Foundations for a Logic of Arguments

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Abstract. This paper lays the foundations of a *logic of argumentation* in which arguments, as well as attacks and supports among arguments are all defined in a unifying formalism. In the latter, an argument is denoted as a pair displaying a *reason* and a *conclusion* but no condition is required to hold relating the reason to the conclusion. We introduce a series of inference rules relating arguments and show how the resulting logic captures important features of argumentation that hitherto have not been captured by existing formalisms. [1]

1 INTRODUCTION

Argumentation is a common activity in everyday life. Indeed, people frequently justify opinions, decisions or actions by *arguments* in order to increase (or to decrease) their acceptability for an audience. Arguments are therefore of great importance. In order to build argumentation systems that are able to capture natural language arguments, it is fundamental to have a clear understanding and a fair representation of this key notion of argument. For that purpose, the following issues need to be investigated:

- what is an argument?
- what may be the conclusion of an argument?
- what may be the premises of an argument?
- what is the nature of the link between premises and conclusion of an argument?

In the AI literature on argumentation, an argument is viewed as identifying a *reason* for concluding some statement. The reason is a set of premises that somehow leads to the conclusion. Hopefully, linguists and philosophers working on argumentation agree with such an idea. In [3], the linguist Apothéloz argued that the reason is *oriented in favour of* the conclusion to which it *propagates its truth*.

The views of the three communities may differ as to the kind of conclusions that can be justified by arguments and the kind of premises to be used in arguments. In the AI literature, arguments are built in favour of a statement or in favour of its contrary. That is, an agent may argue that a statement holds or that the opposing statement holds. However, it is not possible to build an argument in favour of *not concluding* a statement (this does *not* mean that the statement is false). Yet such arguments are common in natural language argumentation as shown by the next example adapted from [3].

[1] This paper reviews and extends two previous papers by the authors on this topic [1, 2]

Adam: *Steve is very smart but didn't work hard this term, so it's unclear whether he will pass his exams.*

Through the above argument, Adam *does not commit* to the conclusion "Steve will fail his exams". He simply means that "Steve will pass his exams" cannot be concluded. Notice that Dung's style argumentation [4] cannot capture such a stand-alone argument: In abstract argumentation, failure can only be expressed by means of an attack from an argument over another argument but there is no such attack in the example, as there is a single argument, and it does *not* attack itself.

Adam does not commit either to the opposite conclusion "Steve will pass his exams". His argument expresses that, in the case of a smart person who did not work hard, it is not possible to predict whether he will fail or pass his exams.

More generally, an argument may justify why a statement:

- is true,
- is false,
- is open to doubt.

The last case actually encompasses two situations: the situation where the opposing statement holds and the situation of *complete indeterminacy* regarding the statement. Importantly, the notion of a statement here is very general as opposed to most proposals for formalizing argumentation, where arguments have a simple format whereby arguments cannot appear in either the premises or conclusions though notable exceptions include [5–7]. Apothéloz offers in [3] natural language arguments whose premises (resp., conclusion) may be arguments. Consider the following argument:

> *The fact that Ryan's car is in the car park is not a reason to conclude that Ryan is in his office. Indeed, his car is broken.*

This example displays an argument embedded in another. The first one says that "Ryan is in his office since his car is in the car park". The second argument concludes that the first fails. The premise used for that purpose is: "Ryan's car is broken". Thus, the argument has a simple premise but its conclusion is a *rejection*.

In short, a rejection is a denial of an argument: While the argument being denied offers a reason for a statement, a rejection of the argument expresses that the reason brought forward cannot serve to conclude the statement. Here is an illustration:

Brian: *Steve will fail his exams. It is raining.*
Craig: *Rain is not a reason to infer that Steve will fail his exams.*

Craig's utterance is a rejection of Brian's argument but is not an argument as it merely expresses –without providing any justification– that Brian's argument should be rejected. Though, a rejection need not make any claims whether the conclusion of the denied argument holds. It can actually be the case that rejection acknowledges truth of the conclusion of the denied argument. For example, such is the case should Craig add:

Craig: *Lack of motivation is the reason.*

That is, Steve will indeed fail his exams according to Craig. Thus, Craig agrees with the conclusion of Brian's argument. Still, Craig disagrees with Brian's argument.

Existing models in computational argumentation must resort to an encoding of Brian's argument in order to capture the fact that Craig's utterance is a rejection of Brian's. Moreover, please notice that Brian's argument is certainly *not self-attacking*.

On a general level, in Dung's style argumentation systems [4], arguments may attack each other. Attacks are viewed as reasons for rejecting arguments (in the sense that acceptability of the attacked arguments fails). In argumentation in natural language, such a phenomenon may be expressed by meta-arguments. Indeed, it is possible to offer an argument in favour of rejecting another argument.

As regards premises, a meaningful distinction is as follows:

- *Factual* reasons: The premises are taken as granted. An example is:
  It has been raining all morning, the outdoor tennis tournament this afternoon will be cancelled.
- *Hypothetical* reasons: The premises are not meant to be endorsed. An example is:
  An economic crisis in Germany would be a reason for a declining value of the euro.

Although there seems to be currently no system having special machinery for dealing with hypotheticals, a number of them (e.g., [8–11]) can deal with hypothetical reasoning by adding hypothetical assumptions to the knowledgebase.

A key feature of an argument is the link between the reason and the conclusion. In existing argumentation systems, the link is *deductive* i.e., the conclusion follows from the reason as an inference in a logic. However, several other kinds of links may be encountered in natural language arguments including *causal*, *analogical*, and others. Here is an illustration:

> *My new phone is the same brand as my former phone. To redial a number, I should probably use the same procedure as with my former phone.*

In the above argument, a reason is given "my new phone is the same brand as my former phone" so that, in order to redial, I should try the same routine as I was used to. In the formalism to be developed in this paper, such an argument is represented with its reason and conclusion, but there is no need to resort to extra premises (presumably rather convoluted) required in a formal derivation of the conclusion.

As to the four questions about the notion of an argument that were listed at the start of the introduction, we can propose the following features to be key to an argument:

- An argument provides a presumptive explanation as to why a statement holds, or why the statement does not hold, or why the statement is doubted.
- A statement is the conclusion of the argument, and can be a simple proposition or a complex one like an argument (or a rejection) or a combination, including nesting, of arguments.
- An argument may resort to factual premises or to hypothetical premises. They play the role of an explanation, and, similarly to the conclusion, range from a simple proposition to a complex one like an argument (or a rejection) or a combination, including nesting, of arguments.

- The nature of the link (relating the premises to the conclusion) can be any among a range of possibilities (e.g. deductive, causal, inductive, analogical, etc).

