Bridging the gap between abstract argumentation systems and logic

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Abstract. Dung's argumentation system takes as input a set of *arguments* and a *binary relation* encoding attacks among these arguments, and returns different *extensions* of arguments. However, no indication is given on how to instantiate this setting, i.e. how to build arguments from a *knowledge base* and how to choose an appropriate attack relation. This leads in some cases to undesirable results like inconsistent extensions (i.e. the set of formulas forming an extension is inconsistent). This is due to the gap between the abstract setting and the knowledge base from which it is defined.

The purpose of this paper is twofold: First it proposes to fill in this gap by extending Dung's system. The idea is to consider all the ingredients involved in an argumentation problem. We start with an abstract monotonic logic which consists of a set of formulas and a consequence operator. We show how to build arguments from a knowledge base using the consequence operator of the logic. Second, we show that the choice of an attack relation is crucial for ensuring consistent results, and should not be arbitrary. In particular, we argue that an attack relation should be at least grounded on the *minimal conflicts* contained in the knowledge base. Moreover, due to the binary character of this relation, some attack relations may lead to unintended results. Namely, symmetric relations are not suitable when ternary (or more) minimal conflicts are in the knowledge base. We propose then the characteristics of attack relations that ensure sound results.

1 Introduction

Argumentation is a reasoning model based on the construction and evaluation of arguments in order to increase or decrease the acceptability of a given standpoint. It is used, for instance, for handling inconsistency in knowledge bases (e.g. [2, 9]) and for decision making (e.g. [1, 3]).

One of the most abstract argumentation systems in existing literature is Dung's one. It consists of a set of arguments and a binary relation encoding attacks among these arguments. Since its original formulation, this system has become very popular and different instantiations of it have been defined. Unfortunately, some of them such as the one presented in [8] can lead to very unintuitive results. In [4], it has been shown that this instantiation violates key postulates like the consistency of extensions. An extension satisfies consistency if the set of formulas used in arguments of that extension is consistent. What is worth noticing is that this postulate refers to the internal structure of an argument (i.e. its formulas) while the link between these formulas and the acceptability semantics is not clear. The link between the inconsistency of a base and the attack relation is also not clear. In summary, there is a gap between the abstract setting and the knowledge base from which it is built. Thus, basic choices like the definition of an argument and of an attack relation are made in an ad hoc way.

The purpose of this paper is twofold: First it proposes to fill in this gap by extending Dung's system. The idea is to consider all the ingredients involved in an argumentation problem ranging from the logical language to the output of the system. We start with an *abstract monotonic logic* as defined in [10]. Tarski defines an abstract monotonic logic as a set of formulas and a consequence operator that satisfies some axiom. We show how to build arguments from any subset of formulas using the consequence operator. Second, we show that the choice of an attack relation is crucial for ensuring consistent results and should not be arbitrary. In particular, an attack relation should be at least grounded on the minimal conflicts contained in the knowledge base. Moreover, due to the binary character of the attack relation in Dung's system, some attack relations may lead to undesirable results. Namely, symmetric relations are not suitable when ternary (or more) minimal conflicts are included in the base. We propose then the characteristics of an attack relation that should be used for ensuring sound results.

Section 2 recalls Tarski's axiomatization of a monotonic logic. Section 3 details our extension of Dung's system. Section 4 presents examples that show the importance of choosing correctly an attack relation. Section 5 studies the properties of such a relation while Section 6 gives recommendations on how to choose one.

2 Tarski's abstract consequence operations

Alfred Tarski [10] defines an *abstract logic* as a pair $(\mathscr{L}, \mathsf{CN})$ where members of $\mathscr{L}$ are called *well-formed formulas*, and CN is a *consequence operator*. CN is any function from $2^{\mathscr{L}}$ to $2^{\mathscr{L}}$ that satisfies the following axioms:

1. $X \subseteq \mathsf{CN}(X)$ **(Expansion)**
2. $\mathsf{CN}(\mathsf{CN}(X)) = \mathsf{CN}(X)$ **(Idempotence)**
3. $\mathsf{CN}(X) = \bigcup_{Y \subseteq_f X} \mathsf{CN}(Y)$ **(Finiteness)**
4. $\mathsf{CN}(\{x\}) = \mathscr{L}$ for some $x \in \mathscr{L}$ **(Absurdity)**
5. $\mathsf{CN}(\emptyset) \neq \mathscr{L}$ **(Coherence)**

Notation: $Y \subseteq_f X$ means that Y is a finite subset of X.

Intuitively, $\mathsf{CN}(X)$ returns the set of formulas that are logical consequences of X according to the logic in question. It can easily be shown from the above axioms that CN is a closure operator, that is, CN enjoys properties such as:

Property 1. Let $X, X', X'' \subseteq \mathscr{L}$.

