Evaluation of Arguments from Support Relations: Axioms and Semantics

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Abstract

This paper focuses on argumentation graphs whose nodes are arguments and edges represent supports, thus positive relations, between arguments. Furthermore, each argument has a weight reflecting its basic or intrinsic strength. For the sake of generality, the internal structure of arguments and the origin of arguments and their weights are unspecified. The paper tackles for the first time the question of evaluating the overall strengths of arguments in such graphs, thus of defining semantics for support graphs. It introduces a set of axioms that any semantics should satisfy. Then, it defines three semantics and evaluates them against the axioms.

1 Introduction

Argumentation is a social activity whose aim is to increase (or decrease) the acceptability of a given standpoint for an audience by putting forward arguments. The standpoint may be a claim which can be true or false, an action to be performed, a goal to be reached, etc. Argumentation has gained great interest in Artificial Intelligence. It is used for decision making (e.g., [Amgoud and Prade, 2009]), defeasible reasoning (e.g., [Bondarenko *et al.*, 1997]), and negotiation (e.g., [Reed, 1998]). Interested reader can find more on applications of argumentation in [Rahwan and Simari(eds.), 2009].

Whatever the application, an argumentation-based formalism, called *argumentation framework*, is generally defined as a set of arguments, attacks amongst the arguments, and a semantics for evaluating the arguments. A semantics assesses to what extent an argument is acceptable. Examples of semantics are those proposed by Dung [1995] and ranking semantics (e.g., [Amgoud and Ben-Naim, 2013; Besnard and Hunter, 2001; da Costa Pereira *et al.*, 2011; Matt and Toni, 2008]).

An attack has a negative effect since its aim is weakening its target. Cayrol and Lagasquie [2005] have pointed out another meaningful relation between arguments, the so-called *support*. Unlike attack, this relation has positive effect and aims at strengthening its target. Several works have thus been done on the evaluation of arguments in argumentation frameworks where supports and attacks coexist [Oren and Norman, 2008; Boella *et al.*, 2010; Brewka and Woltran, 2010; Nouioua, 2013; Polberg and Oren, 2014]. They have extended Dung's semantics [1995] for accounting for supports. Surprisingly enough, when the attack relation is empty, these works consider all the arguments as equally acceptable. Thus, supported arguments are as acceptable as non-supported ones. This means that supports are not fully harnessed. Furthermore, these works assumed that arguments have the same basic strength, an assumption hardly ever satisfied in practice. Each argument has a basic strength representing the weight of its source [Parsons *et al.*, 2011], or the importance degree of the value it promotes [Bench-Capon, 2003], or the importance degrees of the goals it supports [Amgoud and Prade, 2009], or the certainty degrees of its premises [Benferhat *et al.*, 1993], ...

In this paper, we investigate for the first time argumentation frameworks where arguments interact *only* in a positive way (i.e., by supporting each other). Furthermore, arguments may have different basic strengths. Such frameworks have interesting practical applications like recommendation letters. A recommendation letter contains a general recommendation justified by a list of arguments, some of them support others. The evaluation of each argument, by combining its basic strength and the overall strengths of its supporters, gives insights on the weight of the general recommendation. Another application is the evaluation of newspapers' articles, namely argumentative essays. From a theoretical perspective, defining semantics for evaluating arguments on the basis of supports allows a better understanding of the role and impact of supports, and how they should be considered.

The contribution of the paper is threefold. First, we provide a set of axioms that a semantics should satisfy. Most of the axioms are mandatory, except three which represent pairwise incompatible choices. Second, we investigate some properties of semantics that satisfy the axioms. Third, we propose three semantics, each of which satisfies all the mandatory axioms and one of the three optional axioms.

The paper is structured as follows: Section 2 introduces the argumentation graphs we are interested in. Section 3 presents the axioms. Section 4 is devoted to properties of semantics. Section 5 defines three semantics. The last section concludes.

2 Basic Concepts

A *support argumentation framework*, called also *support argumentation graph* throughout the paper, is a set of arguments and a binary relation representing supports amongst arguments. Arguments are abstract entities whose internal structure is not specified. Each argument has an intrinsic strength which is expressed by a numerical value in the interval $[0, 1]$. The value 0 means the argument is worthless while 1 means the argument is very strong. Before introducing the graphs we are dealing with, let us first define the notion of weighting.

Definition 1 (Weighting) *A* weighting *on a set $\mathcal{L}$ is a function from $\mathcal{L}$ to $[0, 1]$.*

Let us now introduce support argumentation graphs.

Definition 2 (Support Argumentation Graph) *A* support argumentation graph *(SAF) is an ordered tuple $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$, where $\mathcal{A}$ is a non empty finite set of arguments, w is a weighting on $\mathcal{A}$, and $\mathcal{S} \subseteq \mathcal{A} \times \mathcal{A}$ is a support relation. For $a, b \in \mathcal{A}$, the notation $a\mathcal{S}b$ means a supports b.*

We define a *semantics* as a function assigning for every argument in a SAF a value between 0 and 1. This value, called *strength degree*, represents the *overall strength* of the argument, i.e., the strength issued from the *aggregation* of the basic strength of the argument and the overall strengths of its supporters. The higher the degree, the stronger the argument.

