Gradual Semantics Accounting for Varied-Strength Attacks

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ABSTRACT

The paper studies how to evaluate arguments in graphs where both arguments and attacks are weighted. It proposes a broad family of gradual semantics that assign to each argument a numerical value representing its *strength*, i.e., how robust is the argument against attacks. It shows that five existing gradual semantics are instances of the family, and extends each of them in various ways for accounting for weights of attacks. The extended versions of each semantics differ in the way they deal with weights of attacks. Furthermore, they are all instances of the family. The paper shows also that the family captures additional semantics, like Euler-Max-based that is investigated in the paper. The new semantics are analyzed against properties from the literature and are compared with existing semantics that deal with weighted attacks.

KEYWORDS

Argumentation; Gradual Semantics; Weighted Attacks

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1 INTRODUCTION

Argumentation is a reasoning approach that has been used for solving numerous and varied Artificial Intelligence problems, like making and explaining decisions (eg. [39]), nonmonotonic reasoning (eg. [22]), classification (eg. [7]), etc. These works have identified the potential benefits of using argumentation in multiagent settings, as a way to implement the capabilities of agents (eg. reasoning, decision making, communication). Some fully integrated argumentation-based agent architectures have even been proposed (eg. [19, 30]). The basic idea behind argumentation is the justification of claims by *arguments*. An argument is a reason for believing or accepting a claim. It has generally a basic weight, which may represent different issues like votes of users [32], certainty degree of the argument's premises [11], trustworthiness degree of its source [20]. It may also be attacked by other arguments, and each attack may have a weight that may represent votes of users [25], or a degree of relevance [24], etc. Arguments and attacks are represented in graphs, called *weighted* when both arguments and attacks are weighted, *semi-weighted* when only arguments are weighted, and *flat* when neither arguments nor attacks have weights.

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A crucial step in an argumentation process is *evaluation* of argument *strength*, i.e., how robust is an argument against attacks. The strength of an argument determines the reliability status of its claim. Indeed, the stronger an argument, the more reliable its claim. Consequently, several semantics were proposed in the literature. The first ones are the *extension* semantics introduced by Dung in [22], which compute acceptable sets of arguments. Extensions are then used in [18] for assigning a three-valued qualitative strength (accepted, rejected, undecided) to each argument. Starting from the observation that the qualitative scale of argument strength is too coarse, in [18] the authors introduced *gradual* semantics that use a richer scale. Another key difference with extension semantics lies in the fact that gradual semantics do not compute extensions. They rather compute directly the strength of each argument using an *evaluation method*, which is a pair of aggregation functions: one for aggregating the values of all direct attackers of an argument, and the other for computing the effect of "direct attack" on its strength. In that way, the strength of an argument depends on its attack structure (direct and indirect attackers and defenders). The authors in [18] introduced a general setting for defining gradual semantics. Indeed, they proposed a general definition of gradual semantics, which relates argument strength to an evaluation method, and specifies some constraints on the latter. Examples of constraints are properties (eg. continuity, monotony) of the two aggregation functions. Furthermore, each evaluation method should uniquely characterize a gradual semantics. This general setting deals with flat graphs, but it was extended to semi-weighted graphs in [32], and to weighted ones in [25].

These developments have led to the introduction of various gradual semantics. Some of them like *h*-Categorizer [12] and Compensation semantics [3] deal only with flat graphs, while others like Weighted *h*-Categorizer [4], Weighted Max-Based [4], Weighted Card-based [4], and Trust-based [20] are devoted to semi-weighted graphs. Following the approach of [18], the authors in [25] proposed an evaluation method, called Simple Product (SP), for defining a semantics that deals with weighted graphs. However, unlike what is conjectured in that paper, it was recently shown in [5] that (SP) does not characterize a single semantics. Hence, there is no gradual semantics dealing with weighted graphs that fits within the setting of [18]. The aim of this paper is to bridge this gap.

The paper proposes five contributions: First, it defines the first family S^* of gradual semantics that take into account weights of attacks. The family instantiates the general setting from [18]. Second, it shows that any semantics of the family satisfies properties identified in [4] for analyzing semantics. Furthermore, it shows how to effectively calculate the values that a semantics from S^* assigns to arguments. Third, it shows that five existing gradual semantics, namely those proposed in [3, 4, 12], are instances of S^*. Fourth, it generalizes each of them in various ways for accounting for weights of attacks. The extended versions of each semantics differ in the way they deal with weights of attacks. Furthermore, they are all instances of the family S^*. Fifth, it shows that S^* encompasses additional gradual semantics that have no counterpart in the literature. It presents one of them, called Euler-Max-based semantics (EMbs), and shows that it satisfies the same properties and leads to the same ordering of arguments as Weighted Max-based semantics from [4]. However, an attack is more harmful under EMbs as strengths of arguments are lower with this semantics. All the above results show that the family S^* is very broad and encompasses semantics that may make different design choices like privileging quality of attackers over their quantity or vice versa.

The paper is organized as follows: Section 2 recalls the general setting of gradual semantics. Section 3 introduces the novel family S^* and Section 4 studies its properties. Section 5 shows that S^* is very broad as it encompasses five existing semantics, their general versions, as well as novel ones. Section 6 compares the new family with other semantics from the literature.

2 BACKGROUND

In the paper, we are interested in *weighted argumentation graphs*. Their nodes are *arguments*, each of which has a *basic weight* representing an aggregation of votes given by users [32], or a certainty degree of the argument's premises [11], or a trustworthiness degree of its source [20], etc. Edges represent *attacks* (i.e., conflicts) between arguments, each attack has a *weight* expressing an aggregation of votes given by users [25], or a relevance degree [24], etc. For the sake of simplicity, weights of both arguments and attacks are elements of the unit interval [0, 1]. The greater the value, the stronger the argument or the attack.

Definition 2.1 (Weighted Graph). A *weighted argumentation graph* is a tuple $\mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R}, \pi \rangle$, where $\mathcal{A}$ is a non-empty finite set of arguments, $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$, $\sigma : \mathcal{A} \to [0,1]$, and $\pi : \mathcal{R} \to [0,1]$. Let $\mathtt{AG}$ denote the set of all weighted graphs.

For $a, b \in \mathcal{A}$, $\sigma(a)$ is the basic weight of a, $(a,b) \in \mathcal{R}$ means *a attacks b*, and $\pi((a,b))$ is the weight of the attack.

Notations: Let $\mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R}, \pi \rangle \in \mathtt{AG}$ and $a \in \mathcal{A}$. We denote by $\sigma \equiv 1$ (resp. $\pi \equiv 1$) the case where all arguments (resp. all attacks) have a weight equal to 1. $\mathtt{Att}_{\mathbf{G}}(a)$ denotes the set $\{b \in \mathcal{A} \mid (b,a) \in \mathcal{R}\}$ of *direct attackers* of a in $\mathbf{G}$. When $\mathbf{G}$ is clear from the context, we write $\mathtt{Att}(a)$ for short. Let $\mathbf{G}' = \langle \mathcal{A}', \sigma', \mathcal{R}', \pi' \rangle \in \mathtt{AG}$ such that $\mathcal{A} \cap \mathcal{A}' = \emptyset$. $\mathbf{G} \oplus \mathbf{G}' = \langle \mathcal{A} \cup \mathcal{A}', \sigma'', \mathcal{R} \cup \mathcal{R}', \pi'' \rangle \in \mathtt{AG}$ such that $\forall x \in \mathcal{A}$ (resp. $x \in \mathcal{A}'$), $\sigma''(x) = \sigma(x)$ (resp. $\sigma''(x) = \sigma'(x)$), and $\forall x \in \mathcal{R}$ (resp. $x \in \mathcal{R}'$), $\pi''(x) = \pi(x)$ (resp. $\pi''(x) = \pi'(x)$).

