Logical equivalences of models

Philippe Balbiani

- Automata

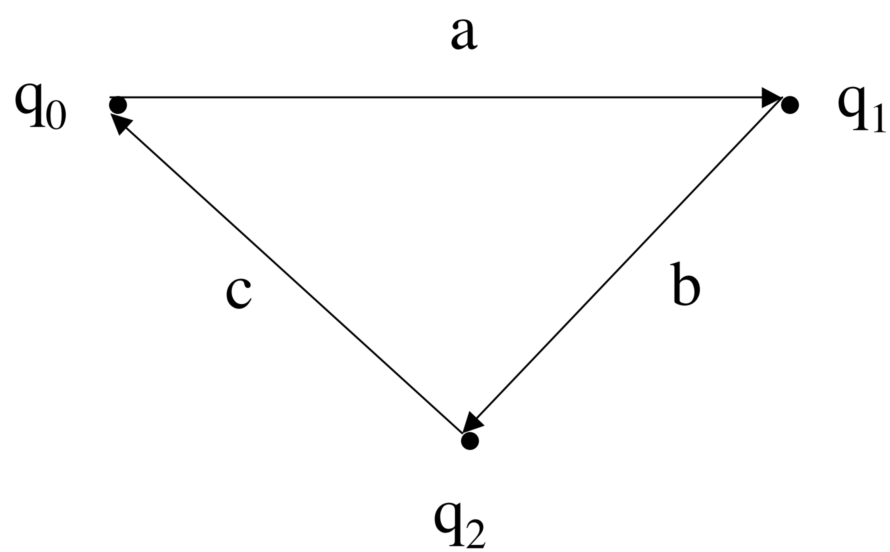
  $A = (Q, q_0, \Sigma, \rightarrow)$

  Q is a nonempty set of states

  q_0 is the initial state

  Σ is a nonempty set of actions

  $\rightarrow$ is a subset of $Q \times \Sigma \times Q$