Definition 3 (Semantics) *A* semantics *is a function $\mathbf{F}$ transforming any SAF $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$ into a weighting on $\mathcal{A}$. For $a \in \mathcal{A}$, $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(a)$ denotes the image of argument a by $\mathbf{F}(\mathbf{A})$, and is called* strength degree *of a.*

Below is the list of all notations used in the paper.

Notations: Let $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$ be a SAF and $a \in \mathcal{A}$. We denote by $\mathtt{Supp}_{\mathbf{A}}(a)$ the set of all *supporters* of a in $\mathbf{A}$ (i.e. $\mathtt{Supp}_{\mathbf{A}}(a) = \{b \in \mathcal{A} \mid b\mathcal{S}a\}$), by $\mathtt{Supp}+_{\mathbf{A}}(a)$ the set of arguments supported by a (i.e. $\mathtt{Supp}+_{\mathbf{A}}(a) = \{b \in \mathcal{A} \mid a\mathcal{S}b\}$). For any two SAFs $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$ and $\mathbf{A}' = \langle \mathcal{A}', w', \mathcal{S}' \rangle$, $\mathbf{A} \oplus \mathbf{A}'$ is the SAF $\langle \mathcal{A} \cup \mathcal{A}', w'', \mathcal{S} \cup \mathcal{S}' \rangle$ where for any $x \in \mathcal{A}$ (resp. $x \in \mathcal{A}'$), $w''(x) = w(x)$ (resp. $w''(x) = w'(x)$).

3 Axiomatic Foundations of Semantics

We propose a set of 17 axioms that shed light on foundational concepts and principles behind a semantics. Some of the axioms are dual to those proposed in [Amgoud and Ben-Naim, 2016] in case of attack graphs. Before presenting the first axiom, let us recall the definition of isomorphism.

Definition 4 (Isomorphism) *Let $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$ and $\mathbf{A}' = \langle \mathcal{A}', w', \mathcal{S}' \rangle$ be two SAFs. An* isomorphism *from $\mathbf{A}$ to $\mathbf{A}'$ is a bijective function f from $\mathcal{A}$ to $\mathcal{A}'$ such that:*

- *$\forall a \in \mathcal{A}$, $w(a) = w'(f(a))$, and*
- *$\forall a, b \in \mathcal{A}$, $a\mathcal{S}b$ iff $f(a)\mathcal{S}'f(b)$.*

The first basic axiom ensures that the strength degree of an argument does not depend on the argument's identity.

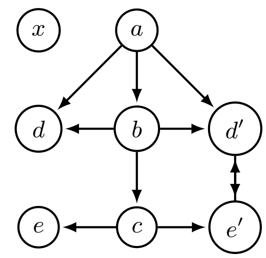
Axiom 1 (Anonymity) *A semantics $\mathbf{F}$ satisfies* anonymity *iff, for any two SAFs $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$ and $\mathbf{A}' = \langle \mathcal{A}', w', \mathcal{S}' \rangle$, for any isomorphism f from $\mathbf{A}$ to $\mathbf{A}'$, the following property holds: $\forall\, a \in \mathcal{A}, \quad \mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(a) = \mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}'}(f(a))$.*

The second basic axiom, called *independence*, states the following: the strength degree of an argument a should be independent of any argument that is not connected to a (i.e., there is no path from that argument to a).

Axiom 2 (Independence) *A semantics $\mathbf{F}$ satisfies* independence *iff, for any two SAFs $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$ and $\mathbf{A}' = \langle \mathcal{A}', w', \mathcal{S}' \rangle$ such that $\mathcal{A} \cap \mathcal{A}' = \emptyset$, the following property holds: $\forall\, a \in \mathcal{A}$, $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(a) = \mathtt{Deg}^{\mathbf{F}}_{\mathbf{A} \oplus \mathbf{A}'}(a)$.*

Let us consider the following running example.

Example 1 *Let $\mathbf{A}$ be the support argumentation graph depicted below.*



Assume that $w(a) = w(b) = w(d) = w(d') = w(e) = w(e') = \frac{1}{3}$, $w(x) = 0.2$ and $w(c) = \frac{3}{4}$. Independence ensures that the degree of a is independent of x's since the two arguments are not linked.

The next axiom, called *non-dilution*, states that supporting arguments has no impact on its own strength degree.

Axiom 3 (Non-Dilution) *A semantics $\mathbf{F}$ satisfies* non dilution *iff, for any two SAFs $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$ and $\mathbf{A}' = \langle \mathcal{A}', w', \mathcal{S}' \rangle$ such that $\mathcal{A} = \mathcal{A}'$, $w = w'$ and $\mathcal{S}' = \mathcal{S} \cup \{(a, b)\}$ with $\mathtt{Supp}+_{\mathbf{A}}(b) = \emptyset$, the following holds: for all $x \in \mathcal{A} \setminus \{b\}$, $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(x) = \mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}'}(x)$.*

Example 1 (Cont) Non-dilution ensures that the degree of a should not take into account the fact that a supports d.

It is worth mentioning that non dilution is not mandatory in reputation systems. PageRank, the reputation system used by Google search engine for ranking web pages, violates the axiom. It considers that the more an agent (a web page) supports other agents, the less it is credible.

