Normative Systems

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Outline

Programming Normative Systems

Monitoring Norms

Analysing Normative Systems

Background (1)

Multi-agent systems as promising candidate to construct distributed software systems that:

- are open and consist of individual autonomous and heterogenous agents
  - Open: agents dynamically enter and exit the system
  - Autonomy: each agent pursues its own objective.
  - Heterogeneity: internal state and operations of an agent may not be known to external entities.
- The overall objective of such systems can be achieved by coordinating the behavior of the involved agents.

Background (2)

Norms (e.g., obligations & prohibitions) are a popular candidate for coordination. Norms are standards of behaviour which prescribe certain behavior.

- Norms can be about states, actions, or behaviors, e.g.,
  - state norms: *Maximum length of papers is 15 pages*.
  - action norms: *PC members should not review own papers*.
  - behavior norms: *Reviews should be delivered in time.*
- Norm can be enforced by means of rewards/sanctions or regimented.
  - Enforcement: *sanction page limit violation by additional fees*.
  - Regimentation: *prevent reviewing conflicting papers*.
  - Enforcement: *blacklist PC members with late reviews*.

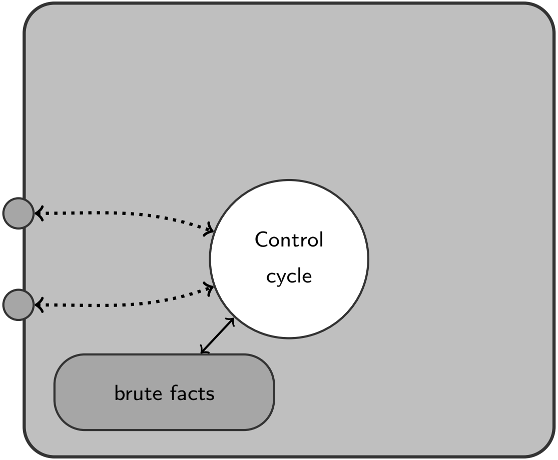
Background (3)

The development of normative systems requires implementation, verification, and analysis of norms.

- Norm implementation: design and develop a programming language that support the implementation of norms and rewards/sanctions.
- Norm Enforcement/Regimentation: monitors to observe norm violations.
- Norm analysis: use game theoretic tools to determine if a norm program can ensure the designer's objectives given some information about the involved agents.

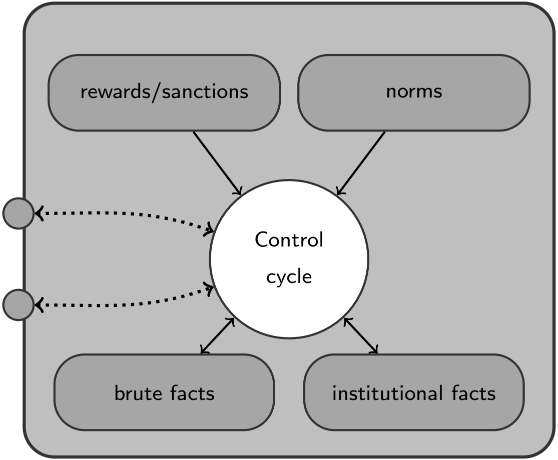
Programming Normative Systems

Coordination Component (IAT 2009, JLC 2011)



- Brute facts model the domain specific (environment) state, including action specifications;
- Agents modify brute facts by performing actions;
- Control cycle monitors agents actions and realizes their effects.

Coordination Component with Norms



- Brute facts model the domain specific (environment) state, including action specifications;
- Agents modify brute facts by performing actions;
- Ideal brute state described by norm;
- Active norms and norm violations stored by institutional facts;
- Norm obedience/violation might lead to rewards/sanctions;
- Control cycle monitors agents actions and realizes their effects in the context of the norms and rewards/sanctions.

Norms

- State-based norms (JLC 2011):

$F(\phi)$	Prohibited to achieve ϕ states
$O(\phi)$	Obliged to achieve ϕ states

- Action-based norms (IJCAI 2011):

$F(\phi, \alpha)$	Prohibited to perform action α in ϕ states
$O(\phi, \alpha)$	Obliged to perform action α in ϕ states

- Behaviour-based norms (IJCAI 2013):

$(cond, F(\phi), d)$	if $cond$, then prohibited to achieve ϕ before d
$(cond, O(\phi), d)$	if $cond$, then obliged to achieve ϕ before d

Examples of Behaviour-based Norms

Norms:

```
reviewdue(R):                                        } label
   < phase(review) and assigned(R,P)                 } condition
     , O(review(R,P))                                } obligation
     , phase(collect)>                               } deadline

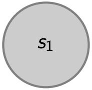
minreviews(P):
   < phase(submission) and paper(P)
     , O( nrReviews(P) >= 2 )
     , phase(collect)>

pagelimit(P):
   < phase(submission) and paper(P),
     , F(pages(P) > 15)
     , phase(review)>
```

Evolution of Obligations

Recall norm:

```
reviewdue(R):
< phase(review) and assigned(R,P), O(review(R,P)), phase(collect) >
```



brute facts	:	`{ phase(review), paper(p5) }`
inst. facts	:	`{ rea(john,chair) }`

Evolution of Obligations

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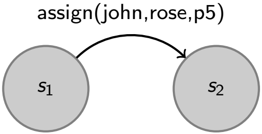

brute facts	:	{ `phase(review)`, `paper(p5)`, **assigned(rose,p5)** }
inst. facts	:	{ `rea(john,chair)` ,
		(reviewdue(rose) , **O(review(rose,p5))** , **phase(collect))** }

