Argumentation-based Ranking Logics

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ABSTRACT

This paper proposes a novel family of argumentation-based logics for handling inconsistency. Starting with a base logic, it builds arguments and attack relations between them. The novelty of the approach lies in the fact that arguments are evaluated using a ranking semantics which rank-orders arguments from the most acceptable to the least acceptable ones. Naturally, a second novelty is that the conclusions to be drawn are ranked with regard to *plausibility*. We provide a couple of axioms that such logics should enjoy and illustrate the approach with a particular ranking semantics. We show that the new logics are more discriminating than existing argumentation-based logics. Moreover, they are good candidates for measuring inconsistency in knowledge bases.

Categories and Subject Descriptors

I.2.3 [**Deduction and Theorem Proving**]: Nonmonotonic reasoning and belief revision; I.2.11 [**Distributed Artificial Intelligence**]: Intelligent agents

General Terms

Human Factors, Theory

Keywords

Handling inconsistency; Argumentation

1. INTRODUCTION

Argumentation is an alternative approach for handling inconsistency (see [11]). Starting from a knowledge base encoded in a particular logical language, it builds arguments and attack relations between them using a consequence operator associated with the language, then it evaluates the arguments using an acceptability semantics. Finally, it draws the conclusions that are supported by the "winning" arguments.

Recently, it was shown in [2] that argumentation systems that are based on Tarskian logics [35] and Dung's acceptability semantics [20] follow the same line of research as well-known syntactic approaches for handling inconsistency, namely the one that computes the maximal (for set inclusion) consistent subbases (MCSs) of a knowledge base, and draws the conclusions that follow from all of them [34]. Indeed, it was shown that there are full correspondences between stable (respectively preferred, naive) extensions and MCSs. This result generalizes the one provided in [16] for argumentation systems that are built on top of propositional logic (thus a Tarskian logic) and that evaluate arguments with stable semantics [20]. These formalisms are intuitive, but have a *sceptical behaviour* towards inconsistency. They only draw conclusions that follow from the formulae of the knowledge base that are not involved in inconsistency. They thus leave inconsistency *unsolved*. Let us consider the following illustrative example.

EXAMPLE 1. *Let $\Phi = \{p, \neg p, q, p \to \neg q\}$ be a propositional knowledge base. This base has three maximal (for set inclusion) consistent subbases:*

- $\Phi_1 = \{p, q\}$,
- $\Phi_2 = \{p, p \to \neg q\}$,
- $\Phi_3 = \{\neg p, q, p \to \neg q\}$.

The common consequences of the three subbases are the tautologies. The same output is reached by any argumentation system that uses grounded, stable, semi-stable, preferred, and naive semantics.

Note that none of the two conflicts $\{p, \neg p\}$ and $\{p, q, p \to \neg q\}$ is solved. Such output may seem unsatisfactory in general and in multi agent systems where one needs an efficient way for solving conflicts between agents.

Let us now have a closer look at the knowledge base Φ. The four formulae in Φ do not have the same responsibility for inconsistency. For instance, the degree of blame of p is higher than the one of q since it is involved in more conflicts. Moreover, p is frontally opposed while q is opposed in an indirect way. Similarly, $\neg q$ is more to blame than q since it follows from the controversial formula p.

This paper proposes a novel family of argumentation-based logics which take advantage of such information in order to handle effectively inconsistency. The new logics are built on top of Tarskian logics. Their main novelty lies in the semantics that are used for evaluating the arguments. Indeed, the new logics use the *ranking* semantics introduced by several scholars (e.g., [1, 10, 17, 32]). These semantics differ from Dung's ones in that they do not compute extensions of arguments, but rather rank-order the arguments from the most

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acceptable to the least acceptable ones. Moreover, they are based on different *principles.* For instance, the number of attackers is taken into account in ranking semantics while it does not play any role in extension-based semantics. The second key feature of our new argumentation-based logics lies in that their conclusions are ranked with regard to *plausibility.* Thus, formulas that appear in minimal conflicts may not only be inferred but also rank-ordered from the most to the least plausible ones.

We provide eight axioms that the new logics should enjoy. These axioms serve as theoretical criteria for validating the logics. For illustration purposes, we present two instantiations of the approach. In the first one, the ranking semantics is *Burden* (one of the semantics proposed in [1]) and the attack relation between arguments is left abstract. This instantiation covers thus a large set of logics. We show that those logics satisfy all the axioms. The second instantiation itself instantiates the first one. Indeed, it uses classical propositional logic, an instance of Tarski's logics, Burden semantics and a particular attack relation, *assumption-attack* [21]. Obviously, the resulting logic satisfies the axioms. It satisfies some other desirable properties. Namely, the logic captures an inconsistency measure discussed in [27].

The paper is structured as follows: We introduce argumentation-based ranking logics (ARLs), followed by a couple of axioms they should satisfy. Next, we define an instance of ARL, *Burden-based ARLs*, and investigate its properties. Then, we introduce Classical Burden-based ARL, an instance of Burden-based ARLs. We compare the new approach with existing work before concluding.

2. RANKING LOGICS

The new family of logics, called *argumentation-based ranking logics* (or ranking logics for short), is built on top of Tarskian logics [35]. According to Tarski, a logic is a set of well-formed *formulae* and a *consequence operator* which returns the set of formulae that follow from another set of formulae. There are no requirements on the connectives used in the language. However, the consequence operator should satisfy some very basic properties.

DEFINITION 1 (LOGIC). *A logic is a tuple $\langle \mathcal{F}, w, \mathcal{C} \rangle$ where $\mathcal{F}$ is a set of well-formed formulae, w is a well-order*[1] *on $\mathcal{F}$, $\mathcal{C}$ is a consequence operator, i.e., a function from $2^{\mathcal{F}}$ to $2^{\mathcal{F}}$ such that for $\Phi \subseteq_f \mathcal{F}$*[2]*,*

- $\Phi \subseteq \mathcal{C}(\Phi)$ *(Expansion)*
- $\mathcal{C}(\mathcal{C}(\Phi)) = \mathcal{C}(\Phi)$ *(Idempotence)*
- $\mathcal{C}(\Phi) = \bigcup_{\Psi \subseteq_f \Phi} \mathcal{C}(\Psi)$ *(Compactness)*
- $\mathcal{C}(\{\phi\}) = \mathcal{F}$ *for some* $\phi \in \mathcal{F}$ *(Absurdity)*
- $\mathcal{C}(\emptyset) \neq \mathcal{F}$ *(Coherence)*
- $\{\mathcal{C}(\{\phi\}) \mid \phi \in \mathcal{F}\}$ *is finite* *(Finiteness)*

[1]A *well-order* relation on a set X is a total order on X with the property that every non-empty subset of X has a least element in this ordering.

[2]$\Psi \subseteq_f \Phi$ means that Ψ is a finite subset of Φ.

The well-ordering w enables to arbitrarily select a *representative* formula among equivalent ones. Its exact definition is not important for the purpose of the paper. Finiteness ensures a finite number of non-equivalent formulae. This condition is not considered by Tarski. However, it is very useful since it will avoid redundant arguments as we will see later. It is worthy to recall that classical logic satisfies this condition when the number of propositional variables is finite, which is a quite common assumption in the literature. Finally, note that any logic that satisfies the first five conditions is *monotonic*. The notion of *consistency* associated with a logic is defined as follows:

DEFINITION 2 (CONSISTENCY). *A set $\Phi \subseteq \mathcal{F}$ is* consistent *wrt logic $\langle \mathcal{F}, w, \mathcal{C} \rangle$ iff $\mathcal{C}(\Phi) \neq \mathcal{F}$. It is* inconsistent *otherwise.*

Before introducing the notion of argument, let us first define when pairs of formulas are equivalent.

DEFINITION 3 (EQUIVALENT FORMULAS). *Let $\langle \mathcal{F}, w, \mathcal{C} \rangle$ be a logic and $\phi, \psi \in \mathcal{F}$. The formula ϕ is* equivalent *to ψ wrt logic $\langle \mathcal{F}, w, \mathcal{C} \rangle$, denoted by $\phi \equiv \psi$, iff $\mathcal{C}(\{\phi\}) = \mathcal{C}(\{\psi\})$.*

The building block of ranking logics is the notion of argument. An argument is a reason for concluding a formula. Thus, it has two main components: a *support* and a *conclusion.* In what follows, two arguments having the same supports and different yet equivalent conclusions are not distinguished, they are rather seen as the same argument. The reason is that those arguments are redundant and increase uselessly and misleadingly the argumentation graph both from a theoretical and computational point of view.

