Towards a unified model of preference-based argumentation

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Abstract. Argumentation is a reasoning model based on the construction and the evaluation of arguments. In his seminal paper, Dung has proposed the most abstract argumentation framework. In that framework, arguments are assumed to have the same strength. This assumption is unfortunately strong and often unsatisfied. Consequently, three extensions of the framework have been proposed in the literature. The first one assumes that an argumentation framework should be equipped with a (partial or total) preorder representing a preference relation between arguments, and capturing a difference of strengths of the arguments. The source of this preference relation is not specified, thus it can be instantiated in different manners. The second extension claims that the strength of an argument depends on the value(s) promoted by this argument. The third extension states that the set of arguments is equipped with several preorders; each of them expresses preferences between arguments in a given context.

The contribution of this paper is two-fold: first, it proposes a comparative study of these extensions of Dung's framework. It clearly shows under which conditions two proposals are equivalent. The second contribution of the paper consists in integrating the three extensions into a common more expressive framework.

1 Introduction

Argumentation is a reasoning model based on the construction and the evaluation of interacting arguments. It has been applied to nonmonotonic reasoning (e.g. [7]), decision making (e.g. [3, 5, 8]), and for modeling different types of dialogues including negotiation (e.g. [10, 12]). Most of the models developed for the above applications are grounded on the abstract argumentation framework proposed by Dung in [7]. That framework consists of a set of arguments and a binary relation on that set, expressing conflicts among arguments. An argument gives a reason for believing a claim, for doing an action. It is worth mentioning that in this framework arguments are assumed to have the same strength. This assumption is quite strong since it is natural to consider an argument built from certain information stronger than another grounded on defeasible information. Consequently, three different extensions of the framework have been proposed in the literature. The first one, proposed in [1], assumes that in addition to the conflict relation among arguments, another binary relation (called preference relation) on the set of arguments is available. This relation captures the differences in strengths of the arguments. The source of this relation is not specified, thus it can be instantiated in different manners. The second extension is proposed in [4] and extended in [9]. It claims that the strength of an argument depends on the value promoted by this argument. Each argument is assumed to promote a value. The values may not have the same importance. Thus, the argument promoting the most important value is considered as stronger than the others. The third extension, proposed in [2], states that the set of arguments is equipped with several preorders; each of them expresses preferences between arguments in a given context. It may be the case that for two arguments α and β, α is preferred to β in a given context and β is preferred to α in another context. This extension aims at generalizing the preference-based model defined in [1]. It is important to compare the three extensions and to highlight the similarities and the differences between them.

The contribution of this paper is two-fold: first, it proposes a comparative study of these extensions of Dung's framework. It clearly shows under which conditions two proposals are equivalent. The second contribution of the paper consists in integrating the three extensions into a common more expressive framework, whose properties are investigated.

The rest of the paper is organized as follows: Section 2 briefly recalls Dung's framework as well as its three extensions. Section 3 presents a comparative study of the three extensions. Section 4 proposes a unifying framework that captures the features of the three extensions.

2 Recalling Abstract Argumentation Frameworks

This section briefly recalls Dung's abstract argumentation framework as well as its three extensions. The three frameworks are illustrated by a running example that shows the power and the weaknesses of each of them.

2.1 Dung's abstract framework

An argumentation process follows three main steps: 1) constructing *arguments* and counter-arguments, 2) evaluating the *acceptability* of the different arguments, and 3) concluding or defining the *justified conclusions*. In [7], an argumentation framework is defined as follows:

Definition 1 (Dung's argumentation framework). *An* argumentation framework *is a pair* $\mathtt{AF} = \langle \mathcal{A}, \mathcal{R} \rangle$ *where* $\mathcal{A}$ *is a set of arguments and* $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ *is an* attack *relation. An argument* α *attacks an argument* β *iff* $(\alpha, \beta) \in \mathcal{R}$.

In the above definition, arguments are abstract entities. Their origin and structure are left unknown. Note that we can associate each argumentation system with a directed graph whose nodes are the different arguments, and the edges represent the attack relation between them.
Among all the conflicting arguments, one has to define which arguments to keep for inferring conclusions or for making decisions. In [7], different semantics for the notion of acceptability have been proposed. For the purpose of this paper, we only recall admissible semantics.

Definition 2 (Conflict-free, Defense, Admissible semantics). *Let* $\mathcal{B} \subseteq \mathcal{A}$.

- $\mathcal{B}$ *is* conflict-free *iff* $\nexists\ \alpha_i,\ \alpha_j \in \mathcal{B}$ *such that* $(\alpha_i, \alpha_j) \in \mathcal{R}$.
- $\mathcal{B}$ defends *an argument* $\alpha_i \in \mathcal{B}$ *iff for each argument* $\alpha_j \in \mathcal{A}$, *if* $(\alpha_j, \alpha_i) \in \mathcal{R}$, *then* $\exists\ \alpha_k \in \mathcal{B}$ *such that* $(\alpha_k, \alpha_j) \in \mathcal{R}$.
- *A conflict-free set* $\mathcal{B}$ *of arguments is an* admissible extension *iff* $\mathcal{B}$ *defends all its elements.*

Let us illustrate the abstract framework through a simple example, describing a multi-criteria decision making situation. A French national nutritional health programme (PNNS), launched in 2001, aims at improving the state of health of the whole population by acting on several major determinants of citizens life, especially bread consumption. A primary objective of this programme is to increase the fraction of complex carbohydrates in the diet, and to reduce the fraction of simple carbohydrates. The part of simple carbohydrates on total carbohydrates is denoted SCP (Simple Carbohydrates Proportion). An action proposed by the decision makers is then to change the type of flour, labeled according to its ash value (mineral content), used in bread. The following table summarizes the performances obtained for two actions (bread type $\mathbb{T}_{65}$ and $\mathbb{T}_{80}$) and for several criteria (ash value, fibers and SCP) [6]. The objective is to choose between two breads, bread obtained with $\mathbb{T}_{65}$ and bread obtained with $\mathbb{T}_{80}$, on the basis of their performance in these criteria.

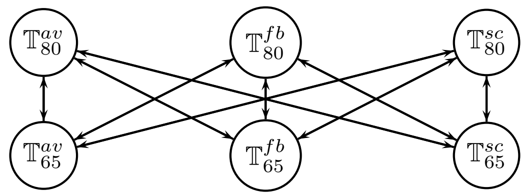
Example 1. Table below summarizes the performances in the different criteria.

	Ash value (av) (%)	Fibers (fb) (g/100g)	SCP (sc) (%)
Bread $\mathbb{T}_{80}$	0.80	4.2	3.85
Bread $\mathbb{T}_{65}$	0.65	3.8	4.11

In this application, an argument gives an information and consequently a reason for choosing a given bread. Note that all the performances are supposed to be in favor of a choice. The following six arguments are thus built:

- $\mathbb{T}_{80}^{av}$ ➟ Bread $\mathbb{T}_{80}$ should be chosen since its ash value is 0.80 %,
- $\mathbb{T}_{80}^{fb}$ ➟ Bread $\mathbb{T}_{80}$ should be chosen since its fibers content is 4.2 g/100g,
- $\mathbb{T}_{80}^{sc}$ ➟ Bread $\mathbb{T}_{80}$ should be chosen since its SCP content is 3.85 %,
- $\mathbb{T}_{65}^{av}$ ➟ Bread $\mathbb{T}_{65}$ should be chosen since its ash value is 0.65 %,
- $\mathbb{T}_{65}^{fb}$ ➟ Bread $\mathbb{T}_{65}$ should be chosen since its fibers content is 3.8 g/100g,
- $\mathbb{T}_{65}^{sc}$ ➟ Bread $\mathbb{T}_{65}$ should be chosen since its SCP content is 4.11 %.

Since only one bread type will be chosen, any pair of arguments that do not support the same option is considered as conflicting (i.e. $\in \mathcal{R}_{ex}$[4]). The following figure summarizes the different conflicts between arguments.
The system has two maximal (for set inclusion) admissible extensions $\{\mathbb{T}_{80}^{av}, \mathbb{T}_{80}^{fb}, \mathbb{T}_{80}^{sc}\}$ and $\{\mathbb{T}_{65}^{av}, \mathbb{T}_{65}^{fb}, \mathbb{T}_{65}^{sc}\}$ each of them supports a bread. Thus, the two breads $\mathbb{T}_{65}$ and $\mathbb{T}_{80}$ are equally preferred in Dung's system.



The above example shows that this framework is not powerful enough for making decisions. Indeed, for some criteria, it is possible to conclude that a bread is better than the other one (e.g. for SCP, bread $\mathbb{T}_{80}$ shows a better performance than bread $\mathbb{T}_{65}$). However, since the framework does not take into account the strengths of arguments, it has only solved the conflicts between arguments and concluded that both options are acceptable.

For comparison purposes, we will define a notion of equivalent frameworks. Two argumentation frameworks are said equivalent if they return exactly the same extensions under a given semantics.

Definition 3 (Equivalent frameworks). *Let* $\texttt{AF}_1$*,* $\texttt{AF}_2$ *be two argumentation frameworks.* $\texttt{AF}_1$ *and* $\texttt{AF}_2$ *are equivalent iff* $Ext(\texttt{AF}_1) = Ext(\texttt{AF}_2)$*, where* $Ext(\texttt{AF}_i)$ *is the set of all extensions of* $\texttt{AF}_i$ *under a given semantics.*[5]

2.2 Preference based Argumentation Framework

In [1], it has been argued that arguments may have different strengths. In the previous example, it is clear that the argument $\mathbb{T}_{80}^{sc}$ is stronger than $\mathbb{T}_{65}^{sc}$. This information should be exploited in the argumentation framework. It allows to reduce the number of attacks among arguments. The idea is that an attack may fail if the attacked argument is stronger than its attacker.