Evolution of Obligations

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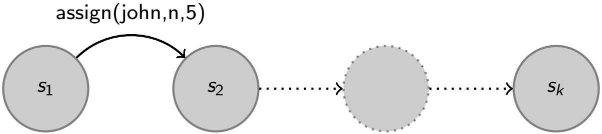
`< phase(review) and assigned(R,P), O(review(R,P)), phase(collect) >`



brute facts : `{ phase(review), paper(p5), assigned(rose,p5) }`

inst. facts : `{ rea(john,chair) ) ,`

(reviewdue(rose) , O(review(rose,p5)) , phase(collect)) }

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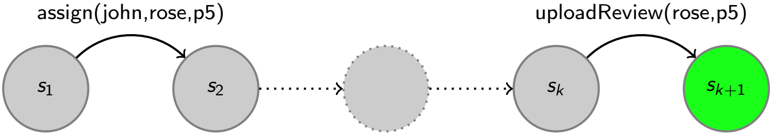

brute facts : { `phase(review)`, `paper(p5)`, `assigned(rose,p5)`, **review(rose,p5)** }

inst. facts : { `rea(john,chair)`, **obey(reviewdue(rose))** }

Evolution of Obligations

Recall norm:

```
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< phase(review) and assigned(R,P), O(review(R,P)), phase(collect) >
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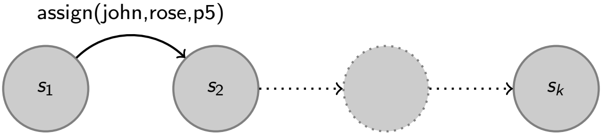

brute facts	:	`{ phase(review), paper(p5), assigned(rose,p5) }`
inst. facts	:	`{ rea(john,chair) ) ,`
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Evolution of Obligations

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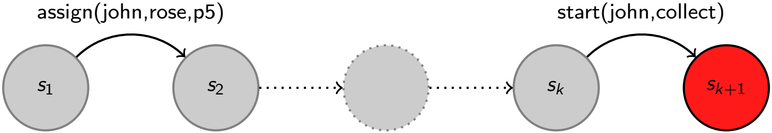
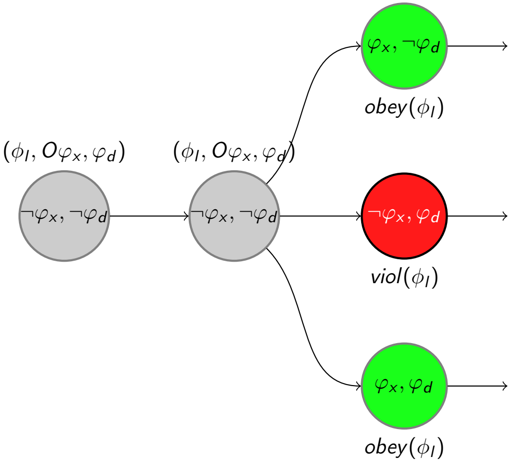


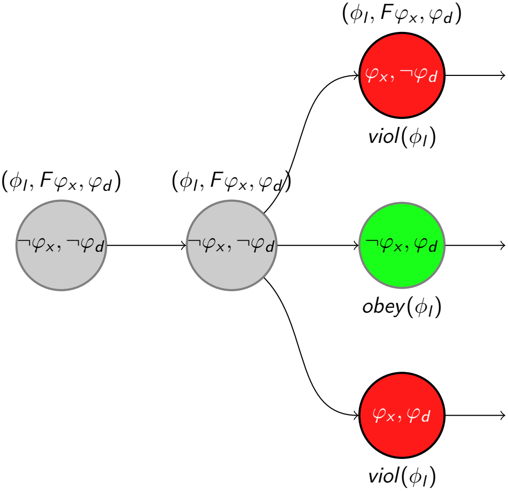
brute facts : { **phase(collect)**, `paper(p5)`, `assigned(rose,p5)` }

inst. facts : { `rea(john,chair)`, **viol(reviewdue(rose))** }

Behavior of an Obligation Summarized



Behavior of a Prohibition Summarized



Normative System Configuration

Definition (Configuration)

The state of a coordination component is a tuple $\langle \sigma_b, \sigma_i, \Delta \rangle$ with:

- σ_b a set of ground first-order atoms, the brute state;
- σ_i a set of ground first-order atoms, the institutional state;
- Δ a set of norms (Static);

For simplicity we ignore static Δ and present a configuration as:

$$\langle \sigma_b, \sigma_i \rangle$$

Triggering Norms

Recall norm:

```
reviewdue(R):
< phase(review) and assigned(R,P), O(review(R,P)), phase(collect) >
```

Definition (Norm Instantiation)

Given:

- $ns = \phi_l(\overline{v_1}) : \langle \varphi_c(\overline{v_2}), \mathbb{P}(\varphi_x(\overline{v_3})), \varphi_d(\overline{v_4}) \rangle$
- $\mathbb{P}$ either O or F
- $\overline{v_1}, \ldots, \overline{v_4}$ the sets of variables occurring in the formulae
- ground substitution θ

The function *inst* for instantiating *ns* with θ is defined as:

$$inst(ns, \theta) = (\ \phi_l(\overline{v_1})\theta\ ,\ \mathbb{P}(\varphi_x(\overline{v_3})\theta)\ ,\ \varphi_d(\overline{v_4})\theta\ )$$

Triggering Norms

Transition Rule

Given ground substitution θ, the rule for triggering of norms is defined as:

$$\frac{ns = (\phi_l : \langle \varphi_c, \mathbb{P}(\varphi_x), \varphi_d \rangle) \in \Delta \quad \sigma_b \models \varphi_c\theta \quad ni = inst(ns, \theta)}{\langle \sigma_b, \sigma_i \rangle \longrightarrow \langle \sigma_b, \sigma_i \cup \{ni\} \rangle}$$

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- When the condition of a norm is satisfied

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- We instantiate it and add it to the institutional facts

Monitoring Obligations

Transition Rule

The rule for violation of an obligation is defined as follows:

$$\frac{(\phi_l, O(\varphi_x), \varphi_d) \in \sigma_i \quad \sigma_b \not\models \varphi_x \quad \sigma_b \models \varphi_d}{\langle \sigma_b, \sigma_i \rangle \longrightarrow \langle \sigma_b, \ (\sigma_i \ \setminus \{ni\}) \ \cup \{viol(\phi_l) \ \}\rangle}$$

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- When an obligation is violated

Monitoring Obligations

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- We remove it from the institutional facts

Monitoring Obligations

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- We record its violation to the institutional facts

Monitoring Obligations

Transition Rule

The rule for fulfillment of an obligation is defined as follows:

$$\frac{(\phi_l, O(\varphi_x), \varphi_d) \in \sigma_i \quad \sigma_b \models \varphi_x}{\langle \sigma_b, \sigma_i \rangle \longrightarrow \langle \sigma_b,\ (\sigma_i \setminus \{ni\})\ \cup \{obey(\phi_l)\} \rangle}$$

Monitoring Obligations

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- When an obligation is fulfilled

Monitoring Obligations

Transition Rule

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Monitoring Obligations

Transition Rule

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- We record its fulfilment to the institutional facts

Properties

Proposition

Detached obligations remain in force until obeyed or violated.

I.e., for every trace $\langle \sigma_b^0, \sigma_i^0 \rangle \rightarrow^* \langle \sigma_b^n, \sigma_i^n \rangle$ *with* $\sigma_b^j \not\models \phi_x$ *and* $\sigma_b^j \not\models \phi_d$ *for* $0 \leq j \leq n$*, if* $(\phi_l, O(\phi_x), \phi_d) \in \sigma_i^0$*, then* $(\phi_l, O(\phi_x), \phi_d) \in \sigma_i^n$*.*

Proposition

Violation of detached obligations are recorded in deadline states.

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Violation is inevitable in case of conflicting norms.

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Properties