The following axiom, called *dummy*, states that arguments that get value 0 have no impact on the arguments they support.

Axiom 4 (Dummy) *A semantics $\mathbf{F}$ satisfies* dummy *iff, for any SAF $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$, for all $a, b \in \mathcal{A}$, if*

- *$w(a) = w(b)$,*
- *$\mathtt{Supp}_{\mathbf{A}}(a) = \mathtt{Supp}_{\mathbf{A}}(b) \cup \{x\}$ such that $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(x) = 0$,*

then $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(a) = \mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(b)$.

The next axiom, called *monotony*, ensures that an argument becomes stronger when its set of supporters gets bigger.

Axiom 5 (Monotony) *A semantics $\mathbf{F}$ satisfies* monotony *iff, for any SAF $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$, for all $a, b \in \mathcal{A}$, if*

- *$w(a) = w(b)$,*
- *$\mathtt{Supp}_{\mathbf{A}}(b) \subseteq \mathtt{Supp}_{\mathbf{A}}(a)$,*

then $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(a) \geq \mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(b)$.

Example 1 (Cont) Monotony ensures $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(d') \geq \mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(d) \geq \mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(b)$, and $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(e') \geq \mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(e)$.

The next axiom, called *equivalence*, ensures that the strength degree of an argument depends on the strength degrees of its direct supporters and its basic strength.

Axiom 6 (Equivalence) *A semantics* $\mathbf{F}$ *satisfies* equivalence *iff, for any SAF* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$*, for all* $a, b \in \mathcal{A}$*, if*

- $w(a) = w(b)$*,*
- $\exists f$*, a bijective function, from* $\mathtt{Supp}_{\mathbf{A}}(a)$ *to* $\mathtt{Supp}_{\mathbf{A}}(b)$ *s.t* $\forall x \in \mathtt{Supp}_{\mathbf{A}}(a)$*,* $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(x) = \mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(f(x))$*,*

then $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(a) = \mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(b)$*.*

The axiom called *coherence*, states that the impact of support is proportional to the basic strength of its target.

Axiom 7 (Coherence) *A semantics* $\mathbf{F}$ *satisfies* coherence *iff, for any SAF* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$*, for all* $a, b \in \mathcal{A}$*, if*

- $w(a) > w(b)$*,*
- $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(b) < 1$*,*
- $\mathtt{Supp}_{\mathbf{A}}(a) = \mathtt{Supp}_{\mathbf{A}}(b)$*,*

then $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(a) > \mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(b)$*.*

The *minimality* axiom ensures that if an argument is not supported, its overall strength is equal to its basic strength.

Axiom 8 (Minimality) *A semantics* $\mathbf{F}$ *satisfies* minimality *iff, for any SAF* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$*, for any argument* $a \in \mathcal{A}$*, if* $\mathtt{Supp}_{\mathbf{A}}(a) = \emptyset$*, then* $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(a) = w(a)$*.*

Example 1 (Cont) From minimality, $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(a) = \frac{1}{3}$ and $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(x) = 0.2$.

The following axiom, called *strengthening*, defines formally the role of supports. It states that a support strengthens its target by increasing its overall strength (possibly only by an infinitesimal amount). This is particularly true when the support emanates from (even slightly) *acceptable* argument (i.e., an argument s.t. $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(.) > 0$). If the degree of the argument is already 1, the supports are useless.

Axiom 9 (Strengthening) *A semantics* $\mathbf{F}$ *satisfies* strengthening *iff, for any SAF* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$*, for any argument* $a \in \mathcal{A}$*, if* $w(a) < 1$ *and* $\exists b \in \mathtt{Supp}_{\mathbf{A}}(a)$ *s.t.* $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(b) > 0$*, then* $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(a) > w(a)$*.*

Example 1 (Cont) Strengthening ensures that $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(b) > \frac{1}{3}$, $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(d) > \frac{1}{3}$, $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(c) > \frac{3}{4}$, $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(d') > \frac{1}{3}$, $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(e) > \frac{1}{3}$ and $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(e') > \frac{1}{3}$.

The next axiom, called *strengthening soundness*, states that the only way of increasing the overall strength of an argument is by supporting the argument with an acceptable one.

Axiom 10 (Strengthening Soundness) *A semantics* $\mathbf{F}$ *satisfies* strengthening soundness *iff, for any SAF* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$*, for any argument* $a \in \mathcal{A}$*, if* $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(a) > w(a)$ *then* $\exists b \in \mathtt{Supp}_{\mathbf{A}}(a)$ *s.t.* $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(b) > 0$*.*

The three previous axioms are about the role of supports, which is strengthening arguments. The following axioms introduce two key factors that may impact the overall strength of an argument: the *number* of supporters and their *quality*. The more numerous the acceptable supporters of an argument, the stronger the argument.

Axiom 11 (Counting) *A semantics* $\mathbf{F}$ *satisfies* counting *iff, for any SAF* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$*, for all* $a, b \in \mathcal{A}$ *such that*

- $w(a) = w(b)$*,* $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(b) < 1$*,*
- $\mathtt{Supp}_{\mathbf{A}}(a) = \mathtt{Supp}_{\mathbf{A}}(b) \cup \{y\}$ *with* $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(y) > 0$*,*

then $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(a) > \mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(b)$*.*

Example 1 (Cont) Counting ensures $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(d') > \mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(d) > \mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(b)$, and $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(e') > \mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(e)$.