Definition 4 (Preference-based argumentation framework (PAF)). *A* preference-based argumentation framework *is a tuple* $\texttt{PAF} = \langle \mathcal{A}, \mathcal{R}, \succeq \rangle$ *where* $\mathcal{A}$ *is a set of arguments,* $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ *is an attack relation, and* $\succeq \subseteq \mathcal{A} \times \mathcal{A}$ *is a (partial or total) preorder*[6]*. For* $\alpha, \beta \in \mathcal{A}$*,* $(\alpha, \beta) \in \succeq$ *(or* $\alpha \succeq \beta$*) means that* α *is at least as strong as* β*.*

[4] $\mathcal{R}_{ex}$ denotes the attack relation used in the example.
[5] In proofs, admissible semantics is used to establish equivalence between framework.
[6] A preorder is a binary relation that is *reflexive* and *transitive*.

The relation $\succeq$ is general and may be instantiated in different manners. In order to evaluate the acceptability of arguments in a preference-based argumentation framework (PAF), a Dung style framework is associated to this PAF. Dung's semantics are then applied to the new framework.

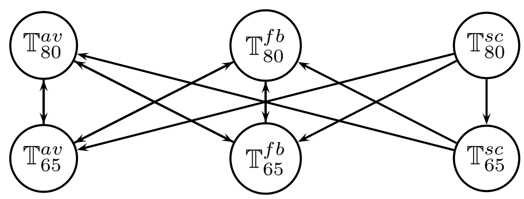
Definition 5. *Let* $\mathtt{PAF} = \langle \mathcal{A}, \mathcal{R}, \succeq \rangle$ *be a preference-based argumentation framework. The* AF *associated with* PAF *is the pair* $\langle \mathcal{A}, \mathtt{Def} \rangle$ *where* $\mathtt{Def} \subseteq \mathcal{A} \times \mathcal{A}$ *such that* $(\alpha, \beta) \in \mathtt{Def}$ *iff* $(\alpha, \beta) \in \mathcal{R}$ *and* $(\beta, \alpha) \notin \succ$[7].

Dung's semantics are applied to the framework $\langle \mathcal{A}, \mathtt{Def} \rangle$ in order to evaluate arguments of $\mathtt{PAF} = \langle \mathcal{A}, \mathcal{R}, \succeq \rangle$.

Property 1. The argumentation frameworks associated respectively with $\mathtt{PAF}_1 = \langle \mathcal{A}, \mathcal{R}, \succ \rangle$ and $\mathtt{PAF}_2 = \langle \mathcal{A}, \mathcal{R}, \succeq \rangle$ (with $\succ$ the strict relation of $\succeq$) are equivalent.

Let us now re-consider Example 1 and see how preferences between arguments will help reduce the number of attacks and possibly return the expected result.

Example 2 (Example 1 cont.). As mentioned above, the following preferences hold between arguments: $\mathbb{T}^{sc}_{80} \succ \mathbb{T}^{sc}_{65}$ ($\mathbb{T}^{av}_{80}$, $\mathbb{T}^{av}_{65}$, $\mathbb{T}^{fb}_{80}$, $\mathbb{T}^{fb}_{65}$ are indifferent).
Another source of preferences between arguments is the importance of the criteria. Let us, for instance, assume that SCP is more important than fibers and ash value content, and that fibers and ash value are equally important. Thus any argument referring to SCP is stronger than any argument referring to ash value or fibers. The graph of $\langle \mathcal{A}, \mathtt{Def} \rangle$ is summarized below.



The framework has only one maximal (for set inclusion) admissible extension which is $\{\mathbb{T}^{av}_{80}, \mathbb{T}^{fb}_{80}, \mathbb{T}^{sc}_{80}\}$. The bread obtained with $\mathbb{T}_{80}$ is preferred to bread obtained with $\mathbb{T}_{65}$ in this PAF. Note that if we change the importance of the criteria and assume that SCP and fibers are equally important and both are more important than ash value, then two extensions $\{\mathbb{T}^{av}_{80}, \mathbb{T}^{fb}_{80}, \mathbb{T}^{sc}_{80}\}$ and $\{\mathbb{T}^{av}_{65}, \mathbb{T}^{fb}_{65}, \mathbb{T}^{sc}_{65}\}$ are obtained.

2.3 Value-based Argumentation Framework

In [4], Bench Capon tried to formalize ideas of Perelman [11]. The latter emphasizes the importance of promoting values through arguments. In other terms, an argument may promote a value like, for instance, health, economy, etc. A value-based argumentation framework (VAF) is defined as follows:

[7] We recall that $(\alpha, \beta) \in \succ$ iff $(\alpha, \beta) \in \succeq$ and $(\beta, \alpha) \notin \succeq$.

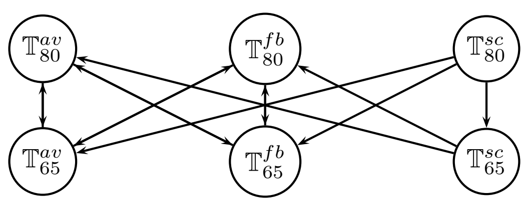
Definition 6 (Value-based argumentation framework). *A* value-based argumentation framework *is a tuple* $\texttt{VAF} = \langle \mathcal{A}, \mathcal{R}, \mathcal{V}, \texttt{val}, \texttt{Pref} \rangle$ *where* $\mathcal{A}$ *is a set of arguments,* $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ *is an attack relation,* $\mathcal{V}$ *is a set of values,* $\texttt{val}\colon \mathcal{A} \mapsto \mathcal{V}$*, and* $\texttt{Pref} \subseteq \mathcal{V} \times \mathcal{V}$ *is an irreflexive, asymmetric and transitive strict relation.*

Like in [1], an argumentation framework à la Dung is associated to each VAF as follows:

Definition 7. *Let* $\texttt{VAF} = \langle \mathcal{A}, \mathcal{R}, \mathcal{V}, \texttt{val}, \texttt{Pref} \rangle$ *be a VAF. The AF associated with* $\texttt{VAF}$ *is* $\langle \mathcal{A}, \texttt{defeats} \rangle$ *where* $\texttt{defeats} \subseteq \mathcal{A} \times \mathcal{A}$ *such that* $(\alpha, \beta) \in \texttt{defeats}$ *iff* $(\alpha, \beta) \in \mathcal{R}$ *and* $(\texttt{val}(\beta), \texttt{val}(\alpha)) \notin \texttt{Pref}$*.*

As for PAFs, Dung's acceptability semantics are applied to the AF for evaluating the different arguments. Let us now illustrate this framework through the running example. For that purpose, one needs to define what the values will be as well as the preference relation between those values. There are several possibilities: the first one consists in considering the different criteria as values. The second solution considers each performance as a possible value. In what follows, we will mix the first and second solutions, and we will show that considering only criteria (or performances) as possible values is not powerful enough to get a meaningful result.

Example 3 (Example 1 cont.). Assume that $\mathcal{V} = \{v^{av}, v^{fb}, v^{sc}_{-}, v^{sc}_{+}\}$ such that $(v^{sc}_{-}, v^{sc}_{+}) \in \texttt{Pref}$ and the two values v^{sc}_{-} and v^{sc}_{+} are preferred to the others (i.e (v^{sc}_{+}, v^{fb}), (v^{sc}_{+}, v^{av}), (v^{sc}_{-}, v^{fb}), $(v^{sc}_{-}, v^{av}) \in \texttt{Pref}$. The function $\texttt{val}$ is defined as follows: $\texttt{val}(\mathbb{T}^{av}_{80}) = \texttt{val}(\mathbb{T}^{av}_{65}) = v^{av}$, $\texttt{val}(\mathbb{T}^{fb}_{80}) = \texttt{val}(\mathbb{T}^{fb}_{80}) = v^{fb}$, $\texttt{val}(\mathbb{T}^{sc}_{80}) = v^{sc}_{-}$ and $\texttt{val}(\mathbb{T}^{sc}_{65}) = v^{sc}_{+}$. The graph associated with the framework $\langle \mathcal{A}, \texttt{defeats} \rangle$ is depicted below:



The framework has only one maximal (for set inclusion) admissible extension which is $\{\mathbb{T}^{av}_{80}, \mathbb{T}^{fb}_{80}, \mathbb{T}^{sc}_{80}\}$. Note that this result is that obtained by the PAF. If we assign the same value v^{sc} for the two arguments $\mathbb{T}^{sc}_{80}$ and $\mathbb{T}^{sc}_{65}$ and assume that this value is preferred to the others, then two extensions $\{\mathbb{T}^{av}_{80}, \mathbb{T}^{fb}_{80}, \mathbb{T}^{sc}_{80}\}$ and $\{\mathbb{T}^{av}_{65}, \mathbb{T}^{fb}_{65}, \mathbb{T}^{sc}_{65}\}$ are obtained.

It seems necessary to distinguish between several values (expressing respectively the considered criteria and its performance) which are here combined for v^{sc}_{-} and v^{sc}_{+}. This implies that it is necessary to allow an argument to support several values, enforcing the expressivity of the framework.

2.4 New value-based argumentation framework

This kind of VAFs introduced in [9] accounts for an extension of the classical VAF introduced by Bench-Capon in [4]. An argument in this framework may promote several values. There are then many ways for comparing pairs of arguments.