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Proposition

Violation is inevitable in case of conflicting norms.

I.e., for every trace $\langle \sigma_b^0, \sigma_i^0 \rangle \rightarrow^* \langle \sigma_b^n, \sigma_i^n \rangle$ *with* $(\phi_l, O(\phi_x), \phi_d) \in \sigma_i^0$ *and* $(\phi_{l'}, F(\phi_x), \phi_{d'}) \in \sigma_i^0$, *if* $\sigma_b^n \models \phi_d$ *and* $\sigma_b^j \not\models \phi_{d'}$ *for* $0 \leq j \leq n$ *then* $\exists 0 \leq k \leq n : \sigma_i^k \models viol(\phi_l)$ *or* $\sigma_i^k \models viol(\phi_{l'})$.

Monitoring Norms

Motivation

Existing work on normative multi-agent systems typically assume perfect monitoring.

We want to develop a very general framework in order to . . .

- . . . characterize monitors.
- . . . reason about monitors.
- . . . study the relation between monitors and norms.

Runs, Norms, Monitors

Given an (interpreted) transition system $(Q, \rightarrow, q_0, v)$, we define:

Definition

$\mathcal{R}$ as the set of runs, where $\mathcal{R} = \{q_0 q_1 \ldots \in Q^\omega \mid \forall n, q_n \rightarrow q_{n+1}\}$

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Definition

$\mathcal{N} \subseteq \mathcal{R}$ as a norm, which denotes a set of desired runs. Moreover, we say that a run r is a $\mathcal{N}$-violation iff $r \notin \mathcal{N}$.

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Definition

Given set $\mathcal{R}$, a monitor m is a function from $\mathcal{R}$ to $\mathcal{P}(\mathcal{R})$.

Monitor Types

Some extreme cases:

- For all $r \in \mathcal{R}$ $m(r) = \{r\}$ (perfect observation).
- For all $r \in \mathcal{R}$ $m(r) = \mathcal{R}$ (no observation).

Monitor Types

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- For all $r \in \mathcal{R}$ $m(r) = \mathcal{R}$ (no observation).

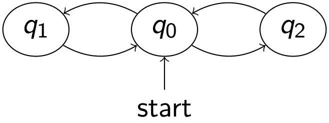
Definition

Let m be a monitor over $\mathcal{R}$. We say that m is

- broken iff there exists a $r \in \mathcal{R}$ such that $m(r) = \emptyset$.
- correct iff for all $r \in \mathcal{R}$ we have $r \in m(r)$.
- ideal iff for all $r \in \mathcal{R}$ we have $m(r) = \{r\}$.

Example

An example:



- $\mathcal{N} = \{(q_0\, q_1)^\omega\}$
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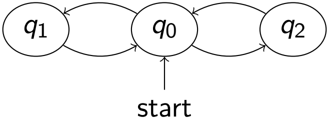

- $\mathcal{N} = \{(q_0\, q_1)^\omega\}$
- $m(r) = \{r' \in \mathcal{R} \mid r[1] = r'[1]\}$

Note that this monitor is correct, but not ideal.

Logical setting

$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \mathcal{U} \varphi \mid \bigcirc\varphi$$

Semantics:

- $\mathfrak{I}, r \models p$ iff $p \in v(r[0])$
- $\mathfrak{I}, r \models \neg\varphi$ iff $\mathfrak{I}, r \not\models \varphi$
- $\mathfrak{I}, r \models \varphi \vee \psi$ iff $\mathfrak{I}, r \models \varphi$ or $\mathfrak{I}, r \models \psi$
- $\mathfrak{I}, r \models \bigcirc\varphi$ iff $\mathfrak{I}, r[1, \infty] \models \varphi$
- $\mathfrak{I}, r \models \varphi \mathcal{U} \psi$ iff $i \geq 0$ such that $\mathfrak{I}, r[i, \infty] \models \psi$ and for all $0 \leq k < i$, $\mathfrak{I}, r[k, \infty] \models \varphi$

Moreover, $\mathfrak{I}, R \models \varphi$ iff $\forall r \in R : \mathfrak{I}, r \models \varphi$.

Definition