Taking advantage of the proposal made by Apothéloz in [3], we introduce a logical setting for *representing* and *reasoning* about arguments that enjoy all the features discussed above. We first give a formal definition of argument and rejection of argument. This gives the language $\mathbb{L}_{RC}$ of our logic (in Section 2). We provide a set of *inference rules* that show how arguments (respectively rejections of arguments) are tied together, in the sense that an agent presenting an argument thereby commits himself to other arguments (so an inference rule $\frac{\alpha}{\beta}$ means that holding argument α entails committing to argument β). That is, the inference rules provide us with a notion of equivalence between arguments, they enable us to express that δ is a counter-argument to α, and so on. This gives us the inference system $\Vdash$ of our logic (Section 3). It is essential to notice that $\Vdash$ has nothing to do with evaluation of arguments and does not deal with acceptability of arguments (as is done by acceptability semantics [4]). Instead, $\Vdash$ expresses what one should expect when committing to a given argument. We investigate some properties of the logic and illustrate how *attacks* and *supports* between arguments are expressed as arguments (Section 4). By the way, although we focus throughout the text on a single system, alternative sets of inference rules are possible depending on various considerations. We feel that the basis of our approach is definitely more important than details about such and such inference rule. As a result of its high level of expressiveness, the new logic captures important features of argumentation that hitherto have not been captured by existing formalisms. Furthermore, it lays the foundations of a fully fledged *argumentation logic*.

2 FORMAL SYNTAX

We present a formalism to represent arguments, inspired by Apothéloz [3]. It is built upon a classical propositional language $\mathbb{L}$ with the classical connectives $\neg, \vee, \wedge, \rightarrow, \leftrightarrow$. The formalism also uses the symbols $\mathcal{R}$ and $\mathcal{C}$, and additional operators, namely $-, |, \&$ (not, or, and), applying to arguments. Thus, two negation operators are needed: $\neg$ for denying propositional formulas ($\neg x$ denotes that x is false), and $-$ for denying $\mathcal{R}(.)$ and $\mathcal{C}(.)$. Please note that $\neg\neg x$ is identified with x and $--\mathcal{R}(.)$ is identified with $\mathcal{R}(.)$ (similarly, $--\mathcal{C}(.)$ is identified with $\mathcal{C}(.)$).

An *argument* gives a reason for concluding a statement. It has two parts: its *premises* (or its reason) and its *conclusion*, following several models (most significantly, [12]) in computational argumentation. An argument is interpreted as follows: its conclusion holds *because* it follows, according to a given notion, from the premises. The notion refers to the nature of the link (for instance, the premises cause the conclusion). Also, a *rejection* is a statement denying an argument. The premises and conclusion occurring in the rejection are those of the denied argument. The difference is that there is "$-$" in front of the (leftmost occurrence of the) $\mathcal{R}$ symbol.

In our formalism, arguments and rejections thereof form the class of *RC-formulas*, denoted $\mathbb{L}_{RC}$.

Definition 1 (RC-formulas). *An* RC-formula *is of the form*

$$(-)\mathcal{R}(y) : (-)\mathcal{C}(x)$$

where x, y are RC-terms, the set of which is defined as the smallest set such that
– a formula of $\mathbb{L}$ is an RC-term,
– an RC-formula is an RC-term,
– if α and β are RC-terms then so are $-\beta$, $\alpha \,|\, \beta$, $\alpha \,\&\, \beta$.

The notation "$(-)$" means that the negation operator "$-$" may, but need not, occur. $\mathcal{R}$ and $\mathcal{C}$ are indicative of the functions of *giving reason* and *concluding*, respectively. Thus, they capture the coupling reason-conclusion. As we will see later, the contents may be true while the functions do not hold and vice versa. Whatever the link between the reason and the conclusion, it is represented by the colon in the definition.

The two symbols $|$ and $\&$ can be used to obtain RC-formulas in a number of ways, examples of RC-formulas include $\mathcal{R}\left(-\mathcal{R}(y) : \mathcal{C}(x)\right) : \mathcal{C}(w), \mathcal{R}(z) : -\mathcal{C}\left(w \,\&\, (-\mathcal{R}(y) : \mathcal{C}(x))\right)$, … Contrariwise, examples of expressions that fail to be RC-formulas include $x \,\&\, \mathcal{R}(y) : \mathcal{C}(z)$, $x \mid \mathcal{R}(y) : \mathcal{C}(z) \ldots$

The simplest RC-terms are formulas of $\mathbb{L}$. Accordingly, we assume that $-\beta$, $\alpha \,|\, \beta$, $\alpha \,\&\, \beta$, $|$ are identified with $\neg\beta$, $\alpha \vee \beta$, and $\alpha \wedge \beta$ when α and β are all in $\mathbb{L}$.

Unlike existing definitions of argument where a conclusion x follows from premises y using a notion of derivation (e.g., [10]), Definition 1 leaves the content of the link unspecified. Accordingly, such a general definition makes it possible to capture links of whatever nature, including non-deductive links, and therefore can offer a way to represent any natural language argument, even somewhat dubious arguments such as:

This paper will be accepted. It's about argumentation.

Taking pa to stand for "this paper will be accepted" and aa to stand for "this paper is about argumentation", $\mathcal{R}(aa) : \mathcal{C}(pa)$ is indeed a representation of the above argument.

Definition 2 (Argument). *An* argument *is an RC-formula of the form*

$$\mathcal{R}(y) : (-)\mathcal{C}(x).$$

The intuitive meaning of the two formal expressions captured by Definition 2 is:

$\mathcal{R}(y) : \mathcal{C}(x)$	means that	"y is a reason for concluding x".
$\mathcal{R}(y) : -\mathcal{C}(x)$	means that	"y is a reason for not concluding x".

Example 1. Let sm stand for "Steve is very smart", and wh stand for "Steve worked hard this term", and pe stand for "Steve will pass his exams". Then, Adam's argument "Steve is very smart but didn't work hard this term, so it's unclear whether he will pass his exams" can be captured by the RC-formula

$$\mathcal{R}(sm \wedge \neg wh) : -\mathcal{C}(pe).$$

Let moreover r stand for "it is raining" and lm stand for "Steve is lacking motivation". Reconstructing Craig's "Lack of motivation is *the* reason" to account for concluding that Steve will fail his exams as well as denying rain to account for it, can then be captured by the RC-formula

$$\mathcal{R}(lm) : \mathcal{C}\left(\neg pe \;\&\; -\mathcal{R}(r) : \mathcal{C}(\neg pe)\right).$$

Accordingly, taking x to be a propositional formula, all this faithfully accounts for the distinctions mentioned in the introduction:

- Arguments *in favour* of x, they are of the form $\mathcal{R}(y) : \mathcal{C}(x)$.
- Arguments *against* x, they are of the form $\mathcal{R}(y) : \mathcal{C}(\neg x)$.
- Arguments justifying why x is *doubted*, they are of the form $\mathcal{R}(y) : -\mathcal{C}(x)$.