Definition 8 (Extended valued-based framework). *An* extended value-based argumentation framework *(VSAF) is a tuple* $\langle \mathcal{A}, \mathcal{R}, \mathcal{V}, \arg, \gg \rangle$ *where* $\mathcal{A}$ *is a set of arguments,* $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ *is an attack relation,* $\mathcal{V}$ *is a set of values,* $\arg : \mathcal{V} \mapsto 2^{\mathcal{A}}$ *such that* $\arg(v)$ *is .the set of arguments promoting value* v*, and gg is a partial order on* $\mathcal{V}$*.*

Since an argument may promote several values, then there are several ways for comparing pairs of arguments. Examples of this relation, denoted by $Pref_{\bigtriangledown}$, are given below.

Definition 9 (Preference relations). *Let* $\langle \mathcal{A}, \mathcal{R}, \mathcal{V}, \arg, \gg \rangle$ *be a VSAF. Let* $\alpha, \beta \in \mathcal{A}$*.*

- $(\alpha, \beta) \in \mathtt{Pref}_M$ *iff* $|\arg^{-1}(\{\alpha\})| > |\arg^{-1}(\{\beta\})|$
- $(\alpha, \beta) \in \mathtt{Pref}_{Bc}$ *iff* $\exists v \in \mathcal{V}$ *such that* $\alpha \in \arg(v)$ *and* $\forall v' \in \mathcal{V}$ *with* $\beta \in \arg(v')$, $(v, v') \in \gg$.

The first relation prefers the argument that promotes most values while the second one privileges the argument that promotes the most important value.

In order to evaluate arguments in a VSAF, a Dung style framework is associated to this extended framework, and thus acceptability semantics are applied.

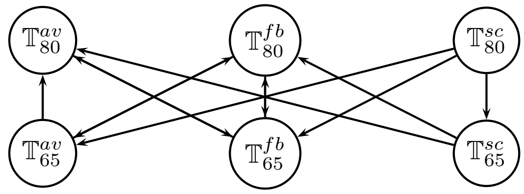
Definition 10. *Let* $\mathtt{VSAF} = \langle \mathcal{A}, \mathcal{R}, \mathcal{V}, \arg, \gg \rangle$ *be a VSAF and* $\mathtt{Pref}_{\bigtriangledown}$ *be particular preference relation between arguments. The argumentation framework associated with* $\mathtt{VSAF}$ *is* $\langle \mathcal{A}, \mathtt{defeats}_{\bigtriangledown} \rangle$ *where* $\mathtt{defeats}_{\bigtriangledown} \subseteq \mathcal{A} \times \mathcal{A}$ *and for* $\alpha, \beta \in \mathcal{A}$, $(\alpha, \beta) \in \mathtt{defeats}_{\bigtriangledown}$ *iff* $(\alpha, \beta) \in \mathcal{R}$ *and* $(\beta, \alpha) \notin Pref_{\bigtriangledown}$*.*

Let us illustrate this approach through the running example.

Example 4 (Example 1 cont.). Assume that $\mathcal{V} = \{v^{av}, v^{fb}, v^{sc}, v^{3.85\%}, v^{4.11\%}, v^{2.0\%}, v^{1.8\%}, v^{4.2g/100g}, v^{3.8g/100g}\}$. Assume also that $\gg$ is defined as follows: $v^{sc} \gg v^{av}, v^{fb} \gg v^{1.8\%} \gg v^{2.0\%} \gg v^{3.85\%} \gg v^{4.11\%}$. The function arg is defined as follows:

$\arg(v^{av}) = \{\mathbb{T}^{av}_{80}, \mathbb{T}^{av}_{65}\}$	$\arg(v^{fb}) = \{\mathbb{T}^{fb}_{80}, \mathbb{T}^{fb}_{65}\}$	$\arg(v^{sc}) = \{\mathbb{T}^{sc}_{80}, \mathbb{T}^{sc}_{65}\}$
$\arg(v^{2.0\%}) = \mathbb{T}^{av}_{80}$	$\arg(v^{1.8\%}) = \mathbb{T}^{av}_{65}$	$\arg(v^{4.2g/100g}) = \mathbb{T}^{fb}_{80}$
$\arg(v^{3.8g/100g}) = \mathbb{T}^{fb}_{65}$	$\arg(v^{3.85\%}) = \mathbb{T}^{sc}_{80}$	$\arg(v^{4.11\%}) = \mathbb{T}^{sc}_{65}$

The graph associated with the framework $\mathtt{VSAF} = \langle \mathcal{A}, \mathtt{defeats}_{Bc} \rangle$ is depicted below:



The framework has only one maximal (for set inclusion) admissible extension which is $\{\mathbb{T}^{av}_{80}, \mathbb{T}^{fb}_{80}, \mathbb{T}^{sc}_{80}\}$, the same result is obtained by using VAF system. Note that if we consider contexts, improving the expressivity of the models, preferences between values (especially $v^{1.8\%} \gg v^{2.0\%}$) can have a contextual validity (v^{sc}) which, for this model, would not necessarily return exactly the same graph depicted with VAF (see example 3).

2.5 Argumentation Framework based on Contextual Preferences

In works on PAFs and VAFs, preferences between arguments are assumed to be not conflicting. However, in real applications this is not always true, for instance when we consider multiple points of view. Let us consider the case of the running example. Assume that two points of view can express different preferences about fibers content: the baker's point of view can be to prefer bread with a lower fibers content (preventing consumer satiety) whereas the miller's point of view can be to prefer a higher fibers content in flour improving yield. Thus, in baker context $\mathbb{T}^{fb}_{65}$ is stronger than $\mathbb{T}^{sc}_{80}$, while in miller context $\mathbb{T}^{sc}_{80}$ is stronger than $\mathbb{T}^{fb}_{65}$. In [2], an extension of PAF has been proposed. The idea is to assume that the set $\mathcal{A}$ of arguments is equipped with several preference relations $\succeq_1, \ldots \succeq_n$, each of them expressing non-conflicting preferences between arguments in a particular *context*. Contexts (e.g. agents, points of view, criteria to be taken into account in a decision choice, etc.) are assumed to be ordered by a complete and strict relation denoted by $\rhd$. Note that for two arguments α and β, it may be the case that α is preferred to β in a given context and β is preferred to α in another one.

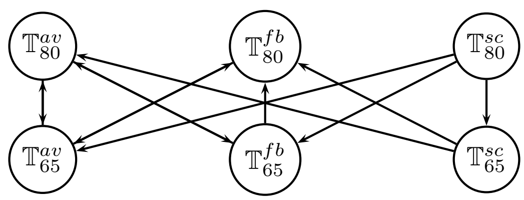
Definition 11 (CPAF). *A contextual preference-based argumentation framework (CPAF) is a tuple* CPAF $= \langle \mathcal{A}, \mathcal{R}, \mathcal{C}, \rhd, \succeq_1, \ldots \succeq_n \rangle$ *where $\mathcal{A}$ is a set of arguments, $\mathcal{R}$ is an attack relation, $\mathcal{C}$ is a finite set of contexts s.t. $|\mathcal{C}| = n$, $\rhd$ is a strict total order on the contexts, and $\succeq_i$ is a (partial or total) preorder associated with context c_i.*

In order to evaluate arguments in a CPAF, again an argumentation framework is associated to this CPAF. For that purpose, the different preference relations $\succeq_i$ are aggregated into a unique relation denoted by $\otimes^{\rhd}(\succeq_1, \ldots \succeq_n)$. An example of such aggregation consists of keeping all the preferences of the strongest context, then to add the preferences of the next important context that are not conflicting with those of the first one. The same process is repeated until there is no remaining context. Note that there are several ways for aggregating preferences. For the purpose of our paper, we keep this aggregation abstract and can thus be instantiated in different manners.

Definition 12. *Let* CPAF $= \langle \mathcal{A}, \mathcal{R}, \mathcal{C}, \rhd, \succeq_1, \ldots \succeq_n \rangle$ *and* $\otimes^{\rhd}(\succeq_1, \ldots \succeq_n)$ *be an aggregated preference relation between arguments. The argumentation framework associated with* CPAF *is* $\langle \mathcal{A}, \mathtt{Def} \rangle$ *where* $\forall \alpha, \beta \in \mathcal{A}$, $(\alpha, \beta) \in \mathtt{Def}$ *iff* $(\alpha, \beta) \in \mathcal{R}$ *and* $(\beta, \alpha) \notin \otimes^{\rhd}(\succeq_1, \ldots, \succeq_n)$.

Dung's acceptability semantics are then applied on the framework $\langle \mathcal{A}, \mathtt{Def} \rangle$ for evaluating the arguments of the set $\mathcal{A}$.