Cayrol and Lagasquie introduced in [18] an abstract setting for defining gradual semantics in case of flat graphs. The setting was later extended by Leite and Martins in [32] for semi-weighted graphs, by Egilmez, Martins and Leite in [25] for weighted ones, where weights of arguments and attacks are aggregations of votes of users, and by Cayrol and Lagasquie in [17] for bipolar graphs. The idea is to define a gradual semantics by an *evaluation method*, which is a tuple of aggregation functions, each of which should satisfy some properties (like continuity, monotony). Such approach offers at least four advantages: First, it makes transparent the different operations made by a semantics (eg., accruing strengths of attackers, adjusting weights, ...) and formalizes them through aggregation functions. Second, it shows the main parameters to be tuned for defining different semantics. Third, it facilitates the study of combinations of functions that lead to reasonable semantics. Fourth, we have shown recently in [6] that properties of aggregation functions are closely related to principles defined in [4].

In what follows, we present a *simplified version* of this general setting. Indeed, we consider the unit interval [0, 1] for all functions. Furthermore, we do not use the exact definitions as given in the original papers. We rather present its main ideas using two novel concepts: *determinative* and *well-behaved* evaluation methods.

Definition 2.2 (Evaluation Method). An *evaluation method* (EM) is a triple $\mathbf{M} = \langle f, g, h \rangle$ such that:

- $h : [0,1] \times [0,1] \to [0,1]$
- $g : \bigcup_{n=0}^{+\infty} [0,1]^n \to [0, +\infty)$ such that g is symmetric
- $f : [0,1] \times \mathsf{Range}(g) \to [0,1]$[1]

The function h calculates the *strength of an attack* by aggregating its weight with the strength of the attacker. The function g evaluates how strongly an argument is attacked. It aggregates the strengths of all attacks (obtained by h) received by the argument. Since the ordering of attackers should not be important, we posed the symmetry condition, i.e.,

$$g(x_1, \ldots, x_n) = g(x_{\rho(1)}, \ldots, x_{\rho(n)}),$$

for any permutation ρ of the set $\{1, \ldots, n\}$. The function f returns the strength of an argument by combining its basic weight with the value returned by g. Table 1 presents some possibilities for the functions f, g and h, including well known T-norms [31] for h and aggregation functions for g. We will see later that most of them are already (implicitly) used in the literature.[2]

Let us now define a gradual semantics. It is a function that assigns a value from a given ordered scale to each argument. The greater the value, the stronger the argument. Different scales can be used, but for simplicity we use the unit interval of reals [0, 1].

Definition 2.3 (Gradual Semantics). A *gradual semantics* is a function S assigning to any $\mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R}, \pi \rangle \in \mathtt{AG}$ a weighting $\mathtt{Deg}^{\mathrm{S}}_{\mathbf{G}}$ on $\mathcal{A}$, i.e., $\mathtt{Deg}^{\mathrm{S}}_{\mathbf{G}} : \mathcal{A} \to [0,1]$. For any $a \in \mathcal{A}$, $\mathtt{Deg}^{\mathrm{S}}_{\mathbf{G}}(a)$ represents the *strength* of a.

A gradual semantics, as defined in [18, 32], should be based on an evaluation method as shown next.

Definition 2.4. A gradual semantics S *is based on* an evaluation method $\mathbf{M} = \langle f, g, h \rangle$ iff $\forall \mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R}, \pi \rangle \in \mathtt{AG}$, $\forall a \in \mathcal{A}$,

$$\mathtt{Deg}^{\mathrm{S}}_{\mathbf{G}}(a) = f(\sigma(a), g(h(\pi((b_1,a)), \mathtt{Deg}^{\mathrm{S}}_{\mathbf{G}}(b_1)), \ldots, h(\pi((b_n,a)), \mathtt{Deg}^{\mathrm{S}}_{\mathbf{G}}(b_n)))), \quad (1)$$

where $\{b_1, \ldots, b_n\} = \mathtt{Att}_{\mathbf{G}}(a)$.

Let us illustrate the definitions with the *Trust-based* (TB) semantics proposed by da Costa et al. in [20]. This semantics deals with semi-weighted graphs (i.e., $\pi \equiv 1$), where the basic weight of an argument represents a degree of trust in the argument's source.

[1] $\mathsf{Range}(g)$ denotes the co-domain of g.

[2] In subscripts of the functions in Table 1, *Ham* stands for Hamacher product, *psum* for probabilistic sum, *comp* for complement and *frac* for fractions. The other subscripts are self-explanatory.

$f_{comp}(x_1, x_2) = x_1(1 - x_2)$	$g_{sum}(x_1, \dots, x_n) = \sum_{i=1}^{n} x_i$	$h_{prod}(x_1, x_2) = x_1 x_2$
$f_{exp}(x_1, x_2) = x_1 e^{-x_2}$	$g_{sum,\alpha}(x_1, \dots, x_n) = (\sum_{i=1}^{n} (x_i)^{\alpha})^{\frac{1}{\alpha}}$	$h_{prod,\alpha}(x_1, x_2) = x_1^{\alpha} x_2,\ \alpha > 0$
$f_{frac}(x_1, x_2) = \frac{x_1}{1+x_2}$	$g_{max}(x_1, \dots, x_n) = \max\{x_1, \dots, x_n\}$	$h_{min}(x_1, x_2) = \min\{x_1, x_2\}$
$f_{min}(x_1, x_2) = \min\{x_1, 1 - x_2\}$	$g_{psum}(x_1, \dots, x_n) = x_1 \oplus \dots \oplus x_n$, where $x_1 \oplus x_2 = x_1 + x_2 - x_1 x_2$	$h_{Ham}(x_1, x_2) = \frac{x_1 x_2}{x_1 + x_2 - x_1 x_2}$; $h_{Ham}(x_1, x_2) = 0$ if $x_1 = x_2 = 0$

Table 1: Examples of functions f, g and h.

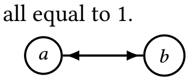
Example 2.5. The TB semantics, that we will denote by fuzzy, assigns to every argument $a \in \mathcal{A}$ in a graph $\mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R}, \pi \equiv 1 \rangle$ the limit of the sequence $\{\alpha_n(a)\}_{n=0}^{+\infty}$, i.e.,

$$\mathtt{Deg}_{\mathbf{G}}^{\mathtt{fuzzy}}(a) = \lim_{n \to +\infty} \alpha_n(a),$$

where $\alpha_0(a) = \sigma(a)$ and

$$\alpha_{n+1}(a) = \frac{1}{2}\alpha_n(a) + \frac{1}{2}\min\{\sigma(a), 1 - \max_{(b,a) \in \mathcal{R}} \alpha_n(b)\}. \quad (2)$$

Consider the graph $\mathbf{G}$ depicted below and whose weights of arguments and attacks are all equal to 1.



It is easy to check that $\mathtt{Deg}_{\mathbf{G}}^{\mathtt{fuzzy}}(a) = \mathtt{Deg}_{\mathbf{G}}^{\mathtt{fuzzy}}(b) = \frac{1}{2}$.
Trust-based semantics is guided by two principles. First, the strength $\alpha(a)$ of an argument a should not be greater than the degree to which the arguments attacking it are unacceptable, second, its strength cannot be greater than its basic weight. These intuitions lead to the equation below:

$$\alpha(a) = \min\{\sigma(a), 1 - \max_{(b,a) \in \mathcal{R}} \alpha(b)\}. \quad (3)$$

It was shown in [20] that Trust-based semantics satisfies equation (3), i.e., $\mathtt{Deg}_{\mathbf{G}}^{\mathtt{fuzzy}}$ is an instance of α. Equation (3) can be decomposed into an evaluation method, called *fuzzy evaluation method*: [3]

$$\mathbf{M}_F = \langle f_{min}, g_{max}, h_{prod} \rangle.$$

So, the semantics $\mathtt{Deg}_{\mathbf{G}}^{\mathtt{fuzzy}}$ is based on the evaluation method $\mathbf{M}_F$.