Please observe that the second item amounts to arguing in favour of $\neg x$ whereas the third item has a sister item, of the form $\mathcal{R}(y) : -\mathcal{C}(\neg x)$, justifying why $\neg x$ is doubted. The case of complete indeterminacy (i.e., when both x and $\neg x$ are doubted) can be identified with both sister items taken together. More generally, $\mathcal{R}(y) : -\mathcal{C}(x)$ is right in two kinds of situations: (1) y is a reason for concluding $\neg x$; *e.g.*, being a penguin is not only a reason for not concluding that Tweety can fly, $\mathcal{R}(p) : -\mathcal{C}(f)$, it furthermore is a reason for concluding that Tweety cannot fly, $\mathcal{R}(p) : \mathcal{C}(\neg f)$ and (2) y is both a reason to refrain concluding x and also a reason to refrain concluding $\neg x$.

Definition 3 (Rejection). *A* rejection *is an RC-formula of the form*

$$-\mathcal{R}(y) : (-)\mathcal{C}(x).$$

The intuitive meaning for these formal expressions is as follows:

$-\mathcal{R}(y) : \mathcal{C}(x)$ means that "y is not a reason for concluding x".
$-\mathcal{R}(y) : -\mathcal{C}(x)$ means that "y is not a reason for not concluding x".

Example 2. Craig's "Rain is not a reason to infer that Steve will fail his exams" can be captured by the RC-formula

$$-\mathcal{R}(r) : \mathcal{C}(\neg pe).$$

As an argument exhibits a reason, a conclusion and a link over them, an argument can be objected by challenging its reason, or its conclusion, or its link. These three possibilities of objecting to an argument $\mathcal{R}(y) : \mathcal{C}(x)$ are rendered by RC-formulas. Assume, for instance, that $x, y \in \mathbb{L}$:

- The *reason* of the argument *is objected to*, which is achieved by an argument of the form $\mathcal{R}(z) : \mathcal{C}(\neg y)$.
- The *conclusion* of the argument *is objected to*, which is achieved by an argument of the form $\mathcal{R}(z) : \mathcal{C}(\neg x)$.
- The *link* in the argument *is objected to*, which is achieved by a rejection of the form $-\mathcal{R}(y) : \mathcal{C}(x)$.

The link can also be objected to by means of more informed items such as arguments of the form $\mathcal{R}(z) : \mathcal{C}\left(-\mathcal{R}(y) : \mathcal{C}(x)\right)$.

Example 3. Again, on whether Ryan is in his office.

- ro stand for "Ryan is in his office",
- cp stand for "Ryan's car is in the car park",
- bc stand for "Ryan's car is broken".

Then,

Dale: *Ryan is in his office. His car is in the car park.*
Earl: *The car is in the car park because it is broken.*

can be formalized as $\mathcal{R}(cp) : \mathcal{C}(ro)$ for Dale's argument and $\mathcal{R}(bc) : \mathcal{C}(cp)$ for Earl's strict utterance. As a response (or objection to Dale's), Earl's argument can be reconstructed as $\mathcal{R}(\mathcal{R}(bc) : \mathcal{C}(cp)) : -\mathcal{C}(ro)$.

3 INFERENCE

The aim of this section is to introduce the consequence operator $\Vdash$ and some of its properties, where $\Vdash$ is the least closure of a set of *inference rules* extended with one *meta-rule*. We investigate a specific combination of inference rules in this section. We have considered alternative combinations of inference rules previously [1, 2].

Of course, w, x, y, z below can be instantiated with RC-terms. These are supposed to obey Boolean identities over $-$ (negation), $|$ (disjunction) and $\&$ (conjunction) such that $- - x = x$, $-(x \,\&\, y) = -x \,|\, -y$, and so on. Also, $-$, $|$ and $\&$ must be understood as $\neg$, $\vee$ and $\wedge$ resp., when applying to RC-terms that are formulas of $\mathbb{L}$.

Importantly, deriving an argument α by means of inference rules does *not* mean that α is accepted. Instead, inferring α means that the argument(s) and/or rejection(s) used as premises for the inference rule(s) applied while deriving α cannot be held without α also being held. Indeed, $\Vdash$ is meant to capture *commitment* between arguments. Hence, if a foolish argument is used as a premise then a foolish α may result: If an agent holds a foolish argument, he henceforth commits to some other foolish arguments.

3.1 Denial of an argument

$\Vdash$ is defined with the requirement that $-(\mathcal{R}(y) : \varPhi)$ is identified with $-\mathcal{R}(y) : \varPhi$, and similarly for $-(-\mathcal{R}(y) : \varPhi)$ with $- - \mathcal{R}(y) : \varPhi$. In doing so, we are faithful to Apothéloz who regards them as equivalent [3]. It seems disputable, though. It could be argued that $-\mathcal{R}(y) : \varPhi$ disqualifies only *one part* of an argument (i.e., its reason) while $-(\mathcal{R}(y) : \varPhi)$ somehow disqualifies the whole argument. Imagine a member of a recruitment committee who presents an argument in favour of a candidate for his own research lab. The argument may be denied by the committee due to conflict of interest. However, such a denial does not (or at least need not) challenge truth of the reason nor its ability to bring about the conclusion of the argument.

3.2 Meta-rule

Rejection $-\mathcal{R}(y) : \mathcal{C}(x)$ means that y is not a reason for x, which is the negation of what $\mathcal{R}(y) : \mathcal{C}(x)$ is supposed to mean, i.e., y is a reason for x. As a consequence, the contrapositive of the fact that $\mathcal{R}(y) : \mathcal{C}(x)$ would entail $-\mathcal{R}(y) : \mathcal{C}(w)$ is that $\mathcal{R}(y) : \mathcal{C}(w)$ would entail $-\mathcal{R}(y) : \mathcal{C}(x)$. Accordingly, the *meta-rule* expresses that we can reverse any inference rule of the form

$$\frac{\mathcal{R}(y) : \Phi}{-\mathcal{R}(y) : \Psi} \qquad \text{into} \qquad \frac{\mathcal{R}(y) : \Psi}{-\mathcal{R}(y) : \Phi}.$$

Of course, the same reversing process takes place whenever "$-$" occurs in front of the leftmost "$\mathcal{R}$" so that, in the general case, an inference rule [2] where $i, j \in \{0, 1\}$

$$\frac{-^{(i)}\mathcal{R}(y) : \Phi \quad \alpha_1 \cdots \alpha_n}{-^{(j)}\mathcal{R}(y) : \Psi} \qquad \text{can be reversed into} \qquad \frac{-^{(1-j)}\mathcal{R}(y) : \Psi \quad \alpha_1 \cdots \alpha_n}{-^{(1-i)}\mathcal{R}(y) : \Phi}$$

whatever the RC-formulas $\alpha_1, \ldots, \alpha_n$.