The quality of acceptable supporters is another factor impacting the overall strength of an argument. The next axiom, called *reinforcement*, states that if the overall strength of a supporter is increased, then its target is strengthened further provided that its strength degree is not already maximal (1).

Axiom 12 (Reinforcement) *A semantics* $\mathbf{F}$ *satisfies* reinforcement *iff, for any SAF* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$*,* $\forall a, b \in \mathcal{A}$*, if*

- $w(a) = w(b)$*,* $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(b) < 1$*,*
- $\mathtt{Supp}_{\mathbf{A}}(a) \setminus \mathtt{Supp}_{\mathbf{A}}(b) = \{x\}$*,*
- $\mathtt{Supp}_{\mathbf{A}}(b) \setminus \mathtt{Supp}_{\mathbf{A}}(a) = \{y\}$*,*
- $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(x) > \mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(y) > 0$*,*

then $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(a) > \mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(b)$*.*

Our next axiom, called *boundedness*, states that an argument which has a maximal degree (1) keeps the same degree if one of its supporters is strengthened.

Axiom 13 (Boundedness) *A semantics* $\mathbf{F}$ *satisfies* boundedness *iff, for any SAF* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$*,* $\forall a, b \in \mathcal{A}$ *such that*

- $w(a) = w(b)$*,*
- $\mathtt{Supp}_{\mathbf{A}}(a) \setminus \mathtt{Supp}_{\mathbf{A}}(b) = \{x\}$*,*
- $\mathtt{Supp}_{\mathbf{A}}(b) \setminus \mathtt{Supp}_{\mathbf{A}}(a) = \{y\}$*,*
- $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(x) > \mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(y)$*,*

if $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(b) = 1$*, then* $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(a) = 1$*.*

The previous axioms are all mandatory and should be satisfied by any semantics. The remaining axioms are optional. The first one, called *imperfection*, states that an argument can never get a maximal value if its basic strength is not maximal.

Axiom 14 (Imperfection) *A semantics* $\mathbf{F}$ *satisfies* imperfection *iff, for any SAF* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$*, for any* $a \in \mathcal{A}$*, if* $w(a) < 1$ *then* $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(a) < 1$*.*

The three last axioms give an overwhelming weight to either the *number* of supporters, or their *quality*, or allow some *compensation*. More precisely, *cardinality precedence* says that an argument a is stronger than an argument b if the acceptable supporters of a are more numerous than those of b.

Axiom 15 (Cardinality Precedence) *A semantics* $\mathbf{F}$ *satisfies* cardinality precedence *iff, for any SAF* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$*, for all* $a, b \in \mathcal{A}$*, if*

- $w(a) = w(b)$*,* $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(b) < 1$*,*
- $0 < |\{x \in \mathtt{Supp}_{\mathbf{A}}(b) \mid \mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(x) > 0\}| < |\{y \in \mathtt{Supp}_{\mathbf{A}}(a) \mid \mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(y) > 0\}|$*,*
- $\exists\ x \in \mathtt{Supp}_{\mathbf{A}}(b)$ *s.t* $\forall\ y \in \mathtt{Supp}_{\mathbf{A}}(a)$*,* $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(x) > \mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(y)$*,*

then $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(a) > \mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(b)$.

The next axiom, *quality precedence*, prefers the quality to the quantity of supporters. It says that an argument a is stronger than an argument b, if some supporter of a is stronger than any supporter of b.

Axiom 16 (Quality Precedence) *A semantics* $\mathbf{F}$ *satisfies* quality precedence *iff, for any SAF* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$*, for all* $a, b \in \mathcal{A}$*, if*

- $w(a) = w(b)$, $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(a) < 1$,
- $0 < |\{x \in \mathtt{Supp}_{\mathbf{A}}(b) \mid \mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(x) > 0\}| < |\{y \in \mathtt{Supp}_{\mathbf{A}}(a) \mid \mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(y) > 0\}|$,
- $\exists x \in \mathtt{Supp}_{\mathbf{A}}(b)$ *s.t* $\forall y \in \mathtt{Supp}_{\mathbf{A}}(a)$, $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(x) > \mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(y)$,

then $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(a) < \mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(b)$.

The very last axiom, called *compensation*, states that a small number of strong supporters compensate a greater number of weak supporters.

Axiom 17 (Compensation) *A semantics* $\mathbf{F}$ *satisfies* compensation *iff, for any SAF* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$*, for all* $a, b \in \mathcal{A}$*, if*

- $w(a) = w(b)$,
- $|\mathtt{Supp}_{\mathbf{A}}(a)| = n$, $|\mathtt{Supp}_{\mathbf{A}}(b)| = m$ *with* $n < m$,
- $\forall x \in \mathtt{Supp}_{\mathbf{A}}(a)$, $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(x) = d$, $\forall y \in \mathtt{Supp}_{\mathbf{A}}(b)$, $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(x) = d'$ *with* $d > d' > 0$

then $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(a) = \mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(b)$.