1. $X \subseteq X' \Rightarrow \mathsf{CN}(X) \subseteq \mathsf{CN}(X')$. **(Monotonicity)**
2. $\mathsf{CN}(X) \cup \mathsf{CN}(X') \subseteq \mathsf{CN}(X \cup X')$.
3. $\mathsf{CN}(X) = \mathsf{CN}(X') \Rightarrow \mathsf{CN}(X \cup X'') = \mathsf{CN}(X' \cup X'')$.

Almost all well-known monotonic logics (classical logic, intuitionistic logic, modal logic, etc.) can be viewed as special cases of Tarski's notion of an abstract logic.

Once $(\mathscr{L}, \mathsf{CN})$ is fixed, we can define a notion of *consistency* as follows:

Definition 1 (Consistency) *Let $X \subseteq \mathscr{L}$. X is* consistent *wrt $(\mathscr{L}, \mathsf{CN})$ iff $\mathsf{CN}(X) \neq \mathscr{L}$. It is* inconsistent *otherwise.*

This says that X is consistent iff its set of consequences is not the set of all formulas. The coherence requirement forces $\emptyset$ to always be consistent - this makes sense for any reasonable logic as saying emptiness should intuitively be consistent.

Property 2. Let $X \subseteq \mathscr{L}$.

1. If X is consistent, then $\mathsf{CN}(X)$ is consistent as well.
2. $\forall X' \subseteq X$, if X is consistent, then X' is consistent.
3. $\forall X' \subseteq X$, if X' is inconsistent, then X is inconsistent.

In what follows we introduce a concept that is useful for the rest of the paper.

Definition 2 (Adjunctive) $(\mathscr{L}, \mathsf{CN})$ *is* adjunctive *iff for all x and y in $\mathscr{L}$, if $\mathsf{CN}(\{x, y\}) \neq \mathsf{CN}(\{x\})$ and $\mathsf{CN}(\{x, y\}) \neq \mathsf{CN}(\{y\})$ then there exists $z \in \mathscr{L}$ such that $\mathsf{CN}(\{z\}) = \mathsf{CN}(\{x, y\})$.*

Intuitively, an adjunctive logic infers, from the union of two formulas $\{x, y\}$, some formula(s) that can be inferred neither from x alone nor from y alone (except, of course, when y ensues from x or vice-versa). In fact, all well-known logics are adjunctive. [1] A logic which is not adjunctive could for instance fail to deny $x \vee y$ from the premises $\{\neg x, \neg y\}$.
Throughout the paper, the following assumption is made:

Assumption 1 *The logic $(\mathscr{L}, \mathsf{CN})$ is adjunctive.*

From now on, we will consider a *knowledge base Σ* which is a subset of the logical language $\mathscr{L}$ ($\Sigma \subseteq \mathscr{L}$). With no loss of generality and for the sake of simplicity, the knowledge base Σ is assumed to be free of tautologies:

Assumption 2 *For all $x \in \Sigma$, $x \notin \mathsf{CN}(\emptyset)$.*

If Σ is inconsistent, then it contains *minimal conflicts*.

Definition 3 (Minimal conflict) *Let Σ be a knowledge base, and $C \subseteq \Sigma$. The set C is a* minimal conflict *iff:*

– *C is inconsistent*
– *$\forall x \in C$, $C \backslash \{x\}$ is consistent*

Let $\mathscr{C}_\Sigma$ denote the set of all minimal conflicts of Σ.

[1] A few *very restricted* fragments of well-known logics fail to be adjunctive, e.g., the pure implicational fragment of classical logic as it is negationless, disjunctionless, and, of course, conjunctionless.

3 An extension of Dung's abstract system

Argumentation is a reasoning paradigm that follows three main steps: i) constructing *arguments* and counterarguments ii) defining the *status* of each argument, and iii) concluding or specifying the *justified conclusions*. In what follows, we refine Dung's system by defining all the above items involved in argumentation without losing generality. We start with an abstract logic $(\mathscr{L}, \mathsf{CN})$ from which the notions of argument and attacks between arguments are defined. More precisely, arguments are built from a knowledge base, say Σ, containing formulas of the language $\mathscr{L}$. An *argument* gives a reason for believing a statement or choosing an action, etc. Formally:

Definition 4 (Argument) *Let Σ be a knowledge base. An* argument *is a pair (X, x) such that:*

1. $X \subseteq \Sigma$
2. X *is consistent*
3. $x \in \mathsf{CN}(X)$
4. $\nexists X' \subset X$ *such that* $x \in \mathsf{CN}(X')$

Notations: $\mathtt{S}upp$ and $\mathtt{C}onc$ denote respectively the *support* X and the *conclusion* x of an argument (X, x). For $\mathcal{S} \subseteq \Sigma$, let $\mathtt{A}rg(\mathcal{S})$ denote the set of all arguments that can be built from $\mathcal{S}$ by means of Definition 4.