DEFINITION 4 (ARGUMENT). *Let $\langle \mathcal{F}, w, \mathcal{C} \rangle$ be a logic and $\Phi \subseteq_f \mathcal{F}$. An* argument *built from Φ is a pair (Ψ, ψ),*

- *$\Psi \subseteq \Phi$ and Ψ is consistent,*
- *ψ is the w-smallest element of $\{\psi' \in \mathcal{F} \mid \psi' \equiv \psi\}$ such that $\psi \in \mathcal{C}(\Psi)$,*
- *$\nexists \Psi' \subset \Psi$ such that $\psi \in \mathcal{C}(\Psi')$.*

Notations: `Supp` and `Conc` denote respectively the *support* Ψ and the *conclusion* ψ of an argument (Ψ, ψ). For all $\Phi \subseteq_f \mathcal{F}$, $\texttt{Arg}(\Phi)$ denotes the set of all arguments that can be built from Φ by means of Definition 4.

PROPOSITION 1. *For all $\Phi \subseteq_f \mathcal{F}$, $\texttt{Arg}(\Phi)$ is finite.*

PROOF. Let $\langle \mathcal{F}, w, \mathcal{C} \rangle$ be a logic and $\Phi \subseteq_f \mathcal{F}$. There is a finite number of consistent subsets of Φ, thus a finite number of supports of arguments. Let us show that the number of possible conclusions of arguments is also finite. From the finiteness condition of the logic, the set of non-equivalent formulae is finite. Moreover, from the definition of argument, in each set of equivalent formulae only one formula may be the conclusion of arguments. Thus, $\texttt{Arg}(\Phi)$ is finite. □

Since information may be inconsistent, arguments may attack each other. In what follows, such attacks are captured by a binary relation, denoted by $\mathcal{R}$. For two arguments a, b, $(a, b) \in \mathcal{R}$ (or $a\mathcal{R}b$) means that *a attacks b.* For the sake of generality, $\mathcal{R}$ is left *unspecified.* It can thus be instantiated in various ways (see [25] for examples of instantiations of $\mathcal{R}$). However, we assume that it is based on inconsistency.

Definition 5 (Conflict-dependency). *Let $\langle \mathcal{F}, w, \mathcal{C}\rangle$ be a logic and $\Phi \subseteq_f \mathcal{F}$. An attack relation $\mathcal{R} \subseteq \texttt{Arg}(\Phi) \times \texttt{Arg}(\Phi)$ is* conflict-dependent *iff for all $a, b \in \texttt{Arg}(\Phi)$, if $(a, b) \in \mathcal{R}$ then $\texttt{Supp}(a) \cup \texttt{Supp}(b)$ is inconsistent. For all $a \in \mathcal{A}$, $\texttt{Att}(a) = \{b \in \mathcal{A} \mid (b, a) \in \mathcal{R}\}$.*

All the attack relations discussed in [25] are conflict-dependent.

Definition 6 (Argumentation function). *An* argumentation function *$\mathcal{G}$ on a logic $\langle \mathcal{F}, w, \mathcal{C}\rangle$ transforms any set $\Phi \subseteq_f \mathcal{F}$ into a finite directed graph*[3] *$\langle \texttt{Arg}(\Phi), \mathcal{R}\rangle$ where $\mathcal{R} \subseteq \texttt{Arg}(\Phi) \times \texttt{Arg}(\Phi)$ is a conflict-dependent attack relation.*

Unlike existing argumentation-based logics which use the family of semantics defined in [20] for the evaluation of arguments, our logics use the family of *ranking semantics* introduced in [1, 10, 17, 32]. Starting with an argumentation graph, these semantics rank the arguments from the most acceptable to the least acceptable ones. It is worth pointing out that the two families of semantics are founded on *different principles and neither one refines the other.*

Definition 7 (Ranking). *A* ranking *on a set X is a binary relation $\preceq$ on X such that: $\preceq$ is* total *(i.e., $\forall x, y \in X$, $x \preceq y$ or $y \preceq x$),* reflexive *(i.e., $\forall\, x \in X$, $x \preceq x$) and* transitive *(i.e., $\forall\, x, y, z \in X$, if $x \preceq y$ and $y \preceq z$, then $x \preceq z$).*
Intuitively, $x \preceq y$ means that x is at least as good *as y. So, $y \not\preceq x$ means that x is* strictly better *than y.*

Definition 8 (Ranking semantics). *A* ranking semantics *is a function $\mathcal{S}$ that transforms any finite directed graph $\langle \mathcal{A}, \mathcal{R}\rangle$ ($\mathcal{A}$ being the set of nodes and $\mathcal{R}$ the set of edges) into a ranking on $\mathcal{A}$.*

We are now ready to introduce *argumentation-based ranking logics* (ARLs). An ARL is a logic (in the sense of Definition 1) which rank-orders with regard to plausibility the formulas drawn using its consequence operator. It is defined upon a *base logic* which is supposed to behave in a rational way when information is consistent but exhibits an irrational behaviour in presence of inconsistency. Propositional logic is an example of such logic. ARL restricts thus the base logic's inference power.

An ARL proceeds as follows: For any set Φ of formulas in the base logic, it defines its corresponding argumentation graph. The conclusions to be drawn from Φ using the consequence operator of the ARL are the formulas that are supported by arguments. The ranking of arguments constructed by a ranking semantics is used in order to rank-order those conclusions from the most plausible to the least plausible ones. The idea is the following: a formula is ranked higher than another formula if it is supported by an argument which is more acceptable than any argument supporting the second formula. It is worth mentioning that this is not the only way for comparing formulae on the basis of their supporting arguments. However, we have chosen an intuitive relation which is, in addition, validated by some inconsistency measure as we will see in a next section.

Definition 9 (ARL). *An* argumentation-based ranking logic *(ARL) is a tuple $\mathcal{L} = \langle \mathcal{F}, w, \mathcal{C}', \mathcal{K}\rangle$ based on base logic $\langle \mathcal{F}, w, \mathcal{C}\rangle$, argumentation function $\mathcal{G}$, ranking semantics $\mathcal{S}$, where:*

- *$\langle \mathcal{F}, w, \mathcal{C}'\rangle$ is a logic such that for all $\Phi \subseteq_f \mathcal{F}$, $\mathcal{C}'(\Phi) = \{\phi \in \mathcal{F} \mid \exists a \in \texttt{Arg}(\Phi)$ with $\mathcal{G}(\Phi) = \langle \texttt{Arg}(\Phi), \mathcal{R}\rangle$ and $\texttt{Conc}(a) \equiv \phi$ wrt logic $\langle \mathcal{F}, w, \mathcal{C}\rangle\}$.*
- *For all $\Phi \subseteq_f \mathcal{F}$, for all $\phi, \psi \in \mathcal{C}'(\Phi)$, $\langle \phi, \psi\rangle \in \mathcal{K}(\Phi)$ iff $\exists a \in \texttt{Arg}(\Phi)$ such that $\texttt{Conc}(a) \equiv \phi$ and $\forall b \in \texttt{Arg}(\Phi)$ such that $\texttt{Conc}(b) \equiv \psi$, $\langle a, b\rangle \in \mathcal{S}(\mathcal{G}(\Phi))$ with $\mathcal{G}(\Phi) = \langle \texttt{Arg}(\Phi), \mathcal{R}\rangle$.*

For a given set Φ of formulas, $\langle \phi, \psi\rangle \in \mathcal{K}(\Phi)$ means that ϕ is at least as plausible as ψ.

Remark 1. *For the sake of simplicity, throughout the paper, we refer to an ARL as a tuple $\mathcal{L} = \langle \mathcal{F}, w, \mathcal{C}, \mathcal{G}, \mathcal{S}, \mathcal{C}', \mathcal{K}\rangle$.*

Remark 2. *Unlike existing argumentation-based logics, an ARL may return an inconsistent set of consequences.*

A straightforward result says that an ARL does not infer conclusions which are not drawn under its base logic.