Definition 2.4 shows that evaluating arguments with a semantics amounts to solving a system of equations (one equation per argument). Indeed, the solutions $v_a = v_a^*$ of the system of equations

$$v_a = f(\sigma(a), g(h(\pi((b_1, a)), v_{b_1}), \dots, h(\pi((b_n, a)), v_{b_n}))), \quad (4)$$

for each argument $a \in \mathcal{A}$ with $\{b_1, \dots, b_n\} = \mathtt{Att}_{\mathbf{G}}(a)$, correspond to semantics $\mathbf{S}$ based on $\mathbf{M}$ with $\mathtt{Deg}_{\mathbf{G}}^{\mathbf{S}}(a) = v_a^*$. The following result ensures that the above system has *at least one solution* when the functions of the evaluation method are continuous. This result simplifies a similar one from [25] that considered additional conditions including monotonicity of the functions.

Theorem 2.6. *If* $\mathbf{M} = \langle f, g, h \rangle$ *is an evaluation method such that* g *is continuous, and* f *and* h *are continuous on the second variable, then there exists a semantics based on* $\mathbf{M}$.

The system of equations (4) may thus have *one* or *several* solutions. Consider the case of the fuzzy evaluation method $\mathbf{M}_F$.

[3]Note that we may consider another function h since Trust-based semantics does not deal with weighted attacks.

Example 2.5 (Cont) Consider the fuzzy evaluation method $\mathbf{M}_F = \langle f_{min}, g_{max}, h_{prod} \rangle$ and the argumentation graph of Example 2.5. Recall that $\mathtt{Deg}_{\mathbf{G}}^{\mathtt{fuzzy}}(a) = \mathtt{Deg}_{\mathbf{G}}^{\mathtt{fuzzy}}(b) = \frac{1}{2}$. This is a solution of the system of equations (5) below.

$$v_a = 1 - v_b, \quad v_b = 1 - v_a. \quad (5)$$

However, this system has infinitely many solutions including $(v_a, v_b) = (0, 1)$. Then, the semantics $\mathbf{S}'$ defined by:

- $\mathtt{Deg}_{\mathbf{G}}^{\mathbf{S}'}(a) = 0$, $\mathtt{Deg}_{\mathbf{G}}^{\mathbf{S}'}(b) = 1$,
- $\mathtt{Deg}_{\mathbf{G}'}^{\mathbf{S}'} \equiv \mathtt{Deg}_{\mathbf{G}'}^{\mathtt{fuzzy}}$ for all graphs $\mathbf{G}' \neq \mathbf{G}$,

is also based on $\mathbf{M}_F$. This means that $\mathbf{M}_F$ does not characterize Trust-based semantics.

In [18], a gradual semantics should be based on an evaluation method which characterizes it. In what follows, we extend the existing general setting by integrating this characterization condition. For that purpose, we introduce the concept of *determinative evaluation methods*, i.e., methods that characterize semantics.

Definition 2.7 (Determinative EM). An evaluation method $\mathbf{M} = \langle f, g, h \rangle$ is *determinative* iff there is a unique semantics $\mathbf{S}$ which is based on $\mathbf{M}$. We denote by $\mathbf{S}(\mathbf{M})$ the semantics characterized by a determinative evaluation method $\mathbf{M}$.

Clearly, the fuzzy evaluation method is not determinative. However, *any* evaluation method is determinative on the class of acyclic graphs, i.e., produces a unique semantics for such graphs.

So far, no constraints are imposed on the functions of an evaluation method, except symmetry of g. In the following definition, we introduce the notion of *well-behaved* evaluation methods, i.e., methods whose functions satisfy some specific properties. We consider a subset of properties from [18, 25], broadening thus the setting. For instance, the parametrized function $h_{prod,\alpha}$, which allows users to give different importance to degrees of attackers and weights of attacks,[4] is excluded from the previous proposals, while it is allowed in our general setting.

Definition 2.8 (Well-Behaved EM). An evaluation method $\mathbf{M} = \langle f, g, h \rangle$ is *well-behaved* iff the following holds:

(1) f is increasing in the first variable, decreasing in the second variable whenever the first variable is not equal to 0, $f(x, 0) = x$, and $f(0, x) = 0$.
(2) $g() = 0$, $g(x) = x$, $g(x_1, \dots, x_n) = g(x_1, \dots, x_n, 0)$, and $g(x_1, \dots, x_n, y) \leq g(x_1, \dots, x_n, z)$ if $y \leq z$.
(3) $h(0, x) = 0$, $h(1, x) = x$, $h(x, y) > 0$ whenever $xy > 0$, and h is non-decreasing in both components.

[4]If $\alpha < \beta$, then h_α gives more importance to weights than h_β.

In [18, 25, 32], a gradual semantics *should* be based on a well-behaved evaluation method.

3 NOVEL FAMILY OF GRADUAL SEMANTICS

In the literature, there are several works on semantics that deal with weighted attacks [16, 23–25, 28, 29]. All of them are extension-based and extend the semantics proposed by Dung in [22]. A notable exception is the gradual semantics proposed in [25], a generalization of the semantics that deals with semi-weighted graphs in [32]. It is based on the evaluation method $\mathbf{M}_p = \langle f_{comp}, g_{psum}, h_{prod} \rangle$. However, unlike what is conjectured in [25], $\mathbf{M}_p$ does not characterize the semantics since the corresponding system of equations may have several solutions. Hence, $\mathbf{M}_p$ is not determinative. Thus, there is *no* specific gradual semantics in the literature that deals with weighted graphs. In what follows, we propose the first broad family of such semantics. It instantiates the general setting recalled in Section 2. For that purpose, we define a large family of determinative and well-behaved evaluation methods. The latter satisfy additional constraints, namely continuity of their functions.

Definition 3.1 ($\mathbf{M}^*$). We define $\mathbf{M}^*$ as the set of all well-behaved evaluation methods $\mathbf{M} = \langle f, g, h \rangle$ such that:

- $\lim_{x_2 \to x_0} f(x_1, x_2) = f(x_1, x_0)$, $\forall x_0 \neq 0$.
- $\lim_{x \to x_0} g(x_1, \ldots, x_n, x) = g(x_1, \ldots, x_n, x_0)$, $\forall x_0 \neq 0$.
- h is continuous on the second variable
- $\lambda f(x_1, \lambda x_2) < f(x_1, x_2)$, $\forall \lambda < 1, x_1 \neq 0$.
- $g(h(y_1, \lambda x_1), \ldots, h(y_n, \lambda x_n)) \geq$ $\lambda g(h(y_1, x_1), \ldots, h(y_n, x_n))$, $\forall \lambda \in [0, 1]$.

Note that the first two conditions of $\mathbf{M}^*$ relax the continuity conditions from Theorem 2.6 by excluding the value 0; we weaken them in order to capture semantics that are sensitive to the number of attackers (like Weighted Card-based [4]), where even a weak attacker can have a significant impact. The last two conditions of $\mathbf{M}^*$ are specific contraction conditions. It turned out that the fifth condition is satisfied for most combinations of aggregation functions (for g) and $T-$norms (for h).

We show that any evaluation method $\mathbf{M} \in \mathbf{M}^*$ characterizes a semantics, ensuring thus a single solution for each weighted graph.

Theorem 3.2. *Any evaluation method* $\mathbf{M} \in \mathbf{M}^*$ *is determinative.*

We are now ready to introduce the novel family of gradual semantics. It contains semantics based on evaluation methods in $\mathbf{M}^*$.

Definition 3.3 ($\mathbf{S}^*$). We define by $\mathbf{S}^*$ the set of all semantics which are based on an evaluation method in $\mathbf{M}^*$, i.e.,

$$\mathbf{S}^* = \{\mathbf{S}(\mathbf{M}) \mid \mathbf{M} \in \mathbf{M}^*\}.$$

Now we turn to the practical question: how to effectively calculate the values that a semantics $\mathbf{S} \in \mathbf{S}^*$ assigns to arguments in a given graph? In the case of acyclic graphs, they can be calculated directly, starting from non-attacked arguments. In the general case, one needs to calculate the solution of the system of equations (4), which is typically obtained by employing an iterative method. In the following result, we propose an uniform iterative way of calculating strengths of arguments, which can be applied to any semantics from $\mathbf{S}^*$. The idea is that at each step, a value is assigned to each argument. In the initial step, the value of an argument is its basic weight. Then, in each step, the value is recomputed on the basis of the weights of arguments and attacks as well as the values of the attackers of the argument at the previous step.