4 Properties

Some axioms are incompatible, that is they cannot be satisfied all together by a semantics.

Proposition 1 *There is no semantics which satisfies Cardinality Precedence and Quality Precedence (respectively Compensation). There is no semantics which satisfies Quality Precedence and Compensation. There is no semantics which satisfies both Quality Precedence and Counting. There is no semantics which satisfies at the same time: Independence, Non-Dilution, Equivalence, Reinforcement, Boundedness and Quality Precedence.*

The other axioms are compatible as shown in Table 1 summarizing the axioms satisfied by the three semantics we will introduce in the next section. Furthermore, we can show that arguments that are supported by only rejected arguments, have an overall strength which is equal to their basic strength.

Proposition 2 *If a semantics* $\mathbf{F}$ *satisfies Independence, Non-Dilution, Dummy and Minimality, then for any SAF* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$*, for any* $a \in \mathcal{A}$*, if* $\forall x \in \mathtt{Supp}_{\mathbf{A}}(a)$, $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(x) = 0$*, then* $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(a) = w(a)$.

From the mandatory axioms, it follows that the strength degree of each argument is at least its basic strength, and at most 1. This means that arguments keep their basic strength if they are not supported or their supporters are all rejected. They are strengthened otherwise.

Theorem 1 *If a semantics satisfies Independence, Monotony, Minimality and Strengthening, then for any SAF* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$*, for any* $a \in \mathcal{A}$*,* $\mathtt{Deg}^{\mathbf{F}}_{\mathbf{A}}(a) \in [w(a), 1]$.

5 Semantics

In this section, we propose three semantics for assessing the overall strength of arguments in support argumentation graphs. In addition to the basic axioms, the first semantics satisfies Quality Precedence, the second satisfies Cardinality Precedence, and the last semantics satisfies Compensation.

5.1 Top-based Semantics

The first semantics satisfies quality precedence, thus it favors the quality of supporters over their cardinality. It is based on a scoring function which follows a multiple steps process. At each step, the function assigns a score to each argument. In the initial step, the score of an argument is its basic strength. Then, in each step, the score is recomputed on the basis of the basic strength as well as the score of the strongest supporter of the argument at the previous step.

Definition 5 (Top-based Function) *Let* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$ *be a support argumentation graph. We define the* top-based function $\mathtt{f_t}$ *from* $\mathcal{A}$ *to* $[0, +\infty)$ *as follows: for any argument* $a \in \mathcal{A}$*, for* $i \in \{0, 1, 2, \dots\}$*, if* $i = 0$ *then* $\mathtt{f^i_t}(a) = w(a)$*, otherwise* $\mathtt{f^i_t}(a) = w(a) + (1 - w(a)) \max_{b \in \mathtt{Supp}_{\mathbf{A}}(a)} \mathtt{f^{i-1}_t}(b)$*. By convention,* $\max_{b \in \mathtt{Supp}_{\mathbf{A}}(a)} \mathtt{f^i_t}(b) = 0$ *if* $\mathtt{Supp}_{\mathbf{A}}(a) = \emptyset$.

The value $\mathtt{f^i_t}(a)$ is the score of the argument a at step i. This value may change at each step, however, it converges to a unique value as i becomes high.

Theorem 2 *The function* $\mathtt{f^i_t}$ *converges.*

The top-based semantics is based on the previous scoring function. The strength degree of each argument is the limit reached using the scoring function $\mathtt{f_t}$.

Definition 6 (Top-based Semantics) *The* top-based semantics *is a function* $\mathtt{Tbs}$ *transforming any support argumentation graph* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$ *into a weighting on* $\mathcal{A}$ *such that for any* $a \in \mathcal{A}$*,* $\mathtt{Deg}^{\mathtt{Tbs}}_{\mathbf{A}}(a) = \lim_{i \to \infty} \mathtt{f^i_t}(a)$.

We show next that the limit scores of arguments satisfy a nice property, namely the equation of Definition 5.

Theorem 3 *For any support argumentation graph* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$*, for any* $a \in \mathcal{A}$*,*

$$\mathtt{Deg}^{\mathtt{Tbs}}_{\mathbf{A}}(a) = w(a) + (1 - w(a)) \max_{b \in \mathtt{Supp}(a)} \mathtt{Deg}^{\mathtt{Tbs}}_{\mathbf{A}}(b).$$

Let us now illustrate the semantics with an example.