Due to Assumption 2 ($x \notin \mathsf{CN}(\emptyset)$ for all $x \in \Sigma$), it can also be shown that each consistent formula in Σ gives birth to an argument:

Property 3. For all $x \in \Sigma$ s.t. the set $\{x\}$ is consistent, there exists $a \in \mathtt{A}rg(\Sigma)$ where $\mathtt{S}upp(a) = \{x\}$.

Since CN is monotonic, constructing arguments is a monotonic process: Additional knowledge never makes the set of arguments to shrink but only gives rise to extra arguments that may interact with the existing arguments.

Property 4. $\mathtt{A}rg(\Sigma) \subseteq \mathtt{A}rg(\Sigma')$ whenever $\Sigma \subseteq \Sigma' \subseteq \mathscr{L}$.

We show next that any proper subset of a minimal conflict is the support of at least one argument. It is a result of utmost importance as regards encoding the attack relation between arguments.

Proposition 1 *Let $(\mathscr{L}, \mathsf{CN})$ be adjunctive. For all non-empty proper subset X of some minimal conflict $C \in \mathscr{C}_\Sigma$, there exists $a \in \mathtt{A}rg(\Sigma)$ s.t. $\mathtt{S}upp(a) = X$.*

Proposition 1 is indeed fundamental because it says that if statements from Σ contradict others then it is always possible to define an argument exhibiting the conflict.

We refine Dung's abstract framework as follows.

Definition 5 (Argumentation system) *Given a knowledge base Σ, an* argumentation system *(AS) over Σ is a pair $(\mathtt{A}rg(\Sigma), \mathcal{R})$ such that $\mathtt{A}rg(\Sigma)$ is a set of arguments defined from Σ and $\mathcal{R} \subseteq \mathtt{A}rg(\Sigma) \times \mathtt{A}rg(\Sigma)$ is an* attack *relation.*

The attack relation captures the different disagreements that may exist between arguments. [6] is silent on how to proceed in order to obtain a reasonable $\mathcal{R}$ in practice. It happens, as pointed out by [4], that it is in fact an error-prone step. We will see in the next section how can things go wrong in this respect, and, in the subsequent section, to what extent our definitions help to circumvent the problem. For the time being, let us focus on what role $\mathcal{R}$ plays in Dung's approach.

Among all the arguments, it is important to know which arguments to rely on for inferring conclusions from a base Σ. In [6], different acceptability semantics have been proposed. The basic idea behind these semantics is the following: for a rational agent, an argument is acceptable if he can defend this argument against all attacks on it. All the arguments acceptable for a rational agent will be gathered in a so-called *extension*. An extension must satisfy a consistency requirement and must defend all its elements.

Definition 6 (Conflict-free, Defence) *Let* $(\mathtt{Arg}(\Sigma), \mathcal{R})$ *be an AS, and* $\mathcal{B} \subseteq \mathtt{Arg}(\Sigma)$.

- $\mathcal{B}$ *is* conflict-free *iff* $\nexists a, b \in \mathcal{B}$ *such that* $(a, b) \in \mathcal{R}$.
- $\mathcal{B}$ defends *an argument* a *iff* $\forall b \in \mathtt{Arg}(\Sigma)$, *if* $(b, a) \in \mathcal{R}$, *then* $\exists c \in \mathcal{B}$ *such that* $(c, b) \in \mathcal{R}$.

The fundamental semantics in [6] is the one that features admissible extensions. The other semantics (i.e., preferred, stable, complete and grounded) are based on it. We only include the definition for the admissible semantics.

Definition 7 (Admissible semantics) *Let* $\mathcal{B}$ *be a conflict-free set of arguments.* $\mathcal{B}$ *is an* admissible *extension iff* $\mathcal{B}$ *defends all its elements.*

Since the notion of the acceptability is defined, we can now characterize the possible conclusions that can be drawn from Σ according to an argumentation system over Σ. The idea is to conclude x if x is the conclusion of at least an argument which belongs to every extension of the system.

Definition 8 (Output of the system) *Let* $(\mathtt{Arg}(\Sigma), \mathcal{R})$ *be an AS over a knowledge base* Σ. *Let* $\mathcal{E}_1, \ldots, \mathcal{E}_n$ *be the extensions of* $(\mathtt{Arg}(\Sigma), \mathcal{R})$ *under a given semantics.*[2] *For* $x \in \mathscr{L}$, x *is a* conclusion *of* Σ *iff* $\exists a \in \mathtt{Arg}(\Sigma)$ *such that* $\mathtt{Conc}(a) = x$ *and* $a \in \mathcal{E}_1 \cap \cdots \cap \mathcal{E}_n$. *We write* $\mathtt{Output}(\Sigma)$ *to denote the set of all conclusions of* Σ.