Let χ be an *LTL*-formula and $\mathfrak{I}$ be a transition system. The χ-norm in $\mathfrak{I}$ is the set $\{r \in \mathcal{R} \mid \mathfrak{I}, r \models \chi\}$. We simply write χ to refer to this norm.

Monitoring LTL-Norms

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Definition

We say that monitor m on input r detects a

- χ-violation iff $\mathfrak{I}, m(r) \models \neg\chi$;
- χ-compliance iff $\mathfrak{I}, m(r) \models \chi$; and
- χ-indifference iff both $\mathfrak{I}, m(r) \not\models \chi$ and $\mathfrak{I}, m(r) \not\models \neg\chi$.

Characterizing Monitors based on LTL-Norms

Definition

Let $\mathfrak{I}$ be a transition system, χ a norm, and m a monitor. We say that m makes a χ-classification error on r iff

- m detects a χ-violation on r and r is not a χ-violation (false negative); or
- m detects a χ-compliance on r and r is a χ-violation (false positive).

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- m detects a χ-compliance on r and r is a χ-violation (false positive).

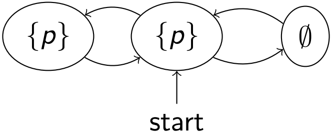
Definition

Let $\mathfrak{I}$ be a transition system, χ a norm and m a monitor. We say that m is

- χ-sound in $\mathfrak{I}$ iff for all $r \in \mathcal{R}$: $\mathfrak{I}, m(r) \models \neg\chi \Rightarrow \mathfrak{I}, r \models \neg\chi$.
- χ-complete in $\mathfrak{I}$ iff for all $r \in \mathcal{R}$: $\mathfrak{I}, r \models \neg\chi \Rightarrow \mathfrak{I}, m(r) \models \neg\chi$.
- χ-sufficient in $\mathfrak{I}$ iff m is χ-sound and χ-complete.

Example

Back to the example:



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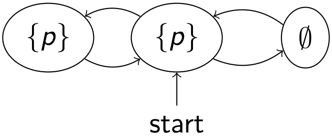

- $\chi = \Box p$
- $m(r) = \{r' \in \mathcal{R} \mid r[1] = r'[1]\}$

m is χ-sound: for all r: $\mathfrak{I}, m(r) \models \neg\Box p \Rightarrow \mathfrak{I}, r \models \neg\Box p$.
m is not χ-complete: exists r: $\mathfrak{I}, r \models \neg\Box p$ and $\mathfrak{I}, m(r) \not\models \neg\Box p$

LTL-Monitors & Composed Monitors

Definition

Let φ be an LTL-formula and $\mathfrak{I}$ be a transition system. The φ-monitor over $\mathfrak{I}$ is the function $m_\varphi : \mathcal{R} \to \mathcal{P}(\mathcal{R})$ defined as follows: $m_\varphi(r) := \{r' \in \mathcal{R} \mid \mathfrak{I}, r \models \varphi \text{ iff } \mathfrak{I}, r' \models \varphi\}$.

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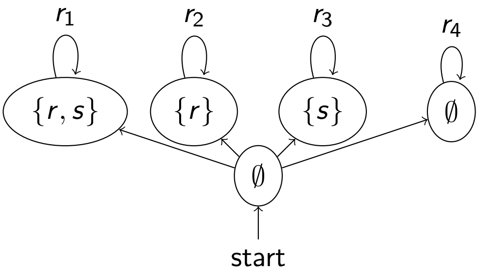
Definition

Let $m_1, m_2 : \mathcal{R} \to \mathcal{P}(\mathcal{R})$ be two monitors. We define the monitor $m \oplus m' : \mathcal{R} \to \mathcal{P}(\mathcal{R})$ as follows: $m \oplus m'(r) := m(r) \cap m'(r)$.

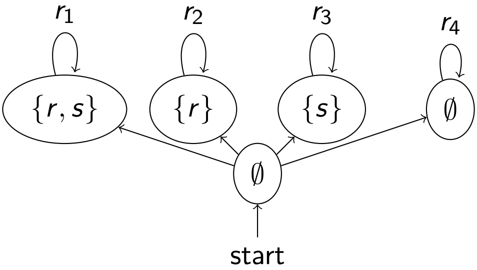
Example



Let $\chi = \bigcirc((r \wedge \neg s) \vee (\neg r \wedge s))$. Let $m_{\bigcirc r}$ and $m_{\bigcirc s}$ be two LTL-monitors. We have the following:

- $(m_{\bigcirc r} \oplus m_{\bigcirc s})(r_2) = \{r_2\}$
- $m_{(\bigcirc r \wedge \bigcirc s)}(r_2) = \{r_2, r_3, r_4\}$

Example



Let $\chi = \bigcirc((r \wedge \neg s) \vee (\neg r \wedge s))$. Let $m_{\bigcirc r}$ and $m_{\bigcirc s}$ be two LTL-monitors. We have the following:

- $(m_{\bigcirc r} \oplus m_{\bigcirc s})(r_2) = \{r_2\}$
- $m_{(\bigcirc r \wedge \bigcirc s)}(r_2) = \{r_2, r_3, r_4\}$