Example 5 (Example 1. cont.). Let $\mathcal{C} = \{\mathfrak{Pnns}, \mathfrak{Baker}, \mathfrak{Miller}\}$ with $\mathfrak{Pnns} \rhd \mathfrak{Baker} \rhd \mathfrak{Miller}$. The set of arguments is equipped with three preference relations, respectively denoted $\succeq_{\mathfrak{B}}$, $\succeq_{\mathfrak{M}}$ and $\succeq_{\mathfrak{P}}$ ($\mathfrak{B}$ stands for $\mathfrak{Baker}$, $\mathfrak{M}$ for $\mathfrak{Miller}$ and $\mathfrak{P}$ for $\mathfrak{Pnns}$). These relations are defined as follows: $\mathbb{T}^{fb}_{65} \succeq_{\mathfrak{B}} \mathbb{T}^{fb}_{80}$; $\mathbb{T}^{fb}_{80} \succeq_{\mathfrak{M}} \mathbb{T}^{fb}_{65}$ and $\mathbb{T}^{sc}_{80} \succeq_{\mathfrak{P}} \mathbb{T}^{sc}_{65} \succeq_{\mathfrak{P}} \{\mathbb{T}^{fb}_{80}, \mathbb{T}^{fb}_{65}, \mathbb{T}^{av}_{80}, \mathbb{T}^{av}_{65}\}$. The aggregated relation is not in this case the union of the three relations because there is a contradiction between preferences (on arguments related to fibers) expressed in contexts $\mathfrak{Miller}$ and $\mathfrak{Baker}$. The order on the contexts induces the aggregated preference $\otimes^{\rhd}(\succeq_{\mathfrak{P}}, \succeq_{\mathfrak{B}}, \succeq_{\mathfrak{M}})$ and the graph associated with this CPAF and depicted in figure below:



The framework has only one maximal (for set inclusion) admissible extension which is $\{\mathbb{T}^{av}_{80}, \mathbb{T}^{fb}_{80}\mathbb{T}^{sc}_{80}\}$. Thus, bread obtained with $\mathbb{T}_{80}$ is preferred to bread obtained with $\mathbb{T}_{65}$.

3 Comparing different abstract argumentation frameworks

This section compares the different argumentation frameworks previously presented in terms of equivalence, on the basis of Definition 3.

3.1 Comparing Dung's framework and PAF

Dung's argumentation framework can be seen as a particular case of a preference-based argumentation framework. Several situations in which PAF and AF are equivalent can be highlighted, in particular when there is no strict preference between arguments, and when all the attacks between arguments succeed (i.e if an argument α attacks an argument β then β is not preferred to α).

Property 2. The argumentation framework $\mathtt{AF} = \langle \mathcal{A}, \mathcal{R} \rangle$ is equivalent to the argumentation framework associated with $\mathtt{PAF} = \langle \mathcal{A}, \mathcal{R}, \succeq \rangle$ iff:

- $\nexists \alpha, \beta \in \mathcal{A}$ such that $(\alpha,\ \beta) \in \succ$, or
- $\nexists \alpha, \beta \in \mathcal{A}$ such that $(\alpha,\beta) \in \mathcal{R}$ and $(\beta,\ \alpha) \in \succ$.

3.2 Comparing Bench Capon's framework and extension

Bench Capon's framework can be seen as a particular case of the extension proposed in [9] and that assumes that an argument may promote more than one value. The following properties describes the situations under which a VAF is equivalent to a VSAF.

Property 3.
- The two argumentation frameworks $\langle \mathcal{A}, \mathtt{defeats} \rangle$ and $\langle \mathcal{A}, \mathtt{defeats}_{Bc} \rangle$ associated respectively with $\mathtt{VAF} = \langle \mathcal{A}, \mathcal{R}, \mathcal{V}, \mathtt{val}, \mathtt{Pref} \rangle$[8] and $\mathtt{VSAF} = \langle \mathcal{A}, \mathcal{R}, \mathcal{V}, \text{arg}, \gg \rangle$ are equivalent iff $\mathtt{val} = \text{arg}^{-1}$ and $\mathtt{Pref} = \gg$.
- The two argumentation frameworks $\langle \mathcal{A}, \mathtt{defeats} \rangle$ and $\langle \mathcal{A}, \mathtt{defeats}_{M} \rangle$ associated respectively with $\mathtt{VAF} = \langle \mathcal{A}, \mathcal{R}, \mathcal{V}, \mathtt{val}, \mathtt{Pref} \rangle$ and $\mathtt{VSAF} = \langle \mathcal{A}, \mathcal{R}, \mathcal{V}', \text{arg}, \gg \rangle$ are equivalent iff $|\text{arg}^{-1}(\mathtt{val}^{-1}(v_i))| = \text{i}$.

3.3 Comparison between the VAF and the PAF

In this section we show that several VAFs can be associated to the same PAF while a unique PAF is associated to a VAF.

Equivalent $\mathtt{PAF}^{V}$ built from VAF:

Definition 13. *Let* $\mathtt{VAF} = \langle \mathcal{A}, \mathcal{R}, \mathcal{V}, \mathtt{val}, \mathtt{Pref} \rangle$ *be a value-based argumentation framework. From a* VAF*, a preference-based argumentation framework can be defined by* $\mathtt{PAF}^V = \langle \mathcal{A}, \mathcal{R}, \succeq^V \rangle$*, with the preference relation* $\succeq^V \subseteq \mathcal{A} \times \mathcal{A}$ *defined as follows:* $\forall \alpha, \beta \in \mathcal{A}\ (\alpha, \beta) \in \succ^V$ *iff* $(\mathtt{val}(\alpha), \mathtt{val}(\beta)) \in \mathtt{Pref}$.

It is easy to show that the relation $\succ^V$ has the same properties as the relation Pref.

Property 4. The relation $\succ^V$ is irreflexive, asymmetric and transitive.

Property 5. The argumentation frameworks associated with VAF and $\mathtt{PAF}^V$ are equivalent.

Definition 13 and Property 5 are illustrated through the following example.

Example 6 (Example 3. cont.). The preference relation extracted from VAF is as follows: $\mathbb{T}^{sc}_{80} \succ^V \mathbb{T}^{sc}_{65} \succ^V \{\mathbb{T}^{av}_{80}, \mathbb{T}^{av}_{65}, \mathbb{T}^{fb}_{80}, \mathbb{T}^{fb}_{65}\}$. The system $\mathtt{PAF}^V$, built with $\succeq^V$, has only one preferred extensions $\{\mathbb{T}^{av}_{80}, \mathbb{T}^{fb}_{80}, \mathbb{T}^{sc}_{80}\}$, similarly to VAF.

[8] $\mathcal{V}=\{v_1, \ldots, v_n\}$ such that for i<j $(v_i,\ v_j) \in \mathtt{Pref}$.

$\mathtt{VAF}$'s equivalence classes built from $\mathtt{PAF}$:

Bijective construction: A value-based argumentation framework can be intuitively built from a $\mathtt{PAF}$ by assigning to each argument a distinct value, and exactly transferring the prioritization of arguments to their corresponding values. This framework, denoted $\mathtt{VAF}_b^P$, is defined as follows:

Definition 14. *Let $\mathtt{PAF} = \langle \mathcal{A}, \mathcal{R}, \succeq \rangle$ be a preference-based system. A $\mathtt{VAF}_b^P$ defined from a $\mathtt{PAF}$ is a tuple $\langle \mathcal{A}, \mathcal{R}, \mathcal{V}_b, \mathtt{val}_b, \mathtt{Pref}_b^P \rangle$ such that:*
$\mathcal{V}_b$ is a set of values with the same cardinality as $\mathcal{A}$ ($|\mathcal{V}_b|$=$|\mathcal{A}|$), $\mathtt{val}_b$ is a bijection from $\mathcal{A}$ to $\mathcal{V}_b$, and $\mathtt{Pref}_b^P \subseteq \mathcal{V}_b \times \mathcal{V}_b$ is defined by: $\forall\ v_\alpha,\ v_\beta \in \mathcal{V}_b$, ($v_\alpha$, v_β) $\in \mathtt{Pref}_b^P$ iff ($\mathtt{val}_b^{-1}(v_\alpha)$, $\mathtt{val}_b^{-1}(v_\beta)$) $\in \succ$, where $\mathtt{val}_b^{-1}$ denotes the inverse function of $\mathtt{val}_b$.

Example 7 (Example 2. cont.). For instance, from the $\mathtt{PAF}$ presented in Example 2, $\mathtt{VAF}_b^P$ can be built by associating each argument ($\mathbb{T}_{65}^{av}$, $\mathbb{T}_{80}^{av}$, ...) with a value, e.g. its name ("$\mathbb{T}_{65}^{av}$", "$\mathbb{T}_{80}^{av}$", ...), keeping the same preferences for the names as for the underlying arguments.

The following property shows that the relation $\mathtt{Pref}_b^P$ has the same properties as the relation $\succ$.

Property 6. The relation $\mathtt{Pref}_b^P$ is irreflexive, asymmetric and transitive.

Property 7. The argumentation frameworks associated with $\mathtt{PAF}$ and $\mathtt{VAF}_b^P$ are equivalent.

In order to show that different $\mathtt{VAF}$s can be mapped from one $\mathtt{PAF}$, we first define a relation between values in the target set $\mathcal{V}_b$ of a value-based argumentation framework $\mathtt{VAF}_b^P$. This relation called typologic equivalence and denoted $\mathtt{Te}$ is defined as follows:

Definition 15. *Two values v_α, $v_\beta \in \mathcal{V}_b$ belongs to the typologic equivalence relation $\mathtt{Te}$, i.e., (v_α, v_β) $\in \mathtt{Te}$ iff:*

- $\forall\ v_\gamma \in \mathcal{V}_b$, ($\mathtt{val}_b^{-1}(v_\gamma)$, $\mathtt{val}_b^{-1}(v_\alpha)$) $\in \succ$ *iff* ($\mathtt{val}_b^{-1}(v_\gamma)$, $\mathtt{val}_b^{-1}(v_\beta)$) $\in \succ$,
- $\forall\ v_\delta \in \mathcal{V}_b$, ($\mathtt{val}_b^{-1}(v_\alpha)$, $\mathtt{val}_b^{-1}(v_\delta)$) $\in \succ$ *iff* ($\mathtt{val}_b^{-1}(v_\beta)$, $\mathtt{val}_b^{-1}(v_\delta)$) $\in \succ$.