Example 1 (Cont) The strength degrees of the arguments of graph $\mathbf{A}$ under semantics $\mathtt{Tbs}$ are: $\mathtt{Deg}^{\mathtt{Tbs}}_{\mathbf{A}}(a) = \frac{1}{3}$, $\mathtt{Deg}^{\mathtt{Tbs}}_{\mathbf{A}}(b) = \frac{5}{9}$, $\mathtt{Deg}^{\mathtt{Tbs}}_{\mathbf{A}}(c) = \frac{8}{9}$, $\mathtt{Deg}^{\mathtt{Tbs}}_{\mathbf{A}}(d) = \frac{19}{27}$, $\mathtt{Deg}^{\mathtt{Tbs}}_{\mathbf{A}}(d') = \frac{77}{81}$, $\mathtt{Deg}^{\mathtt{Tbs}}_{\mathbf{A}}(e) = \frac{25}{27}$, $\mathtt{Deg}^{\mathtt{Tbs}}_{\mathbf{A}}(e') = \frac{77}{81}$, and $\mathtt{Deg}^{\mathtt{Tbs}}_{\mathbf{A}}(x) = 0.2$. Notice that $\mathtt{Supp}_{\mathbf{A}}(e) = \{c\}$ and $\mathtt{Supp}_{\mathbf{A}}(d) = \{a, b\}$. The argument e has thus less supporters but its supporter c is stronger than both supporters of d. Since $\mathtt{Tbs}$ satisfies quality precedence, then e is stronger than d. Furthermore, $d' \approx e' \succ e \succ c \succ d \succ b \succ a \succ x$, where $e \succ c$ means e is stronger than c and $d' \approx e'$ means d' is as strong as e'.

The Top-based semantics satisfies quality precedence as well as all the mandatory axioms which are compatible with it.

Theorem 4 *Table 1 summarizes the axioms satisfied (violated) by top-based semantics.*

From Theorems 1 and 4, it follows that the strength degree of each argument a is in the interval $[w(a), 1]$.

Corollary 1 *For any support argumentation graph* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$, *for any* $a \in \mathcal{A}$, $\mathtt{Deg}^{\mathtt{Tbs}}_{\mathbf{A}}(a) \in [w(a), 1]$.

5.2 Reward-based Semantics

The second semantics, called *reward-based semantics*, favours the number of supporters over their quality. Its basic idea is the following: an argument receives a reward for each of its supporters. The greater the number of supporters, the smaller the amount of the reward. The reward concerning the last supporter takes into account the quality of the supporters. Note that no particular order of arguments is needed. Furthermore, since arguments having degree 0 have no impact on the arguments they support (Dummy axiom), such supporters are not taken into account. We thus consider only *founded* supporters. An argument is founded if there exists at least one path leading to it such that the basic strength of the source of the path is not 0. As we will see later, a founded argument has necessarily a positive degree.

Definition 7 (Founded Argument) *Let* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$ *be a support argumentation graph and* $a \in \mathcal{A}$. *The argument* a *is* founded *iff there exists a finite sequence* $\langle a_0, a_1, \ldots, a_n \rangle$ *of arguments such that* $w(a_0) > 0$, $a_n = a$ *and for all* $i = 0, 1, \ldots, n-1$, $a_i \mathcal{S} a_{i+1}$. *It is* unfounded *otherwise. Let* $\mathtt{SuppF}_{\mathbf{A}}(a)$ *denote the set of founded supporters of* a.

It is easy to show that the basic strength of an unfounded argument is 0. Furthermore, if at least one supporter of an argument is founded, then the argument is itself founded.

Proposition 3 *Let* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$ *be a support argumentation graph and* $a \in \mathcal{A}$.

- *If* $\mathtt{SuppF}_{\mathbf{A}}(a) \neq \emptyset$, *then* a *is founded.*
- a *is unfounded iff* $\mathtt{SuppF}_{\mathbf{A}}(a) = \emptyset$ *and* $w(a) = 0$.

The reward-based semantics is based on a scoring function which assigns a numerical value to each argument. If an argument is not founded, then it receives a score 0. Otherwise, the function proceeds in multiple steps. In the initial step, it assigns to each argument its basic strength. Then in each step, it recomputes all the scores by taking into account the basic strength, the number of founded supporters and their scores in the previous step.

Definition 8 (Reward-based Function) *Let* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$ *be a support argumentation graph. We define the* reward-based function $\mathtt{f_r}$ *from* $\mathcal{A}$ *to* $[0, +\infty)$ *as follows: for any argument* $a \in \mathcal{A}$, *if* a *is unfounded, then* $\mathtt{f^i_r}(a) = 0$ *for any* $i \in \{0, 1, 2, \ldots\}$. *If* a *is founded, then for* $i \in \{0, 1, 2, \ldots\}$, *if* $i = 0$ *then* $\mathtt{f^i_r}(a) = w(a)$, *otherwise*

$$\mathtt{f^i_r}(a) = w(a) + (1 - w(a))\left(\sum_{j=1}^{n-1} \frac{1}{2^j} + \frac{m}{2^n}\right),$$

where $n = |\mathtt{SuppF}_{\mathbf{A}}(a)|$ *and* $m = \frac{\sum_{b \in \mathtt{SuppF}_{\mathbf{A}}(a)} \mathtt{f^{i-1}_r}(b)}{n}$. *By convention,* $\sum_{j=1}^{n-1} \frac{1}{2^j} + \frac{m}{2^n} = 0$ *if* $\mathtt{SuppF}_{\mathbf{A}}(a) = \emptyset$.

The value $\mathtt{f^i_r}(a)$ is the score of the argument a at step i. This value converges to a unique value as i becomes high.

Theorem 5 *The function* $\mathtt{f^i_r}$ *converges.*

The reward-based semantics assigns for each argument a score which is equal to the limit reached by the reward-based function f_r.