Only composed monitor $(m_{\bigcirc r} \oplus m_{\bigcirc s})$ is χ-sufficient.

Theorem

Let m_φ and m_ψ be two (or more) LTL-monitors, χ be an LTL-norm, and $\mathfrak{I}$ be a transition system. Let $\Sigma = \{\varphi \wedge \psi, \varphi \wedge \neg\psi, \neg\varphi \wedge \psi, \neg\varphi \wedge \neg\psi\}$. Then, the following statements are equivalent:

(a) *$m_\varphi \oplus m_\psi$ is χ-sufficient over $\mathfrak{I}$.*

(b) *for all $\xi \in \Sigma$, if $\neg\chi \wedge \xi$ is satisfiable on $\mathfrak{I}$ then $\mathfrak{I} \models \xi \rightarrow \neg\chi$.*

(c) *exists $\Sigma' \subseteq \Sigma$ such that $\mathfrak{I} \models (\bigvee_{\xi \in \Sigma'} \xi) \leftrightarrow \neg\chi$.*

Analysing Normative Systems

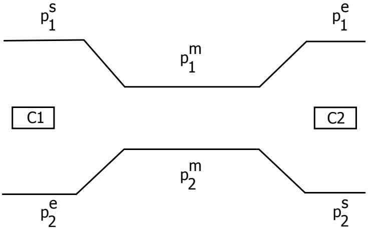
Norms as Mechanism Design (AAMAS 2011, IJCAI 2011)

Can specific behaviours be enforced by a normative environment program if agents follow their subjective preferences?

I.e.,

Does a set of norms and sanctions/rewards implements specific social choice functions (designer's objectives) in specific equilibria?

Example: A Road Scenario



- $s0$: the cars are at their starting positions, i.e., $p_1^s \wedge p_2^s$
- $s1$: car $C1$ is at ending and car $C2$ at starting position,i.e., $p_1^e \wedge p_2^s$
- $s2$: car $C1$ is at starting and car $C2$ at ending position,i.e., $p_1^s \wedge p_2^e$
- $s3$: the cars are jammed in the middle of the road,i.e., $p_1^m \wedge p_2^m$
- $s4$: the cars are at their ending positions, i.e., $p_1^e \wedge p_2^e$

Action Specification

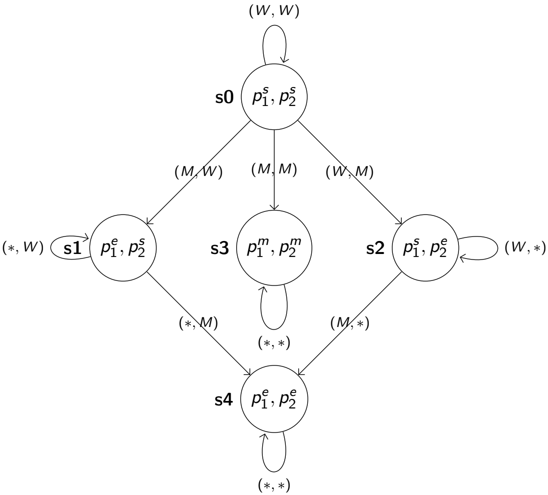
- Each car has two actions.
  - Move (M)
  - Wait (W), and
  - $* \in \{M, W\}$.
- Joint actions are specified in terms of their pre- and post-conditions.

$$
\begin{array}{lll}
\{p_1^s, p_2^s\} & (M, W) & \{p_1^e, p_2^s\} \\
\{p_1^s, p_2^s\} & (W, M) & \{p_1^s, p_2^e\} \\
\{p_1^s, p_2^s\} & (M, M) & \{p_1^m, p_2^m\} \\
\{p_1^s, p_2^e\} & (M, *) & \{p_1^e, p_2^e\} \\
\{p_1^e, p_2^s\} & (*, M) & \{p_1^e, p_2^e\}
\end{array}
$$

Other actions do not cause state change.

Transition System without Norms

Possible behaviours of the cars can be described by the following transition system.



Preferences: Cars & System Designer

Preferences profiles, λ_1 (egoistic) and λ_2 (social), are represented as lists of LTL formulae.

$$\begin{aligned}
\lambda_1 = \{ \quad & [(Xp_1^e \wedge \Box\neg s_1, 3), (\Diamond p_1^e \wedge \Box\neg s_1, 2), (\Box\neg s_1, 1), (\top, 0)] \ , \\
& [(Xp_2^e \wedge \Box\neg s_2, 3), (\Diamond p_2^e \wedge \Box\neg s_2, 2), (\Box\neg s_2, 1), (\top, 0)] \ \}
\end{aligned}$$

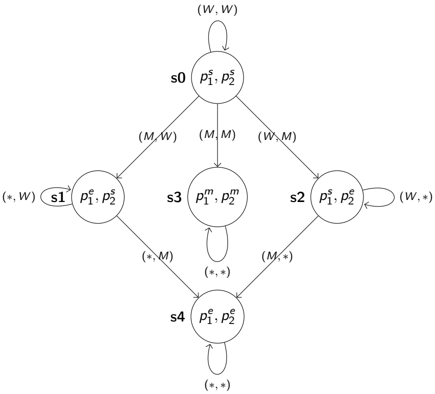
$$\begin{aligned}
\lambda_2 = \{ \quad & [(X(p_1^e \wedge p_2^e) \wedge \Box\neg s_1, 3), (Xp_1^e \wedge \Box\neg s_1, 2), (\Box\neg s_1, 1), (\top, 0)] \ , \\
& [(X(p_1^e \wedge p_2^e) \wedge \Box\neg s_2, 3), (Xp_2^e \wedge \Box\neg s_2, 2), (\Box\neg s_2, 1), (\top, 0)] \ \}
\end{aligned}$$

The preference of the system designer is represented by the following social choice function:

$$SCF(\lambda_i) = Xp_1^e \vee (Xp_1^e \wedge \Diamond p_2^e)$$

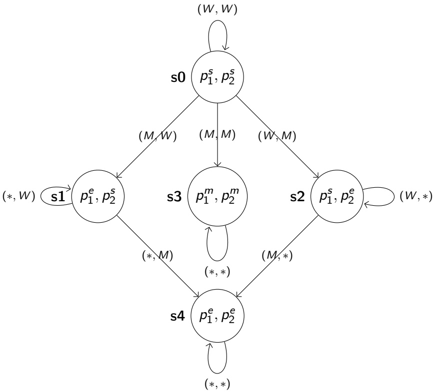
The designer prefers that first car reaches its end position directly.

Equilibrium Analysis



$1 \backslash 2$	$M**$	$WM*$	$WW*$