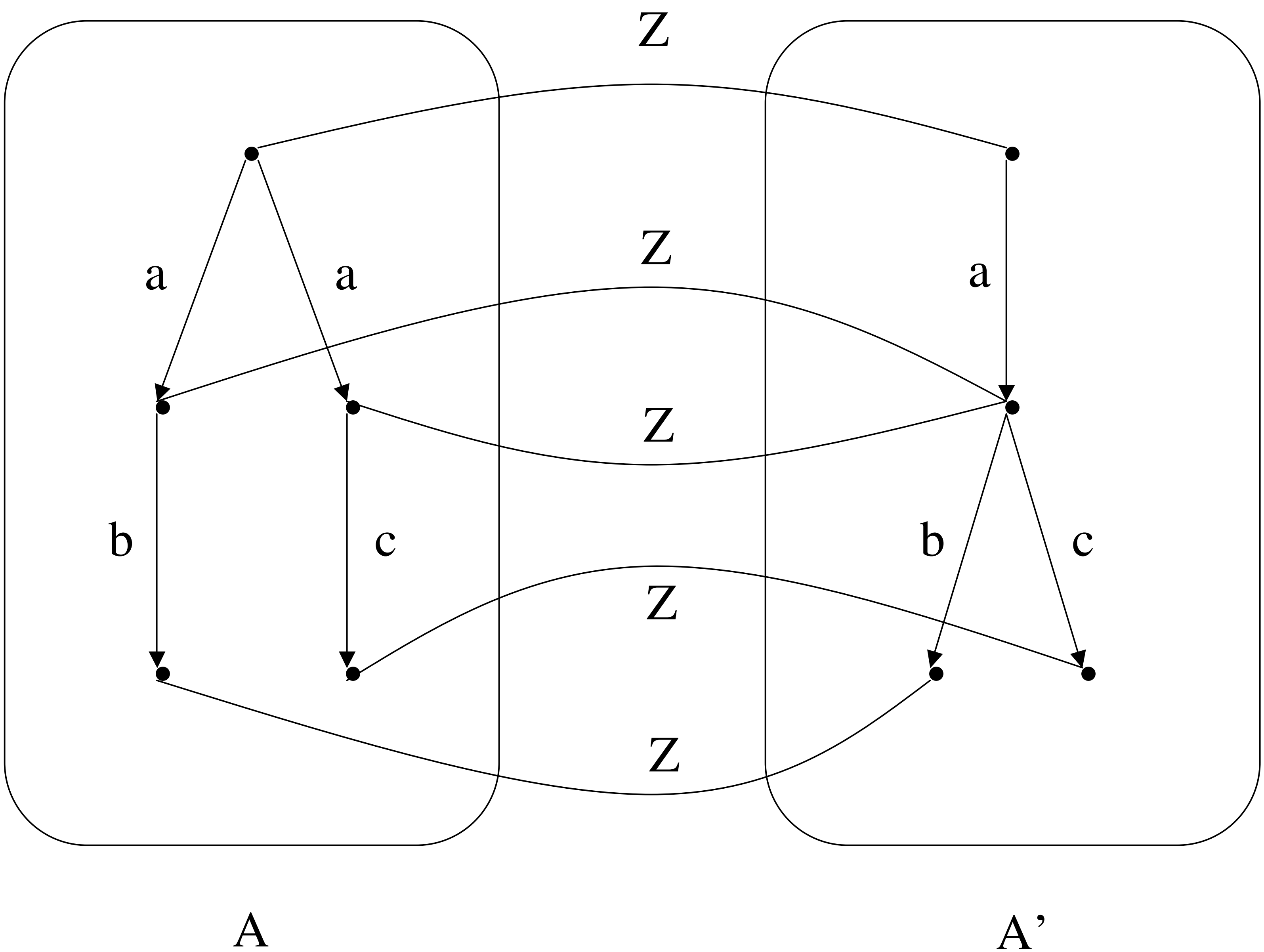


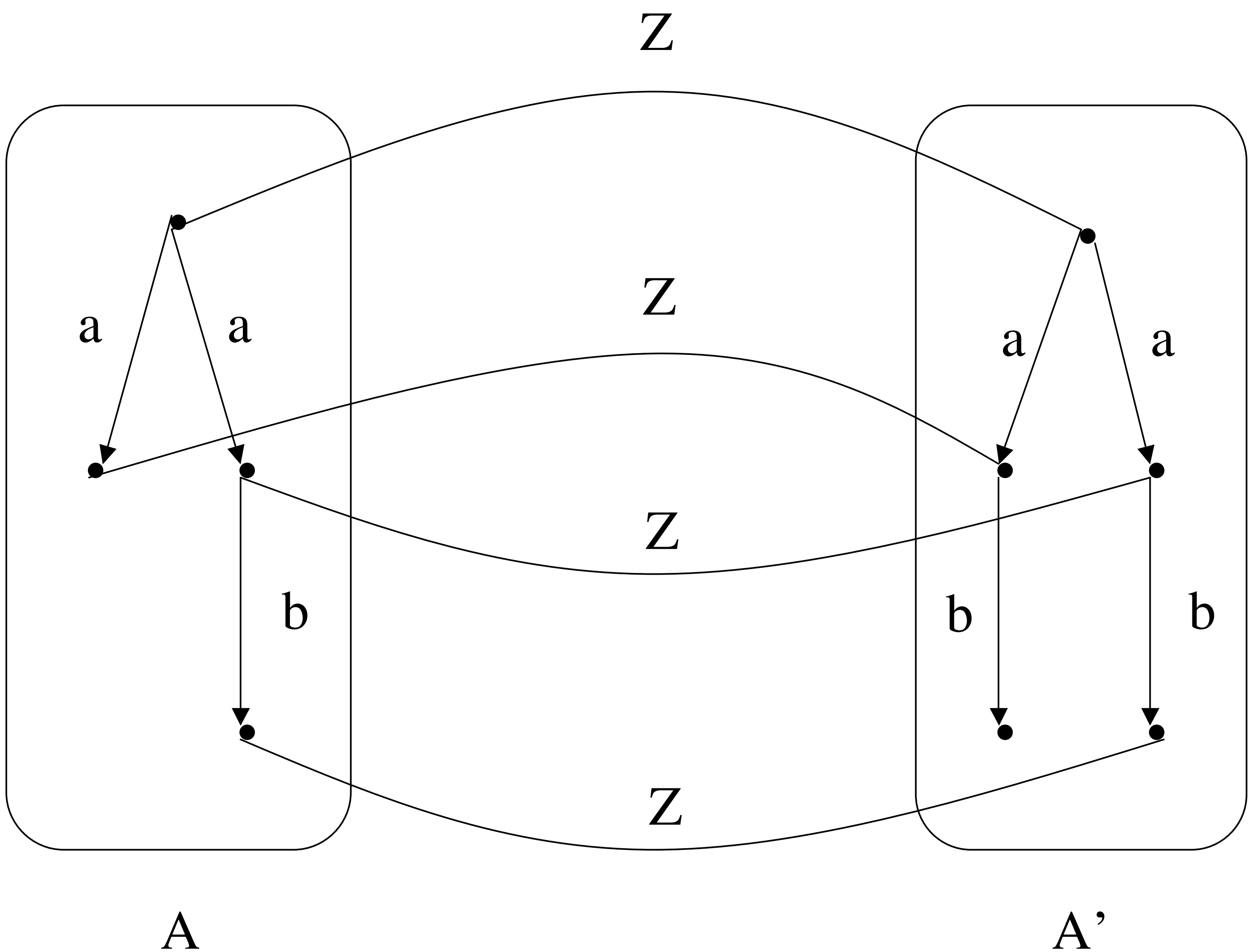
a
q0
q1
c
b
q2

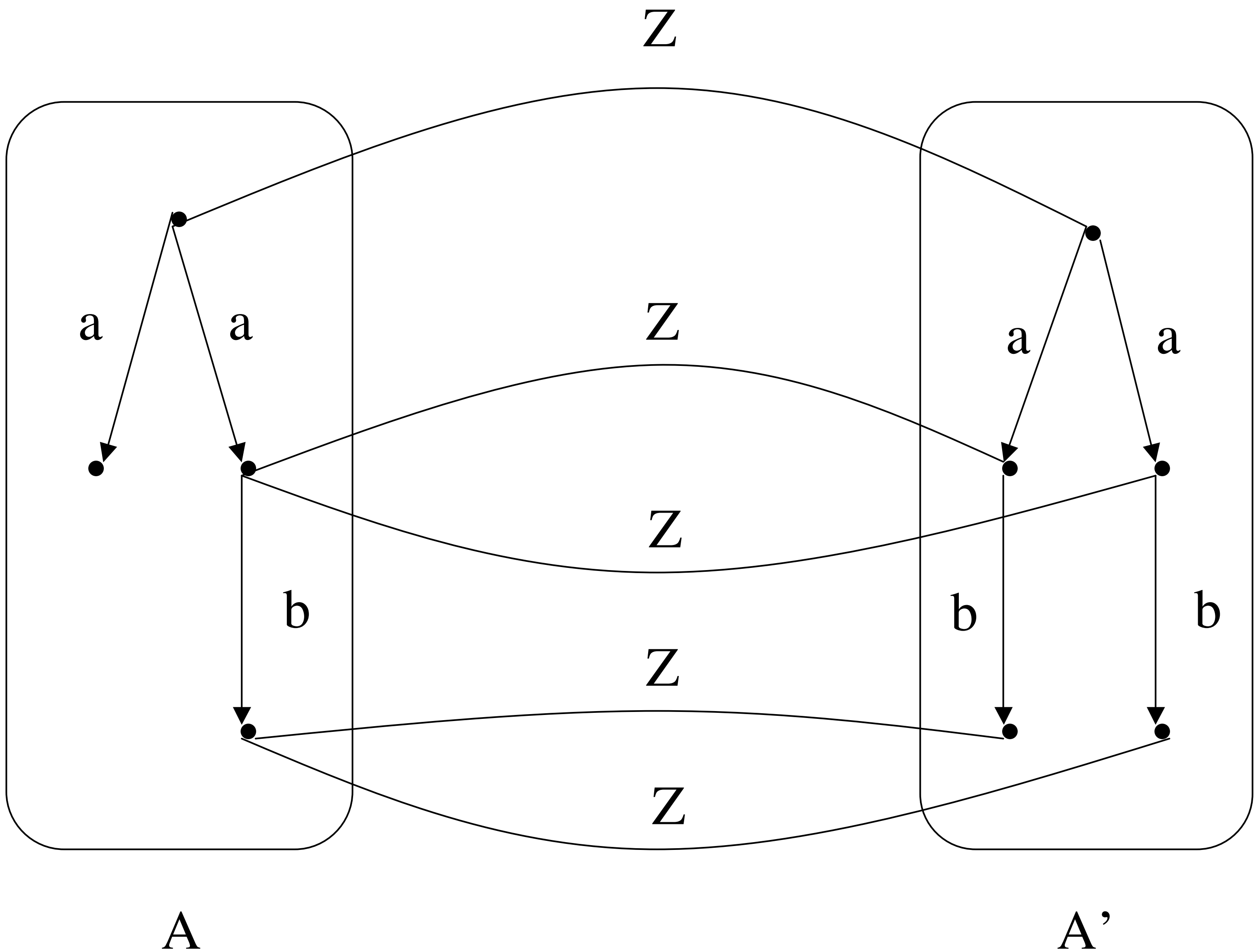
- Let $A = (Q, q_0, \Sigma, \rightarrow)$, $A' = (Q', q_0', \Sigma', \rightarrow')$ be automata
- $Z \subseteq Q \times Q'$ is a simulation between A and A' iff