In [4], it has been argued that the extensions of an argumentation system should ensure consistent results. This means that the base built from the supports of arguments of each extension should be consistent.

Definition 9 (Consistency of extensions) *Let* $(\mathtt{Arg}(\Sigma), \mathcal{R})$ *be an AS. An extension* $\mathcal{E}$ *(under a given semantics)* satisfies consistency *iff* $\bigcup_{a \in \mathcal{E}} \mathtt{Supp}(a)$ *is consistent.*

It can be shown that if all the extensions of an argumentation system enjoy consistency, then the output of the system is consistent as well.

Proposition 2 *Let* $(\mathtt{Arg}(\Sigma), \mathcal{R})$ *be an AS over a knowledge base* Σ. *Let* $\mathcal{E}_1, \ldots, \mathcal{E}_n$ *be the extensions of* $(\mathtt{Arg}(\Sigma), \mathcal{R})$ *under a given semantics. If* $\forall \mathcal{E}_{i=1,\ldots,n}$, $\mathcal{E}_i$ *satisfies consistency, then* $\mathtt{Output}(\Sigma)$ *is consistent.*

[2] One can use any semantics: complete, stable, preferred, or grounded. However, admissible semantics is not recommended since the empty set is admissible. Thus, no argument can belong to all the extensions.

4 Some problematic cases

In the previous section we have provided a clear definition of an argument and how it is built from a knowledge base Σ. However, there still is no indication on how $\mathcal{R}$ is defined and how it is related to Σ. Moreover, in [4] it has been shown that there are some instantiations of Dung's system that violate extension consistency. Does this mean that the notion of being conflict-free is not sufficient to ensure consistency? It is sufficient provided that the attack relation is defined in an appropriate way. In this section, we present some problematic examples that shed light on the minimal requirements for defining an attack relation. Throughout the section, we consider propositional logic.

Let us start with an attack relation that is not related to inconsistency in Σ.

Example 1 (Independence from minimal conflict) *Let $\Sigma = \{x, \neg x\}$. So, $\mathscr{C}_\Sigma = \{\Sigma\}$. Take $\mathtt{A}rg(\Sigma) = \{a_1, a_2\}$ where $a_1 = (\{x\}, x)$ and $a_2 = (\{\neg x\}, \neg x)$. Assume that $\mathcal{R} = \emptyset$. This means that $\mathcal{R}$ does not depend at all on minimal conflicts in Σ. In this case, $(\mathtt{A}rg(\Sigma), \mathcal{R})$ has an admissible extension that violates consistency, it is $\{a_1, a_2\}$.*

Requirement 1 *An attack relation should "capture" at least the minimal conflicts of the knowledge base at hand.*

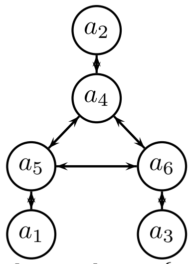
Let us now consider an attack relation that captures all the minimal conflicts of Σ. We start with a *symmetric* relation, due to [7], called *rebut*. An argument a *rebuts* an argument b iff $\mathtt{Conc}(a) \equiv \neg\mathtt{Conc}(b)$ (or vice-versa).

Example 2 (Binary minimal conflict) *The two arguments $a_1 = (\{x\}, x)$ and $a_2 = (\{\neg x\}, \neg x)$ rebut each other. So, the system $(\mathtt{A}rg(\Sigma), Rebut)$ has two admissible extensions $\{a_1\}$ and $\{a_2\}$. They each satisfy consistency.*

Unfortunately, the fact that an attack relation captures the minimal conflicts of a knowledge base does not always ensure consistency of the extensions. Let us consider a knowledge base displaying a ternary minimal conflict.

Example 3 (Ternary conflict) *Let $\Sigma = \{x, y, x \to \neg y\}$. Let $\mathtt{A}rg(\Sigma)$ consist of the following arguments:*

- $a_1 = (\{x\}, x)$
- $a_2 = (\{y\}, y)$
- $a_3 = (\{x \to \neg y\}, x \to \neg y)$
- $a_4 = (\{x, x \to \neg y\}, \neg y)$
- $a_5 = (\{y, x \to \neg y\}, \neg x)$
- $a_6 = (\{x, y\}, x \wedge y)$



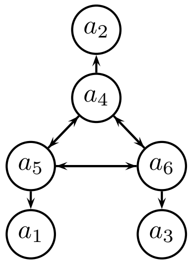
Let $\mathcal{R}$ be such as depicted in figure above. The set $\{a_1, a_2, a_3\}$ is an admissible extension of $(\mathtt{A}rg(\Sigma), \mathcal{R})$. However, $\mathtt{Supp}(a_1) \cup \mathtt{Supp}(a_2) \cup \mathtt{Supp}(a_3)$ is inconsistent.