3.3 Inference rules

Certainly, the feature most expected is consistency in terms of arguments:

$$\frac{\mathcal{R}(y) : \mathcal{C}(x)}{-\mathcal{R}(y) : -\mathcal{C}(x)} \qquad \frac{\mathcal{R}(y) : \mathcal{C}(x)}{\mathcal{R}(y) : -\mathcal{C}(-x)} \qquad \textbf{(Consistency)}$$

The leftmost inference rule means that if y is a reason for x then y is not a reason to doubt x. The rightmost inference rule means that if y is a reason for x then it is also a reason to doubt $-x$.

Property 1. The inference rules below derive from (Consistency) and the meta-rule.

$$\frac{\mathcal{R}(y) : \mathcal{C}(x)}{-\mathcal{R}(y) : \mathcal{C}(-x)} \quad \frac{\mathcal{R}(y) : -\mathcal{C}(x)}{-\mathcal{R}(y) : \mathcal{C}(x)} \quad \frac{\mathcal{R}(y) : \mathcal{C}(-x)}{\mathcal{R}(y) : -\mathcal{C}(x)} \quad \frac{\mathcal{R}(y) : \mathcal{C}(-x)}{-\mathcal{R}(y) : \mathcal{C}(x)}$$

Please observe that an instance of the third rule in Property 1 is:

$$\frac{\mathcal{R}(z) : \mathcal{C}(-\mathcal{R}(y) : \mathcal{C}(x))}{\mathcal{R}(z) : -\mathcal{C}(\mathcal{R}(y) : \mathcal{C}(x))}$$

A similar rule, related to $\mathcal{R}$ instead of $\mathcal{C}$, is:

$$\frac{\mathcal{R}(-\mathcal{R}(y) : \mathcal{C}(x)) : \mathcal{C}(w)}{\mathcal{R}(\mathcal{R}(y) : -\mathcal{C}(x)) : \mathcal{C}(w)} \qquad \textbf{(A Fortiori)}$$

The inference rules below are concerned with various principles permitting to infer arguments from other arguments. One such principle is the idea that, if y is a reason for

[2] $-^{(1)}$ denotes a single occurrence of the hyphen and $-^{(0)}$ the absence of it.

z and vice-versa, then z is a reason for whatever y is a reason for. This motivates the following inference rule.

$$\frac{\mathcal{R}(y) : \mathcal{C}(z) \quad \mathcal{R}(z) : \mathcal{C}(y) \quad \mathcal{R}(y) : \mathcal{C}(x)}{\mathcal{R}(z) : \mathcal{C}(x)} \qquad \textbf{(Mutual Support)}$$

Another principle is that if each of y and z is a reason for x, then the disjunction y or z is a reason for x. Conversely, if y or z is a reason for x then any of y and z must be a reason for x. All this can be expressed by the next rules, as follows.

$$\frac{\mathcal{R}(y) : \mathcal{C}(x) \qquad \mathcal{R}(z) : \mathcal{C}(x)}{\mathcal{R}(y\,|\,z) : \mathcal{C}(x)} \qquad \frac{\mathcal{R}(y\,|\,z) : \mathcal{C}(x)}{\mathcal{R}(y) : \mathcal{C}(x)} \qquad \textbf{(Or)}$$

There is also the idea that if a reason can be decomposed into parts, of which one, say y, is a reason for the others (namely, the other parts), then y is enough of a reason. The next inference rule takes care of this.

$$\frac{\mathcal{R}(y\,\&\,z) : \mathcal{C}(x) \qquad \mathcal{R}(y) : \mathcal{C}(z)}{\mathcal{R}(y) : \mathcal{C}(x)} \qquad \textbf{(Cut)}$$

The next rule turns an argument whose claim is itself an argument into an argument with decreased depth of nesting in $\mathcal{C}(.)$, as follows.

$$\frac{\mathcal{R}(y) : \mathcal{C}(\mathcal{R}(z) : \mathcal{C}(x))}{\mathcal{R}(y\,\&\,z) : \mathcal{C}(x)} \qquad \textbf{(Exportation)}$$

The last rule expresses how permutation of reasons can take place.

$$\frac{\mathcal{R}(y) : \mathcal{C}(\mathcal{R}(z) : \mathcal{C}(x))}{\mathcal{R}(z) : \mathcal{C}(\mathcal{R}(y) : \mathcal{C}(x))} \qquad \textbf{(Permutation)}$$

From now on, $\Vdash$ denotes the system consisting of (Consistency) together with the seven rules above from (A Fortiori) to (Permutation), closed under substitution and the meta-rule. Similarly, "derive" will refer to the usual notion for $\Vdash$ thus defined.

We show that $-\mathcal{R}(y) : \mathcal{C}(x)$ cannot be schematically derived from $\mathcal{R}(y) : \mathcal{C}(x)$ and that $-\mathcal{R}(y) : -\mathcal{C}(x)$ cannot be schematically derived from $\mathcal{R}(y) : -\mathcal{C}(x)$, so that a basic kind of consistency for the consequence operator $\Vdash$ is ensured.

Property 2. There is no $i, j \in \{0, 1\}$ such that

$$\frac{-^{(i)}\mathcal{R}(y) : -^{(j)}\mathcal{C}(x)}{-^{(1-i)}\mathcal{R}(y) : -^{(j)}\mathcal{C}(x)}$$

is a derived inference rule.

Property 2 furthermore expresses (using inference rules in Property 1) that neither $\mathcal{R}(y) : \mathcal{C}(-x)$ nor $\mathcal{R}(y) : -\mathcal{C}(x)$ can be schematically derived from $\mathcal{R}(y) : \mathcal{C}(x)$.

The Boolean identity $\alpha\,\&\,\alpha = \alpha$ yields an instance of (Exportation) worth mentioning, that is

$$\frac{\mathcal{R}(y) : \mathcal{C}(\mathcal{R}(y) : \mathcal{C}(x))}{\mathcal{R}(y) : \mathcal{C}(x)}.$$

The converse rule

$$\frac{\mathcal{R}(y) : \mathcal{C}(x)}{\mathcal{R}(y) : \mathcal{C}(\mathcal{R}(y) : \mathcal{C}(x))}$$

is acceptable as well, in which case $\mathcal{R}(y) : \mathcal{C}(\mathcal{R}(y) : \mathcal{C}(x))$ could be identified with $\mathcal{R}(y) : \mathcal{C}(x)$.