  $q_0 Z q_0'$

  if $q_1 Z q_1'$ and (q_1, a, q_2) is in $\rightarrow$ then there exists q_2' such that $q_2 Z q_2'$ and (q_1', a, q_2') is in $\rightarrow'$


Z
a
a
a
Z
Z
b
c
b
c
Z
Z
A
A’


Z
Z
Z
Z
a
a
a
a
b
b
b
A
A’

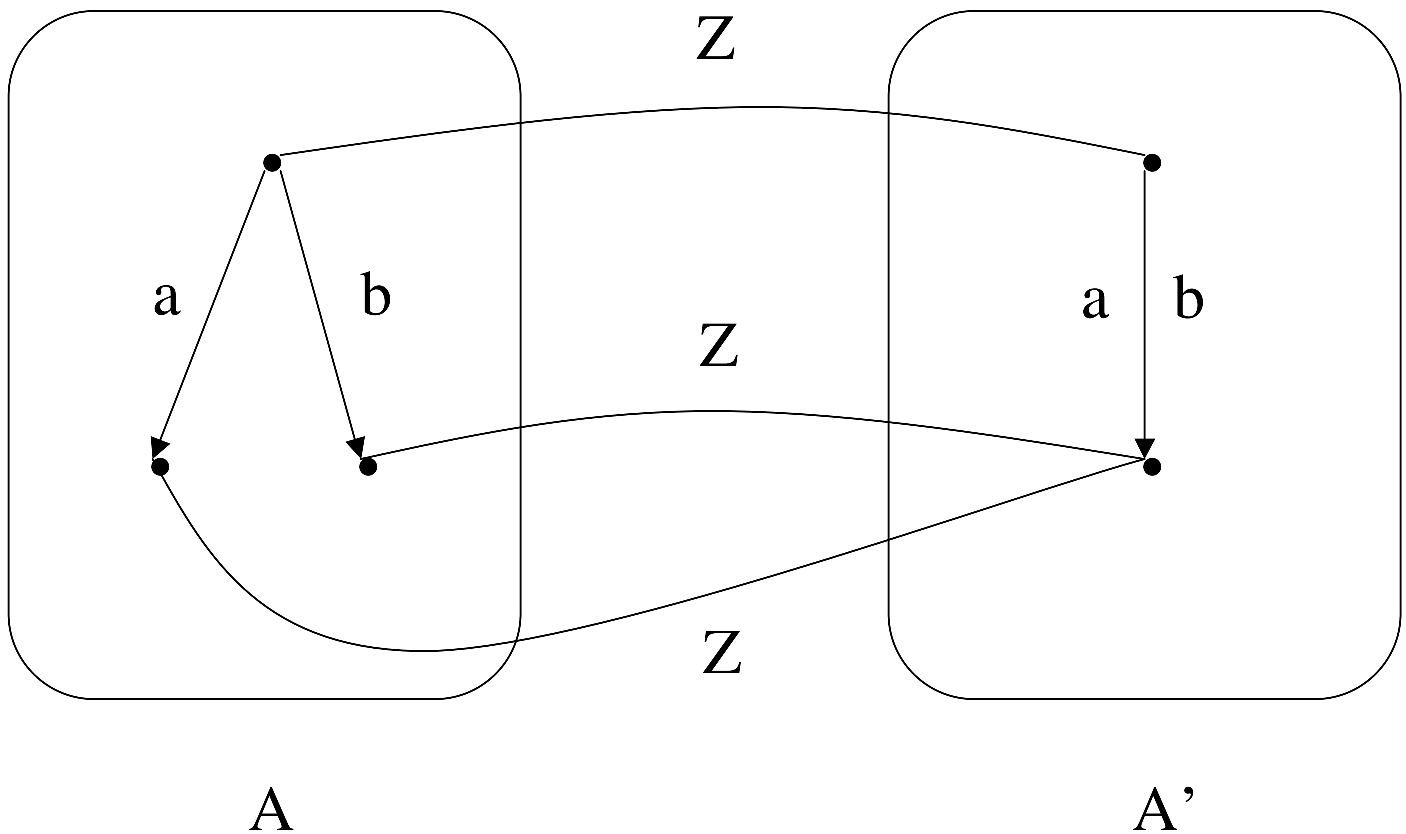

Z
a
a
Z
a
a
Z
b
b
b
Z
Z
A
A’

- Let $A = (Q,q_0,\Sigma,\rightarrow)$, $A' = (Q',q_0',\Sigma',\rightarrow')$ be automata
- $Z \subseteq Q\text{x}Q'$ is a bisimulation between A and A' iff

  $q_0 Z q_0'$

  if $q_1 Z q_1'$ and (q_1,a,q_2) is in $\rightarrow$ then there exists q_2' such that $q_2 Z q_2'$ and (q_1',a,q_2') is in $\rightarrow'$