This example shows that a conflict-free set of arguments may fail consistency. This is due to the fact that $\mathcal{R}$ is binary, in compliance with Dung's definitions imposing the attack relation to be binary. Thus, the ternary conflict between a_1, a_2 and a_3 is not captured.

Requirement 2 *If a knowledge base has ternary (or more) minimal conflicts, the attack relation should be asymmetric.*

Indeed, [5] shows that an argumentation system which uses the asymmetric, *assumption attack* defined in [7] yields consistent extensions. An argument a *undercuts* b iff $\exists h' \in$ $\texttt{Supp}(b)$ such that $\texttt{Conc}(a) \equiv \neg h'$ (or $h' \equiv \neg\texttt{Conc}(a)$).

Example 4 (Ternary conflict cont.) *The attack relation $\mathcal{R}$, in the sense of undercut, between arguments of $\texttt{A}rg(\Sigma)$ from Example 3 is now such as depicted in the figure below:*



There are three maximal (wrt set inclusion) admissible extensions: $\{a_1, a_2, a_6\}$, $\{a_2, a_3, a_5\}$, and $\{a_1, a_3, a_4\}$. All three extensions satisfy consistency.

5 Properties of attack relations

In this section, we discuss primitive properties that may be expected from an attack relation. The first of these is about the origin of $\mathcal{R}$. As argued in the previous section, an attack relation should capture at least the minimal conflicts arising from the knowledge base Σ under consideration.

Definition 10 (Capturing conflicts) *Let $C \in \mathscr{C}_\Sigma$. A pair (a, b) in $\mathcal{R}$* captures *C if C $\subseteq \texttt{Supp}(a) \cup \texttt{Supp}(b)$.*

Alas, that an attack relation captures all the minimal conflicts is not sufficient to ensure consistency of extensions.

Example 5 *Let $\texttt{A}rg(\Sigma) = \{a, b, c\}$, $\mathscr{C}_\Sigma = \{C\}$ and $\mathcal{R} = \{(a, b)\}$. If $C \subseteq \texttt{Supp}(a) \cup$ $\texttt{Supp}(b)$ and $C \subseteq \texttt{Supp}(a) \cup \texttt{Supp}(c)$, then $\mathcal{R}$ captures (not "faithfully", though) the minimal conflict C but the admissible extension $\{a, c\}$ violates consistency.*

Definition 11 (Conflict-sensitive) *An attack relation $\mathcal{R}$ is* conflict-sensitive *iff for all a and b in $\texttt{A}rg(\Sigma)$ such that there exists a minimal conflict $C \subseteq \texttt{Supp}(a) \cup \texttt{Supp}(b)$ then either $(a, b) \in \mathcal{R}$ or $(b, a) \in \mathcal{R}$.*

Being conflict-sensitive means this: If Σ provides evidence (according to CN) that the supports of a and b conflict with each other, then this conflict shows in $\mathcal{R}$ (i.e., either $(a, b) \in \mathcal{R}$ holds or $(b, a) \in \mathcal{R}$ holds). In other words, being conflict-sensitive ensures that, when passing from Σ to $(\texttt{A}rg(\Sigma), \mathcal{R})$, no conflict is "forgotten" in $\mathcal{R}$.

Example 6 *The attack relation $\mathcal{R}$ of Example 5 is not conflict-sensitive since neither (a, c) nor (c, a) is in $\mathcal{R}$.*

In case the knowledge base does not contain an inconsistent formula, if $\mathcal{R}$ is conflict-sensitive then $\mathcal{R}$ captures all minimal conflicts.

Proposition 3 *Let $\mathscr{C}_\Sigma$ s.t. $\forall C \in \mathscr{C}_\Sigma$, $|C| > 1$. If $\mathcal{R}$ is sensitive, then $\mathcal{R}$ captures all minimal conflicts of $\mathscr{C}_\Sigma$.*

That an attack relation is conflict-sensitive and captures all the minimal conflicts need not mean that it is strictly based on minimal conflicts. Next is an illustration:

Example 7 *Consider $\mathtt{Arg}(\Sigma) = \{a, b, c\}$, $\mathscr{C}_\Sigma = \{C\}$ and $\mathcal{R} = \{(a, b), (a, c)\}$. Assume $C \subseteq \mathtt{Supp}(a) \cup \mathtt{Supp}(b)$. Then, $\mathcal{R}$ is conflict-sensitive and captures the minimal conflicts in $\mathscr{C}_\Sigma$. But $\mathcal{R}$ contains an attack, (a, c), which is unrelated to $\mathscr{C}_\Sigma$.*

Definition 12 (Conflict-dependent) *An attack relation $\mathcal{R}$ is* conflict-dependent *iff for all a and b in $\mathtt{Arg}(\Sigma)$, $(a, b) \in \mathcal{R}$ implies that there exists a minimal conflict $C \subseteq \mathtt{Supp}(a) \cup \mathtt{Supp}(b)$.*

Being conflict-dependent means this: $\mathcal{R}$ shows no attack from a to b unless Σ provides evidence (according to CN) that the supports of a and b conflict with each other. That is, being conflict-dependent ensures that, when passing from Σ to $(\mathtt{Arg}(\Sigma), \mathcal{R})$, no conflict is "invented" in $\mathcal{R}$.