3.4 Non-Inference

Now we consider some inference rules that do not hold for the consequence relation we are presenting in this paper. First, if there were any axiom, the most likely candidate would be

$$-\mathcal{R}(\top) : \mathcal{C}(\bot).$$

The reader may find it surprising that the list above includes no inference rule induced by logical consequence. Nonetheless, most of the expected rules fail as detailed now.

$$\frac{}{\mathcal{R}(x) : \mathcal{C}(x)}\ x \in \mathbb{L} \qquad \textbf{(Reflexivity)}$$

Key is the difference between being an argument syntactically and being an argument that is held. $\mathcal{R}(x) : \mathcal{C}(x)$ is identified with an argument, by the mere fact that it does conform with Definition 2. Taking $\mathcal{R}(x) : \mathcal{C}(x)$ as an axiom would mean that any agent would be regarded as committed to holding $\mathcal{R}(x) : \mathcal{C}(x)$ for every x. Depending on the nature of the link in the argument, (i.e., the reading of the colon), this might be inappropriate. Think of a recruitment committee member who holds that "*Tracy should be given the position*". Taking x to stand for the statement that Tracy should be given the position, the argument $\mathcal{R}(x) : \mathcal{C}(x)$ is certainly not acceptable. Indeed, what is expected in such committees is to bring independent evidence in favour of candidates.

$$\frac{\models y \to x}{\mathcal{R}(y) : \mathcal{C}(x)}\ x, y \in \mathbb{L} \qquad \textbf{(Logical Consequence)}$$

Inhibiting (Reflexivity) as just argued implies that (Logical Consequence) must also be left out, because (Reflexivity) follows from (Logical Consequence).

$$\frac{\mathcal{R}(y) : \mathcal{C}(x) \qquad \models y \leftrightarrow z}{\mathcal{R}(z) : \mathcal{C}(x)}\ y, z \in \mathbb{L} \qquad \textbf{(Left Logical Equivalence)}$$

(Left Logical Equivalence) must be left out, again on the grounds that the nature of the link need not conform with logical consequence. Most notably, an effect need not be caused by something logically equivalent to its cause. However, (Mutual Support) can be viewed as a restricted substitute to this purported rule.

$$\frac{\mathcal{R}(y) : \mathcal{C}(x) \qquad \models z \to y}{\mathcal{R}(z) : \mathcal{C}(x)}\ y, z \in \mathbb{L} \qquad \textbf{(Left Logical Consequence)}$$

This is even more dubious, it actually entails (Left Logical Equivalence) and is then not worth considering any further.

$$\frac{\mathcal{R}(y) : \mathcal{C}(x) \qquad \models x \to w}{\mathcal{R}(y) : \mathcal{C}(w)}\ w, x \in \mathbb{L} \qquad \textbf{(Right Logical Consequence)}$$

(Right Logical Consequence) cannot be adopted either because being a reason for x is in general more restrictive than having x as a logical consequence. Consider for instance the causal argument in [1]: flu is a reason for your body temperature to be in the range 39° C$-$41° C. However, the fact that being in the range 36° C$-$41° C is a logical consequence of being in the range 39° C$-$41° C does not make flu a reason for your body temperature to be in the range 36° C$-$41° C (it is the only possible range unless you are dead!).

Interestingly, failure of (Right Logical Consequence) dismisses the seemingly harmless rule below

$$\frac{-\mathcal{R}(y) : \mathcal{C}(w)}{-\mathcal{R}(y) : \mathcal{C}(x\&w)}$$

which is nothing but the contrapositive of an instance of (Right Logical Consequence) —let x be $x \,\&\, w$.

$$\frac{\mathcal{R}(y) : \mathcal{C}(x) \qquad \mathcal{R}(y) : \mathcal{C}(z)}{\mathcal{R}(y) : \mathcal{C}(x \wedge z)} \qquad \textbf{(And)}$$

(And) is inappropriate, too. To start with, (And) opposes most cases dealing with limited resources. Certainly, from the fact that I have one Euro is a reason for me to buy a chocolate bar and is also a reason for me to buy a pastry, it cannot sensibly be held that the fact that I have one Euro is a reason for me to buy *both*. Assume that y stands for “I have one Euro” while x and z stand for “I am to buy a chocolate bar” and “I am to buy a pastry”. Definitely, it would be wrong to derive $\mathcal{R}(y) : \mathcal{C}(x \wedge z)$ from $\mathcal{R}(y) : \mathcal{C}(x)$ together with $\mathcal{R}(y) : \mathcal{C}(z)$. Another case against (And), that does not involve limited resources, can be found in [1].

$$\frac{\mathcal{R}(z) : \mathcal{C}(y) \qquad \mathcal{R}(y) : \mathcal{C}(x)}{\mathcal{R}(z) : \mathcal{C}(x)} \qquad \textbf{(Transitivity)}$$

(Transitivity) can be challenged by means of the Ryan example in the introduction: $\mathcal{R}(cp) : \mathcal{C}(ro)$ (Ryan’s car is in the carpark hence Ryan is in his office) and $\mathcal{R}(bc) : \mathcal{C}(cp)$ (Ryan’s car is in the carpark because it is broken) do not give $\mathcal{R}(bc) : \mathcal{C}(ro)$. The fact that Ryan’s car is broken does not support the conclusion “Ryan is in his office” but precludes it instead. (Transitivity) fails mainly due to $\mathcal{R}$ being nonmonotonic in the following sense: It can be the case that y is generally a reason for x although there are some special circumstances where this breaks down.

$$\frac{\mathcal{R}(y) : \mathcal{C}(x) \qquad \mathcal{R}(y) : \mathcal{C}(z)}{\mathcal{R}(y \wedge z) : \mathcal{C}(x)} \qquad \textbf{(Cautious Monotonicity)}$$

(Cautious Monotony), which is adapted from the study of non-monotonic consequence relations [13], is the controversial principle that the reason y in an argument for an x can be expanded with any statement z for which y is a reason. There is an interest in such a principle because it is the converse of (Cut) when both are viewed as principles applying in the context of $\mathcal{R}(y) : \mathcal{C}(z)$. Dismissal of (Cautious Monotony) cannot be escaped, if only from the fact that $\mathcal{R}(y) : \mathcal{C}(x)$ together with $\mathcal{R}(y) : \mathcal{C}(z)$ would yield $\mathcal{R}(y) : \mathcal{C}(\mathcal{R}(z) : \mathcal{C}(x))$ (and $\mathcal{R}(y) : \mathcal{C}(\mathcal{R}(x) : \mathcal{C}(z))$ as well) in the presence of the converse of (Exportation), namely (Importation).

$$\frac{\mathcal{R}(y \wedge z) : \mathcal{C}(x)}{\mathcal{R}(y) : \mathcal{C}(\mathcal{R}(z) : \mathcal{C}(x))} \qquad \textbf{(Importation)}$$

Actually, in the case that both (Importation) and (Cautious Monotony) were adopted, for every z for which y is a reason, $\mathcal{R}(y) : \mathcal{C}(\mathcal{R}(z) : \mathcal{C}(x))$ would ensue. In particular, $\mathcal{R}(y) : \mathcal{C}(\mathcal{R}(x) : \mathcal{C}(x))$ would hold for every x for which y is a reason.