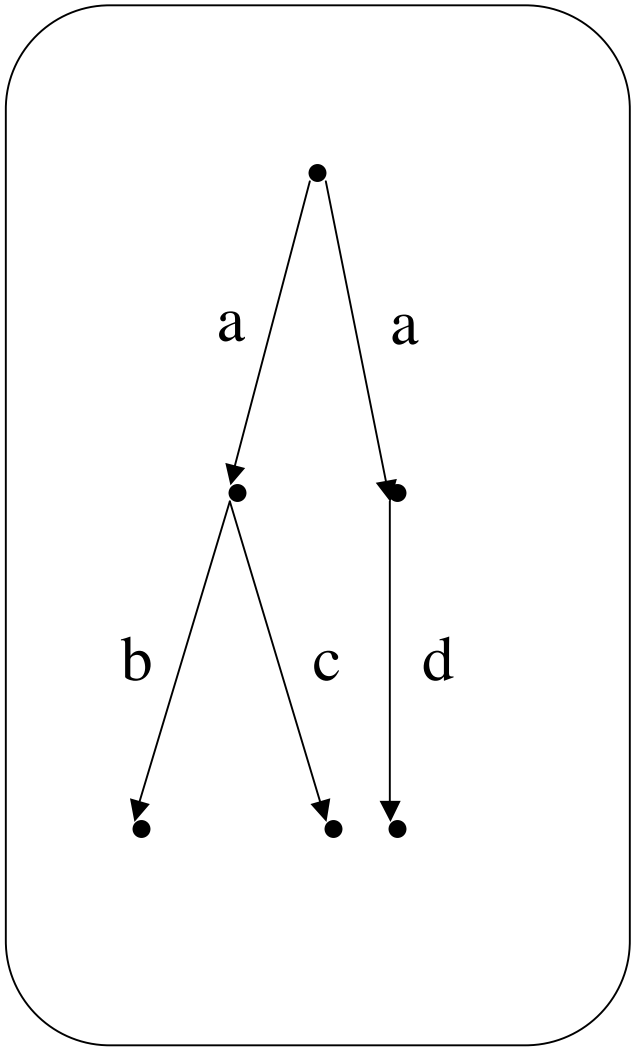
  if $q_1 Z q_1'$ and (q_1',a,q_2') is in $\rightarrow'$ then there exists q_2 such that $q_2 Z q_2'$ and (q_1,a,q_2) is in $\rightarrow$


Z
a
b
a
b
Z
Z
A
A'

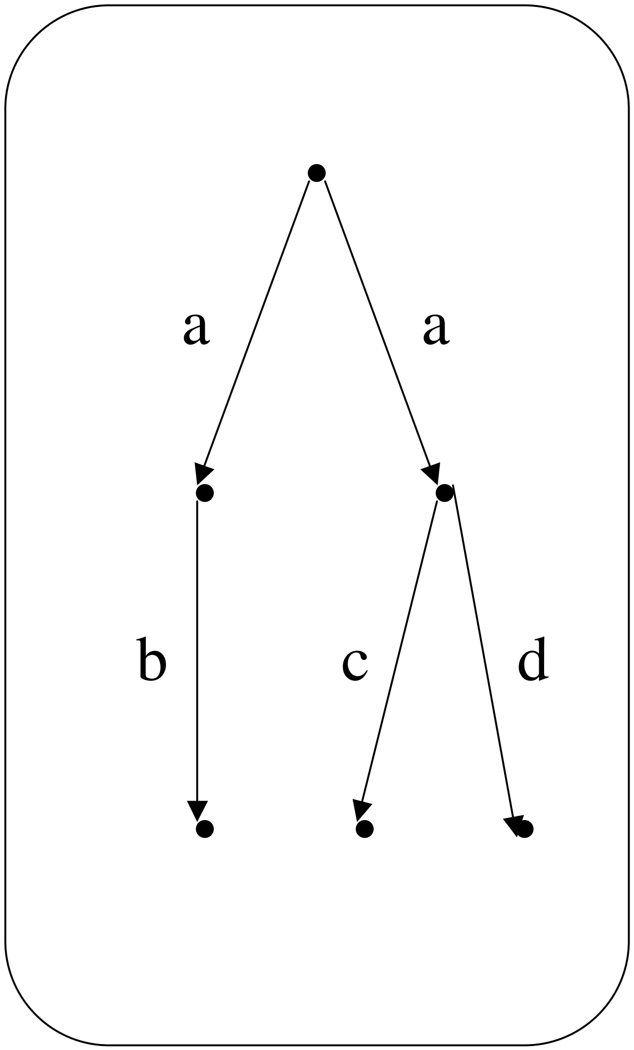
- Let $A = (Q,q_0,\Sigma,\rightarrow)$ be an automaton and $(a_1,\ldots,a_n)$ be in Σ^*
- $(a_1,\ldots,a_n)$ is recognized by A iff
  there exists $q_1, \ldots, q_n$ such that for all $i=1..n$, (q_{i-1},a_i,q_i) is in $\rightarrow$
- We write trace(A) for the set of all $(a_1,\ldots,a_n)$ in Σ^* recognized by A

- Let $A = (Q, q_0, \Sigma, \rightarrow)$, $A' = (Q', q_0', \Sigma', \rightarrow')$ be automata
- A and A' are trace equivalent iff

  trace(A) = trace(A')



A



A’

- Let C be a class of automata
- Decision problem SIM(C)

  Input : finite automata A, A’ in C

  Output : determine if A and A’ are linked by some simulation
- Decision problem BIS(C)

  Input : finite automata A, A’ in C

  Output : determine if A and A’ are linked by some bisimulation
- Decision problem TRA(C)

  Input : finite automata A, A’ in C

  Output : determine if A and A’ are trace equivalent

- If C is the class of all automata then
  SIM(C) is P-complete
  BIS(C) is in P
  TRA(C) is PSPACE-complete

- Open problem
  How hard is it to solve SIM(C), BIS(C) and TRA(C) when C is a restricted class of automata?