Theorem 3.4. *Let* $\mathbf{M} = \langle f, g, h \rangle \in \mathbf{M}^*$, $\mathbf{S} = \mathbf{S}(\mathbf{M})$, *and* $\mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R}, \pi \rangle \in \mathtt{AG}$. *For every* $a \in \mathcal{A}$, *we define the sequence* $\{s(a)^{(n)}\}_{n=1}^{+\infty}$ *in the following way:*

- $s(a)^{(1)} = \sigma(a)$,
- $s(a)^{(n+1)} = f(\sigma(a), g(h(\pi((a_1, a)), s(a_1)^{(n)}), \ldots, h(\pi((a_k, a)), s(a_k)^{(n)})))$, *where* $\{a_1, \ldots a_k\} = \mathtt{Att}(a)$.

Then, for every $a \in \mathcal{A}$:

(1) $\{s(a)^{(n)}\}_{n=1}^{+\infty}$ *converges, and*
(2) $\lim_{n \to +\infty} s(a)^{(n)} = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a)$.

This result can be implemented for an arbitrary semantics from $\mathbf{S}^*$, as an algorithm that computes the approximations of strengths.

4 PROPERTIES OF THE NOVEL SEMANTICS

Recently, there is great interest in defining formal properties that can be used for analyzing individual semantics, and comparing distinct ones. Some properties were proposed for semantics that evaluate arguments of flat graphs (eg. [2]) while others (eg. [4, 9]) were proposed for semantics that deal with semi-weighted graphs. However, there are no such properties for analyzing semantics that deal with weighted graphs. In what follows, we bridge this gap.

We consider some properties proposed in [4], and extend them for accounting for weights of attacks. Those properties state that the strength of an argument does not depend on its identity (*Anonymity*); it depends only on arguments that are related to it with a path (*Independence*); it does not depend on argument's outgoing arrows (*Directionality*); it depends only on the argument's basic strength, the weights of its direct attacks and the strengths of its direct attackers (*Equivalence*); it is equal to the argument's basic weight if the argument is not attacked (*Maximality*); it does not take into account neither worthless attackers (those whose strength is equal to 0) nor worthless attacks (*Neutrality*); it is less than the argument's basic weight if the argument has at least one serious attack from a serious attacker (*Weakening*); it is sensitive to the argument's basic weight (*Proportionality*), to the quality of attackers (*Reinforcement*) and to the number of non-worthless attacks and attackers (*Counting*). The formal definitions are below. Let S be a gradual semantics.

Anonymity: S satisfies *anonymity* iff $\forall \mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R}, \pi \rangle$, $\mathbf{G}' = \langle \mathcal{A}', \sigma', \mathcal{R}', \pi' \rangle \in \mathtt{AG}$, for any isomorphism f from $\mathbf{G}$ to $\mathbf{G}'$, the following holds: $\forall\, a \in \mathcal{A}, \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}'}(f(a))$.

Independence: S satisfies *independence* iff $\forall \mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R}, \pi \rangle, \mathbf{G}' = \langle \mathcal{A}', \sigma', \mathcal{R}', \pi' \rangle \in \mathtt{AG}$ s.t $\mathcal{A} \cap \mathcal{A}' = \emptyset$, the following holds: $\forall\, a \in \mathcal{A},\ \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G} \oplus \mathbf{G}'}(a)$.

Directionality: S satisfies *directionality* iff $\forall \mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R}, \pi \rangle \in \mathtt{AG}$, $\forall a, b \in \mathcal{A}$, $\forall \mathbf{G}' = \langle \mathcal{A}', \sigma', \mathcal{R}', \pi' \rangle \in \mathtt{AG}$, s.t. $\mathcal{A}' = \mathcal{A}$, $\sigma' = \sigma$, $\mathcal{R}' = \mathcal{R} \cup \{(a, b)\}$, $\forall x \in \mathcal{R}, \pi'(x) = \pi(x)$, it holds that: $\forall x \in \mathcal{A}$, if there is no path from b to x, then $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(x) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}'}(x)$.

Equivalence: S satisfies equivalence iff $\forall \mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R}, \pi \rangle \in \mathtt{AG}$, $\forall a, b \in \mathcal{A}$, if i) $\sigma(a) = \sigma(b)$, and ii) there exists a bijective function

f from $\texttt{Att}(a)$ to $\texttt{Att}(b)$ s.t. $\forall x \in \texttt{Att}(a)$, $\texttt{Deg}^{\mathrm{S}}_{\mathbf{G}}(x) = \texttt{Deg}^{\mathrm{S}}_{\mathbf{G}}(f(x))$ and $\pi((x,a)) = \pi((f(x),b))$, then $\texttt{Deg}^{\mathrm{S}}_{\mathbf{G}}(a) = \texttt{Deg}^{\mathrm{S}}_{\mathbf{G}}(b)$.

Maximality: S satisfies maximality iff $\forall \mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R}, \pi \rangle \in \mathsf{AG}$, $\forall a \in \mathcal{A}$, if $\texttt{Att}(a) = \emptyset$, then $\texttt{Deg}^{\mathrm{S}}_{\mathbf{G}}(a) = \sigma(a)$.

Neutrality: S satisfies neutrality iff $\forall \mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R}, \pi \rangle \in \mathsf{AG}$, $\forall a, b \in \mathcal{A}$, if i) $\sigma(a) = \sigma(b)$, and ii) $\texttt{Att}(b) = \texttt{Att}(a) \cup \{x\}$ s.t. $x \in \mathcal{A} \setminus \texttt{Att}(a)$ and ($\texttt{Deg}^{\mathrm{S}}_{\mathbf{G}}(x) = 0$ or $\pi((x,b)) = 0$), then $\texttt{Deg}^{\mathrm{S}}_{\mathbf{G}}(a) = \texttt{Deg}^{\mathrm{S}}_{\mathbf{G}}(b)$.

Weakening: S satisfies weakening iff $\forall \mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R}, \pi \rangle \in \mathsf{AG}$, $\forall a \in \mathcal{A}$, if $\sigma(a) > 0$, and $\exists b \in \texttt{Att}(a)$ s.t. $\sigma(b) > 0$ and $\pi((a,b)) > 0$, then $\texttt{Deg}^{\mathrm{S}}_{\mathbf{G}}(a) < \sigma(a)$.

Proportionality: S satisfies proportionality iff $\forall \mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R}, \pi \rangle \in \mathsf{AG}$, $\forall a, b \in \mathcal{A}$, if

- $\texttt{Att}(a) = \texttt{Att}(b)$,
- $\forall x \in \texttt{Att}(a)$, $\pi((x,a)) = \pi((x,b))$,
- $\sigma(a) > \sigma(b)$,
- $\texttt{Deg}^{\mathrm{S}}_{\mathbf{G}}(a) > 0$,

then $\texttt{Deg}^{\mathrm{S}}_{\mathbf{G}}(a) > \texttt{Deg}^{\mathrm{S}}_{\mathbf{G}}(b)$.

Resilience: S satisfies resilience iff $\forall \mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R}, \pi \rangle \in \mathsf{AG}$, $\forall a \in \mathcal{A}$, $\texttt{Deg}^{\mathrm{S}}_{\mathbf{G}}(a) = 0$ iff $\sigma(a) = 0$.

Reinforcement: S satisfies reinforcement iff $\forall \mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R}, \pi \rangle \in \mathsf{AG}$, $\forall a, b \in \mathcal{A}$, if

- $\sigma(a) = \sigma(b)$,
- $\texttt{Deg}^{\mathrm{S}}_{\mathbf{G}}(a) > 0$,
- $\texttt{Att}(a) \setminus \texttt{Att}(b) = \{x\}$, $\texttt{Att}(b) \setminus \texttt{Att}(a) = \{y\}$,
- $\texttt{Deg}^{\mathrm{S}}_{\mathbf{G}}(y) > \texttt{Deg}^{\mathrm{S}}_{\mathbf{G}}(x)$,
- $\pi((x,a)) = \pi((y,b))$,

then $\texttt{Deg}^{\mathrm{S}}_{\mathbf{G}}(a) > \texttt{Deg}^{\mathrm{S}}_{\mathbf{G}}(b)$.