$M**$	03^*	014^*	011^*
$W*M$	024^*	00^*	00^*
$W*W$	022^*	00^*	00^*

Equilibrium Analysis

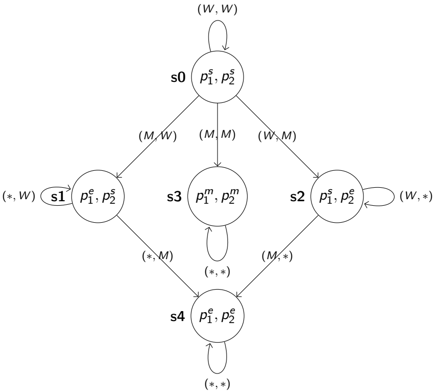


$1\backslash 2$	$M**$	$WM*$	$WW*$
$M**$	03^*	014^*	011^*
$W*M$	024^*	00^*	00^*
$W*W$	022^*	00^*	00^*

$1\backslash 2$	$M**$	$WM*$	$WW*$
$M**$	$1\backslash 1$	$3\backslash 2$	$3\backslash 1$
$W*M$	$2\backslash 3$	$1\backslash 1$	$1\backslash 1$
$W*W$	$1\backslash 3$	$1\backslash 1$	$1\backslash 1$

$$\lambda_1 = \{ \quad \begin{array}{l} [(Xp_1^e \wedge \Box\neg s_1, 3), (\Diamond p_1^e \wedge \Box\neg s_1, 2), (\Box\neg s_1, 1), (\top, 0)] \ , \\ [(Xp_2^e \wedge \Box\neg s_2, 3), (\Diamond p_2^e \wedge \Box\neg s_2, 2), (\Box\neg s_2, 1), (\top, 0)] \ \} \end{array}$$

Equilibrium Analysis

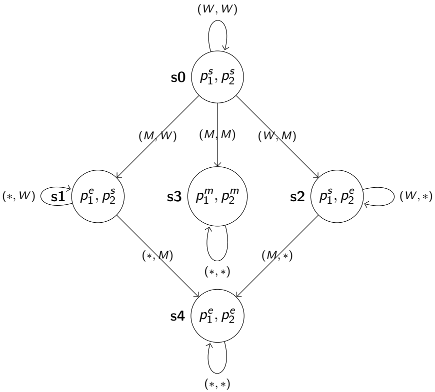


$1\backslash 2$	$M**$	$WM*$	$WW*$
$M**$	03^*	014^*	011^*
$W*M$	024^*	00^*	00^*
$W*W$	022^*	00^*	00^*

$1\backslash 2$	$M**$	$WM*$	$WW*$
$M**$	$1\backslash 1$	$3\backslash 2$	$3\backslash 1$
$W*M$	$\cancel{2\backslash 3}$	$1\backslash 1$	$1\backslash 1$
$W*W$	$1\backslash 3$	$1\backslash 1$	$1\backslash 1$

$$SCF(\lambda_i) = Xp_1^e \vee (Xp_1^e \wedge \Diamond p_2^e)$$

Equilibrium Analysis

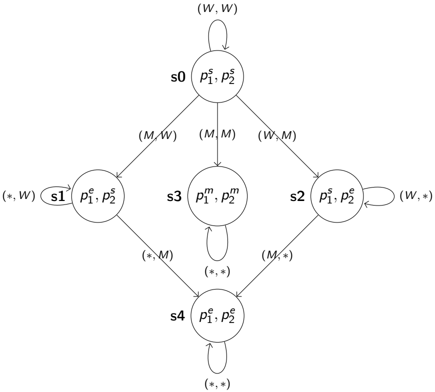


$1\backslash 2$	$M**$	$WM*$	$WW*$
$M**$	03^*	014^*	011^*
$W*M$	024^*	00^*	00^*
$W*W$	022^*	00^*	00^*

$1\backslash 2$	$M**$	$WM*$	$WW*$
$M**$	$1\backslash 1$	$2\backslash 1$	$2\backslash 1$
$W*M$	$1\backslash 2$	$1\backslash 1$	$1\backslash 1$
$W*W$	$1\backslash 2$	$1\backslash 1$	$1\backslash 1$

$$\lambda_2 = \{ \quad [(X(p_1^e \wedge p_2^e) \wedge \Box\neg s_1, 3), (Xp_1^e \wedge \Box\neg s_1, 2), (\Box\neg s_1, 1), (\top, 0)]\ ,$$
$$[(X(p_1^e \wedge p_2^e) \wedge \Box\neg s_2, 3), (Xp_2^e \wedge \Box\neg s_2, 2), (\Box\neg s_2, 1), (\top, 0)]\ \}$$

Equilibrium Analysis

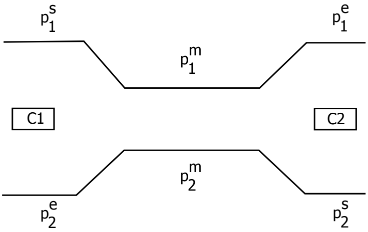


$1\backslash 2$	$M**$	$WM*$	$WW*$
$M**$	03^*	014^*	011^*
$W*M$	024^*	00^*	00^*
$W*W$	022^*	00^*	00^*

$1\backslash 2$	$M**$	$WM*$	$WW*$
$M**$	$1\backslash 1$	$2\backslash 1$	$2\backslash 1$
$W*M$	$\cancel{1\backslash 2}$	$1\backslash 1$	$1\backslash 1$
$W*W$	$\cancel{1\backslash 2}$	$1\backslash 1$	$1\backslash 1$

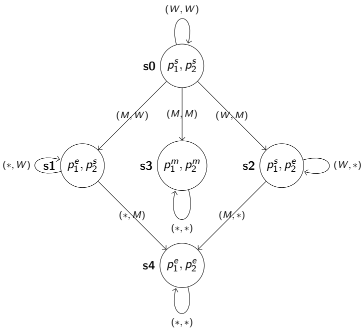
$$SCF(\lambda_i) = Xp_1^e \vee (Xp_1^e \wedge \Diamond p_2^e)$$

Example: Introducing Norms in the Road Scenario



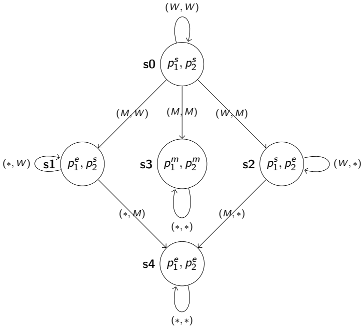
- The second car is prohibited to move in the start position, otherwise sanction s_2 will be imposed, i.e.,
$$F(p_1^s \wedge p_2^s \ , \ (*, W) \ , \ s_2)$$
- It is prohibited that cars wait on each other in the start position, otherwise sanction s_1 will be imposed, i.e.,
$$F(p_1^s \wedge p_2^s \ , \ (W, W) \ , \ s_1)$$

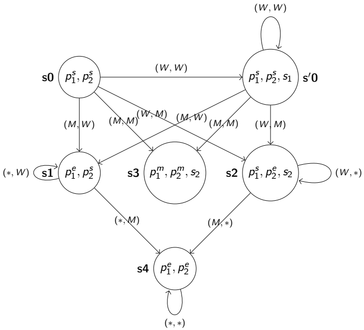
Norms and Norm Updates



- The second car is prohibited to move in the start position, otherwise sanction s_2 will be imposed, i.e.,
$$F(p_1^s \wedge p_2^s \ , \ (*, W) \ , \ s_2)$$
- It is prohibited that cars wait on each other in the start position, otherwise sanction s_1 will be imposed, i.e.,
$$F(p_1^s \wedge p_2^s \ , \ (W, W) \ , \ s_1)$$

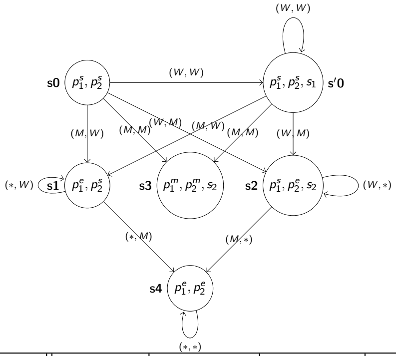
Norms and Norm Updates





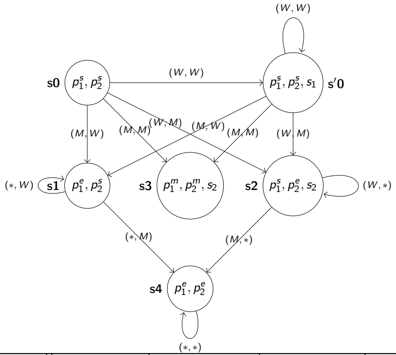
- The second car is prohibited to move in the start position, otherwise sanction s_2 will be imposed, i.e.,