Example 8 *$\mathcal{R}$ in Example 7 is not conflict-dependent since the attack (a, c) does not depend on the minimal conflict C.*

Clearly, an attack relation that is conflict-sensitive need not be conflict-dependent. Conversely, in Example 5, $\mathcal{R}$ illustrates the fact that an attack relation that is conflict-dependent is not necessarily conflict-sensitive. Note that an attack relation which is conflict-dependent exhibits no self-attack:

Proposition 4 *Let $(\mathtt{Arg}(\Sigma), \mathcal{R})$ be s. t. $\mathcal{R}$ is conflict-dependent. For all $a \in \mathtt{Arg}(\Sigma)$, $(a, a) \notin \mathcal{R}$.*

When the attack relation is conflict-dependent, if a set of arguments is such that its corresponding base (set-theoretic union of supports) is consistent then it is a conflict-free set:

Proposition 5 *Let $(\mathtt{Arg}(\Sigma), \mathcal{R})$ be s. t. $\mathcal{R}$ is conflict-dependent. $\forall \mathcal{B} \subseteq \mathtt{Arg}(\Sigma)$, if $\bigcup_{a \in \mathcal{B}} \mathtt{Supp}(a)$ is consistent, $\mathcal{B}$ is conflict-free.*

When $\mathcal{R}$ is both conflict-sensitive and conflict-dependent, $\mathcal{R}$ then essentially makes no difference between arguments that have equivalent supports:

Proposition 6 *Let $(\mathtt{Arg}(\Sigma), \mathcal{R})$ such that $\mathcal{R}$ is conflict-sensitive and conflict-dependent. For all a and b in $\mathtt{Arg}(\Sigma)$, if $\mathsf{CN}(\mathtt{Supp}(a)) = \mathsf{CN}(\mathtt{Supp}(b))$, then for all $c \in \mathtt{Arg}(\Sigma)$*
$(a, c) \in \mathcal{R}$ *or* $(c, a) \in \mathcal{R}$ *iff* $(b, c) \in \mathcal{R}$ *or* $(c, b) \in \mathcal{R}$.

Definitions 10-12 characterize the "origin" of the attack relation $\mathcal{R}$ by relating it to the minimal conflicts of the knowledge base Σ. It is then natural that the attack relation conforms with the minimal conflicts. In what follows, we present rules about $\mathcal{R}$ that relate to the minimal conflicts.

Definition 13 (Homogeneous relation) *Let a and b in $\mathtt{Arg}(\Sigma)$ such that $\mathtt{Supp}(a) \subseteq \mathtt{Supp}(b)$. For all $c \in \mathtt{Arg}(\Sigma)$,*

R1: $(a, c) \in \mathcal{R} \Rightarrow (b, c) \in \mathcal{R}$.
R2: $(c, a) \in \mathcal{R} \Rightarrow (c, b) \in \mathcal{R}$.

$\mathcal{R}$ is homogeneous *if it satisfies both $R1$ and $R2$.*

The above two rules capture exactly the idea of Property 2 which says that if a set of formulas is inconsistent, then all its supersets are inconsistent as well. This property is captured by two rules since an attack relation is not necessarily symmetric whereas inconsistency is not oriented. We show that, when the attack relation is symmetric, if it satisfies one of the above two rules, then it also satisfies the other.

Property 5. Let $\mathcal{R}$ be symmetric. If $\mathcal{R}$ satisfies rule $R1$ (resp. $R2$) then it satisfies $R2$ (resp. $R1$).

Finally, according to the definition of a minimal conflict C, any partition of C into two subsets X_1 and X_2, it holds that X_1 and X_2 are consistent. Since Proposition 1 ensures that X_1 and X_2 are the supports of arguments, then it is natural to consider that those arguments are conflicting.