Proposition 2. *Let $\mathcal{L} = \langle \mathcal{F}, w, \mathcal{C}, \mathcal{G}, \mathcal{S}, \mathcal{C}', \mathcal{K}\rangle$ be an ARL. For all $\Phi \subseteq_f \mathcal{F}$, $\mathcal{C}'(\Phi) \subseteq \mathcal{C}(\Phi)$.*

Proof. Let $\mathcal{L} = \langle \mathcal{F}, w, \mathcal{C}, \mathcal{G}, \mathcal{S}, \mathcal{C}', \mathcal{K}\rangle$ be an ARL and $\Phi \subseteq_f \mathcal{F}$. $\mathcal{G}(\Phi) = \langle \texttt{Arg}(\Phi), \mathcal{R}\rangle$ such that for all $a \in \texttt{Arg}(\Phi)$, $\texttt{Conc}(a) \in \mathcal{C}(\Phi)$ (from Definition 4). □

3. AXIOMS

An ARL should satisfy some axioms, each of which expresses an intuitive and mandatory property. Below, we propose two categories of axioms: axioms of the first category describe properties of ARL's consequence operator $\mathcal{C}'$ whereas axioms of the second category constrain the ranking function $\mathcal{K}$.

Since the base logic behaves in a rational way when information is consistent, it is natural that both logics coincide in this case.

Axiom 1 (Consistency). *Let $\mathcal{L} = \langle \mathcal{F}, w, \mathcal{C}, \mathcal{G}, \mathcal{S}, \mathcal{C}', \mathcal{K}\rangle$ be an ARL. We say $\mathcal{L}$* satisfies consistency *iff, for all $\Phi \subseteq_f \mathcal{F}$, if Φ is consistent wrt logic $\langle \mathcal{F}, w, \mathcal{C}\rangle$, then $\mathcal{C}'(\Phi) = \mathcal{C}(\Phi)$.*

While this axiom may seem obvious, it is surprisingly not satisfied by several many-valued logics as we will see in Section 6. The next axiom expresses that when information is consistent, all formulae are equally plausible.

Axiom 2 (Flatness). *Let $\mathcal{L} = \langle \mathcal{F}, w, \mathcal{C}, \mathcal{G}, \mathcal{S}, \mathcal{C}', \mathcal{K}\rangle$ be an ARL. We say $\mathcal{L}$* satisfies flatness *iff, for all $\Phi \subseteq_f \mathcal{F}$, if Φ is consistent wrt logic $\langle \mathcal{F}, w, \mathcal{C}\rangle$, then $\mathcal{K}(\Phi) = \mathcal{C}'(\Phi) \times \mathcal{C}'(\Phi)$.*

ARLs should also avoid absurd inferences.

Axiom 3 (Non-Trivialization). *Let $\mathcal{L} = \langle \mathcal{F}, w, \mathcal{C}, \mathcal{G}, \mathcal{S}, \mathcal{C}', \mathcal{K}\rangle$ be an ARL. We say $\mathcal{L}$* satisfies non-triviality *iff, for all $\Phi \subseteq_f \mathcal{F}$, $\mathcal{C}'(\Phi) \neq \mathcal{F}$.*

The next axiom concerns the *free formulae* wrt the base logic of an argumentation-based ranking logic. The free formulae of a given set Φ of formulae are those that are not involved in any *conflict*, i.e., minimal (for set inclusion) inconsistent subset of Φ.

Definition 10 (Conflicts–Free formulae). *Let $\langle \mathcal{F}, w, \mathcal{C}\rangle$ be a logic and $\Phi \subseteq_f \mathcal{F}$.*

[3] In the literature, the pair $\langle \texttt{Arg}(\Phi), \mathcal{R}\rangle$ is also called *argumentation system.*

- *A* conflict *of Φ is any $\Psi \subseteq \Phi$ such that Ψ is inconsistent and for all $\phi \in \Psi$, $\Psi \setminus \{\phi\}$ is consistent.*
- *A formula $\phi \in \Phi$ is* free *iff there does not exist a conflict Ψ of Φ such that $\phi \in \Psi$.*

$\texttt{Free}(\Phi)$ *denotes the set of free formulae of Φ.*

The logical consequences of free formulae should be inferred by the consequence operator $\mathcal{C}'$.

Axiom 4 (Free Recovering). *Let $\mathcal{L} = \langle \mathcal{F}, w, \mathcal{C}, \mathcal{G}, \mathcal{S}, \mathcal{C}', \mathcal{K}\rangle$ be an ARL. We say $\mathcal{L}$* satisfies free recovering *iff, for all $\Phi \subseteq_f \mathcal{F}$, $\mathcal{C}(\texttt{Free}(\Phi)) \subseteq \mathcal{C}'(\Phi)$.*

The following property is straightforward.

Proposition 3. *If an ARL $\mathcal{L} = \langle \mathcal{F}, w, \mathcal{C}, \mathcal{G}, \mathcal{S}, \mathcal{C}', \mathcal{K}\rangle$ satisfies free recovering axiom, then for all $\Phi \subseteq_f \mathcal{F}$, $\texttt{Free}(\Phi) \subseteq \mathcal{C}'(\Phi)$.*

Proof. Let $\mathcal{L} = \langle \mathcal{F}, w, \mathcal{C}, \mathcal{G}, \mathcal{S}, \mathcal{C}', \mathcal{K}\rangle$ be an ARL which satisfies free recovering. Thus, for all $\Phi \subseteq_f \mathcal{F}$, $\mathcal{C}(\texttt{Free}(\Phi)) \subseteq \mathcal{C}'(\Phi)$. Since $\mathcal{C}$ satisfies the Expansion axiom (in Definition 1), then $\texttt{Free}(\Phi) \subseteq \mathcal{C}(\texttt{Free}(\Phi))$. Consequently, $\texttt{Free}(\Phi) \subseteq \mathcal{C}'(\Phi)$. $\square$

Logical consequences of free formulae are all equally plausible. Furthermore, they are at least as plausible as any non-free formula.

Axiom 5 (Free Precedence). *Let $\mathcal{L} = \langle \mathcal{F}, w, \mathcal{C}, \mathcal{G}, \mathcal{S}, \mathcal{C}', \mathcal{K}\rangle$ be an ARL. We say $\mathcal{L}$* satisfies free precedence *iff, for all $\Phi \subseteq_f \mathcal{F}$, for any $\phi \in \mathcal{C}(\texttt{Free}(\Phi)) \cap \mathcal{C}'(\Phi)$, for any $\psi \in \mathcal{C}'(\Phi)$, $\langle \phi, \psi\rangle \in \mathcal{K}(\Phi)$.*

The strict version of the previous axiom gives precedence to free formulae over any other formula.

Axiom 6 (Strict Free Precedence). *Let $\mathcal{L} = \langle \mathcal{F}, w, \mathcal{C}, \mathcal{G}, \mathcal{S}, \mathcal{C}', \mathcal{K}\rangle$ be an ARL. We say $\mathcal{L}$* satisfies strict free precedence *iff, for all $\Phi \subseteq_f \mathcal{F}$, for any $\phi \in \mathcal{C}(\texttt{Free}(\Phi)) \cap \mathcal{C}'(\Phi)$, for any $\psi \in \mathcal{C}'(\Phi) \setminus \mathcal{C}(\texttt{Free}(\Phi))$, $\langle \psi, \phi\rangle \notin \mathcal{K}(\Phi)$.*

The next axioms concern the ranking of arbitrary formulae (i.e., formulae that are not free). The first one states that any formula is at most as reliable as its logical consequences. This means that logically stronger formulae bring potentially more blame. A similar axiom is defined in [27] for inconsistency measures.

Axiom 7 (Dominance). *Let $\mathcal{L} = \langle \mathcal{F}, w, \mathcal{C}, \mathcal{G}, \mathcal{S}, \mathcal{C}', \mathcal{K}\rangle$ be an ARL. We say $\mathcal{L}$* satisfies dominance *iff, for all $\Phi \subseteq_f \mathcal{F}$, for any $\phi, \psi \in \mathcal{C}'(\Phi)$, if $\phi \in \mathcal{C}(\{\psi\})$ then $\langle \phi, \psi\rangle \in \mathcal{K}(\Phi)$.*

ARLs that satisfy dominance rank equally equivalent formulae.

Proposition 4. *Let $\mathcal{L} = \langle \mathcal{F}, w, \mathcal{C}, \mathcal{G}, \mathcal{S}, \mathcal{C}', \mathcal{K}\rangle$ be an ARL. If $\mathcal{L}$ satisfies dominance, then for all $\Phi \subseteq_f \mathcal{F}$, for any $\phi, \psi \in \mathcal{C}'(\Phi)$, if $\phi \equiv \psi$ wrt logic $\langle \mathcal{F}, w, \mathcal{C}\rangle$, then $\langle \phi, \psi\rangle \in \mathcal{K}(\Phi)$.*

The next axiom captures the following idea: Assume three formulae $\phi, \psi, \delta \notin \mathcal{C}(\texttt{Free}(\Phi))$ such that $\mathcal{C}(\{\phi, \psi\}) = \mathcal{C}(\{\delta\})$. If ϕ and ψ are independent, then the blame of δ is the sum of blame of both ϕ and ψ, and thus δ is weaker than the two other formulae.