Counting: S satisfies counting iff $\forall \mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R}, \pi \rangle \in \mathsf{AG}$, $\forall a, b \in \mathcal{A}$, if $\sigma(a) = \sigma(b)$, $\texttt{Att}(b) = \texttt{Att}(a) \cup \{x\}$ with $\pi((x,b)) > 0$, and $\texttt{Deg}^{\mathrm{S}}_{\mathbf{G}}(a) > 0$, then $\texttt{Deg}^{\mathrm{S}}_{\mathbf{G}}(a) > \texttt{Deg}^{\mathrm{S}}_{\mathbf{G}}(b)$.

Since graphs are semi-weighted in [4], there is no property that deals with weights of attacks. We propose next *Attack-Sensitivity*, which states that the stronger the weight of an attack, the greater its impact on the targeted argument.

Attack-sensitivity: S is *attack sensitive* iff $\forall \mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R}, \pi \rangle \in \mathsf{AG}$, $\forall a, b \in \mathcal{A}$, if

- $\sigma(a) = \sigma(b)$,
- $\texttt{Att}(a) \setminus \texttt{Att}(b) = \{x\}$, $\texttt{Att}(b) \setminus \texttt{Att}(a) = \{y\}$,
- $\texttt{Deg}^{\mathrm{S}}_{\mathbf{G}}(x) = \texttt{Deg}^{\mathrm{S}}_{\mathbf{G}}(y)$, and
- $\pi((y,b)) > \pi((x,a))$,
- $\texttt{Deg}^{\mathrm{S}}_{\mathbf{G}}(a) > 0$,

then $\texttt{Deg}^{\mathrm{S}}_{\mathbf{G}}(a) > \texttt{Deg}^{\mathrm{S}}_{\mathbf{G}}(b)$.

The above properties are *compatible*, i.e., they can be satisfied all together by a semantics.

Theorem 4.1. *The twelve properties are compatible.*

Let us now analyze semantics of S^* with respect to the above twelve properties. The next result shows that any semantics in S^* satisfies all the first nine properties.

Theorem 4.2. *For any gradual semantics* $\mathrm{S} \in \mathrm{S}^*$, S *satisfies Anonymity, Independence, Directionality, Equivalence, Maximality, Neutrality, Weakening, Proportionality, and Resilience.*

Note that Reinforcement, Counting and Attack-sensitivity are not guaranteed for the whole family of semantics (S^*). However, we show that when the functions (g and h) of the evaluations methods in $\mathbf{M}^*$ satisfy monotony, then the semantics they characterize satisfy the three properties.

Definition 4.3 ($\mathbf{M}^*_e$). We define $\mathbf{M}^*_e$ (e stands for extended) as the set of all evaluation methods $\mathbf{M} = \langle f, g, h \rangle$ such that:

- $\mathbf{M} \in \mathbf{M}^*$,
- $g(x_1, \ldots, x_n, y) < g(x_1, \ldots, x_n, z)$ whenever $y < z$,
- $h(x_1, y) > h(x_2, y)$ whenever $x_1 > x_2$, $y \neq 0$.

Definition 4.4 (S^*_e). We define by S^*_e the set of all semantics which are based on an evaluation method in $\mathbf{M}^*_e$, i.e.,

$$\mathrm{S}^*_e = \{\mathrm{S}(\mathbf{M}) \mid \mathbf{M} \in \mathbf{M}^*_e\}.$$

Proposition 4.5. *The inclusions* $\mathbf{M}^*_e \subseteq \mathbf{M}^*$ *and* $\mathrm{S}^*_e \subset \mathrm{S}^*$ *hold.*

Any semantics in S^*_e satisfies *all* the twelve properties.

Theorem 4.6. *For any gradual semantics* $\mathrm{S} \in \mathrm{S}^*_e$, S *satisfies all the 12 properties.*

Theorems 4.2 and 4.6 can be seen as big steps towards the ultimate goals of *fully characterizing* the whole family of all gradual semantics that satisfy the first nine properties and the whole family of semantics that satisfy the twelve properties. By saying that, we claim that S^* is very broad and encompasses many semantics, however we also recognize that S^* does not cover *all* the possible gradual semantics that can be defined for evaluating arguments in weighted graphs.

5 SOME INSTANCES OF THE FAMILY S^*

The novel family of semantics is general in that it only specifies constraints on the evaluation methods underlying its semantics. The aim of this section is to provide examples of specific semantics covered by S^*. We provide three groups of such semantics. The first one contains five semantics that were proposed in the literature and that deal with flat/semi-weighted argumentation graphs. The second group contains generalizations of the five previous semantics for accounting for weights of attacks. Indeed, we generalize each of the five semantics in various ways, by considering different possible functions h. The last group contains one semantics, which has no counterpart in the literature.

5.1 Five existing semantics as instances of S^*

There are several gradual semantics in the literature. Some of them like h-Categorizer (defined for acyclic graphs in [12] and for graphs with cycles in [36]) and compensation-based semantics [3] deal with flat graphs, while others like weighted h-Categorizer, Weighted Max-based, Weighted Card-based [4], Trust-based [20] deal with semi-weighted graphs (only arguments are weighted). Table 2 recalls the formal definitions of the first five semantics. In what follows, we show that those five semantics are instances of the new family (S^*) while Trust-based (TB) is not. Indeed, we have seen previously that (TB) is based on an evaluation method, the fuzzy method $\mathbf{M}_F$, which is neither determinative nor well-behaved in the sense of definitions 2.7 and 2.8.

Semantics	Formal definition	Type of graphs
h-Categorizer [12]	$\mathtt{Deg}_{\mathbf{G}}^{h}(a) = \frac{1}{1+\sum\limits_{b\mathcal{R}a} \mathtt{Deg}_{\mathbf{G}}^{h}(b)}$	Flat
Compensation-based [3]	$s_{\mathbf{G}}^{\alpha-\mathtt{BBS}}(a) = 1 + \left(\sum\limits_{b\mathcal{R}a} \frac{1}{(s(b))^{\alpha}}\right)^{1/\alpha}, \alpha \in (0, +\infty)$	Flat
Weighted h-Categorizer [4]	$\mathtt{Deg}_{\mathbf{G}}^{\mathtt{Hbs}}(a) = \frac{\sigma(a)}{1+\sum\limits_{b\mathcal{R}a} \mathtt{Deg}_{\mathbf{G}}^{\mathtt{Hbs}}(b)}$	Semi-weighted
Weighted Max-based [4]	$\mathtt{Deg}_{\mathbf{G}}^{\mathtt{Mbs}}(a) = \frac{\sigma(a)}{1+\max\limits_{b\mathcal{R}a} \mathtt{Deg}_{\mathbf{G}}^{\mathtt{Mbs}}(b)}$	Semi-weighted
Weighted Card-based [4]	$\mathtt{Deg}_{\mathbf{G}}^{\mathtt{Cbs}}(a) = \frac{\sigma(a)}{1+\lvert\mathtt{AttF}_{\mathbf{G}}(a)\rvert + \frac{\sum_{b \in \mathtt{AttF}_{\mathbf{G}}(a)} \mathtt{Deg}_{\mathbf{G}}^{\mathtt{Cbs}}(b)}{\lvert\mathtt{AttF}_{\mathbf{G}}(a)\rvert}}$	Semi-weighted

Table 2: Existing Gradual Semantics.

THEOREM 5.1. *The Trust-based semantics is based on the evaluation method* $\mathbf{M}_F = \langle f_{min}, g_{max}, h_{prod} \rangle$. *However,* $\mathbf{M}_F$ *is neither determinative nor well-behaved.*

Weighted Max-based, denoted by Mbs in [4], is a semantics that favors the quality of attackers over their quantity. The next result shows that it is based on an evaluation method, which is both determinative and well-behaved. This means that Mbs is an instance of the general setting. Since Mbs does not deal with weighted attacks, we use h_{prod} for h in its evaluation method, however it may be any function of those recalled in Table 1. This holds for the evaluation methods that define the other semantics.