The following properties for the typologic equivalence $\mathtt{Te}$ are satisfied.

Property 8. $\mathtt{Te}$ is reflexive, symmetric and transitive.

Considering properties of this relation, $\mathtt{Te}$ defines an equivalence relation on the set $\mathcal{V}_b$. It can be also defined equivalence classes partitioning the set $\mathcal{V}_b$ into several disjoint subsets, all the elements in a given equivalence class being equivalent among themselves.

Definition 16. *The equivalence class of an element v_α in $\mathcal{V}_b$ equipped by the equivalence relation $\mathtt{Te}$, denoted $\mathtt{Te}(v_\alpha)$, is the subset of all images of v_α by $\mathtt{Te}$:*
$\mathtt{Te}(v_\alpha) = \{v_\beta \in \mathcal{V}_b \mid (v_\alpha,\ v_\beta) \in \mathtt{Te}\}$.

The set of all equivalence classes in $\mathcal{V}_b$ given by the equivalence relation $\mathtt{Te}$ is called quotient set of $\mathcal{V}_b$ by $\mathtt{Te}$.

Definition 17. *The quotient set of $\mathcal{V}_b$ by $\mathtt{Te}$, denoted $V_b/\mathtt{Te}$ is the set of all equivalence classes of $\mathcal{V}_b$ according to $\mathtt{Te}$. It is defined as follows:*
$\mathcal{V}_b/\mathtt{Te} = \{\mathtt{Te}(v) \mid v \in \mathcal{V}_b\}$.

Surjective construction: Another way to represent a $\mathtt{PAF}$ with a value-based argumentation framework can be to assign a same value for the set of arguments being themselves indifferent according to the preference relation $\succeq$. This framework, denoted $\mathtt{VAF}_s^P$, is defined as follows:

Definition 18. *Let $\mathtt{PAF} = \langle \mathcal{A}, \mathcal{R}, \succeq \rangle$ be a PAF. A $\mathtt{VAF}_s^P$ defined from a $\mathtt{PAF}$ is a tuple $\langle \mathcal{A}, \mathcal{R}, \mathcal{V}_s, \mathtt{val}_s, \mathtt{Pref}_s^P \rangle$ such that:*
$\mathcal{V}_s$ is a set of values with a cardinality at most equal to the cardinality of $\mathcal{A}$, $\mathtt{val}_s$ is a function from $\mathcal{A}$ to $\mathcal{V}_s$.
$\mathtt{Pref}_s^P \subseteq \mathcal{V}_s \times \mathcal{V}_s$ is such that:

1. *$\forall\ \alpha,\ \beta \in \mathcal{A}$, if $\mathtt{val}_s(\alpha) = \mathtt{val}_s(\beta)$ then $\forall\ \gamma \in \mathcal{A}$ it holds that $(\alpha,\ \gamma) \in \succ$ iff $(\beta,\ \gamma) \in \succ$ and it holds that $(\gamma,\ \alpha) \in \succ$ iff $(\gamma,\ \beta) \in \succ$,*
2. *$(v_\alpha,\ v_\beta) \in \mathtt{Pref}_s^P$ iff $\forall\ \alpha \in \mathtt{val}_s^{-1}(v_\alpha)$ and $\forall\ \beta \in \mathtt{val}_s^{-1}(v_\beta)$ it holds that $(\alpha,\ \beta) \in \succ$, where $\mathtt{val}_s^{-1}$ denotes the inverse function of $\mathtt{val}_s$.*

Example 8 (Example 2. cont.). For instance, from the $\mathtt{PAF}$ presented in Example 2, a $\mathtt{VAF}_s^P$ can be built by associating with the arguments $\mathbb{T}_{65}^{av}$ and $\mathbb{T}_{80}^{av}$ a value v_α, with the arguments $\mathbb{T}_{65}^{fb}$ and $\mathbb{T}_{80}^{fb}$ a value v_β, with the argument $\mathbb{T}_{80}^{sc}$ a value v_γ and with the argument $\mathbb{T}_{65}^{sc}$ a value v_δ, such that $(v_\gamma,\ v_\delta) \in \mathtt{Pref}_s^P$, $(v_\delta,\ v_\alpha) \in \mathtt{Pref}_s^P$ and $(v_\delta,\ v_\beta) \in \mathtt{Pref}_s^P$.

The relation $\mathtt{Pref}_s^P$ has the same properties as the relation $\succ$.

Property 9. The relation $\mathtt{Pref}_s^P$ is irreflexive, asymmetric and transitive.

Property 10. $\mathtt{PAF}$ and $\mathtt{VAF}_s^P$ are equivalent.

There is one surjective construction denoted $\mathtt{VAF}_{s(min)}^P$, giving a minimal target set of value denoted $\mathcal{V}_s^{min}$ (in the sense of cardinality) for which the related $\mathtt{VAF}_s^P$ holds in Definition 18 and a $\mathtt{VAF}_b^P$ satisfies Definition 14 for a same $\mathtt{PAF}$.

Example 9 (Example 2. cont.). From the $\mathtt{PAF}$ presented in Example 2, the $\mathtt{VAF}_{s(min)}^P$ can be built by associating with the arguments $\mathbb{T}_{65}^{av}$, $\mathbb{T}_{80}^{av}$, $\mathbb{T}_{65}^{fb}$ and $\mathbb{T}_{80}^{fb}$ a common value, (e.g. v_α), with the argument $\mathbb{T}_{80}^{sc}$ a value v_γ and with the argument $\mathbb{T}_{65}^{sc}$ a value v_δ, such that $(v_\gamma,\ v_\delta) \in \mathtt{Pref}_s^P$ and $(v_\delta,\ v_\alpha) \in \mathtt{Pref}_s^P$.

Property 11. $|\mathcal{V}_s^{min}| = |\mathcal{V}_b\ /\ \mathtt{Te}|$

It is worth mentioning that this mapping does not necessarily distinguish indifferent arguments and incomparable arguments.

3.4 Related work

Previous works, in particular [9], have focused on the comparison of the argumentation frameworks $\texttt{AF}$, $\texttt{PAF}$, $\texttt{VAF}$, and $\texttt{VSAF}$. However this comparison is purely syntactical, since no formal definition of the equivalence between frameworks was proposed.

Comparison of $\texttt{PAF}$ and $\texttt{VAF}$. Moreover, [9] indicates that each $\texttt{PAF}$ can be represented by various $\texttt{VAF}$s, which is correct, but then claims that all of these $\texttt{VAF}$s have the same topology, that is, each of them is a renaming of the others. Although stated by the Lemma 4 of [9], this statement is not correct. Indeed, a counter-example is the following: Let a $\texttt{PAF}$ be defined by $\langle \mathcal{A}, \mathcal{R}, \succeq \rangle$ with $\mathcal{A} = \{\alpha, \beta, \gamma\}$ and $\succeq = \{(\alpha, \beta), (\alpha,\gamma)\}$. Consider the two following $\texttt{VAF}$s. $\texttt{VAF}_1 = \langle \mathcal{A}, \mathcal{R}, \mathcal{V}_1, \texttt{val}_1, \texttt{Pref}_1 \rangle$, with $\mathcal{V}_1 = \{v_1, v_2, v_3\}$, $\texttt{val}_1(\alpha) = v_3$, $\texttt{val}_1(\beta) = v_2$ and $\texttt{val}_1(\gamma) = v_1$, and $\texttt{Pref}_1 = \{(v_3, v_2), (v_3, v_1)\}$. $\texttt{VAF}_2 = \langle \mathcal{A}, \mathcal{R}, \mathcal{V}_2, \texttt{val}_2, \texttt{Pref}_2 \rangle$, with $\mathcal{V}_2 = \{v_1, v_2\}$, $\texttt{val}_2(\alpha) = v_2$, $\texttt{val}_2(\beta) = v_1$ and $\texttt{val}_2(\gamma) = v_1$, and $\texttt{Pref}_1 = \{(v_2, v_1)\}$. Although $\texttt{VAF}_1$ and $\texttt{VAF}_2$ are both equivalent to the $\texttt{PAF}$, they do not have the same topology. Actually, the above statement would stand if $\texttt{Pref}_i$ were total orders, which is not assumed.

Comparison of $\texttt{PAF}$ and $\texttt{VSAF}$. For each value specification argumentation framework, there is at most one preference-based argumentation framework it represents. From definition, $\langle \mathcal{A}, \mathcal{R}, \mathcal{V}, \arg, \gg, MM \rangle$ represents $\langle \mathcal{A}, \mathcal{R}, \texttt{Pref}_{MM} \rangle$ if and only if $\texttt{Pref}_{MM}$ is the least specific relation among the $\texttt{Pref}'_{MM}$ such that $\langle \mathcal{A}, \mathcal{R}, \texttt{Pref}'_{MM} \rangle$ satisfies $\langle \mathcal{A}, \mathcal{R}, \mathcal{V}, \arg, \gg, MM \rangle$. Let v_1, $v_2 \in \mathcal{V}$ such that $\arg(v_1) = \{\alpha, \beta, \gamma\}$ and $\arg(v_2) = \{\gamma, \delta\}$, the two systems $\langle \mathcal{A}, \mathcal{R}, \texttt{Pref}^1_{MM} \rangle$ and $\langle \mathcal{A}, \mathcal{R}, \texttt{Pref}^2_{MM} \rangle$ satisfy $\langle \mathcal{A}, \mathcal{R}, \mathcal{V}, \arg, \gg, MM \rangle$ with $\texttt{Pref}^1_{MM} = \{(\alpha,\gamma), (\gamma,\delta), (\delta,\gamma)\}$ and $\texttt{Pref}^2_{MM} = \{(\beta,\gamma), (\gamma,\delta), (\delta,\gamma)\}$. There is no more specific relation between $\texttt{Pref}^1_{MM}$ and $\texttt{Pref}^2_{MM}$ according to Definition presented in [13], and contradicting Theorem 5 presented in [9].