Axiom 8 (Conjunction Dominance). *Let $\mathcal{L} = \langle \mathcal{F}, w, \mathcal{C}, \mathcal{G}, \mathcal{S}, \mathcal{C}', \mathcal{K}\rangle$ be an ARL. We say $\mathcal{L}$* satisfies conjunction dominance *iff, for all $\Phi \subseteq_f \mathcal{F}$, for any $\phi, \psi, \delta \in \mathcal{C}'(\Phi) \setminus \mathcal{C}(\texttt{Free}(\Phi))$, if $\mathcal{C}(\{\phi, \psi\}) = \mathcal{C}(\{\delta\})$, $\delta \notin \Phi$, and $\forall (X, \phi) \in \texttt{Arg}(\Phi)$, $\forall (X', \psi) \in \texttt{Arg}(\Phi)$, $\mathcal{C}(X) \cap \mathcal{C}(X') = \mathcal{C}(\emptyset)$, then $\langle \phi, \delta\rangle \notin \mathcal{K}(\Phi)$.*

The axioms are *compatible*, i.e., they can be satisfied all together.

Theorem 1. *The eight axioms are compatible.*

Most of them are independent, i.e., none of them implies the others. Notable exceptions are: strict free precedence which implies free precedence, and conjunction dominance which implies dominance.

Proposition 5. *Strict free precedence implies free precedence. Conjunction dominance implies dominance.*

4. BURDEN-BASED RANKING LOGICS

The aim of this section is to show that there are ARLs for which all axioms hold. For that purpose, we propose an instantiation in which the parameter $\mathcal{S}$ (i.e., the ranking semantics) is specified. Note that the resulting instance is still relatively general since it covers a broad range of base logics and attack relations.

Our aim is not to propose a novel ranking semantics, but to use an existing one. There are several alternatives (e.g., [1, 10, 32]). We choose the *Burden* semantics which has been introduced in [1]. The main reason is that this semantics has been axiomatized in [1]. In other words, its underlying principles and its properties are known.

Burden semantics (Bbs) assigns a *burden number* to every argument. This number represents the weight of the attacks on an argument. The semantics follows a multiple steps process. In the initial step, the burden number is 1 for all arguments. Then, in each step, all the burden numbers are simultaneously recomputed on the basis of the *number of attackers* and their burden numbers in the previous step. More precisely, for every argument a, its burden number is set back to 1, then, for every argument b attacking a, the burden number of a is increased by a quantity *inversely* proportional to the burden number of b in the previous step.

Definition 11 (Burden numbers). *Let $\mathbf{A} = \langle \mathcal{A}, \mathcal{R}\rangle$ be a finite directed graph, $i \in \{0, 1, \ldots\}$, and $a \in \mathcal{A}$. We denote by $\texttt{Bur}_i(a)$ the* burden number *of a in the i^{th} step:*

$$\texttt{Bur}_i(a) = \begin{cases} 1 & \textit{if } i = 0; \\ 1 + \sum_{b \in \texttt{Att}(a)} \frac{1}{\texttt{Bur}_{i-1}(b)} & \textit{otherwise.} \end{cases}$$

By convention, if $\texttt{Att}(a) = \emptyset$, then $\sum_{b \in \texttt{Att}(a)} \frac{1}{\texttt{Bur}_{i-1}(b)} = 0$.

Example 2. *Assume an argumentation graph made of three arguments: a, b, and c such that $a\mathcal{R}b$ and $b\mathcal{R}c$. The burden numbers of each argument are summarized in the table below. These numbers do not change beyond step 2.*

Step i	*a*	*b*	*c*
0	*1*	*1*	*1*
1	*1*	*2*	*2*
2	*1*	*2*	*1.5*

It is worthy to recall that the burden numbers of arguments always converge. Arguments are compared lexicographically on the basis of their burden numbers.

DEFINITION 12 (BBS). *The* burden-based semantics $\texttt{Bbs}$ *transforms any argumentation graph* $\mathbf{A} = \langle \mathcal{A}, \mathcal{R} \rangle$ *into the ranking* $\texttt{Bbs}(\mathbf{A})$ *on* $\mathcal{A}$ *such that* $\forall\, a, b \in \mathcal{A}$, $\langle a, b \rangle \in \texttt{Bbs}(\mathbf{A})$ *iff one of the two following cases holds:*

- $\forall\, i \in \{0, 1, \ldots\}$, $\texttt{Bur}_i(a) = \texttt{Bur}_i(b)$*;*
- $\exists\, i \in \{0, 1, \ldots\}$ *such that* $\texttt{Bur}_i(a) < \texttt{Bur}_i(b)$ *and* $\forall\, j \in \{0, 1, \ldots, i-1\}$, $\texttt{Bur}_j(a) = \texttt{Bur}_j(b)$*.*

$\langle a, b \rangle \in \texttt{Bbs}(\mathbf{A})$ means that a is *at least as acceptable* as b.

Example 2 (Cont) The argument a is more acceptable than c which is itself strictly more acceptable than b.

It is easy to check that this semantics privileges arguments that have less attackers.

PROPOSITION 6. *Let* $\mathbf{A} = \langle \mathcal{A}, \mathcal{R} \rangle$ *be a finite directed graph. For all* $a, b \in \mathcal{A}$*, if* $|\texttt{Att}(a)| < |\texttt{Att}(b)|$*, then* $\langle b, a \rangle \notin \texttt{Bbs}(\mathbf{A})$*.*

We are now ready to define burden-based ranking logics.

DEFINITION 13 (BURDEN-BASED LOGICS). *A* burden-based ranking logic *is an ARL* $\mathcal{L} = \langle \mathcal{F}, w, \mathcal{C}, \mathcal{G}, \mathcal{S}, \mathcal{C}', \mathcal{K} \rangle$ *such that* $\mathcal{S}$ *is* $\texttt{Bbs}$*.*

EXAMPLE 3. *Assume a set* $\Phi \subseteq \mathcal{F}$ *from which the argumentation graph of Example 2 is generated. Thus,* $\texttt{Conc}(a)$ *is more plausible than* $\texttt{Conc}(c)$ *and* $\texttt{Conc}(c)$ *is more plausible than* $\texttt{Conc}(b)$*. Each conclusion is as plausible as any of its equivalent formulae.*

We show next that all the eight axioms are satisfied by burden-based ARLs. This provides a theoretical validation for this family of logics. Six out of eight axioms hold for any attack relation.

THEOREM 2. *Burden-based ARLs satisfy consistency, non trivialization, free-recovering, flatness, equivalence and conjunction dominance.*

PROOF. Let $\langle \mathcal{F}, w, \mathcal{C} \rangle$ be a logic, $\Phi \subseteq_f \mathcal{F}$ and $\mathbf{A} = \langle \texttt{Arg}(\Phi), \mathcal{R} \rangle$ be the corresponding argumentation graph. Assume that Φ is consistent and let $\phi \in \mathcal{C}(\Phi)$. Thus, $\exists \Psi \subseteq \Phi$ such that Ψ is the minimal, for set inclusion, subset of Φ such that $\phi \in \mathcal{C}(\Psi)$. Then, $\langle \Psi, \phi \rangle \in \texttt{Arg}(\Phi)$ and $\phi \in \mathcal{C}'(\Phi)$. So, $\mathcal{C}(\Phi) \subseteq \mathcal{C}'(\Phi)$.

Let $\phi*$ be the formula such that $\mathcal{C}(\{\phi*\}) = \mathcal{F}$. Thus, the set $\{\phi*\}$ is inconsistent. However, since for any argument $\langle \Psi, \psi \rangle$, the set Ψ is consistent, then so is for ψ. Consequently, there is no argument in $\texttt{Arg}(\Phi)$ whose conclusion is $\phi*$. Thus, $\mathcal{C}'(\Phi) \neq \mathcal{F}$.

By definition, $\texttt{Free}(\Phi)$ is consistent, so is for $\mathcal{C}(\texttt{Free}(\Phi))$ since by idempotence, $\mathcal{C}(\texttt{Free}(\Phi)) = \mathcal{C}(\mathcal{C}(\texttt{Free}(\Phi)))$. Thus, for all $\phi \in \mathcal{C}(\texttt{Free}(\Phi))$, $\exists \langle \Psi, \phi \rangle \in \texttt{Arg}(\texttt{Free}(\Phi))$. Since $\texttt{Arg}(\texttt{Free}(\Phi)) \subseteq \texttt{Arg}(\Phi)$, then $\phi \in \mathcal{C}'(\Phi)$ and $\mathcal{C}(\texttt{Free}(\Phi)) \subseteq \mathcal{C}'(\Phi)$.