Property 3. (Mutual Support) is a restricted version of (Transitivity).

Blocking a reason (in the form $\mathcal{R}(y) : -\mathcal{C}(\mathcal{R}(z) : \mathcal{C}(x))$ i.e. y justifies not to hold that z is a reason for x) is different from blocking a conclusion (in the form $\mathcal{R}(y) : -\mathcal{C}(x)$ i.e. y justifies not to hold the conclusion x). In symbols:

- $\mathcal{R}(y) : -\mathcal{C}(\mathcal{R}(z) : \mathcal{C}(x)) \quad \not\Vdash \quad \mathcal{R}(y) : -\mathcal{C}(x)$.
- $\mathcal{R}(y) : -\mathcal{C}(x) \quad \not\Vdash \quad \mathcal{R}(y) : -\mathcal{C}(\mathcal{R}(z) : \mathcal{C}(x))$.

Consider the following argument.

> *The fact that several European countries have a good economy* (ge) *is a reason for not concluding that an economic crisis (ec) in Spain is a reason for a declining value of the euro* (de).

This has the form $\mathcal{R}(ge) : -\mathcal{C}(\mathcal{R}(ec) : \mathcal{C}(de))$. Please note that $\mathcal{R}(ge) : -\mathcal{C}(de)$ does not necessarily hold since an economic crisis in Germany may lead to a declining value of the euro.

Consider now the informal argument:

> *The fact that Steve did not follow the course (fc) is a reason for his failing his exams.*

It is formally captured as $\mathcal{R}(\neg fc) : \mathcal{C}(fe)$. That this argument is doubted on the grounds that Steve is smart can then be written $\mathcal{R}(sm) : -\mathcal{C}(\mathcal{R}(\neg fc) : \mathcal{C}(fe))$. However, the latter argument, $\mathcal{R}(sm) : -\mathcal{C}(\mathcal{R}(\neg fc) : \mathcal{C}(fe))$, need not hold even in the presence of $\mathcal{R}(sm) : -\mathcal{C}(fe)$ (Steve being smart is a reason not to conclude his failing his exams).

3.5 Properties of the consequence relation

We show that the consequence relation $\Vdash$ meets the minimum requirements as argued by Tarski [14].

Property 4. The following are properties of the $\Vdash$ relation where Δ is a set of RC-formulas, and α and β are RC-formulas.

$$\begin{array}{lr} \Delta \Vdash \alpha \text{ if } \alpha \in \Delta & \textbf{(Reflexivity)} \\ \Delta \cup \{\alpha\} \Vdash \beta \text{ if } \Delta \Vdash \beta & \textbf{(Monotonicity)} \\ \Delta \Vdash \beta \text{ if } \Delta \cup \{\alpha\} \Vdash \beta \text{ and } \Delta \Vdash \alpha & \textbf{(Cut)} \end{array}$$

In addition, the $\Vdash$ consequence relation is *paraconsistent* in the following sense.

Property 5. The following non-trivialization property holds for the $\Vdash$ relation:

$$\{-^{(i)}\mathcal{R}(y) : -^{(j)}\mathcal{C}(x), -^{(1-i)}\mathcal{R}(y) : -^{(j)}\mathcal{C}(x)\} \nVdash \mathbb{L}_{RC}.$$

The properties of reflexivity, monotonicy, and cut, mean that with the $\Vdash$ consequence relation, the manipulation of arguments by the inference rules is well-founded. The non-trivialization property means that contradictory arguments can be handled in a straightforward way.

It is worth pointing out that, even though $\Vdash$ is monotonic, it does exhibits non-monotonicity through its object language in the guise of $\mathcal{R}$. Indeed, "being a reason" is a non-monotonic inference relation $\mid\!\sim$ as witnessed by failure of transitivity. However, the fact that $\mathcal{R}$ plays the role of $\mid\!\sim$ in our formalism makes the non-monotonicity confined to failure of inferring $\mathcal{R}(y \wedge z) : \mathcal{C}(x)$ from $\mathcal{R}(y) : \mathcal{C}(x)$. Therefore, this has no effect on the logic. As an aside, the situation is similar to conditional logics because an operator capturing a counterfactual conditional must be non-monotonic (still, conditional logics are monotonic). E.g., "were I to scratch this match, it would ignite" denoted $y \rightarrowtail x$ may hold while "were I to scratch this match, that is wet, it would ignite" denoted $y \wedge z \rightarrowtail x$ fails to hold.

4 EXPRESSIVENESS OF THE LANGUAGE

This section discusses the expressive power of the language, namely the effects of allowing nesting of $\mathcal{R}(.)$ and $\mathcal{C}(.)$ on

- encoding *meta-arguments*,
- expressing various forms of *attacks*, and
- expressing *supports* between arguments.

4.1 Meta-arguments

The next table displays various forms of arguments allowed by Definition 1. Of course, the table is not exhaustive.

Basic arguments	F_1	$\mathcal{R}(y) : \mathcal{C}(x)$
	F_2	$\mathcal{R}(y) : \mathcal{C}(\neg x)$
	F_3	$\mathcal{R}(y) : -\mathcal{C}(x)$
Single-embedding meta-arguments (in reason)	F_4	$\mathcal{R}(\mathcal{R}(z) : \mathcal{C}(y)) : \mathcal{C}(x)$
	F_5	$\mathcal{R}(\mathcal{R}(z) : \mathcal{C}(y)) : \mathcal{C}(\neg x)$
	F_6	$\mathcal{R}(\mathcal{R}(z) : \mathcal{C}(y)) : -\mathcal{C}(x)$
Single-embedding meta-arguments (in conclusion)	F_7	$\mathcal{R}(y) : \mathcal{C}(\mathcal{R}(z) : \mathcal{C}(x))$
	F_8	$\mathcal{R}(y) : \mathcal{C}(-\mathcal{R}(z) : \mathcal{C}(x))$
	F_9	$\mathcal{R}(y) : -\mathcal{C}(\mathcal{R}(z) : \mathcal{C}(x))$
Double-embedding meta-arguments	F_{10}	$\mathcal{R}(\mathcal{R}(z) : \mathcal{C}(y)) : \mathcal{C}(\mathcal{R}(t) : \mathcal{C}(x))$
	F_{11}	$\mathcal{R}(\mathcal{R}(z) : \mathcal{C}(y)) : \mathcal{C}(-\mathcal{R}(t) : \mathcal{C}(x))$
	F_{12}	$\mathcal{R}(\mathcal{R}(z) : \mathcal{C}(y)) : -\mathcal{C}(\mathcal{R}(t) : \mathcal{C}(x))$

Next is a list of arguments showing that each form F_i makes sense.