THEOREM 5.2. *Let* $\mathbf{M}_M = \langle f_{frac}, g_{max}, h_{prod} \rangle$. *For any* $\mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R}, \pi \rangle \in \mathtt{AG}$ *such that* $\pi \equiv 1$, *the following properties hold.*

- $\mathtt{Deg}_{\mathbf{G}}^{\mathtt{Mbs}} \equiv \mathtt{Deg}_{\mathbf{G}}^{\mathtt{S}(\mathbf{M}_M)}$.
- $\mathbf{M}_M$ *is both determinative and well-behaved.*

Let us switch to the compensation-based semantics ($\alpha-\mathtt{BBS}$) from [3], where $\alpha \in (0, +\infty)$ is a parameter allowing compensation between quality and quantity of attackers. With this semantics, the stronger the argument, the smaller its value. Thus, the general setting discussed in the paper cannot capture $\alpha-\mathtt{BBS}$ in a direct way. However, we define an evaluation method that defines a class of semantics that is equivalent to $\alpha-\mathtt{BBS}$. Interestingly, the same evaluation method defines also Weighted h−Categorizer [4] which is denoted Hbs, and h−Categorizer [12], denoted by $h-\mathtt{Cat}$. We show that the evaluation method is determinative and well-behaved, meaning that the three semantics are also instances of $\mathbf{S}^*$.

THEOREM 5.3. *Let* $\mathbf{M}_\alpha = \langle f_{frac}, g_{sum,\alpha}, h_{prod} \rangle$. *For any* $\mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R}, \pi \rangle \in \mathtt{AG}$, *the following properties hold:*

(1) *If* $\sigma \equiv 1$ *and* $\pi \equiv 1$, *then* $s_{\mathbf{G}}^{\alpha-\mathtt{BBS}} \equiv \frac{1}{\mathtt{Deg}_{\mathbf{G}}^{\mathtt{S}(\mathbf{M}_\alpha)}}$.

(2) *If* $\sigma \equiv 1$, $\pi \equiv 1$, *and* $\alpha = 1$, *then* $\mathtt{Deg}_{\mathbf{G}}^{h-\mathtt{Cat}} \equiv \mathtt{Deg}_{\mathbf{G}}^{\mathtt{S}(\mathbf{M}_\alpha)}$.

(3) *If* $\pi \equiv 1$, *and* $\alpha = 1$, *then* $\mathtt{Deg}_{\mathbf{G}}^{\mathtt{Hbs}} \equiv \mathtt{Deg}_{\mathbf{G}}^{\mathtt{S}(\mathbf{M}_\alpha)}$.

(4) $\mathbf{M}_\alpha$ *is both determinative and well-behaved.*

Weighted Card-Based semantics [4], denoted by Cbs, is a semantics which favors the quantity of attackers over their quality. In its formal definition in Table 2, $\mathtt{AttF}_{\mathbf{G}}(a)$ is the set of attackers of a whose basic weight is strictly positive. We show that it is another instance of the new family. Indeed, it is based on a determinative and well-behaved evaluation method.

Definition 5.4 ($\mathbf{M}_C$). The *Card-based evaluation method* is the tuple $\mathbf{M}_C = \langle f_{frac2}, g_{card}, h_{prod} \rangle$ such that

- $f_{frac2}(x_1, 0) = 0$,
- $f_{frac2}(x_1, x_2) = \frac{x_1}{2+x_2}$ for $x_2 > 0$,
- $g_{card}(0, \dots, 0) = 0$,
- if there is $x_i \neq 0$ for some $i \leq n$,

$$g_{card}(x_1, \dots, x_n) = |\{i \; | x_i \neq 0\}| - 1 + \frac{\sum\limits_{1 \leq i \leq n} x_i}{|\{i \; | x_i \neq 0\}|}.$$

THEOREM 5.5. *The following properties hold:*

- *For all* $\mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R}, \pi \rangle \in \mathtt{AG}$ *such that* $\pi \equiv 1$,

$$\mathtt{Deg}_{\mathbf{G}}^{\mathtt{Cbs}} \equiv \mathtt{Deg}_{\mathbf{G}}^{\mathtt{S}(\mathbf{M}_C)}.$$

- $\mathbf{M}_C$ *is both determinative and well-behaved.*

We have seen that the three evaluation methods $\mathbf{M}_\alpha, \mathbf{M}_C, \mathbf{M}_M$ are both determinative and well-behaved. In what follows, we show that they are also instances of $\mathbf{M}^*$, and the semantics they characterize are instances of $\mathbf{S}^*$.

THEOREM 5.6. *The following inclusions hold:*

- $\{\mathbf{M}_\alpha, \mathbf{M}_C\} \subset \mathbf{M}_e^*$ *and* $\mathbf{M}_M \in \mathbf{M}^*$.
- $\{\mathtt{S}(\mathbf{M}_\alpha), \mathtt{S}(\mathbf{M}_C)\} \subset \mathbf{S}_e^*$ *and* $\mathtt{S}(\mathbf{M}_M) \in \mathbf{S}^*$.

5.2 Generalizations of existing semantics

We have seen in the previous section that the five gradual semantics recalled in Table 2 are instances of the new family. However, those semantics deal only with flat or semi-weighted argumentation graphs. In what follows, we show that $\mathbf{S}^*$ encompasses several instances that *generalize* the five semantics for accounting for weights of attacks. Indeed, we generalize each of them in various ways by considering different functions aggregating the weights of attacks and degrees of attackers. For that purpose, we consider the following broad class $\mathbf{H}$ of functions h.

Definition 5.7 ($\mathbf{H}$). We define $\mathbf{H}$ as the set of all functions h : $[0, 1] \times [0, 1] \to [0, 1]$ such that:

- $h(0, x) = 0$, $h(1, x) = x$,
- $\forall \lambda \in [0, 1]$, $h(x, \lambda y) \geq \lambda h(x, y)$, and
- h is non-decreasing in both variables and continuous on the second variable.

It is worth mentioning that the class $\mathbf{H}$ includes the functions presented in Table 1 except h_{min}.

Let us now start by generalizing Weighted Max-based semantics Mbs for dealing with weighted graphs. Recall that this semantics is based on the evaluation method $\mathbf{M}_M = \langle f_{frac}, g_{max}, h_{prod} \rangle$, where the choice of h_{prod} was arbitrary as the semantics does not consider weights of attacks. The idea of generalizing Mbs is to allow in its evaluation method $\mathbf{M}_M$ any $h \in \mathbf{H}$. We show that each $\mathbf{M}^h_M = \langle f_{frac}, g_{max}, h \rangle$ is an element of $\mathbf{M}^*$, hence it characterizes a semantics of the set $\mathbf{S}^*$. We provide thus as many semantics that extend Mbs as elements in the $\mathbf{H}$.

Theorem 5.8. *For any $h \in \mathbf{H}$, the following hold:*

- $\mathbf{M}^h_M = \langle f_{frac}, g_{max}, h \rangle \in \mathbf{M}^*$.
- $\mathbf{S}(\mathbf{M}^h_M) \in \mathbf{S}^*$.
- *For any* $\mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R}, \pi \rangle \in \mathsf{AG}$ *such that* $\pi \equiv 1$,

$$\mathsf{Deg}^{\mathsf{Mbs}}_{\mathbf{G}} \equiv \mathsf{Deg}^{\mathbf{S}(\mathbf{M}^h_M)}_{\mathbf{G}}.$$

From Theorem 4.2, it follows that any semantics generalizing Mbs satisfies all the principles except Reinforcement, Counting and Attack-Sensitivity. While the violation of Reinforcement and Counting is due to the fact that these semantics use only one attacker, Attack-Sensitivity can be ensured if the function h of the underlying evaluation method is strictly monotonic.