Assume that Φ is consistent. Since $\mathcal{R}$ is conflict-dependent, then $\mathcal{R} = \emptyset$. Consequently, for all $a \in \texttt{Arg}(\Phi)$, for all $i = 0, 1, ...$, $\texttt{Bur}_i(a) = 1$. Thus, $\texttt{Bbs}(\mathbf{A}) = \texttt{Arg}(\Phi) \times \texttt{Arg}(\Phi)$. Consequently, $\mathcal{K}(\Phi) = \mathcal{C}'(\Phi) \times \mathcal{C}'(\Phi)$.

Let $\phi \in \mathcal{C}'(\Phi)$ and $X = \{x \in \mathcal{F} \mid x \equiv \phi\}$. From the definition of argument, only one element of X, say $x*$, is supported by arguments. Let $\mathcal{A}* = \{a \in \texttt{Arg}(\Phi) \mid \texttt{Conc}(a) = x*\}$. Since $\texttt{Bbs}$ ensures a total order on $\texttt{Arg}(\Phi)$, then $\exists a \in \mathcal{A}*$ such that for all $b \in \mathcal{A}*$, $\langle a, b \rangle \in \texttt{Bbs}(\texttt{Arg}(\Phi))$. Thus, for all $\psi \in \mathcal{C}'(\Phi)$ such that $\phi \equiv \psi$, $\langle \phi, \psi \rangle \in \mathcal{K}(\Phi)$ and $\langle \psi, \phi \rangle \in \mathcal{K}(\Phi)$.

Let $\phi, \psi, \delta \in \mathcal{C}'(\Phi) \setminus \mathcal{C}(\texttt{Free}(\Phi))$. Assume that $\mathcal{C}(\{\phi, \psi\}) = \mathcal{C}(\{\delta\})$, $\delta \notin \Phi$, and $\forall (X, \phi) \in \texttt{Arg}(\Phi)$, $\forall (X', \psi) \in \texttt{Arg}(\Phi)$, $\mathcal{C}(X) \cap \mathcal{C}(X') = \mathcal{C}(\emptyset)$. Let $\mathcal{A} = \{a \in \texttt{Arg}(\Phi) \mid \texttt{Conc}(a) \equiv \phi\}$, $\mathcal{B} = \{b \in \texttt{Arg}(\Phi) \mid \texttt{Conc}(b) \equiv \psi\}$, $\mathcal{C} = \{c \in \texttt{Arg}(\Phi) \mid \texttt{Conc}(c) \equiv \delta\}$. $\mathcal{A} \neq \emptyset$, $\mathcal{B} \neq \emptyset$ and $\mathcal{C} \neq \emptyset$ (since $\phi, \psi, \delta \in \mathcal{C}'(\Phi)$). Since $\delta \notin \Phi$ and (3), then for all $c \in \mathcal{C}$, $\texttt{Supp}(c) = \texttt{Supp}(a) \cup \texttt{Supp}(b)$ for $a \in \mathcal{A}$ and $b \in \mathcal{B}$. Moreover, $|\texttt{Att}(c)| = |\texttt{Att}(a)| + |\texttt{Att}(b)|$. Since $\phi, \psi \in \mathcal{C}'(\Phi) \setminus \mathcal{C}(\texttt{Free}(\Phi))$, then $|\texttt{Att}(a)| \geq 1$ and $|\texttt{Att}(b)| \geq 1$. Thus, $|\texttt{Att}(c)| > |\texttt{Att}(a)|$ and $|\texttt{Att}(c)| > |\texttt{Att}(b)|$. From Prop. 6, $\langle c, a \rangle \notin \texttt{Bbs}(\mathbf{A})$ and $\langle c, b \rangle \notin \texttt{Bbs}(\mathbf{A})$ for all $a \in \mathcal{A}$ and for all $b \in \mathcal{B}$. Then $\langle \phi, \delta \rangle \notin \mathcal{K}(\Phi)$ and $\langle \psi, \delta \rangle \notin \mathcal{K}(\Phi)$. □

Free precedence is also satisfied by Burden-based ARLs. This is mainly due to the fact that any argument whose support contains only free formulae has no attackers.

PROPOSITION 7. *Let* $\mathbf{A} = \langle \texttt{Arg}(\Phi), \mathcal{R} \rangle$ *be an argumentation graph built on a logic* $\langle \mathcal{F}, w, \mathcal{C} \rangle$ *and* $\Phi \subseteq_f \mathcal{F}$*. For all* $a \in \texttt{Arg}(\texttt{Free}(\Phi))$*,* $\texttt{Att}(a) = \emptyset$*.*

A consequence of the previous result is that arguments built from the free part of a set of formulae have all the same burden number 1. This means also that they are all equally acceptable with respect to $\texttt{Bbs}$, and are at least as acceptable as any other argument built from the same set.

PROPOSITION 8. *Let* $\mathbf{A} = \langle \texttt{Arg}(\Phi), \mathcal{R} \rangle$ *be an argumentation graph built on a logic* $\langle \mathcal{F}, w, \mathcal{C} \rangle$ *and* $\Phi \subseteq_f \mathcal{F}$*.*

- *For all* $a \in \texttt{Arg}(\texttt{Free}(\Phi))$, $\forall i \in \{0, 1, \ldots\}$, $\texttt{Bur}_i(a) = 1$*.*
- *For all* $a \in \texttt{Arg}(\texttt{Free}(\Phi))$*, for all* $b \in \texttt{Arg}(\Phi)$*,* $\langle a, b \rangle \in \texttt{Bbs}(\mathbf{A})$*.*

PROOF. Let $\mathbf{A} = \langle \texttt{Arg}(\Phi), \mathcal{R} \rangle$ be an argumentation graph built on $\Phi \subseteq_f \mathcal{F}$. From Prop. 7, for all $a \in \texttt{Arg}(\texttt{Free}(\Phi))$, $\texttt{Att}(a) = \emptyset$. From Def. 11, it follows that $\forall\, i \in \{0, 1, \ldots\}$, $\texttt{Bur}_i(a) = 1$. Let $b \in \texttt{Arg}(\Phi)$. For all $i \in \{0, 1, \ldots\}$, $\texttt{Bur}_i(b) \geq 1$. Thus, $\langle a, b \rangle \in \texttt{Bbs}(\mathbf{A})$. □

From this result, we can show that burden-based ARLs satisfy free precedence.

THEOREM 3. *Burden-based ARLs satisfy free-precedence.*

PROOF. Let $\mathbf{A} = \langle \texttt{Arg}(\Phi), \mathcal{R} \rangle$ be built on a logic $\langle \mathcal{F}, w, \mathcal{C} \rangle$ and $\Phi \subseteq_f \mathcal{F}$. Assume that $\mathcal{C}(\texttt{Free}(\Phi)) \neq \emptyset$. Since $\mathcal{C}(\texttt{Free}(\Phi))$ is consistent, then for each $\phi \in \mathcal{C}(\texttt{Free}(\Phi))$, $\exists a \in \texttt{Arg}(\texttt{Free}(\Phi))$ such that $\texttt{Conc}(a) \equiv \phi$. From Prop. 8 $\forall\, i \in \{0, 1, \ldots\}$, $\texttt{Bur}_i(a) = 1$. Thus, there is no argument in favour of ϕ which is more acceptable than a. Thus, for all $\phi, \psi \in \mathcal{C}(\texttt{Free}(\Phi))$, $\langle \phi, \psi \rangle \in \mathcal{K}(\Phi)$.

Let $\phi \in \mathcal{C}(\texttt{Free}(\Phi))$ and $a \in \texttt{Arg}(\texttt{Free}(\Phi))$ such that $\texttt{Conc}(a) \equiv \phi$. Let $\psi \in \mathcal{C}'(\Phi) \setminus \mathcal{C}(\texttt{Free}(\Phi))$, and let $b \in \texttt{Arg}(\Phi)$ be one of the most acceptable arguments in favor of ψ. From Prop. 8, $\langle a, b \rangle \in \texttt{Bbs}(\mathbf{A})$, thus $\langle \phi, \psi \rangle \in \mathcal{K}(\Phi)$. □

Free precedence holds for any conflict-dependent attack relation. Things are different for strict precedence axiom. We show that it holds when the attack relation satisfies a very basic property. The idea is that an argument whose support contains at least one non-free formula should be attacked. To say it differently, any argument that uses a controversial formula in its support should be attacked. This is a natural requirement.