- Automata

  $A^1 = (Q^1, q_0^{\,1}, \Sigma^1, \rightarrow^1)$, …, $A^k = (Q^k, q_0^{\,k}, \Sigma^k, \rightarrow^k)$

  $A^1 \times \ldots \times A^k = (Q, q_0, \Sigma, \rightarrow)$

  $Q = Q^1 \times \ldots \times Q^k$

  $q_0 = (q_0^{\,1}, \ldots, q_0^{\,k})$

  $\Sigma = \Sigma^1 \cup \ldots \cup \Sigma^k$

  $((q_1^{\,1}, \ldots, q_1^{\,k}), a, (q_2^{\,1}, \ldots, q_2^{\,k}))$ is in $\rightarrow$ iff there exists i in $\{1,\ldots,k\}$ such that $(q_1^{\,i}, a, q_2^{\,i})$ is in $\rightarrow^i$ and for all j in $\{1,\ldots,k\}$, if $i \neq j$ then $q_1^{\,j} = q_2^{\,j}$

- Let C be a class of automata
- Decision problem $SIM^{\times}(C)$

  Input : finite automata A, B^1, …, B^k in C

  Output : determine if A and $B^1 \times \ldots \times B^k$ are linked by some simulation
- Decision problem $BIS^{\times}(C)$

  Input : finite automata A, B^1, …, B^k in C

  Output : determine if A and $B^1 \times \ldots \times B^k$ are linked by some bisimulation
- Decision problem $TRA^{\times}(C)$

  Input : finite automata A, A’ in C

  Output : determine if A and $B^1 \times \ldots \times B^k$ are trace equivalent

- If C is the class of all automata then

  $SIM^{\times}(C)$ is EXPTIME-complete

  $BIS^{\times}(C)$ is PSPACE-hard and in EXPTIME

  $TRA^{\times}(C)$ is EXPSPACE-complete

- Open problem

  How hard is it to solve $SIM^{\times}(C)$, $BIS^{\times}(C)$ and $TRA^{\times}(C)$ when C is a restricted class of automata?

- Let C be a class of automata
- Decision problem $SIM^{\times\exists}(C)$

  Input : a finite automaton A in C and a finite set U of automata in C

  Output : determine if there exists a nonempty subset $\{B^1, \ldots, B^k\}$ of U such that A and $B^1 \times \ldots \times B^k$ are linked by some simulation
- Decision problem $BIS^{\times\exists}(C)$

  Input : a finite automaton A in C and a finite set U of automata in C

  Output : determine if there exists a nonempty subset $\{B^1, \ldots, B^k\}$ of U such that A and $B^1 \times \ldots \times B^k$ are linked by some bisimulation
- Decision problem $TRA^{\times\exists}(C)$

  Input : a finite automaton A in C and a finite set U of automata in C

  Output : determine if there exists a nonempty subset $\{B^1, \ldots, B^k\}$ of U such that A and $B^1 \times \ldots \times B^k$ are trace equivalent

- If C is the class of all automata then
  $SIM^{\times\exists}(C)$ is EXPTIME-complete
  $BIS^{\times\exists}(C)$ is PSPACE-hard and in EXPTIME
  $TRA^{\times\exists}(C)$ is EXPSPACE-complete

- Open problem
  How hard is it to solve $SIM^{\times\exists}(C)$, $BIS^{\times\exists}(C)$ and $TRA^{\times\exists}(C)$ when C is a restricted class of automata?