3.5 Comparing the $\texttt{PAF}$ and the $\texttt{CPAF}$

A $\texttt{CPAF}$ can be viewed as several $\texttt{PAF}$s completely ordered with a relation r_{PAF}, and aggregated using the operator denoted $\otimes^{r_{PAF}}$ and defined as follows:

Definition 19. *Let* $\{\texttt{PAF}_1 = \langle \mathcal{A}, \mathcal{R}, \succeq_1 \rangle, \ldots, \texttt{PAF}_n = \langle \mathcal{A}, \mathcal{R}, \succeq_n \rangle\}$ *be a set of preference-based argumentation frameworks, totally ordered by a relation denoted* r_{PAF}. $\otimes^{r_{PAF}}(\texttt{PAF}_1, \ldots, \texttt{PAF}_n)$ *is a* $\texttt{CPAF} = \langle \mathcal{A}, \mathcal{R}, \mathcal{C}, \rhd, \succeq_{c_1}, \ldots \succeq_{c_n} \rangle$ *such that:*

- $\mathcal{C}$ *is a set of* n *contexts, each* c_i *associated with* $\texttt{PAF}_i$*,*
- $\rhd$ *is a total preorder on* $\mathcal{C} \times \mathcal{C}$*, such that* $(c_i, c_j) \in \rhd$ *iff* $(\texttt{PAF}_i, \texttt{PAF}_j) \in r_{PAF}$*,*
- $\succeq_{c_i} = \succeq_i$.

It is clear according to this definition that $\texttt{PAF}$ can be viewed as a particular case of $\texttt{CPAF}$ with $n = 1$.

On the other hand, the evaluation of a CPAF relies on an aggregation function (see Definition 11), in order to provide a unique defeat relation, which leads to the computation of a PAF.

Definition 20. *Given a* CPAF $= \langle \mathcal{A}, \mathcal{R}, \mathcal{C}, \rhd, \succeq_1, \ldots \succeq_n \rangle$*, an aggregated preference-based argumentation framework can be defined as follows:* $\texttt{PAF}_{ag} = \langle \mathcal{A}, \mathcal{R}, \otimes^{\rhd}(\succeq_1, \ldots \succeq_n) \rangle$.

4 The unifying framework

The aim of this section is to propose an argumentation framework generalizing the previous preference-based argumentation frameworks and improving their expressivity.

4.1 An argument can be expressed in one or several contexts

The extensions of Dung's framework are integrated into a common more expressive framework that can be used in a multicriteria decision situation.

Definition 21. *An* argumentation framework based on multi-contextual preferences *(*MCPAF*) is a pair* $\langle \mathcal{A}, \texttt{Def} \rangle$*, where* Def *is defined as follows:* $\forall \alpha, \beta \in \mathcal{A}$, $(\alpha, \beta) \in \texttt{Def}$ *iff* $(\alpha, \beta) \in \oplus^{\trianglerighteq}(\mathcal{R}_1, \ldots, \mathcal{R}_n)$ *and* $(\beta, \alpha) \notin \otimes^{\trianglerighteq}(\succeq_1, \ldots \succeq_n)$ *such that:*

- $\mathcal{C} = c_1, \ldots, c_n$ *is the set of contexts,*
- $\trianglerighteq$ *is a complete preordering on* $\mathcal{C} \times \mathcal{C}$,
- $\mathcal{A}_1, \ldots, \mathcal{A}_n$ *are sets of arguments,* $\mathcal{A}_i \subseteq \mathcal{A}$ *(with* $\mathcal{A} = \cup_{i \in [1,n]} \mathcal{A}_i$*) is the set of arguments which are expressed in the context* c_i,
- $\mathcal{R}_1, \ldots, \mathcal{R}_n$ *are binary relations representing contextual attacks,* $\mathcal{R}_i \subseteq \mathcal{A}_i \times \mathcal{A}_i$ *concerns the attack of arguments expressed in context* c_i,
- $\succeq_1, \ldots, \succeq_n$ *is the set of contextual preferences,* $\succeq_i \subseteq \mathcal{A}_i \times \mathcal{A}_i$ *is a partial preordering and concerns preferences of argument expressed in context* c_i,
- $\oplus^{\trianglerighteq}$[9] *(resp.* $\otimes^{\trianglerighteq}$*) is an aggregation operator of contextual attacks (resp. preferences).*

MCPAF *will be represented as a tuple:* $\langle \mathcal{A}_1, \ldots, \mathcal{A}_n, \mathcal{R}_1, \ldots, \mathcal{R}_n, \mathcal{C}, \trianglerighteq, \succeq_1, \ldots, \succeq_n \rangle$.

This system allows one to relate that an argument can be expressed in one or several contexts, to compare two arguments in the same context with a preference relation or to express an attack between two arguments in a given context.

[9] For comparison purpose, $\oplus^{\trianglerighteq}$ can be axiomatized as follows: $\oplus^{\trianglerighteq}(\mathcal{R}_1, \ldots, \mathcal{R}_n) = \mathcal{R}_1$ if $\mathcal{R}_2 = \ldots = \mathcal{R}_n$ and $\oplus^{\trianglerighteq}(\mathcal{R}_c, \ldots, \mathcal{R}_c) = \mathcal{R}_c$

4.2 CPAF is a particular case of a MCPAF

CPAF is an argumentation system which can be seen as a particular case of a MCPAF reduced to a strict order between contexts and where all sets of arguments $\mathcal{A}_i$ and contextual attacks $\mathcal{R}_i$ are similar. In the following definition a multi-contextual preferences argumentation framework is built from a CPAF (denoted $\texttt{MCPAF}^C$).

Definition 22. *Let a* CPAF *be a tuple such that* $\texttt{CPAF} = \langle \mathcal{A}, \mathcal{R}, \mathcal{C}, \triangleright, \succeq_1, \dots \succeq_n \rangle$, $\texttt{MCPAF}^C$ *is also a tuple built from* CPAF *such that*
$\texttt{MCPAF}^C = \langle \mathcal{A}_1^C, \dots, A_n^C, \mathcal{R}_1^C, \dots, \mathcal{R}_n^C, \mathcal{C}, \trianglerighteq^C, \succeq_1, \dots, \succeq_n \rangle$ *with:*
$\mathcal{A}_1^C = \dots = A_n^C = \mathcal{A}$, $\mathcal{R}_1^C = \dots = R_n^C = \mathcal{R}$, $\trianglerighteq^C = \triangleright$.

Property 12. $\texttt{MCPAF}^C$ and CPAF are equivalent.

4.3 Aggregation operator of contextual preferences

Since the set of contexts in a MCPAF is equipped with a complete preordering, there are indifferent contexts w.r.t $\trianglerighteq$. It thus possible to stratify the set $\mathcal{C}$ of contexts $\mathcal{C}^1$, ..., $\mathcal{C}^m$ such that for all $c, c' \in \mathcal{C}^i$, $(c, c') \in \trianglerighteq$ and $(c', c) \in \trianglerighteq$. Moreover, for any $c \in \mathcal{C}^i$ and $c' \in \mathcal{C}^j$ with $j > i$, it holds that $(c', c) \in \triangleright$ (meaning that $(c', c) \in \trianglerighteq$ and $(c, c') \notin \trianglerighteq$).

In each subset of preferences $\bigcup_i \succeq^i$, there may be contradictory preferences whose set is denoted $\texttt{CP}^i$.

Definition 23. $\texttt{CP}^i \subseteq \mathcal{A} \times \mathcal{A}$ *is the set of contradictory preferences between arguments expressed in equivalent contexts of level at most equal to i.*
$\texttt{CP}^1 = \{(\alpha, \beta) \mid \exists\, c_k, c_l \in \mathcal{C}^1 \text{ s.t. } (\alpha, \beta) \in \succeq_k \text{ and } (\beta, \alpha) \in \succeq_l\}$
$\texttt{CP}^i = \{(\alpha, \beta) \mid \exists\, c_k, c_l \in \mathcal{C}^i \text{ s.t. } (\alpha, \beta) \in \succeq_k \text{ and } (\beta, \alpha) \in \succeq_l\} \cup_{r \in [1, i-1]} \texttt{CP}^r$

Definition 24. *An aggregation operator of contextual preferences for* MCPAF *can be defined as follows:* $\otimes^{\trianglerighteq}(\succeq_1, \dots \succeq_n) = \Pi_n$*:*

$$\Pi_1 = \{(\alpha, \beta) \in \succeq^1 \text{ and } (\alpha, \beta) \notin \texttt{CP}^1\}$$
$$\Pi_{k+1} = \Pi_k \cup \{(\alpha, \beta) \in \succeq^{k+1} \text{ and } (\beta, \alpha) \notin \texttt{CP}^{k+1} \cup \Pi_k\}$$

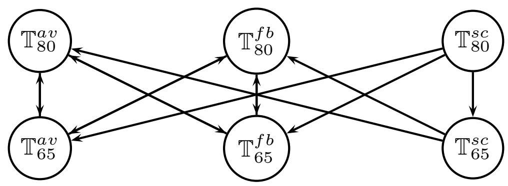
In CPAF, the set of contexts is equipped with a total order, therefore after discretization of preferences about the fiber criterion within two audiences (represented as the contexts $\mathfrak{Miller}$ and $\mathfrak{Baker}$), this framework doesn't allow to consider contexts as indifferent. The benefit generated by enforced expressivity is counterbalanced by a limitation in the ordering of contexts, involving an important difference for Def relation. Moreover, even though a preference can be expressed in a given context, arguments lose the expressivity obtained in VSAF (where an argument can promote one or several values). MCPAF allows to encompass advantages included in VSAF and CPAF without a loss of PAF generality.