Definition 14 (Conflict-exhaustive) *Let $(\mathtt{Arg}(\Sigma), \mathcal{R})$ be an AS over a knowledge base Σ.*

- *$\mathcal{R}$ is* strongly conflict-exhaustive *iff for all $C \in \mathscr{C}_\Sigma$ s.t. $|C| > 1$, for all non-empty proper subset X of C, there exist a and b in $\mathtt{Arg}(\Sigma)$ s.t. $\mathtt{S}upp(a) = X$, $\mathtt{S}upp(b) = C \setminus X$, $(a, b) \in \mathcal{R}$ and $(b, a) \in \mathcal{R}$.*
- *$\mathcal{R}$ is* conflict-exhaustive *iff for all $C \in \mathscr{C}_\Sigma$ s.t. $|C| > 1$, for all non-empty proper subset X of C, there exist a and b in $\mathtt{Arg}(\Sigma)$ s.t. $\mathtt{S}upp(a) = X$, $\mathtt{S}upp(b) = C \setminus X$ and either $(a, b) \in \mathcal{R}$ or $(b, a) \in \mathcal{R}$.*
- *$\mathcal{R}$ is* weakly conflict-exhaustive *iff $\forall C \in \mathscr{C}_\Sigma$ s.t. $|C| > 1$, for all $x \in C$, there exist a and b in $\mathtt{Arg}(\Sigma)$ s.t. $\mathtt{S}upp(a) = \{x\}$, $\mathtt{S}upp(b) = C \setminus \{x\}$ and $(b, a) \in \mathcal{R}$.*

The following property highlights the links between the different notions presented in this section.

Property 6. Let $\mathcal{R} \subseteq \mathtt{Arg}(\Sigma) \times \mathtt{Arg}(\Sigma)$.

1. $\mathcal{R}$ is strongly conflict-exhaustive, then $\mathcal{R}$ is weakly conflict-exhaustive and $\mathcal{R}$ is conflict-exhaustive.
2. If $\mathcal{R}$ is homogeneous and conflict-exhaustive, then $\mathcal{R}$ is conflict-sensitive.
3. If $\mathcal{R}$ is conflict-sensitive, then it is conflict-exhaustive.

Finally, one can characterize symmetric relations using the above properties.

Proposition 7 *If $\mathcal{R}$ is conflict-dependent, homogeneous, and strongly conflict-exhaustive, then $\mathcal{R}$ is symmetric.*

6 Choosing an attack relation

This section studies the appropriate attack relation of an argumentation system. By appropriate relation, we mean a relation that ensures at least extension consistency. We will study two cases: the case where all the minimal conflicts that are contained in a base are binary, and the case of a base containing ternary or more minimal conflicts.

6.1 Case of binary minimal conflicts

Throughout this section, we will consider a knowledge base Σ whose minimal conflicts are all *binary*. This means that any minimal conflict contains exactly two formulas. Thus, there is no inconsistent formula in the base.

The following result is of great importance since it provides a class of attack relations that ensure extension consistency.

Proposition 8 *Let* $(\mathtt{Arg}(\Sigma), \mathcal{R})$ *be an AS over* Σ *s.t. all minimal conflicts of* Σ *are binary. If* $\mathcal{R}$ *is conflict-sensitive, then for all* $\mathcal{B} \subseteq \mathtt{Arg}(\Sigma)$*, that* $\mathcal{B}$ *is conflict-free implies that* $\bigcup_{a \in \mathcal{B}} \mathtt{Supp}(a)$ *is consistent.*

The above result is not surprising. As shown in the previous section, in order for a conflict-free set of arguments to ensure consistency, the attack relation should capture the minimal conflicts. However, it is not necessary to be conflict-dependent. This latter is however important to show that any set of arguments that satisfies consistency is conflict-free (see Proposition 5).

Corollary 1. *Let* $(\mathtt{Arg}(\Sigma), \mathcal{R})$ *be an AS over* Σ *s.t. all minimal conflicts of* Σ *are binary. If* $\mathcal{R}$ *is conflict-sensitive, then* $(\mathtt{Arg}(\Sigma), \mathcal{R})$ *satisfies extension consistency.*

From this corollary and Property 6, the following result holds.

Corollary 2. *Let* $(\mathtt{Arg}(\Sigma), \mathcal{R})$ *be an AS over* Σ *s.t. all minimal conflicts of* Σ *are binary. If* $\mathcal{R}$ *is homogeneous and conflict-exhaustive then* $(\mathtt{Arg}(\Sigma), \mathcal{R})$ *satisfies consistency extension.*

Another direct consequence of Proposition 5, and Proposition 8 is the following:

Corollary 3. *Let* $(\mathtt{Arg}(\Sigma), \mathcal{R})$ *be an AS over* Σ *s.t. all minimal conflicts of* Σ *are binary. Let* $\mathcal{R}$ *be conflict-sensitive and conflict-dependent. For all* $\mathcal{B} \subseteq \mathtt{Arg}(\Sigma)$*,* $\mathcal{B}$ *is conflict-free iff* $\bigcup_{a \in \mathcal{B}} \mathtt{Supp}(a)$ *is consistent.*

It is worth noticing that symmetric relations can be used in the binary case. This means that it is possible to have an attack relation that is both sensitive and symmetric. Indeed, according to Proposition 7, when $\mathcal{R}$ is conflict-dependent, homogeneous, and strongly conflict-exhaustive, then $\mathcal{R}$ is symmetric. From Property 6 it is clear that when $\mathcal{R}$ is strongly exhaustive, then it is also conflict-exhaustive. Thus, $\mathcal{R}$ is sensitive.