- Conditional automata

  $A = (Q, q_0, \Sigma, At, \rightarrow)$

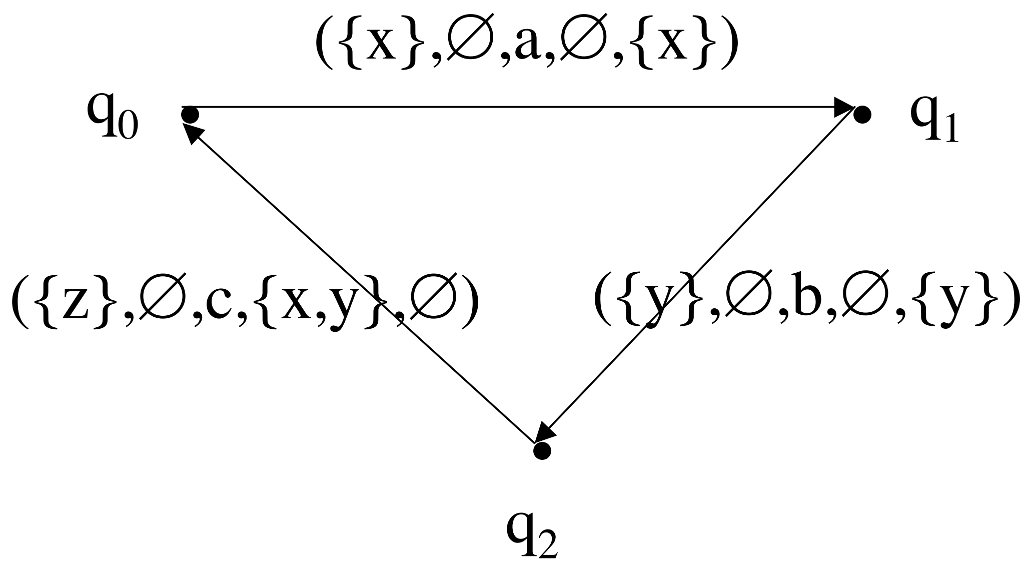
  Q is a nonempty set of states

  q_0 is the initial state

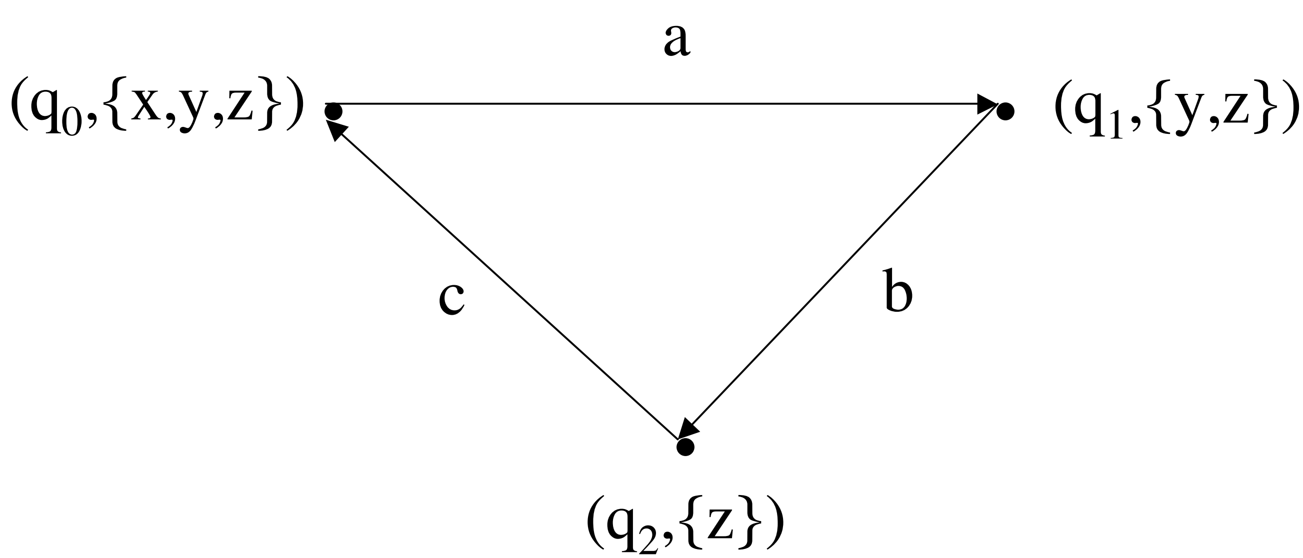
  Σ is a nonempty set of actions

  At is a nonempty set of atoms

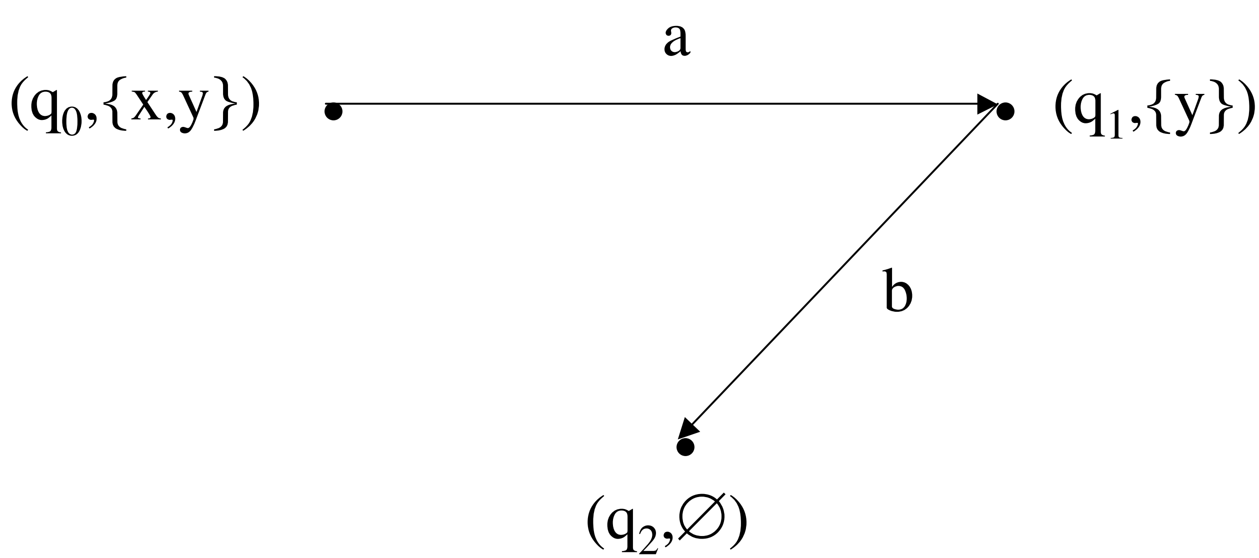
  $\rightarrow$ is a subset of $Q \times 2^{At} \times 2^{At} \times \Sigma \times 2^{At} \times 2^{At} \times Q$ where 2^{At} denotes the set of all subsets of At


({x},∅,a,∅,{x})
q0
q1
({z},∅,c,{x,y},∅)
({y},∅,b,∅,{y})
q2

- We will say that $((q_0,\{x,y,z\}),a,(q_1,\{y,z\}))$, $((q_1,\{y,z\}),b,(q_2,\{z\}))$ and $((q_2,\{z\}),c,(q_0,\{x,y,z\}))$ are in $\rightarrow$



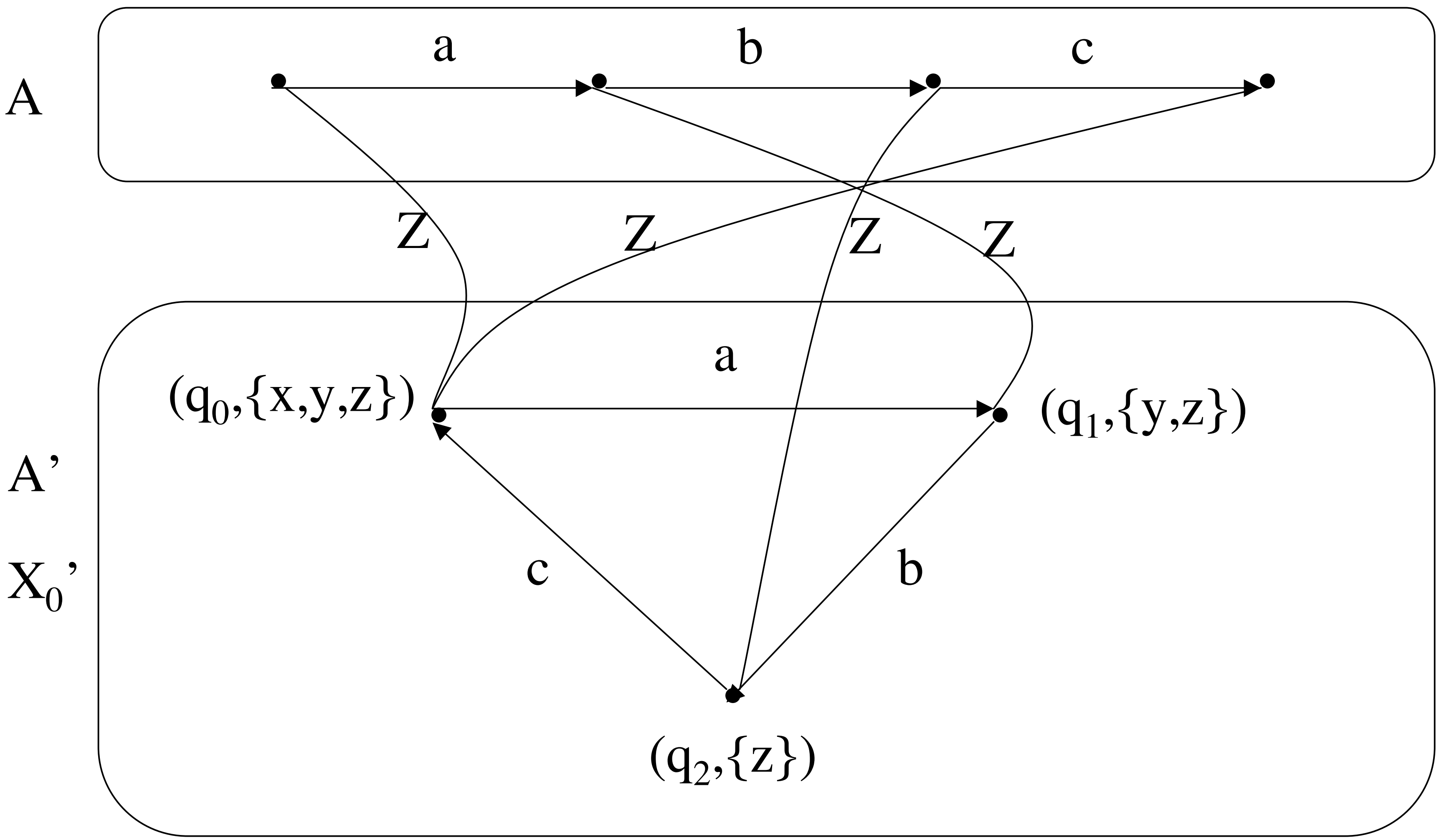
- We will say that $((q_0,\{x,y\}),a,(q_1,\{y\}))$ and $((q_1,\{y\}),b,(q_2,\varnothing))$ are in $\rightarrow$