Definition 14 (Conflict-sensitiveness). *Let* $\langle\mathcal{F}, w, \mathcal{C}\rangle$ *be a logic and* $\Phi \subseteq_f \mathcal{F}$. *An attack relation* $\mathcal{R} \subseteq \mathtt{Arg}(\Phi) \times \mathtt{Arg}(\Phi)$ *is* conflict-sensitive *iff for all* $a \in \mathtt{Arg}(\Phi)$, *if there exists a conflict* $C \subseteq \Phi$ *such that* $C \cap \mathtt{Supp}(a) \neq \emptyset$, *then* $\mathtt{Att}(a) \neq \emptyset$.

We show next that strict precedence is satisfied when the attack relation is conflict-sensitive.

Theorem 4. *For any burden-based ARL* $\mathcal{L} = \langle\mathcal{F}, \mathcal{C}, \mathcal{G}, \mathcal{S}, \mathcal{C}', \mathcal{K}\rangle$, *for all* $\Phi \subseteq_f \mathcal{F}$, *if the attack relation of* $\mathcal{G}(\Phi)$ *is conflict-sensitive, then* $\mathcal{L}$ *satisfies strict free-precedence.*

Proof. Let $\mathbf{A} = \langle\mathtt{Arg}(\Phi), \mathcal{R}\rangle$ be built on a logic $\langle\mathcal{F}, w, \mathcal{C}\rangle$ and $\Phi \subseteq_f \mathcal{F}$. Assume that $\mathcal{C}(\mathtt{Free}(\Phi)) \neq \emptyset$. Let $\phi, \psi \in \mathcal{C}'(\Phi)$ such that $\phi \in \mathcal{C}(\mathtt{Free}(\Phi))$ and $\psi \notin \mathcal{C}(\mathtt{Free}(\Phi))$. Thus, $\exists a \in \mathtt{Arg}(\mathtt{Free}(\Phi))$ such that $\mathtt{Conc}(a) \equiv \phi$ and from Prop. 7, $\mathtt{Att}(a) = \emptyset$. Since $\psi \notin \mathcal{C}(\mathtt{Free}(\Phi))$, then $\forall b \in \mathtt{Arg}(\Phi)$ such that $\mathtt{Conc}(b) \equiv \psi$, $\exists C \subseteq \Phi$ such that C is a conflict and $C \cap \mathtt{Supp}(b) \neq \emptyset$. Since $\mathcal{R}$ is conflict-sensitive, $\mathtt{Att}(b) \neq \emptyset$. From Prop. 6, $\langle b, a\rangle \notin \mathtt{Bbs}(\mathbf{A})$, thus $\langle\psi, \phi\rangle \notin \mathcal{K}(\Phi)$. □

Let us now introduce another property of attack relations.

Definition 15 (Monotonic attack relation). *Let* $\langle\mathcal{A}, \mathcal{R}\rangle$ *be an argumentation graph. The relation* $\mathcal{R}$ *is* monotonic *iff for all* $a, b, c \in \mathcal{A}$,

$$(\mathtt{Supp}(a) \subseteq \mathtt{Supp}(b)) \Rightarrow (c\mathcal{R}a \Rightarrow c\mathcal{R}b)$$

Dominance is guaranteed for logics that use monotonic attack relations.

Theorem 5. *For any burden-based ARL* $\mathcal{L} = \langle\mathcal{F}, \mathcal{C}, \mathcal{G}, \mathcal{S}, \mathcal{C}', \mathcal{K}\rangle$, *for all* $\Phi \subseteq_f \mathcal{F}$, *if the attack relation of* $\mathcal{G}(\Phi)$ *is monotonic, then* $\mathcal{L}$ *satisfies dominance.*

Proof. Let $\mathbf{A} = \langle\mathtt{Arg}(\Phi), \mathcal{R}\rangle$ be built on a logic $\langle\mathcal{F}, w, \mathcal{C}\rangle$ and $\Phi \subseteq_f \mathcal{F}$. Let $\phi, \psi \in \mathcal{C}'(\Phi)$ such that $\phi \in \mathcal{C}(\{\psi\})$. Let $\mathcal{A} = \{a \in \mathtt{Arg}(\Phi) \mid \mathtt{Conc}(a) \equiv \psi\}$. Note that $\mathcal{A} \neq \emptyset$ since $\psi \in \mathcal{C}'(\Phi)$. For all $a \in \mathcal{A}$, $\psi \in \mathcal{C}(\mathtt{Supp}(a))$ (by definition of argument). From monotonicity of $\mathcal{C}$, $\phi \in \mathcal{C}(\mathtt{Supp}(a))$. So, for all $a \in \mathcal{A}$, $\exists a' \in \mathtt{Arg}(\Phi)$ such that $\mathtt{Supp}(a') \subseteq \mathtt{Supp}(a)$ and $\mathtt{Conc}(a') \equiv \phi$. Since $\mathcal{R}$ is monotonic, then $\mathtt{Att}(a') \subseteq \mathtt{Att}(a)$. There are two cases: i) $\mathtt{Att}(a) = \mathtt{Att}(a')$. Thus, $\forall\, i \in \{0, 1, \ldots,\}$, $\mathtt{Bur}_i(a) = \mathtt{Bur}_i(a')$. This means that $\langle a, a'\rangle \in \mathtt{Bbs}(\mathbf{A})$ and $\langle a', a\rangle \in \mathtt{Bbs}(\mathbf{A})$. ii) $\mathtt{Att}(a') \subset \mathtt{Att}(a)$. Thus, $\forall i \in \{0, 1, \ldots,\}$, $\mathtt{Bur}_i(a) = \mathtt{Bur}_i(a') + \Sigma_{c \in \mathtt{Att}(a) \setminus \mathtt{Att}(a')} \frac{1}{\mathtt{Bur}_{i-1}(c)}$. It follows that $\mathtt{Bur}_i(a) > \mathtt{Bur}_i(a')$ which means that $\langle a, a'\rangle \notin \mathtt{Bbs}(\mathbf{A})$. This means that for all $a \in \mathcal{A}$, $\exists a' \in \mathtt{Arg}(\Phi)$ such that $\mathtt{Conc}(a') \equiv \phi$ and $\langle a', a\rangle \in \mathtt{Bbs}(\mathbf{A})$. Since $\mathtt{Bbs}(\mathbf{A})$ returns a total preorder, then $\exists a' \in \mathtt{Arg}(\Phi)$ such that for all $a \in \mathcal{A}$, $\langle a', a\rangle \in \mathtt{Bbs}(\mathbf{A})$. Thus, $\langle\phi, \psi\rangle \in \mathcal{K}(\Phi)$ □

Note that equivalence which follows from dominance is satisfied for any attack relation, i.e., even for those that violate monotonicity.

5. CLASSICAL BURDEN-BASED LOGICS

The aim of this section is twofold: i) to fully illustrate the approach introduced so far by fixing all the parameters of an ARL, and ii) to show that the new approach is more discriminating than existing argumentation based logics.

Burden-based logics are still general and two of its parameters (the base logic and the attack relation) are not specified. In this section, we study one particular instance of burden-based logics, the so-called *classical burden-based logic*. The latter uses propositional logic with a *finite number of variables* in the language. This assumption, very common in the literature, ensures the finiteness condition of Definition 1. The attack relation between arguments is *assumption-attack* introduced for the first time in [21].

Definition 16 (Assumption attack). *Let* $\langle\mathcal{F}, w, \mathcal{C}\rangle$ *be propositional logic. An argument* $\langle\Psi, \psi\rangle$ attacks *an argument* $\langle\Psi', \psi'\rangle$, *denoted by* $\langle\Psi, \psi\rangle\ \mathcal{R}_{as}\ \langle\Psi', \psi'\rangle$, *iff* $\exists\phi \in \Psi'$ *s.t.* $\psi \equiv \neg\phi$.

This relation is conflict-dependent, monotonic and conflict-sensitive.

Proposition 9. *The relation* $\mathcal{R}_{as}$ *is conflict-dependent, monotonic and conflict-sensitive.*

Let us define the classical burden-based ranking logic.