$$F(p_1^s \wedge p_2^s\ ,\ (*, W)\ ,\ s_2)$$
- It is prohibited that cars wait on each other in the start position, otherwise sanction s_1 will be imposed, i.e.,
$$F(p_1^s \wedge p_2^s\ ,\ (W, W)\ ,\ s_1)$$

Equilibrium Analysis



$1\backslash 2$	$M***$	$WWW*$	$WMM*$	$WMW*$
$M***$	03^*	011^*	014^*	011^*
$WW*M$	024^*	$00'0'^*$	$00'24^*$	$00'24^*$
$WW*W$	022^*	$00'0'^*$	$00'22^*$	$00'22^*$
$WM*M$	024^*	$00'11^*$	$00'3^*$	$00'3^*$
$WM*W$	022^*	$00'11^*$	$00'3^*$	$00'3^*$

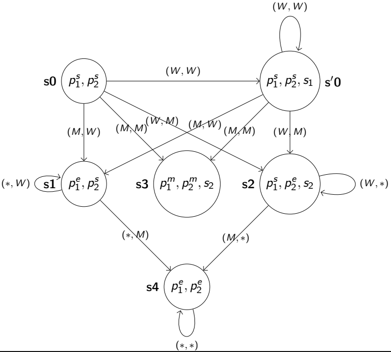
Equilibrium Analysis



$1\backslash 2$	$M{*}{*}{*}$	$WWW{*}$	$WMM{*}$	$WMW{*}$
$M{*}{*}{*}$	$1\backslash 0$	$3\backslash 1$	$3\backslash 2$	$3\backslash 1$
$WW{*}M$	$2\backslash 0$	$0\backslash 1$	$0\backslash 0$	$0\backslash 0$
$WW{*}W$	$1\backslash 0$	$0\backslash 1$	$0\backslash 0$	$0\backslash 0$
$WM{*}M$	$2\backslash 0$	$0\backslash 1$	$0\backslash 0$	$0\backslash 0$
$WM{*}W$	$1\backslash 0$	$0\backslash 1$	$0\backslash 0$	$0\backslash 0$

$$\lambda_1 = \{ \quad [(Xp_1^e \wedge \Box\neg s_1, 3), (\Diamond p_1^e \wedge \Box\neg s_1, 2), (\Box\neg s_1, 1), (\top, 0)] \;,$$
$$[(Xp_2^e \wedge \Box\neg s_2, 3), (\Diamond p_2^e \wedge \Box\neg s_2, 2), (\Box\neg s_2, 1), (\top, 0)] \; \}$$

Equilibrium Analysis



$1\backslash 2$	$M***$	$WWW*$	$WMM*$	$WMW*$
$M***$	$1\backslash 0$	$2\backslash 1$	$2\backslash 1$	$2\backslash 1$
$WW*M$	$1\backslash 0$	$0\backslash 1$	$0\backslash 0$	$0\backslash 0$
$WW*W$	$1\backslash 0$	$0\backslash 1$	$0\backslash 0$	$0\backslash 0$
$WM*M$	$1\backslash 0$	$0\backslash 1$	$0\backslash 0$	$0\backslash 0$
$WM*W$	$1\backslash 0$	$0\backslash 1$	$0\backslash 0$	$0\backslash 0$

$$\lambda_2 = \{ \quad [(X(p_1^e \wedge p_2^e) \wedge \Box\neg s_1, 3), (Xp_1^e \wedge \Box\neg s_1, 2), (\Box\neg s_1, 1), (\top, 0)] \ ,$$
$$[(X(p_1^e \wedge p_2^e) \wedge \Box\neg s_2, 3), (Xp_2^e \wedge \Box\neg s_2, 2), (\Box\neg s_2, 1), (\top, 0)] \ \}$$

Verification Problems

Let $\mathcal{M}$ be a transition system, N be a set of norms, $\mathcal{M} \upharpoonright N$ be the transition system $\mathcal{M}$ updated with N.

- ($Nash$, N)-implementation problem ($\mathrm{IP}_{\mathrm{N}}^{\mathrm{Nash}}$)
  Given a norm set N do the outcome paths of $\mathcal{M} \upharpoonright N$ satisfy SCF if agents follow $Nash$-equilibria strategy profiles?
- ($Nash$)-synthesis problem ($\mathrm{SP}^{\mathrm{Nash}}$)
  Is there a norm set M such that . . . ?

Verification Results

The following results are about state based norms and Nash equilibria!

Theorem (($Nash$, N)-implementation problem)

The problem $\mathrm{IP}_{\mathrm{N}}^{\mathrm{Nash}}$ *is* Π_2^P*-complete.*

Verification Results

The following results are about state based norms and Nash equilibria!

Theorem (($Nash$, N)-implementation problem)

The problem $\mathrm{IP}_{\mathrm{N}}^{\mathrm{Nash}}$ is Π_2^P-complete.

Theorem (($Nash$)-Synthesis problem)

The problem $\mathrm{SP}^{\mathrm{Nash}}$ is Σ_3^P-complete.

Conclusions and Future Research

- Development of normative systems.
- Monitors and Norms.
- Game theoretic analysis of normative systems.
- Norms in standard software technology.
- Monitoring, imperfect monitors, and approximated norms.
- Distributed normative systems.