- Let $A = (Q, q_0, \Sigma, \rightarrow)$ be an automaton, $A' = (Q', q_0', \Sigma', At', \rightarrow')$ be a conditional automaton and X_0' be a subset of At'
- $Z \subseteq Q x (Q' \times 2^{At'})$ is a simulation between A and A' with respect to X_0' iff

  $q_0 Z (q_0', X_0')$

  if $q_1 Z (q_1', X_1')$ and (q_1, a, q_2) is in $\rightarrow$ then there exists q_2' and X_2' such that $q_2 Z (q_2', X_2')$ and $((q_1', X_1'), a, (q_2', X_2'))$ is in $\rightarrow'$

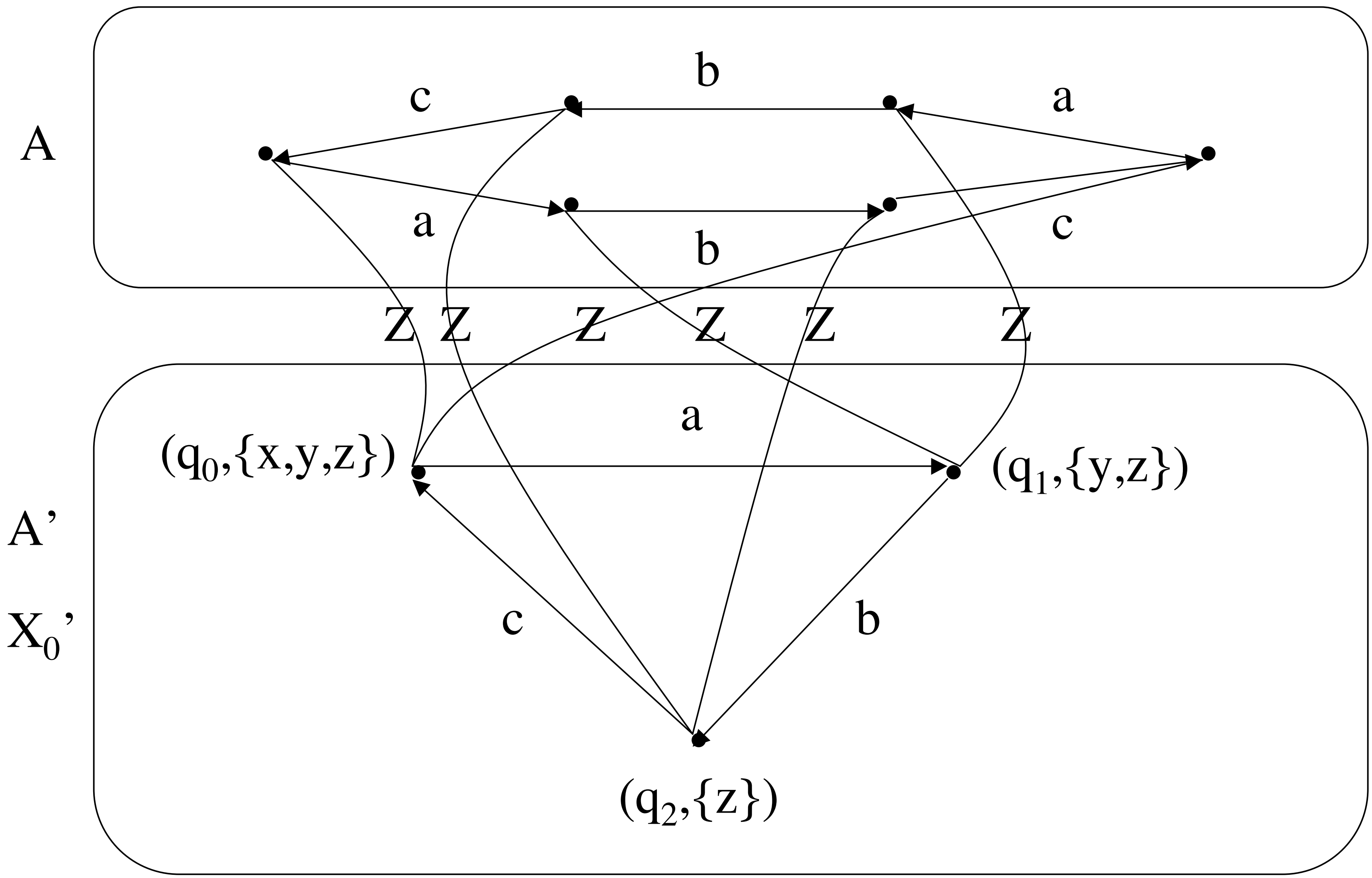

A
a
b
c
Z
Z
Z
Z
A'
X_0'
a
$(q_0,\{x,y,z\})$
$(q_1,\{y,z\})$
c
b
$(q_2,\{z\})$

- Let $A = (Q,q_0,\Sigma,\rightarrow)$ be an automaton, $A' = (Q',q_0',\Sigma',At',\rightarrow')$ be a conditional automaton and X_0' be a subset of At'
- $Z \subseteq Qx(Q' \times 2^{At'})$ is a bisimulation between A and A' with respect to X_0' iff