Definition 17 (Classical burden-based logic). Classical burden-based logic *is the burden-based ARL* $\mathcal{L} = \langle\mathcal{F}, w, \mathcal{C}, \mathcal{G}, \mathcal{S}, \mathcal{C}', \mathcal{K}\rangle$ *such that:*

- $\langle\mathcal{F}, w, \mathcal{C}\rangle$ *is the classical propositional logic with a finite number of propositional variables*
- *For all* $\Phi \subseteq_f \mathcal{F}$, *the attack relation of* $\mathcal{G}(\Phi)$ *is* $\mathcal{R}_{as}$

Example 1 (Cont) Recall the propositional knowledge base $\Phi = \{p, \neg p, q, p \to \neg q\}$. Assume that the non equivalent formulae selected by the well-ordering w are as follows:

$p \wedge \neg p$	$\neg p \wedge \neg q$	$\neg p$	$p \to \neg q$	$p \vee \neg p$
	$\neg p \wedge q$	$\neg q$	$p \to q$	
	$p \wedge \neg q$	$p \leftrightarrow q$	$q \to p$	
	$p \wedge q$	$p \leftrightarrow \neg q$	$\neg p \to q$	
		q		
		p		

The set $\mathtt{Arg}(\Phi)$ contains the 21 following arguments.

a	$\langle\{\neg p, q\}, \neg p \wedge q\rangle$	k	$\langle\{q, p \to \neg q\}, p \leftrightarrow \neg q\rangle$
b	$\langle\{q, p \to \neg q\}, \neg p \wedge q\rangle$	l	$\langle\{q\}, q\rangle$
c	$\langle\{p, p \to \neg q\}, p \wedge \neg q\rangle$	m	$\langle\{p\}, p\rangle$
d	$\langle\{p, q\}, p \wedge q\rangle$	n	$\langle\{\neg p\}, p \to \neg q\rangle$
e	$\langle\{\neg p\}, \neg p\rangle$	o	$\langle\{p \to \neg q\}, p \to \neg q\rangle$
f	$\langle\{q, p \to \neg q\}, \neg p\rangle$	y	$\langle\{\neg p\}, p \to q\rangle$
g	$\langle\{p, p \to \neg q\}, \neg q\rangle$	z	$\langle\{q\}, p \to q\rangle$
h	$\langle\{p, q\}, p \leftrightarrow q\rangle$	r	$\langle\{p\}, q \to p\rangle$
i	$\langle\{\neg p, q\}, p \leftrightarrow \neg q\rangle$	s	$\langle\{p\}, \neg p \to q\rangle$
j	$\langle\{p, p \to \neg q\}, p \leftrightarrow \neg q\rangle$	t	$\langle\{q\}, \neg p \to q\rangle$
		u	$\langle\emptyset, p \vee \neg p\rangle$

The set $\mathcal{C}'(\Phi)$ contains the conclusions of the arguments and their equivalent formulae. Due to space limitation and the large number of attacks, we do not give all of them here. Examples of attacks are $\langle f, r\rangle$ and $\langle g, l\rangle$. From the graph of attacks, the following burden numbers are computed (table on the left). The ranking on $\mathtt{Arg}(\Phi)$ is as shown in the table on the right.

	$i=0$	$i=1$	$i=2$	$i=3$
u	1	1	1	1
m, s, r	1	3	1.83	2.41
e, n, y	1	2	1.33	1.54
l, z, t	1	2	1.25	1.48
o	1	2	1.25	1.48
d, h	1	4	2.05	2.89
b, f, k	1	3	1.50	1.96
c, g, j	1	4	2.05	2.89
a, i	1	3	1.58	2.02

u
o, l, z, t
e, n, y
b, f, k
a, i
m, r, s
c, d, g, h, j

The conclusions of the arguments are ranked as follows:

$p \vee \neg p$
$p \to \neg q,\ q,\ p \to q,\ \neg p \to q$
$\neg p$
$\neg p \wedge q,\ p \leftrightarrow \neg q$
$p,\ q \to p$
$p \wedge \neg q,\ \neg q,\ p \wedge q,\ p \leftrightarrow q$

Due to the equivalence property (see Theorem 2), any formula which is equivalent to a conclusion x of an argument will be ranked at the same level as x. For instance, $p \wedge p$ is as plausible as p. Note that $\neg p$ is more plausible than p, and q is more plausible than $\neg q$. Thus, unlike existing argumentation approaches, our approach solves both conflicts of the base Φ.

Let us now investigate the properties of classical burden-based logic. Obviously, it satisfies all the axioms satisfied by burden-based ARLs.

THEOREM 6. *Classical burden-based logic satisfies all the axioms.*

We show that the ranking produced by classical burden-based logic captures in some cases a well-known *inconsistency measure* [27]. The latter assigns a degree of blame to each formula of a knowledge base. This degree is the number of conflicts in which the formula is involved. Before presenting the formal result, let us first estimate the number of attacks an argument may receive. It is the number of conflicts in which the formulae of the support of the argument are involved.

Notation: Let $\langle \mathcal{F}, w, \mathcal{C} \rangle$ be a logic and $\Phi \subseteq_f \mathcal{F}$. $\mathtt{MIC}(\Phi)$ denotes the set of conflicts of Φ (see Def. 10).

PROPOSITION 10. *Let $\mathbf{A} = \langle \mathtt{Arg}(\Phi), \mathcal{R}_{as} \rangle$ be an argumentation graph built over Φ. For all $a \in \mathtt{Arg}(\Phi)$,*

$$|\mathtt{Att}(a)| = \sum_{\phi \in \mathtt{Supp}(a)} |\{\Psi \in \mathtt{MIC}(\Phi) \mid \phi \in \Psi\}|$$

PROOF. Let $\mathbf{A} = \langle \mathtt{Arg}(\Phi), \mathcal{R}_{as} \rangle$ be an argumentation graph built over Φ, $a \in \mathtt{Arg}(\Phi)$ and $\mathtt{Supp}(a) = \{\psi_1, \ldots, \psi_n\}$. Let $\mathtt{Def}(\psi)$ denote the set $\{b \in \mathtt{Arg}(\Phi) \mid \mathtt{Conc}(b) \equiv \neg\psi\}$. From the definition of $\mathcal{R}_{as}$, $|\mathtt{Att}(a)| = |\mathtt{Def}(\psi_1)| + \ldots + |\mathtt{Def}(\psi_n)|$. For each $\psi_i \in \mathtt{Supp}(a)$, we show that there is a bijection between $\mathtt{Def}(\psi_i)$ and $\{\Psi \in \mathtt{MIC}(\Phi) \mid \psi_i \in \Psi\}$. Let $b \in \mathtt{Def}(\psi_i)$. Thus, $\mathtt{Supp}(b) \vdash$ [4]$\neg\psi_i$. From the minimality of $\mathtt{Supp}(b)$, it follows that $\mathtt{Supp}(b) \cup \{\psi_i\} \in \mathtt{MIC}(\Phi)$. Let now $b_k, b_j \in \mathtt{Def}(\psi_i)$. Assume that $\mathtt{Supp}(b_k) \cup \{\psi_i\} = \mathtt{Supp}(b_j) \cup \{\psi_i\}$, then $b_k = b_j$. Thus, each argument in $\mathtt{Def}(\psi_i)$ refers to one conflict and two arguments refer to distinct conflicts. Let $\Psi \in \mathtt{MIC}(\Phi)$ such that $\psi_i \in \Psi$. Thus, $\Psi \setminus \{\psi_i\}$ is a minimal (for set inclusion) consistent set that infers $\neg\psi_i$. Thus, $\langle \Psi \setminus \{\psi_i\}, \neg\psi_i \rangle \in \mathtt{Def}(\psi_i)$. □

We show now that if a formula ϕ of a knowledge base (a set of formulae) is involved in more conflicts than another formula ψ of the base, then ψ is more plausible than ϕ. This result is only true in case each formula in the base cannot be inferred from another consistent subset of the base.

THEOREM 7. *Let $\mathcal{L} = \langle \mathcal{F}, w, \mathcal{C}, \mathcal{G}, \mathcal{S}, \mathcal{C}', \mathcal{K} \rangle$ be the classical burden-based logic, and $\Phi \subseteq \mathcal{F}$ such that for all $\phi \in \Phi$, $\nexists \Psi \subseteq \Phi \setminus \{\phi\}$ such that Ψ is consistent and $\phi \in \mathcal{C}(\Psi)$.*
For all $\phi, \psi \in \mathcal{C}'(\Phi) \cap \Phi$, if $|\{\Psi \in \mathtt{MIC}(\Phi) \mid \phi \in \Psi\}| > |\{\Psi' \in \mathtt{MIC}(\Phi) \mid \psi \in \Psi'\}|$, then $\langle \phi, \psi \rangle \notin \mathcal{K}(\Phi)$.