Theorem 5.9. *For any $h \in \mathbf{H}$ such that*

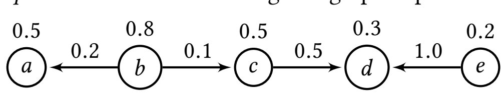
$$h(x_1, y) > h(x_2, y) \text{ whenever } x_1 > x_2, y \neq 0$$

it holds that $\mathbf{S}(\mathbf{M}^h_M)$ satisfies Attack-Sensitivity.

Let us illustrate the family of extended Mbs semantics. For that purpose, we consider $h = h_{prod}$. Hence, the evaluation method of the extended semantics is $\mathbf{M}_M = \langle f_{frac}, g_{max}, h_{prod} \rangle$ and the corresponding semantics will be denoted by Mbs_{prod} (Mbs_{prod} is $\mathbf{S}(\mathbf{M}^{h_{prod}}_M)$). For any $\mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R}, \pi \rangle \in \mathsf{AG}$, for any $a \in \mathcal{A}$,

$$\mathsf{Deg}^{\mathsf{Mbs}_{prod}}_{\mathbf{G}}(a) = \frac{\sigma(a)}{1 + \max\limits_{b\mathcal{R}a}(\pi((b,a))\mathsf{Deg}^{\mathsf{Mbs}_{prod}}_{\mathbf{G}}(b))}. \quad (6)$$

Example 5.10. Consider the weighted graph depicted below.



It is easy to check that $\mathsf{Deg}^{\mathsf{Mbs}_{prod}}_{\mathbf{G}}(b) = \sigma(b) = 0.8$ and $\mathsf{Deg}^{\mathsf{Mbs}_{prod}}_{\mathbf{G}}(e) = \sigma(e) = 0.2$ since b and e are not attacked. Furthermore, $\mathsf{Deg}^{\mathsf{Mbs}_{prod}}_{\mathbf{G}}(a) = \frac{25}{58} \approx 0.431$, $\mathsf{Deg}^{\mathsf{Mbs}_{prod}}_{\mathbf{G}}(c) = \frac{25}{54} \approx 0.463$, and $\mathsf{Deg}^{\mathsf{Mbs}_{prod}}_{\mathbf{G}}(d) = 0.24$.

Let us now generalize h-Categorizer, Weighted h-Categorizer, and compensation-based semantics for dealing with weighted attacks. For that purpose, we will use the same broad set of functions h, namely $\mathbf{H}$. As for Mbs, each of the three semantics will be generalized by several semantics.

Theorem 5.11. *For any $h \in \mathbf{H}$, $\forall \alpha \in (0, +\infty)$, the following hold:*

- $\mathbf{M}^h_\alpha = \langle f_{frac}, g_{sum,\alpha}, h \rangle \in \mathbf{M}^*_e$.
- $\mathbf{S}(\mathbf{M}^h_\alpha) \in \mathbf{S}^*_e$.
- *For any* $\mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R}, \pi \rangle \in \mathsf{AG}$,
  - *If* $\sigma \equiv 1$ *and* $\pi \equiv 1$, *then* $s^{\alpha-\mathsf{BBS}}_{\mathbf{G}} \equiv \frac{1}{\mathsf{Deg}^{\mathbf{S}(\mathbf{M}^h_\alpha)}_{\mathbf{G}}}$.
  - *If* $\pi \equiv 1$ *and* $\alpha = 1$, *then* $\mathsf{Deg}^{\mathsf{Hbs}}_{\mathbf{G}} \equiv \mathsf{Deg}^{\mathbf{S}(\mathbf{M}^h_\alpha)}_{\mathbf{G}}$.
  - *If* $\sigma \equiv 1$, $\pi \equiv 1$ *and* $\alpha = 1$, *then* $\mathsf{Deg}^{h}_{\mathbf{G}} \equiv \mathsf{Deg}^{\mathbf{S}(\mathbf{M}^h_\alpha)}_{\mathbf{G}}$.

Let us illustrate the family of extended Weighted $h-$Categorizer semantics. We consider the $\mathbf{S}(\mathbf{M}^{h_{prod}}_1)$, denoted by Hbs_{prod} in what follows. For any $\mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R}, \pi \rangle \in \mathsf{AG}$, for any $a \in \mathcal{A}$,

$$\mathsf{Deg}^{\mathsf{Hbs}_{prod}}_{\mathbf{G}}(a) = \frac{\sigma(a)}{1 + \sum\limits_{b\mathcal{R}a} \pi((b,a))\mathsf{Deg}^{\mathsf{Hbs}_{prod}}_{\mathbf{G}}(b)}. \quad (7)$$

Example 5.10 (Cont) $\mathsf{Deg}^{\mathsf{Hbs}_{prod}}_{\mathbf{G}}(a) \approx 0.431$, $\mathsf{Deg}^{\mathsf{Hbs}_{prod}}_{\mathbf{G}}(b) = 0.8$, $\mathsf{Deg}^{\mathsf{Hbs}_{prod}}_{\mathbf{G}}(c) \approx 0.463$, $\mathsf{Deg}^{\mathsf{Hbs}_{prod}}_{\mathbf{G}}(d) \approx 0.207$ and $\mathsf{Deg}^{\mathsf{Hbs}_{prod}}_{\mathbf{G}}(e) = 0.2$. Note that the degrees of the arguments a and c are the same for both semantics, since they have only one attacker. On the other hand, $\mathsf{Deg}^{\mathsf{Hbs}_{prod}}_{\mathbf{G}}(d) < \mathsf{Deg}^{\mathsf{Mbs}_{prod}}_{\mathbf{G}}(d)$, since g_c, unlike g_m, also takes into account the weaker attacker e of d.

We next extend Card-based semantics for considering weighted attacks. As for the previous semantics, we consider any function $h \in \mathbf{H}$.

Theorem 5.12. *For any $h \in \mathbf{H}$, the following hold:*

- $\mathbf{M}^h_C = \langle f_{frac2}, g_{card}, h \rangle \in \mathbf{M}^*_e$.
- $\mathbf{S}(\mathbf{M}^h_C) \in \mathbf{S}^*_e$.
- $\forall \mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R}, \pi \rangle \in \mathsf{AG}$ *s.t.* $\pi \equiv 1$, $\mathsf{Deg}^{\mathsf{Cbs}}_{\mathbf{G}} \equiv \mathsf{Deg}^{\mathbf{S}(\mathbf{M}^h_C)}_{\mathbf{G}}$.

5.3 Euler-Max-based semantics

We have seen in the previous section that all the semantics recalled in Table 2 use a function of the form $\frac{x_1}{a+x_2}$ for f. Interestingly enough, our Theorem 3.2 offers more alternatives for f. As an illustration, we use the function $f_{exp}(x_1, x_2) = x_1 e^{-x_2}$.

Definition 5.13 (Euler-Max-based EM). The Euler-Max-based evaluation method is the tuple $\mathbf{M}_e = \langle f_{exp}, g_{max}, h_{prod} \rangle$.

We show that $\mathbf{M}_e$ is determinative.

Theorem 5.14. $\mathbf{M}_e \in \mathbf{M}^*$. *Consequently,* $\mathbf{S}(\mathbf{M}_e) \in \mathbf{S}^*$.

Let us denote $\mathbf{S}(\mathbf{M}_e)$ by EMbs (for Euler-Max-based semantics).

Definition 5.15 (Euler-Max-based Semantics). Let $\mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R}, \pi \rangle \in \mathsf{AG}$. For any $a \in \mathcal{A}$,

$$\mathsf{Deg}^{\mathsf{EMbs}}_{\mathbf{G}}(a) = \sigma(a) \cdot e^{-\max\limits_{b\mathcal{R}a} \pi((b,a))\mathsf{Deg}^{\mathsf{EMbs}}_{\mathbf{G}}(b)}$$

Example 5.10 (Cont) In graph $\mathbf{G}$ of the running example, we obtain $\mathsf{Deg}^{\mathsf{EMbs}}_{\mathbf{G}}(a) \approx 0.426$, $\mathsf{Deg}^{\mathsf{EMbs}}_{\mathbf{G}}(b) = 0.8$, $\mathsf{Deg}^{\mathsf{EMbs}}_{\mathbf{G}}(c) \approx 0.462$, $\mathsf{Deg}^{\mathsf{EMbs}}_{\mathbf{G}}(d) \approx 0.234$ and $\mathsf{Deg}^{\mathsf{EMbs}}_{\mathbf{G}}(e)) = 0.2$.