F_1: Tweety can fly (f). It is a bird (b). $\mathcal{R}(b) : \mathcal{C}(f)$

F_2: Tweety cannot fly. It is a penguin (p). $\mathcal{R}(p) : \mathcal{C}(\neg f)$

F_3: Steve is smart. Thus, it is not possible to conclude that he will fail his exams. $\mathcal{R}(sm) : -\mathcal{C}(fe)$

F_4: That Tweety can fly because it is a bird, is a reason to conclude that Tweety has wings (w). $\mathcal{R}(\mathcal{R}(b) : \mathcal{C}(f)) : \mathcal{C}(w)$

F_5: That Steve will fail his exams because he did not work hard is a reason to conclude that he is not so smart.
$\mathcal{R}(\mathcal{R}(\neg wh) : \mathcal{C}(fe)) : \mathcal{C}(\neg sm)$

F_6: Paul's car is in the park (pr) because it is broken (br), hence we cannot conclude that Paul is in his office (of).
$\mathcal{R}(\mathcal{R}(br) : \mathcal{C}(pr)) : -\mathcal{C}(of)$

F_7: The weather is sunny (su). Thus, rain (ra) will lead to rainbow (rb). $\mathcal{R}(su) : \mathcal{C}(\mathcal{R}(ra) : \mathcal{C}(rb))$

F_8: The fact that Tweety is a penguin is a reason to conclude that being a bird is not a sufficient reason for Tweety being able to fly. $\mathcal{R}(p) : \mathcal{C}(-\mathcal{R}(b) : \mathcal{C}(f))$

F_9: The fact that all European countries have a strong economy (se) is a reason for not concluding that an economic crisis (ec) in Germany is a reason for a declining value of the euro (de). $\mathcal{R}(se) : -\mathcal{C}(\mathcal{R}(ec) : \mathcal{C}(de))$

F_{10}: CFCs (cfc) cause damage to the ozone layer of the atmosphere (do). Man-made pollution (mp) causes global warming (gw). $\mathcal{R}(\mathcal{R}(cfc) : \mathcal{C}(do)) : \mathcal{C}(\mathcal{R}(mp) : \mathcal{C}(gw))$

F_{11}: Stress is the reason that Steve will fail his exams, hence it is not the fact that he did not work hard (st).
$\mathcal{R}(\mathcal{R}(st) : \mathcal{C}(fe)) : \mathcal{C}(-\mathcal{R}(\neg wh) : \mathcal{C}(fe))$

F_{12}: The object looks red (lr). It is illuminated by red light (il). Thus, we cannot conclude that looking red implies the object being indeed red (re). $\mathcal{R}(\mathcal{R}(il) : \mathcal{C}(lr)) : -\mathcal{C}(\mathcal{R}(lr) : \mathcal{C}(re))$

4.2 Expressing attacks

An argument $\mathcal{R}(y) : \mathcal{C}(x)$ may be attacked on any one of its components: conclusion, premises or the function of reason. For instance, the RC-formula below

$$-\mathcal{R}(\mathcal{R}(y) : \mathcal{C}(x)) : \mathcal{C}(x).$$

attacks the premise y in the argument $\mathcal{R}(y) : \mathcal{C}(x)$ because $-\mathcal{R}(\mathcal{R}(y) : \mathcal{C}(x)) : \mathcal{C}(x)$ states that y being a reason for concluding x is not enough to conclude x; therefore y must fail: if y were the case then, that y is a reason for concluding x would lead to conclude x. By contrast, attacking the link in the argument $\mathcal{R}(y) : \mathcal{C}(x)$ is simply the rejection

$$-\mathcal{R}(y) : \mathcal{C}(x).$$

We propose below a set of inference rules which not only show the various forms of attack that may hold between arguments, but also how to detect attacks (the rules themselves) and how to express attacks as arguments (the part β of a rule α/β).

$$\frac{\mathcal{R}(y) : \mathcal{C}(x) \qquad \mathcal{R}(z) : \mathcal{C}(\neg x)}{\mathcal{R}(\mathcal{R}(z) : \mathcal{C}(\neg x)) : \mathcal{C}(-\mathcal{R}(y) : \mathcal{C}(x))} \qquad \textbf{(Strong Rebuttal)}$$

$$\frac{\mathcal{R}(y) : \mathcal{C}(x) \qquad \mathcal{R}(z) : -\mathcal{C}(x)}{\mathcal{R}(\mathcal{R}(z) : -\mathcal{C}(x)) : \mathcal{C}(-\mathcal{R}(y) : \mathcal{C}(x))} \qquad \textbf{(Weak Rebuttal)}$$

$$\frac{\mathcal{R}(y) : \mathcal{C}(x) \qquad \mathcal{R}(z) : \mathcal{C}(\neg y)}{\mathcal{R}(\mathcal{R}(z) : \mathcal{C}(\neg y)) : \mathcal{C}(-\mathcal{R}(y) : \mathcal{C}(x))} \qquad \textbf{(Strong Premise Attack)}$$

$$\frac{\mathcal{R}(y) : \mathcal{C}(x) \qquad \mathcal{R}(z) : -\mathcal{C}(y)}{\mathcal{R}(\mathcal{R}(z) : -\mathcal{C}(y)) : \mathcal{C}(-\mathcal{R}(y) : \mathcal{C}(x))} \qquad \textbf{(Weak Premise Attack)}$$

$$\mathcal{R}(z) : \mathcal{C}(-\mathcal{R}(y) : \mathcal{C}(x)) \qquad \textbf{(Strong Reason Attack)}$$

$$\mathcal{R}(z) : -\mathcal{C}(\mathcal{R}(y) : \mathcal{C}(x)) \qquad \textbf{(Weak Reason Attack)}$$

$$-\mathcal{R}(y) : \mathcal{C}(x) \qquad \textbf{(Pure Reason Attack)}$$

Note that the three attack relations that are distinguished in existing argumentation formalisms are captured in the new setting: *rebuttal* is captured by (Strong Rebuttal), *assumption attack* corresponds to (Strong Premise Attack) and Pollock undercutting is reflected by our notion of (Weak Rebuttal). However, since in those formalisms it is not possible to build arguments for blocking conclusions, the blocking is done in an indirect way as explained through Adam's example in the introduction. Therefore, with our logic of arguments, we can formalize and manipulate attacks explicitly within the logic (which is not possible in other formal systems of argumentation), and we have a wider range of attacks than are considered in other formal proposals for argumentation. For instance, in our formalism the *argumentative orientation* of the reason y towards the conclusion x of an argument $\mathcal{R}(y) : \mathcal{C}(x)$ can be attacked. Consider the following example borrowed from [15].