PROOF. Let $\mathcal{L} = \langle \mathcal{F}, w, \mathcal{C}, \mathcal{G}, \mathcal{S}, \mathcal{C}', \mathcal{K} \rangle$ be the classical burden-based logic, and $\Phi \subseteq \mathcal{F}$ such that for all $\phi \in \Phi$, $\nexists \Psi \subseteq \Phi \setminus \{\phi\}$ such that Ψ is consistent and $\phi \in \mathcal{C}(\Psi)$. Thus, for all $\phi \in \Phi$ such that $\{\phi\}$ is consistent, there exists a single argument supporting it, $\langle \{\phi\}, \phi \rangle$. Assume now two consistent formulae $\phi, \psi \in \mathcal{C}'(\Phi) \cap \Phi$. Let $a = \langle \{\phi\}, \phi \rangle$ and $b = \langle \{\psi\}, \psi \rangle$. Assume also that $x = |\{\Psi \in \mathtt{MIC}(\Phi) \mid \phi \in \Psi\}| > y = |\{\Psi' \in \mathtt{MIC}(\Phi) \mid \psi \in \Psi'\}|$. From Proposition 10, $|\mathtt{Att}(a)| = x$ and $|\mathtt{Att}(b)| = y$. From Proposition 6, it follows that $\langle a, b \rangle \notin \mathtt{Bbs}(\mathbf{A})$. Consequently, $\langle \phi, \psi \rangle \notin \mathcal{K}(\Phi)$. □

Works on inconsistency measures focus only on the formulae of the base and completely neglect their logical consequences. Our approach focuses on both. That's why the two approaches may not find the same results in the general case. Indeed, it may be the case that a formula ϕ of a base is involved in more conflicts than another formula ψ of the same base, but ϕ follows logically from a subset of the base and this subset constitutes a more acceptable argument than the one supporting ψ. Thus, our approach will rank ϕ higher than ψ while the inconsistency measure will prefer ψ.

6. RELATED WORK

In what follows, we compare our approach with existing works on handling inconsistency in knowledge bases.

6.1 Argumentation-based approaches

Most of existing argumentation-based logics (ALs) use Dung's semantics [20] for the evaluation of arguments. Those semantics compute sets of acceptable arguments, called *extensions*. There are mainly two families of ALs: the ALs that are built on top of a Tarskian logic (e.g., [16, 25]) and the ALs that use a rule-based logic (e.g., [3, 13, 22]). Rule-based logics do not satisfy Tarski's axioms, thus the corresponding ALs are not concerned by our study. Regarding the first family of ALs, it was shown recently in [2] that such ALs coincide with the syntactic approach which computes the maximal consistent subbases of a knowledge base. Such logics do not solve conflicts unlike our approach which does. The validation of the classical burden-based ARL by a well-known inconsistency measure testifies this point.

6.2 Inconsistency measures

Works on inconsistency measures look for measuring the degree of inconsistency of a knowledge base (e.g. [18, 26, 27, 28, 29]). The very basic measure considers the number of minimal conflicts of a base as the degree of inconsistency of

[4]The symbol $\vdash$ denotes the classical inference.

that base. Such measures are not relevant to our approach since they do not focus on individual formulae. Other works (e.g., [27]) defined ways for measuring the degree of blame of the formulae of a base. Such degrees make it possible to compare pairs of formulae. However, these works focused only on the formulae of the base and completely neglect their logical consequences. Our approach focuses on both. When the formulae of the knowledge base do not follow from consistent parts of the base, our approach returns the same results as some measures, but in the general case it does not. We believe that our approach is more natural since it focuses on the different ways of getting a formula (being an element of a base or a logical consequence of the base). It returns thus more accurate results. Moreover, it analyses more deeply the conflicts that raise in a knowledge base thanks to the attack relation. Our approach can be seen as a *rich* inconsistency measure. Finally, it is worthy to recall that works on inconsistency measures considered propositional knowledge bases while our approach considers a larger class of logics.

6.3 Many-valued logics

A large branch of logics for handling inconsistency consists of *many-valued logics*, i.e., logics based on interpretations that can assign more than two values to formulae. Such values may represent *degrees of truth* or *information states* indicating whether the truth or falsity of a formula is supported by some information. The crucial point is that a classically inconsistent set of formulae is always satisfied by some many-valued interpretations, so these interpretations can be used to derive non-trivial conclusions.

An important many-valued logic is the three-valued one from [19]. A formula can be assigned to one of the three following information states: $\{1\}$ (the truth of the formula is supported by some information, but not its falsity), $\{0\}$ (its falsity is supported, but not its truth), and $\{0, 1\}$ (both its truth and falsity are supported). This logic was introduced to answer a question posed in 1948 by S. Jaśkowski, who was interested in systematizing theories capable of containing contradictions. Further investigations of this logic can be found in e.g. [5] and [15].

Another important many-valued logic is the four-valued one motivated in [7, 8]. The three first values are the same as above, i.e., $\{0\}$, $\{1\}$, and $\{0, 1\}$. The fourth one $\{\}$ means that neither the truth nor the falsity of the formula is supported by some information. As a consequence, the main difference with the three-valued logic described previously is that classical tautologies are no longer automatically derived. This logic has been investigated in e.g. [4], where it is shown in particular that the algebraic structure of the present four-valued interpretations plays a central role in the *bilattices* proposed in [23, 24]. The present four-valued logic has also been extended in [14], where a general framework, called society semantics, was introduced with the aim of providing various ways of processing information from multiple sources, each leading to a particular many-valued logic.

Another four-valued logic was introduced in [6]. The values are $\{\}$, $\{0\}$, $\{1\}$, and $\{0, 1\}$ with the same meanings. The main difference with the two previous logics is the following: information can be provided directly about compound formulae, while, in the two previous logics, information can be provided only about atomic formulae and then propagated to the compound ones. As a consequence, an interpretation is no longer *truth-functional*. In particular, if $v(\varphi) = \{0\}$ and $v(\psi) = \{0\}$, then $v(\varphi \vee \psi)$ can be either $\{0\}$ or $\{0, 1\}$ (instead of only $\{0\}$). This non-determinism reflects the fact that 1 belongs or does not belong to $v(\varphi \vee \psi)$ depending on the direct information about $\varphi \vee \psi$, while 0 is indirectly forced into $v(\varphi \vee \psi)$ by the information about φ and ψ.

The three logics described above have two points in common. First, all the conclusions have the same level of reliability. In particular, they do not formally distinguish between free conclusions and the other conclusions. Second, they do not coincide with classical logic in case of classically consistent databases (at least without an additional mechanism). Two advantages of our present ranking logics is that they always rank at the top the elements of the free part of the premisses and they naturally draws exactly the classical conclusions from consistent premisses.

6.4 Works on nonmonotonic reasoning

The idea of ranking formulae can also be found in works on nonmonotonic reasoning. In some works, the ranking is an input like in [12] where a knowledge base is equipped with a partial or total preordering. The latter reflects certainty degrees of formulae. In our approach the ranking is an output and reflects at what extent inferences are plausible. In works like [9, 30, 31] or even system Z [33], *defaults* are ranked from the most general to the most specific ones. In our approach no distinction is made between the formulae of a knowledge base.

7. CONCLUSION

In this paper, we tackled the important problem of handling inconsistency in knowledge bases. Starting from the observation that most existing argumentation-based logics have at their heart the idea of computing the maximal consistent subbases of an inconsistent knowledge base, then choosing the conclusions that follow from all the subbases, we argued that such formalisms do not really solve the inconsistency. They rather choose all what is out of inconsistency and forget the rest. We proposed a novel approach which effectively solves inconsistency. The formalism presents various other features: first, it defines a logic which returns a ranking of the conclusions, unlike existing logics that compute a flat set of conclusions from a knowledge base. Second, the logic avoids absurd inferences and in case of consistent knowledge bases, it coincides with classical logics, as desired. Third, it enjoys many other desirable properties expressed by the axioms. Last but not least, it makes an elegant bridge between works on inconsistency measures and formalisms for handling inconsistency. To the best of our knowledge, this is the first work in this direction.

This work can be extended in different ways. First, we would like to find a complete characterization of the ranking produced by classical-burden logics. Another line of research consists of investigating other logics produced using other ranking semantics among the ones proposed in [1, 10, 17]. We plan also to investigate the impact of the attack relation on the final ranking. Finally, we plan to investigate the computational issues of the approach.

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