbour's wife tells me that her husband told her that his friend, who lives in the moutains, told him that the temperature has risen there. (third-hand information)

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Second question:
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On which conditions (about my neighbour, her wife, his friend) can I conclude that the temperature has risen ?

Logical framework

A propositional modal logic with two families of operators: B_i (beliefs) and R_i (reporting).

- Axioms schemata of classical propositional logic
- $(D_B)\quad B_i\neg p \rightarrow \neg B_i p$
- $(K_B)\quad B_i p \wedge B_i(p \rightarrow q) \rightarrow B_i q$
- (MP) $\frac{p \quad p\rightarrow q}{q}$
- (Nec_{B_i}) $\frac{p}{B_i p}$
- (RE_{R_i}) $\frac{p\leftrightarrow q}{R_i p\leftrightarrow R_i q}$

Properties of agents

[Demolombe 2004]: *valid* agents and *complete* agents

$$valid(i, \varphi) \equiv R_i\varphi \rightarrow \varphi$$
$$complete(i, \varphi) \equiv \varphi \rightarrow R_i\varphi$$

Properties of agents

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$$\begin{aligned} valid(i,\varphi) &\equiv R_i\varphi \rightarrow \varphi \\ complete(i,\varphi) &\equiv \varphi \rightarrow R_i\varphi \\ misinformer(i,\varphi) &\equiv R_i\varphi \rightarrow \neg\varphi \\ falsifier(i,\varphi) &\equiv \varphi \rightarrow R_i\neg\varphi \end{aligned}$$

Answering question (Q_1)

$$
\begin{gathered}
B_i R_j R_k \varphi \wedge B_i valid(j, R_k \varphi) \wedge B_i valid(k, \varphi) \rightarrow B_i \varphi \\
B_i R_j R_k \varphi \wedge B_i valid(j, R_k \varphi) \wedge B_i misinformer(k, \varphi) \rightarrow B_i \neg\varphi \\
B_i R_j R_k \varphi \wedge B_i misinformer(j, R_k \varphi) \wedge B_i complete(k, \varphi) \rightarrow B_i \neg\varphi \\
B_i R_j R_k \varphi \wedge B_i misinformer(j, R_k \varphi) \wedge B_i falsifier(k, \neg\varphi) \rightarrow B_i \varphi
\end{gathered}
$$

My neighbour told me that his friend told him that the temperature has risen. I know that when my neighbour says such a sentence, it is false i.e, I can infer that his friend did not tell him that the temperature has risen. But suppose that I know that his friend always inform him when the temperature rises. I can conclude that the temperature had not risen.

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What are the formulas H:

1. $\mathcal{B}_i \cup \{H\} \models B_i A$
2. H is composed with atomic formulas of the form $B_a valid(.,.)$, $B_a misinformer(.,.)$, $B_a complete(.,.)$ or $B_a falsifier(.,.)$.
3. $\mathcal{B}_i \cup \{H\}$ is consistent.
4. H is minimal, i.e, if H' statisfies the two previous constraints and $\models H \rightarrow H'$ then $\models H' \rightarrow H$.

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Find premisses which miss for deducing a given conclusion: **abductive reasoning** or equiv. **logical consequence generation** problem.

SOL-resolution (Skip Ordered Linear Resolution) is an inference rule sound and complete for generating

1. logical consequences of a set of clauses

4. minimal for the subsumption

2. belonging to a given language

3. satisfying some constraints

When *2, 3* satisfy some condition (**stable production field.**)

Inference rule defined in First Order Logic

Reformulate Q_2 in First Order Logic

$$
\begin{aligned}
Def_i = \{ & \forall x \forall y \quad B(i, valid(x,y)) \rightarrow (B(i, R(x,y)) \rightarrow B(i,y)), \\
& \forall x \forall y \quad B(i, misinformer(x,y) \rightarrow (B(i, R(x,y)) \rightarrow B(i, not(y))), \\
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(q_2): what are the clauses h:

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Conjecture: answering (q_2) is “equivalent” to answering (Q_2)

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Conjecture: answering (q_2) is "equivalent" to answering (Q_2)

We can apply SOL-resolution (constraints *2, 3* are OK)
... but with $ground(Def_i, n)$ instead of Def_i.

Conclusion

- Find the conditions under which the conjecture is OK
- Extend the model to topics
- Extend the model to uncertainty
- How can we decide that an agent is valid/misinformer/... ?