Definition 9 (Reward-based Semantics) *The* reward-based semantics *is a function* $\mathtt{Rbs}$ *transforming any support argumentation graph* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$ *into a weighting on* $\mathcal{A}$ *such that for any* $a \in \mathcal{A}$, $\mathtt{Deg}^{\mathtt{Rbs}}_{\mathbf{A}}(a) = \lim_{i \to \infty} \mathtt{f^i_r}(a)$.

We show next that the limit scores of arguments satisfy the equation of Definition 8.

Theorem 6 *For any support argumentation graph* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$, *for any* $a \in \mathcal{A}$, $\mathtt{Deg}^{\mathtt{Rbs}}_{\mathbf{A}}(a) = 0$ *if* a *is unfounded. Otherwise,*

$$\mathtt{Deg}^{\mathtt{Rbs}}_{\mathbf{A}}(a) = w(a) + (1 - w(a))\left(\sum_{j=1}^{n-1} \frac{1}{2^j} + \frac{m}{2^n}\right)$$

where $n = |\mathtt{SuppF}_{\mathbf{A}}(a)|$ *and*

$$m = \frac{\sum_{b \in \mathtt{SuppF}_{\mathbf{A}}(a)} \mathtt{Deg}^{\mathtt{Rbs}}_{\mathbf{A}}(b)}{n}.$$

Let us illustrate the semantics with an example.

Example 1 (Cont) It is worth noticing that all the eight arguments are founded. Their strength degrees under semantics $\mathtt{Rbs}$ are: $\mathtt{Deg}^{\mathtt{Rbs}}_{\mathbf{A}}(a) = \frac{1}{3}$, $\mathtt{Deg}^{\mathtt{Rbs}}_{\mathbf{A}}(b) = \frac{4}{9}$, $\mathtt{Deg}^{\mathtt{Rbs}}_{\mathbf{A}}(c) = \frac{29}{36}$, $\mathtt{Deg}^{\mathtt{Rbs}}_{\mathbf{A}}(d) = \frac{79}{108}$, $\mathtt{Deg}^{\mathtt{Rbs}}_{\mathbf{A}}(d') = \frac{31}{36}$, $\mathtt{Deg}^{\mathtt{Rbs}}_{\mathbf{A}}(e) = \frac{13}{27}$, $\mathtt{Deg}^{\mathtt{Rbs}}_{\mathbf{A}}(e') = \frac{109}{144}$, and $\mathtt{Deg}^{\mathtt{Rbs}}_{\mathbf{A}}(x) = 0.2$. According to $\mathtt{Rbs}$, $d' \succ c \succ e' \succ d \succ e \succ b \succ a \succ x$.

Theorem 7 *Table 1 summarizes the axioms satisfied (violated) by reward-based semantics.*

From Theorems 1 and 4, it follows that the strength degree of each argument a is in the interval $[w(a), 1]$.

Corollary 2 *For any support argumentation graph* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$, *for any* $a \in \mathcal{A}$, $\mathtt{Deg}^{\mathtt{Rbs}}_{\mathbf{A}}(a) \in [w(a), 1]$.

The following proposition shows additional basic properties of reward-based semantics.

Proposition 4 *Let* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S} \rangle$ *be a support argumentation graph and* $a \in \mathcal{A}$.

- *If* a *is founded, then* $\mathtt{Deg}^{\mathtt{Rbs}}_{\mathbf{A}}(a) > 0$.
- *If* $\mathtt{SuppF}_{\mathbf{A}}(a) = \emptyset$, *then* $\mathtt{Deg}^{\mathtt{Rbs}}_{\mathbf{A}}(a) = w(a)$.

5.3 Aggregation-based Semantic

Our last semantics satisfies compensation axiom, i.e., allows a small number of strong supporters to compensate a large number of weaker supporters. This semantics makes use of a scoring function which assigns a numerical value to each argument. Like the two previous functions, the new function proceeds in several steps. It starts by assigning to each argument its basic strength. Then at each step, it recomputes the score of the argument by adding to the basic strength a certain percentage of the scores of its supporters at the previous step.

Axioms - Semantics	Tbs	Rbs	Gbs
Anonymity	•	•	•
Independence	•	•	•
Non-Dilution	•	•	•
Monotony	•	•	•
Equivalence	•	•	•
Dummy	•	•	•
Minimality	•	•	•
Strengthening	•	•	•
Strengthening Soundness	•	•	•
Coherence	•	•	•
Counting	◦	•	•
Boundedness	•	•	•
Reinforcement	◦	•	•
Imperfection	◦	•	•
Cardinality Precedence	◦	•	◦
Quality Precedence	•	◦	◦
Compensation	◦	◦	•

Table 1: *Axioms satisfaction by the three semantics.* The symbol • (resp. ◦) means the axiom is satisfied (resp. violated).