Example 10 (Example 1 cont.). Table below summarizes contexts, sets of arguments expressed in these contexts, contextual preferences and attacks, represented in MCPAF through the running example. The ordering of the set of contexts is described as follows:

$\mathfrak{Pnns} \rhd \mathfrak{Baker} \sim \mathfrak{Miller} \rhd \mathfrak{Ash\ Value} \sim \mathfrak{Fibers} \sim \mathfrak{SCP} \sim \% \sim \mathfrak{g}/100\mathfrak{g} \sim 0.80 \sim \ldots \sim 42.$

c_i	$\mathfrak{Pnns}$	$\mathfrak{Baker}$	$\mathfrak{Miller}$	$\mathfrak{Ash\ Value}$	$\ldots$	$\%$	$\ldots$	42
$\succeq_i$	$\mathbb{T}^{sc}_{80} \succ_{\mathfrak{P}} \mathbb{T}^{sc}_{65} \succ_{\mathfrak{P}} \{\mathbb{T}^{fb}_{80}, \ldots, \mathbb{T}^{av}_{65}\}$	$\mathbb{T}^{fb}_{65} \succeq_{\mathfrak{B}} \mathbb{T}^{fb}_{80}$	$\mathbb{T}^{fb}_{80} \succeq_{\mathfrak{M}} \mathbb{T}^{fb}_{65}$	$\emptyset$	$\ldots$	$\emptyset$	$\ldots$	$\emptyset$
$\mathcal{A}_i$	$\mathbb{T}^{sc}_{80}, \mathbb{T}^{sc}_{65}, \mathbb{T}^{fb}_{80}, \mathbb{T}^{fb}_{65}, \mathbb{T}^{av}_{80}, \mathbb{T}^{av}_{65}$	$\mathbb{T}^{fb}_{65}, \mathbb{T}^{fb}_{80}$	$\mathbb{T}^{fb}_{80}, \mathbb{T}^{fb}_{65}$	$\mathbb{T}^{av}_{80}, \mathbb{T}^{av}_{65}$	$\ldots$	$\mathbb{T}^{sc}_{80}, \mathbb{T}^{sc}_{65}, \mathbb{T}^{av}_{80}, \mathbb{T}^{av}_{65}$	$\ldots$	$\mathbb{T}^{fb}_{80}$
$\mathcal{R}_i$	$\mathcal{R}_{ex}$	$\emptyset$	$\emptyset$	$\emptyset$	$\ldots$	$\emptyset$	$\ldots$	$\emptyset$

MCPAF framework has only one maximal (for set inclusion) admissible extension which is $\{\mathbb{T}^{av}_{80}, \mathbb{T}^{fb}_{80}, \mathbb{T}^{sc}_{80}\}$. The bread obtained with $\mathbb{T}_{80}$ is preferred to bread obtained with $\mathbb{T}_{65}$ in this system on the basis of the aim of the nutritional program and no preferences between actors point of views (e.g. bakers and millers).



5 Conclusion

Comparing different argumentation framework can be a hard task, especially since there are few propositions in literature on the ways to achieve this task. In this paper, we have proposed to compare frameworks on the basis of their extensions under a given semantics. We have considered two argumentation frameworks as equivalent if they return exactly the same extensions. Then, we have compared well-known frameworks (AF, VAF, VSAF, PAF, CPAF) under the light of this comparison method. It is also clearly shown that these frameworks can be considered as equivalent under particular conditions.

We have then proposed a more general framework (MCPAF) generalizing the others as special cases, and allowing for fine representations of contextual specificities. Although its benefits have to be evaluated on more complex real-world problems, we think that it will prove useful in multiple criteria decision problems in presence of multiple actors. We therefore plan to apply it to agronomical issues.

Indeed a case study recently investigated covers questions related to policy decisions concerning public health. In a first step, area describes knowledge base from bread nutritional formulation using different types of flour, then it combines arguments (coming from the actors points of view) put into a decisional system. Finally, the ambition is to refine a consensus decision that satisfies both public authorities and consumers through all the actors involved in the bread transformation process.

Appendix

Proof. of Property 1. Assume that Ext($\mathtt{PAF}_1$), Ext($\mathtt{PAF}_2$) be the sets of admissible extensions in these two abstract frameworks $\mathtt{PAF}_1$ and $\mathtt{PAF}_2$. Let us show that $Ext(\mathtt{PAF}_2) \subseteq Ext(\mathtt{PAF}_1)$ and $Ext(\mathtt{PAF}_1) \subseteq Ext(\mathtt{PAF}_2)$:

1. Ext($\mathtt{PAF}_2$) $\subseteq$ Ext($\mathtt{PAF}_1$). Let $\mathcal{E} \in$ Ext($\mathtt{PAF}_1$). Assume that $\mathcal{E} \notin$ Ext($\mathtt{PAF}_2$). This means that $\mathcal{E}$ is not an admissible extension in $\mathtt{PAF}_2$. According to Definition 2, there are two possibilities:

   ∗Case 1: $\mathcal{E}$ is not conflict-free in $\mathtt{PAF}_2$. $\exists\ \alpha,\ \beta \in \mathcal{E}$ such that $(\alpha,\ \beta) \in \mathtt{Def}_2$ (where $\mathtt{Def}_2$ is built from $\mathcal{R}$ and $\succ$, the strict relation of $\succeq$, by Definition 4). This means that $(\alpha,\ \beta) \in \mathcal{R}$ and $(\beta,\ \alpha) \notin\ \succ$. It holds also that $(\alpha,\ \beta) \in \mathtt{Def}_1$ (where $\mathtt{Def}_1$ is defined from $\mathcal{R}$ and $\succ$). Thus, $\mathcal{E}$ is not conflict-free in $\mathtt{PAF}_1$. This contradicts the fact that $\mathcal{E}$ is an admissible extension in $\mathtt{PAF}_1$.

   ∗Case 2: $\mathcal{E}$ does not defend its elements in $\mathtt{PAF}_2$. This means that $\exists\ \alpha \in \mathcal{E}$, $\exists\ \beta \in \mathcal{A}$ such that $(\beta,\ \alpha) \in \mathtt{Def}_2$ and $\nexists\ \gamma \in \mathcal{E}$ such that $(\gamma,\ \beta) \in \mathtt{Def}_2$. According to Definition 4, since $(\beta,\ \alpha) \in \mathtt{Def}_2$ then $(\beta,\ \alpha) \in \mathcal{R}$ and $(\alpha,\ \beta) \notin\ \succ$, i.e $(\beta,\ \alpha) \in \mathtt{Def}_1$. But, $\mathcal{E}$ is an admissible extension in $\mathtt{PAF}_1$, i.e $\exists\ \gamma \in \mathcal{E}$ such that $(\gamma,\ \beta) \in \mathtt{Def}_1$, i.e $(\gamma,\ \beta) \in$ R and $(\beta,\ \gamma) \notin\ \succ$, i.e $(\gamma,\ \beta) \in \mathtt{Def}_2$. This conclusion contradicts the fact that $\mathcal{E}$ does not defend its arguments in $\mathtt{PAF}_2$.
2. Ext($\mathtt{PAF}_1$) $\subseteq$ Ext($\mathtt{PAF}_2$). Let $\mathcal{E} \in$ Ext($\mathtt{PAF}_2$). Assume that $\mathcal{E} \notin$ Ext($\mathtt{PAF}_1$). This means that $\mathcal{E}$ is not an admissible extension in $\mathtt{PAF}_1$. According to Definition 2, there are two possibilities:

   ∗Case 1: $\mathcal{E}$ is not conflict-free in $\mathtt{PAF}_1$. $\exists\ \alpha,\ \beta \in \mathcal{E}$ such that $(\alpha,\ \beta) \in \mathtt{Def}_1$ ($\mathtt{Def}_1$ is built from $\mathcal{R}$ and $\succ$). This means that $(\alpha,\ \beta) \in \mathcal{R}$ and $(\beta,\ \alpha) \notin\ \succ$. By Definition 4, it holds also that $(\alpha,\ \beta) \in \mathtt{Def}_2$ (where $\mathtt{Def}_2$ is defined from $\mathcal{R}$ and $\succ$, the strict relation of $\succeq$). Thus, $\mathcal{E}$ is not conflict-free in $\mathtt{PAF}_2$. This contradicts the fact that $\mathcal{E}$ is an admissible extension in $\mathtt{PAF}_2$.