EMbs is correctly defined (from Theorem 3.2) and satisfies all the principles except Reinforcement and Counting.

Theorem 5.16. EMbs *satisfies all the principles except Reinforcement and Counting.*

Note that, like Weighted Max-based semantics (Mbs) and its extended version Mbs_{prod}, EMbs considers only the strongest attacker. Indeed, it uses the function g_{max} as those semantics. We show that for each attacked argument, the value given by EMbs is weaker than the one assigned by $\mathtt{Mbs}_{prod}$ to the same argument. Indeed, the strongest attacker is more harmful with EMbs than with $\mathtt{Mbs}_{prod}$.

Theorem 5.17. *For any* $\mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R}, \pi \rangle \in \mathtt{AG}$*, for any* $a \in \mathcal{A}$ *s.t. there exists* $b \in \mathtt{Att}(a)$ *with* $\sigma(b) > 0$*,* $\mathtt{Deg}^{\mathtt{EMbs}}_{\mathbf{G}}(a) < \mathtt{Deg}^{\mathtt{Mbs}_{prod}}_{\mathbf{G}}(a)$.

EMbs compensates the small number of attackers that is considered for the evaluation of the strength of an argument. Indeed, since it considers only one attacker (among maybe several ones), it gives a lot of power to that chosen attacker. Mbs is less requiring.

6 RELATED WORK

There are several semantics in the literature that deal with weighted attacks, and all of them are extension-based [16, 23, 24, 28, 29]. They extend Dung's ([22]) semantics for accounting for varied-strengths of attacks. When all attacks have the same weight, they coincide with Dung's ones in the corresponding flat graph. It was shown in [4] that Dung's semantics do not satisfy all the properties discussed in this paper. Any property that is violated in case of flat/semi-weighted graphs, it is also violated in case of weighted ones. Thus, the existing semantics that deal with weighted graphs are different from those of the family we proposed. This shows also that those semantics are not instances of $\mathbf{S}^*$. In what follows, we go further by showing that extension semantics cannot be based on evaluation methods. For that purpose, we consider complete, grounded, preferred, and stable semantics as proposed in [22]. Those semantics were defined for flat graphs (i.e., $\mathbf{G} = \langle \mathcal{A}, \sigma \equiv 1, \mathcal{R}, \pi \equiv 1 \rangle$). They start first by identifying subsets of arguments that are collectively acceptable, called *extensions*. Let $\mathbf{G} \in \mathtt{AG}$ be an arbitrary but fixed flat graph, and let $\mathtt{Ext}_x(\mathbf{G})$ denote the set of all extensions of $\mathbf{G}$ under semantics $x \in \{c, p, s, g\}$, where c, p, s, g stand respectively for *complete*, *preferred*, *stable*, and *grounded* semantics.

Once extensions are computed, the second step consists of assigning a single overall strength to each argument by checking its membership in extensions. In the argumentation literature (e.g., [8, 18, 27, 35]), three qualitative values are used: *skeptically accepted* (the argument belongs to all extensions), *credulously accepted* (the argument is in some but not all extensions), and *rejected* otherwise. In what follows, we *refine* this definition by considering more values and numerical ones from the interval $[0, 1]$. Indeed, the overall strength of an argument is the proportion of extensions containing it. In case there is no extension, the value of each argument is 0. Obviously, an skeptically accepted argument gets the maximal value 1 and a rejected argument gets 0.

Definition 6.1 (Extension Semantics). Let $x \in \{c, p, s, g\}$, $\mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R}, \pi \rangle \in \mathtt{AG}$, and $a \in \mathcal{A}$.

$$\mathtt{Deg}^{x}_{\mathbf{G}}(a) = \begin{cases} 0 & \text{if } \mathtt{Ext}_x(\mathbf{G}) = \emptyset \\ \frac{|\mathcal{E} \in \mathtt{Ext}_x(\mathbf{G}) \ | \ a \in \mathcal{E}|}{|\mathtt{Ext}_x(\mathbf{G})|} & \text{otherwise} \end{cases}$$

Example 2.5 (Cont) The graph $\mathbf{G}$ has two stable/preferred extensions ($\mathcal{E}_1 = \{a\}$ and $\mathcal{E}_2 = \{b\}$) and three complete ones ($\mathcal{E}_1$, $\mathcal{E}_2$, and $\mathcal{E}_3 = \emptyset$). Thus, $\mathtt{Deg}^{x}_{\mathbf{G}}(a) = \mathtt{Deg}^{x}_{\mathbf{G}}(b) = \frac{1}{2}$, for $x \in \{p, s\}$ and $\mathtt{Deg}^{c}_{\mathbf{G}}(a) = \mathtt{Deg}^{c}_{\mathbf{G}}(b) = \frac{1}{3}$. Finally, $\mathtt{Deg}^{g}_{\mathbf{G}}(a) = \mathtt{Deg}^{g}_{\mathbf{G}}(b) = 0$.

The following result shows that there is no evaluation method that can return the evaluations of (stable, complete, preferred, grounded) semantics. This means that these semantics *cannot* be expressed in the general setting of [18].

Theorem 6.2. *Let* $\mathbf{S} \in \{s, p, c, g\}$*. There is no evaluation method* $\mathbf{M}$ *such that* $\mathbf{S}$ *is based on* $\mathbf{M}$.

Gabbay and Rodrigues ([26]) developed a method, called Iterative Schema (IS), for evaluating arguments in semi-weighted graphs. Like Trust-based, IS cannot be based on a well-behaved evaluation method. Furthermore, it does not satisfy some properties including Maximality. It is thus different from semantics of $\mathbf{S}^*$.

The two gradual semantics (QuAD, DF-QuAD) were proposed respectively in [10, 37] for dealing with acyclic bipolar graphs, i.e., graphs where arguments may have basic weights and may be attacked and supported. Both QuAD and DF-QuAD use evaluation methods. However, those methods are not determinative (thus are not in the class $\mathtt{M}^*$) for the class of all possible graphs.

In [38], another gradual semantics was defined for bipolar graphs. It uses an evaluation method, however the authors did not investigate its properties, and more precisely whether it is determinative.

In [1], the family of ranking semantics was introduced. Unlike gradual semantics, ranking semantics do not necessarily assign a single value to each argument. They rather focus on defining a total ordering on arguments. Examples of such semantics are those defined in [1, 13–15, 21, 27].

In [33], evaluation of arguments in graphs whose topology is uncertain (probabilities are assigned to arguments and attacks) is studied. In our case, graphs are fixed. In [34] probabilities are assigned to attacks and express belief in attacks. Both approaches are different from ours since they derive a probability of each argument from probability distributions over sets of arguments.

7 CONCLUSION

The paper investigates semantics for argumentation graphs where both arguments and attacks may have varied strengths. It proposes a fine grained definition of gradual semantics using the evaluation method approach. It defines the first family of gradual semantics in the literature that deal with weighted attacks. The proposed family is broad enough to encompass five semantics that were proposed in the literature for dealing with flat or semi-weighted graphs. It also generalizes in several ways each of those semantics for accounting for weights of attacks, and covers a large class of other semantics including Euler-Max-based. The new semantics are theoretically analyzed against the set of principles that were proposed in [4]. For that purpose, those principles were first extended for weighted graphs and a novel principle was proposed.

This work can be extended in several ways. First, we plan to characterize the whole family of gradual semantics that satisfy (the first nine, all the twelve) properties discussed in Section 4. Such results would give a complete view on all the possible gradual semantics that can be defined and that have the same behavior as those we proposed. As existing gradual and ranking semantics share similar properties, another line of research would consist of reformulating ranking semantics in terms of determinative evaluation methods, unifying thus their definitions with those of gradual semantics.

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