Definition 10 *Let $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S}\rangle$ be a support argumentation graph. We define the* aggregation function $\mathtt{f_g}$ *from $\mathcal{A}$ to $[0, +\infty)$ as follows: for any argument $a \in \mathcal{A}$, for $i \in \{0, 1, 2, \ldots\}$, if $i = 0$, then $\mathtt{f}^{\mathtt{i}}_{\mathtt{g}}(a) = w(a)$ otherwise $\mathtt{f}^{\mathtt{i}}_{\mathtt{g}}(a) = w(a) + (1 - w(a))\frac{\sum_{b \in \mathtt{Supp}_{\mathbf{A}}(a)} \mathtt{f}^{\mathtt{i-1}}_{\mathtt{g}}(b)}{1+\sum_{b \in \mathtt{Supp}_{\mathbf{A}}(a)} \mathtt{f}^{\mathtt{i-1}}_{\mathtt{g}}(b)}$. By convention, $\sum_{b\in \mathtt{Supp}_{\mathbf{A}}(a)} \mathtt{f}^{\mathtt{i-1}}_{\mathtt{g}}(b) = 0$ if $\mathtt{Supp}_{\mathbf{A}}(a) = \emptyset$.*

The value $\mathtt{f}^{\mathtt{i}}_{\mathtt{g}}(a)$ is the score of the argument a at step i. This value may change at each step, however, it converges to a unique value as i becomes high.

Theorem 8 *The function $\mathtt{f}^{\mathtt{i}}_{\mathtt{g}}$ converges.*

The aggregation-based semantics is based on the previous scoring function. The strength degree of each argument is the limit reached using the scoring function.

Definition 11 (Aggregation-based Semantics) *The* aggregation-based semantics *is a function* Gbs *transforming any support argumentation graph $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S}\rangle$ into a weighting on $\mathcal{A}$ such that for any $a \in \mathcal{A}$, $\mathtt{Deg}^{\mathtt{Gbs}}_{\mathbf{A}}(a) = \lim_{i\to\infty} \mathtt{f}^{\mathtt{i}}_{\mathtt{g}}(a)$.*

We show next that the limit scores of arguments satisfy a nice property, namely the equation of Definition 10.

Theorem 9 *For any support argumentation graph $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S}\rangle$, for any $a \in \mathcal{A}$,*

$$\mathtt{Deg}^{\mathtt{Gbs}}_{\mathbf{A}}(a) = w(a) + (1 - w(a))\frac{\sum_{b \in \mathtt{Supp}_{\mathbf{A}}(a)} \mathtt{Deg}^{\mathtt{Gbs}}_{\mathbf{A}}(b)}{1 + \sum_{b \in \mathtt{Supp}_{\mathbf{A}}(a)} \mathtt{Deg}^{\mathtt{Gbs}}_{\mathbf{A}}(b)}.$$

Consider again our running example.

Example 1 (Cont) The strength degrees of the arguments of graph $\mathbf{A}$ under semantics Gbs are: $\mathtt{Deg}^{\mathtt{Gbs}}_{\mathbf{A}}(a) = \frac{1}{3}$, $\mathtt{Deg}^{\mathtt{Gbs}}_{\mathbf{A}}(b) = \frac{1}{2}$, $\mathtt{Deg}^{\mathtt{Gbs}}_{\mathbf{A}}(c) = \frac{5}{6}$, $\mathtt{Deg}^{\mathtt{Gbs}}_{\mathbf{A}}(d) = \frac{21}{33}$, $\mathtt{Deg}^{\mathtt{Gbs}}_{\mathbf{A}}(d') = 0.74$, $\mathtt{Deg}^{\mathtt{Gbs}}_{\mathbf{A}}(e) = \frac{21}{33}$, $\mathtt{Deg}^{\mathtt{Gbs}}_{\mathbf{A}}(e') = 0.72$, and $\mathtt{Deg}^{\mathtt{Gbs}}_{\mathbf{A}}(x) = 0.2$. Recall that $\mathtt{Supp}_{\mathbf{A}}(e) = \{c\}$ and $\mathtt{Supp}_{\mathbf{A}}(d) = \{a, b\}$. The argument e has less supporters but its supporter c is stronger than both supporters of d. Since Gbs satisfies compensation, then d is as strong as e. Tbs promotes the quality, thus it declares e stronger than d. Finally, Rbs promotes the cardinality, thus declares d stronger than e.

Aggregation-based semantics satisfies compensation as well as all the mandatory axioms.

Theorem 10 *Table 1 summarizes the axioms satisfied (violated) by aggregation-based semantics.*

From Theorems 1 and 4, it follows that the strength degree of each argument a is in interval $[w(a), 1]$.

Corollary 3 *For any support argumentation graph $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{S}\rangle$, for any $a \in \mathcal{A}$, $\mathtt{Deg}^{\mathtt{Gbs}}_{\mathbf{A}}(a) \in [w(a), 1]$.*

It is worth noticing from Table 1, that Tbs is the only semantics that violates Imperfection. Indeed, according to this semantics, an argument whose intrinsic strength is 0 may become very strong (i.e., gets degree 1) due to its supporters. This scenario is not possible with the two other semantics.

6 Conclusion

This paper presented the first study on support argumentation graphs. It tackled the problem of assessing the overall strengths of arguments in such graphs. It proposed a set of axioms guiding the well definition of semantics. Then, it proposed three semantics that satisfy most of the axioms.

Our future work consists of extending these semantics for evaluating arguments in bipolar graphs (graphs containing attacks and supports). Recall that the existing semantics in the argumentation literature do not treat properly supports. For instance, if the attack relation is empty, then they declare all the arguments as acceptable, neglecting thus the supports.

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