Floyd:	*"A World Apart" is not a good movie. It does not teach us anything new about apartheid.*
Gary:	*That's precisely what makes it good.*

Let

- gm stand for "*A World Apart* is a good movie",
- $\neg ta$ stand for "It does not teach us anything new about apartheid",

Then, Floyd's utterance can be captured by $\mathcal{R}(\neg ta) : \mathcal{C}(\neg gm)$. Gary's can be expressed by $\mathcal{R}(\neg ta) : \mathcal{C}(gm)$ and his argument reconstructed as:

$$\mathcal{R}(\mathcal{R}(\neg ta) : \mathcal{C}(gm)) : \mathcal{C}(-\mathcal{R}(\neg ta) : \mathcal{C}(\neg gm)).$$

4.3 Expressing supports

Unlike attacks which express negative links between arguments, supports express positive links. In the existing literature (e.g., [16, 17]), such links are captured by a binary relation defined on the set of arguments. In our formalism, such an external relation is not needed since supports can be expressed by arguments of the form

$$\mathcal{R}(\mathcal{R}(y) : \mathcal{C}(x)) : \mathcal{C}(\mathcal{R}(z) : \mathcal{C}(w))$$

or

$$\mathcal{R}(v) : \mathcal{C}(\mathcal{R}(z) : \mathcal{C}(w)).$$

Let us return to Steve and his exams:

Hugh:	*Steve will pass his exams. He is very smart.*
Ian:	*He is well prepared.*

Letting wp stand for "Steve is well prepared", Ian's argument can be formalized as $\mathcal{R}(wp) : \mathcal{C}(\mathcal{R}(sm) : \mathcal{C}(pe))$ (Hugh's is $\mathcal{R}(sm) : \mathcal{C}(pe)$). From $\mathcal{R}(wp) : \mathcal{C}(\mathcal{R}(sm) : \mathcal{C}(pe))$, using the reduction rule, the argument $\mathcal{R}(wp \wedge sm) : \mathcal{C}(pe)$ ensues.

$$(\mathcal{R}(z) : \mathcal{C}(y)) : \mathcal{C}(x)$$

is an even more direct form expressing that $\mathcal{R}(y) : \mathcal{C}(x)$ is supported by $\mathcal{R}(z) : \mathcal{C}(y)$. It is obtained from the more general form above, using reduction. Also, rejection of support has the form

$$\mathcal{R}(z) : \mathcal{C}(-\mathcal{R}(y) : \mathcal{C}(x)).$$

5 Cube of opposition

The use of a cube of opposition is an interesting way of organizing complementary notions in the study of logics (e.g. [18]). The idea of opposition plays also an important role in argumentation [19]. Indeed, Apothéloz [3] has pointed out the existence of four basic argumentative forms, where two negations are at work: i) “y is a reason for concluding x", ii) “y is not a reason for concluding x", iii) “y is a reason for not concluding x", and iv) “y is not a reason for not concluding x". These four statements were organized by Salavastru [20] in a square of opposition, which was slightly corrected in [21] (the vertical entailments were put in the wrong way). The following cube of oppositions summarizes the different links between basic forms of arguments and rejections and adapts the previous proposal [21].

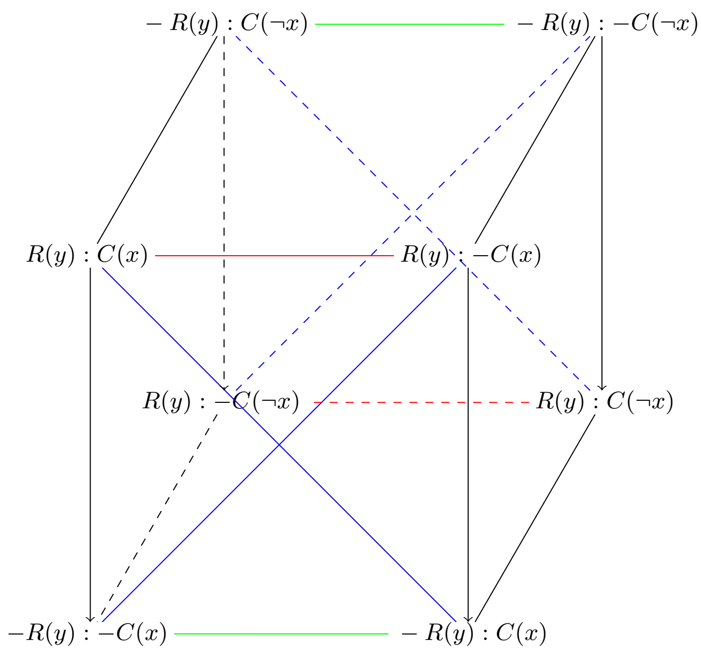

Fig. 1. Cube of opposition for RC-formulas.

A blue link (whether dashed or not) expresses a contradiction, i.e., its two endpoints cannot be true together and cannot be false together. A red link (whether dashed or not) expresses contrariness, i.e., its two endpoints cannot be true together —but they can be false together. A green link expresses sub-contrariness, i.e., its two endpoints cannot be false together —but they can be true together. The cube contains some subaltern relations represented by arrows, providing a direction. For instance, the arrow from $\mathcal{R}(y) : \mathcal{C}(x)$ to $-\mathcal{R}(y) : -\mathcal{C}(x)$ means that if the former holds, then so does the latter.

6 CONCLUSION

This paper proposes a novel logic for representing and reasoning about arguments in a way that is just not possible with the existing formalisms. The logical language is made of arguments and rejections of arguments. The definition of arguments encompasses different roles of reasons (concluding and blocking statements), various forms of reasons (factual and hypothetical) and different kinds of links (deductive, abductive, inductive, ...). Unlike the existing computational models of argumentation where attacks and supports between arguments are expressed by external relations on the set of arguments, in the new logic they are elements of the language. Indeed, every attack (respectively support) is expressed as an argument.

The logic offers key advantages. First, it respects the nature of argument. Indeed, it does not reduce the *meaning* of the statements to a formal derivation between the reason and the conclusion. To say this differently, not any such derivation is a natural argument. Importantly, $\Vdash$ does *not* serve to handle, or cure, inconsistency between arguments, but it provides, in a logical setting, a basis for reasoning between arguments. Second, it can be parameterized for several purposes like reasoning about causal arguments, analogical arguments, decision arguments, etc. Third, it lends itself to encoding *fairly* natural language dialogues. Indeed, one may pass directly from natural language dialogue to the logical setting without intermediate encodings which are often convoluted. Moreover, preferences between arguments could be captured as meta-arguments. Fourth, it provides the basis for a *logic of argumentation*, i.e., a logic in which arguments are represented and *evaluated*. Indeed, in the future, we plan to define on top of $\langle \mathbb{L}_{RC}, \Vdash \rangle$, a logic $\langle \mathbb{L}_{RC}, \Vvdash \rangle$ dedicated to acceptability of arguments (i.e., $\Vvdash$ will return the accepted arguments).

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