6.2 Case of general minimal conflicts

In the previous section, we have shown that when all the minimal conflicts of a knowledge base are binary, then a symmetric relation can be used and ensures extension consistency. Unfortunately, this is not the case when ternary or more minimal conflicts are present in a base. The following result shows that symmetric relations lead to the violation of consistency.

Proposition 9 *Let* $\Sigma = \{x_1, \ldots, x_n\}$ *where* $n > 2$ *and* $\mathscr{C}_\Sigma = \{\Sigma\}$*. Let* $a_1, \ldots, a_n \in \mathtt{Arg}(\Sigma)$ *s.t.* $\mathtt{Supp}(a_i) = \{x_i\}$*. If* $\mathcal{R}$ *is conflict-dependent and symmetric, then* $\mathcal{E} = \{a_1, \ldots, a_n\}$ *is an admissible extension of* $(\mathtt{Arg}(\Sigma), \mathcal{R})$*.*

A direct consequence of the above result is the following:

Corollary 4. *Let* $\Sigma = \{x_1, \ldots, x_n\}$ *where* $n > 2$ *and* $\mathscr{C}_\Sigma = \{\Sigma\}$*. If* $\mathcal{R}$ *is conflict-dependent and symmetric, then the AS* $(\mathtt{Arg}(\Sigma), \mathcal{R})$ *over* Σ *violates extension consistency.*

The previous result can be generalized as follows:

Corollary 5. *Let* $\mathscr{C}_\Sigma$ *s.t.* $\exists C \in \mathscr{C}_\Sigma$ *and* $|C| > 2$*. If* $\mathcal{R}$ *is conflict-dependent and symmetric, then the AS* $(\mathtt{Arg}(\Sigma), \mathcal{R})$ *over* Σ *violates extension consistency.*

Let us now present a class of attack relations that ensure consistency. The following result states that when the relation $\mathcal{R}$ satisfies both $R2$ and $R4$ and is weakly exhaustive, then the corresponding argumentation system satisfies extension consistency.

Proposition 10 *If* $\mathcal{R}$ *satisfies R2, R4 and is weakly exhaustive, then* $(\mathtt{Arg}(\Sigma), \mathcal{R})$ *satisfies extension consistency.*

Note that there may exist other classes of asymmetric attack relations that ensure extensions consistency.

7 Conclusion

The paper extended Dung's argumentation framework by taking into account the logic from which arguments are built. The new framework is general since it is grounded on an abstract monotonic logic. Thus, a wide variety of logics can be used even those that are not yet considered in argumentation like temporal logic, modal logic. The extension has two main advantages: First, it enforces the framework to make safe conclusions. Second, it relates the different notions of Dung's system, like the attack relation and conflict-free, to the knowledge base that is under study. The paper also presented a formal methodology for defining arguments from a knowledge base, and for choosing an appropriate attack relation.

There are a number of ways to extend this work. One future direction is to complete the study of attack relations that ensure consistency. Another direction that we want to pursue is to consider the case of non-adjunctive logics. Recall that all the results presented in this paper hold in the case of an adjunctive logic.

References

1. L. Amgoud and H. Prade. Using arguments for making and explaining decisions. *Artificial Intelligence J.*, 173:413–436, 2009.
2. P. Besnard and A. Hunter. *Elements of Argumentation*. MIT Press, 2008.
3. B. Bonet and H. Geffner. Arguing for decisions: A qualitative model of decision making. In *Proc. of UAI'96*, pages 98–105, 1996.
4. M. Caminada and L. Amgoud. On the evaluation of argumentation formalisms. *Artificial Intelligence J.*, 171 (5-6):286–310, 2007.
5. C. Cayrol. On the relation between argumentation and non-monotonic coherence-based entailment. In *Proc. of IJCAI'95*, pages 1443–1448, 1995.
6. P. M. Dung. On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. *Artificial Intelligence J.*, 77:321–357, 1995.
7. M. Elvang-Gøransson, J. Fox, and P. Krause. Acceptability of arguments as 'logical uncertainty. In *Proc. of ECSQARU'93*, pages 85–90, 1993.
8. H. Prakken and G. Sartor. Argument-based extended logic programming with defeasible priorities. *J. of Applied Non-Classical Logics*, 7:25–75, 1997.
9. G. Simari and R. Loui. A mathematical treatment of defeasible reasoning and its implementation. *Artificial Intelligence J.*, 53:125–157, 1992.
10. A. Tarski. *On Some Fundamental Concepts of Metamathematics*. Oxford Uni. Press, 1956.