  $q_0Z(q_0',X_0')$

  if $q_1Z(q_1',X_1')$ and (q_1,a,q_2) is in $\rightarrow$ then there exists q_2' and X_2' such that $q_2Z(q_2',X_2')$ and $((q_1',X_1'),a,(q_2',X_2'))$ is in $\rightarrow'$

  if $q_1Z(q_1',X_1')$ and $((q_1',X_1'),a,(q_2',X_2'))$ is in $\rightarrow'$ then there exists q_2 such that $q_2Z(q_2',X_2')$ and (q_1,a,q_2) is in $\rightarrow$



A
c
b
a
a
b
c
Z Z Z Z Z Z
a
(q0,{x,y,z})
(q1,{y,z})
A’
X0’
c
b
(q2,{z})

- Let $A = (Q,q_0,\Sigma,At,\rightarrow)$ be a conditional automaton, X_0 be a subset of At and $(a_1,\ldots,a_n)$ be in Σ^*
- $(a_1,\ldots,a_n)$ is recognized by A with respect to X_0 iff

  there exists $q_1, \ldots, q_n$ and there exists $X_1, \ldots, X_n$ such that

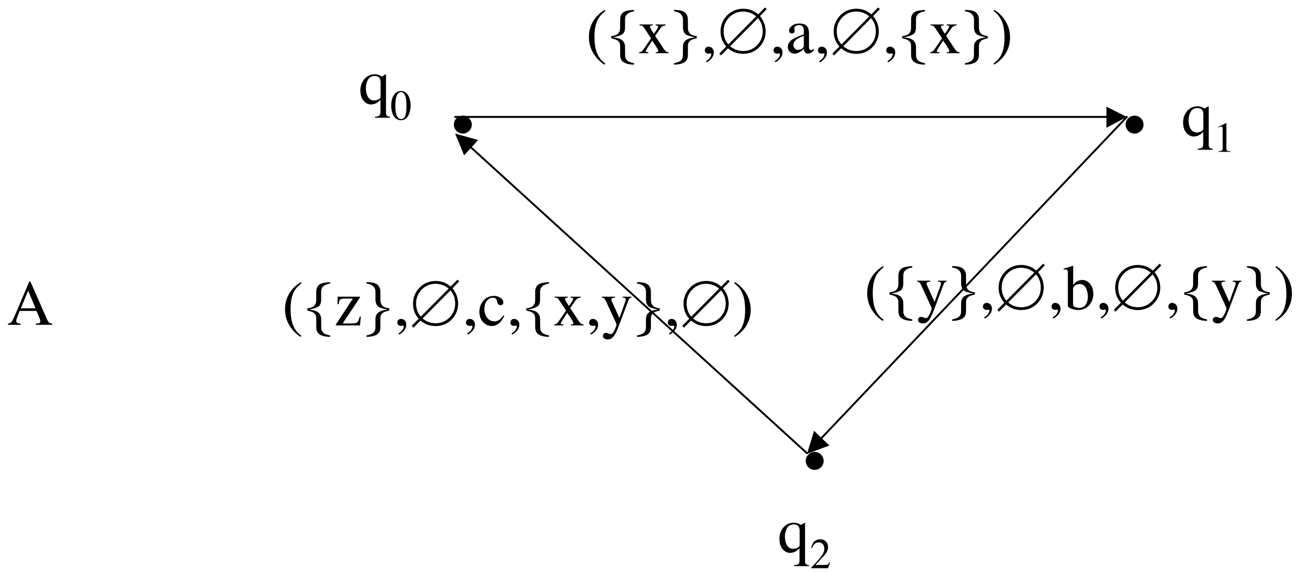
  for all i=1..n, $((q_{i-1},X_{i-1}),a_i,(q_i,X_i))$ is in $\rightarrow$
- We write trace(A,X_0) for the set of all $(a_1,\ldots,a_n)$ in Σ^* recognized by A with respect to X_0

$\text{trace}(A,\{x,y,z\}) = \{(),(a),(a,b),(a,b,c),\ldots\}$

$\text{trace}(A,\{x,y\}) = \{(),(a),(a,b)\}$

$\text{trace}(A,\{x\}) = \{(),(a)\}$

$\text{trace}(A,\emptyset) = \{()\}$



- Let $A = (Q, q_0, \Sigma, \rightarrow)$ be an automaton, $A' = (Q', q_0', \Sigma', At', \rightarrow')$ be a conditional automaton and X_0' be a subset of At'
- A and A' are trace equivalent with respect to X_0' iff

  $trace(A) = trace(A', X_0')$

- Let C be a class of automata and D be a class of conditional automata
- Decision problem SIM(C,D)

  Input : a finite automaton A in C, a finite conditional automaton A’ in D and a set X’ of atoms

  Output : determine if A and A’ are linked by some simulation with respect to X’
- Decision problem BIS(C,D)

  Input : a finite automaton A in C, a finite conditional automaton A’ in D and a set X’ of atoms

  Output : determine if A and A’ are linked by some bisimulation with respect to X’
- Decision problem TRA(C,D)

  Input : a finite automaton A in C, a finite conditional automaton A’ in D and a set X’ of atoms

  Output : determine if A and A’ are trace equivalent with respect to X’

- If C is the class of all automata and D is the class of all conditional automata then
  SIM(C,D) is EXPTIME-complete
  BIS(C,D) is PSPACE-hard and in EXPTIME
  TRA(C,D) is EXPSPACE-complete

- Open problem
  How hard is it to solve SIM(C,D), BIS(C,D) and
  TRA(C,D) when C is a restricted class of automata and D is a restricted class class of conditional automata?