   ∗Case 2: $\mathcal{E}$ does not defend its elements in $\mathtt{PAF}_1$. This means that $\exists\ \alpha \in \mathcal{E}$, $\exists\ \beta \in \mathcal{A}$ such that $(\beta,\ \alpha) \in \mathtt{Def}_1$ and $\nexists\ \gamma \in \mathcal{E}$ such that $(\gamma,\ \beta) \in \mathtt{Def}_1$. However, since $(\beta,\ \alpha) \in \mathtt{Def}_1$ then $(\beta,\ \alpha) \in \mathcal{R}$ and $(\alpha,\ \beta) \notin\ \succ$. By Definition 4, $\mathtt{Def}_2$ is built from $\mathcal{R}$ and $\succ$, the strict relation of $\succeq$, i.e $(\beta,\ \alpha) \in \mathtt{Def}_2$. But, $\mathcal{E}$ is an admissible extension in $\mathtt{PAF}_2$, i.e $\exists\ \gamma \in \mathcal{E}$ such that $(\gamma,\ \beta) \in \mathtt{Def}_2$, i.e $(\gamma,\ \beta) \in \mathcal{R}$ and $(\beta,\ \gamma) \notin\ \succ$, i.e $(\gamma,\ \beta) \in \mathtt{Def}_1$. This conclusion contradicts the fact that $\mathcal{E}$ does not defend its arguments in $\mathtt{PAF}_1$.

Proof. of Property 2. Assume that Ext($\mathtt{AF}$), Ext($\mathtt{PAF}$) be the sets of admissible extensions in these two abstract frameworks $\mathtt{AF}$ and $\mathtt{PAF}$. Let us show that $Ext(\mathtt{AF}) \subseteq Ext(\mathtt{PAF})$ and $Ext(\mathtt{PAF}) \subseteq Ext(\mathtt{AF})$:

1. Ext($\mathtt{AF}$) $\subseteq$ Ext($\mathtt{PAF}$). Let $\mathcal{E} \in$ Ext($\mathtt{PAF}$). Assume that $\mathcal{E} \notin$ Ext($\mathtt{AF}$). This means that $\mathcal{E}$ is not an admissible extension in $\mathtt{AF}$. According to Definition 2, there are

two possibilities:

$*$Case 1: $\mathcal{E}$ is not conflict-free in $\texttt{AF}$. $\exists\ \alpha,\ \beta \in \mathcal{E}$ such that $(\alpha,\ \beta) \in \mathcal{R}$. But $(\beta,\ \alpha) \notin \succ$ since $\nexists\ (\beta,\ \alpha) \in \succ$, it holds also that $(\alpha,\ \beta) \in \texttt{Def}$ ($\texttt{Def}$ is built from $\mathcal{R}$ and $\succ$ according to Definition 4). This contradicts the fact that $\mathcal{E}$ is conflict-free in $\texttt{PAF}$.

$*$Case 2: $\mathcal{E}$ does not defend its elements in $\texttt{AF}$. This means that $\exists\ \alpha \in \mathcal{E}$, $\exists\ \beta \in \mathcal{A}$ such that $(\beta,\ \alpha) \in \mathcal{R}$ and $\nexists\ \gamma \in \mathcal{E}$ such that $(\gamma,\ \beta) \in \mathcal{R}$. However, since $(\beta,\ \alpha) \in \mathcal{R}$ and $(\alpha,\ \beta) \notin \succ$, i.e $(\beta,\ \alpha) \in \texttt{Def}$. But, $\mathcal{E}$ is an admissible extension in $\texttt{PAF}$, i.e $\exists\ \gamma \in \mathcal{E}$ such that $(\gamma,\ \beta) \in \texttt{Def}$, i.e $(\gamma,\ \beta) \in \mathcal{R}$ and $(\beta,\ \gamma) \notin \succ$ (by property), i.e $(\gamma,\ \beta) \in \mathcal{R}$. This conclusion contradicts the fact that $\mathcal{E}$ does not defend its arguments in $\texttt{AF}$.

2. Ext($\texttt{PAF}$) $\subseteq$ Ext($\texttt{AF}$). Let $\mathcal{E} \in$ Ext($\texttt{AF}$). Assume that $\mathcal{E} \notin$ Ext($\texttt{PAF}$). This means that $\mathcal{E}$ is not an admissible extension in $\texttt{PAF}$. According to Definition 2, there are two possibilities:

   $*$Case 1: $\mathcal{E}$ is not conflict-free under $\texttt{PAF}$. On the other hand, $\mathcal{E}$ is conflict-free in $\texttt{AF}$, i.e $\nexists\ \alpha,\ \beta \in \mathcal{E}$ such that $(\alpha,\ \beta) \in \mathcal{R}$. It holds also that $(\alpha,\ \beta) \notin \texttt{Def}$. ($\texttt{Def}$ is built from $\mathcal{R}$ and $\succ$ according to Definition 4). This contradicts the fact that $\mathcal{E}$ is not conflict-free in $\texttt{PAF}$.

   $*$Case 2: $\mathcal{E}$ does not defend its elements in $\texttt{PAF}$. $\mathcal{E}$ is an admissible extension in $\texttt{AF}$, i.e if $\exists\ \alpha \in \mathcal{E}$, $\exists\ \beta \in \mathcal{A}$ such that $(\beta,\ \alpha) \in \mathcal{R}$ then $\exists\ \gamma \in \mathcal{E}$ such that $(\gamma,\ \beta) \in \mathcal{R}$. By property $(\alpha,\ \beta) \notin \succ$ and $(\beta,\ \gamma) \notin \succ$. This means that $\exists\ \alpha \in \mathcal{E}$, $\exists\ \beta \in \mathcal{A}$ such that $(\beta,\ \alpha) \in \texttt{Def}$ and $\exists\ \gamma \in \mathcal{E}$ such that $(\gamma,\ \beta) \in \texttt{Def}$ ($\texttt{Def}$ is built from $\mathcal{R}$ and $\succ$ according to Definition 4). This conclusion contradicts the fact that $\mathcal{E}$ does not defend its arguments in $\texttt{PAF}$.

- If $\nexists\ \alpha,\ \beta \in \mathcal{A}$, such that $(\alpha,\beta) \in \mathcal{R}$ and $(\beta,\ \alpha) \in \succ$.

Assume that Ext($\texttt{AF}$), Ext($\texttt{PAF}$) be the sets of admissible extensions in these two abstract frameworks $\texttt{AF}$ and $\texttt{PAF}$. Let us show that $Ext(\texttt{AF}) \subseteq Ext(\texttt{PAF})$ and $Ext(\texttt{PAF}) \subseteq Ext(\texttt{AF})$:

1. Ext($\texttt{AF}$) $\subseteq$ Ext($\texttt{PAF}$). Let $\mathcal{E} \in$ Ext($\texttt{PAF}$). Assume that $\mathcal{E} \notin$ Ext($\texttt{AF}$). This means that $\mathcal{E}$ is not an admissible extension in $\texttt{AF}$. According to Definition 2, there are two possibilities:

   $*$Case 1: $\mathcal{E}$ is not conflict-free under $\texttt{AF}$. $\exists\ \alpha,\ \beta \in \mathcal{E}$ such that $(\alpha,\ \beta) \in \mathcal{R}$. But $(\beta,\ \alpha) \notin \succ$ since $\nexists\ (\beta,\ \alpha)$ such that $(\beta,\ \alpha) \in \succ$ and $(\alpha,\ \beta) \in \mathcal{R}$, it holds also that $(\alpha,\ \beta) \in \texttt{Def}$ ($\texttt{Def}$ is built from $\mathcal{R}$ and $\succ$ according to Definition 4). This contradicts the fact that $\mathcal{E}$ is conflict-free in $\texttt{PAF}$.

   $*$Case 2: $\mathcal{E}$ does not defend its elements in $\texttt{AF}$. This means that $\exists\ \alpha \in \mathcal{E}$, $\exists\ \beta \in \mathcal{A}$ such that $(\beta,\ \alpha) \in \mathcal{R}$ and $\nexists\ \gamma \in \mathcal{E}$ such that $(\gamma,\ \beta) \in \mathcal{R}$. However, since $(\beta,\ \alpha) \in \mathcal{R}$ and by property $(\alpha,\ \beta) \notin \succ$, i.e $(\beta,\ \alpha) \in \texttt{Def}$ ($\texttt{Def}$ is built from $\mathcal{R}$ and $\succ$ according to Definition 4). But, $\mathcal{E}$ is an admissible extension in $\texttt{PAF}$, i.e $\exists\ \gamma \in \mathcal{E}$ such that $(\gamma,\ \beta) \in \texttt{Def}$, i.e $(\gamma,\ \beta) \in \mathcal{R}$ and $(\beta,\ \gamma) \notin \succ$ (according to Definition 4), i.e $(\gamma,\ \beta) \in \mathcal{R}$. This conclusion contradicts the fact that $\mathcal{E}$ does not defend its arguments in $\texttt{AF}$.

2. Ext(PAF) $\subseteq$ Ext(AF). Let $\mathcal{E} \in$ Ext(AF). Assume that $\mathcal{E} \notin$ Ext(PAF). This means that $\mathcal{E}$ is not an admissible extension in PAF. According to Definition 2, there are two possibilities:

   ∗Case 1: $\mathcal{E}$ is not conflict-free in PAF. On the other hand, $\mathcal{E}$ is conflict-free in AF, i.e $\nexists\ \alpha, \beta \in \mathcal{E}$ such that $(\alpha, \beta) \in \mathcal{R}$. It holds also that $(\alpha, \beta) \notin$ Def. (Def is built from $\mathcal{R}$ and $\succ$ according to Definition 4). This contradicts the fact that $\mathcal{E}$ is not conflict-free in PAF.

   ∗Case 2: $\mathcal{E}$ does not defend its elements in PAF. $\mathcal{E}$ is an admissible extension in AF, i.e if $\exists\ \alpha \in \mathcal{E}, \exists\ \beta \in \mathcal{A}$ such that $(\beta, \alpha) \in \mathcal{R}$ then $\exists\ \gamma \in \mathcal{E}$ such that $(\gamma, \beta) \in \mathcal{R}$. By property if $\exists\ \alpha, \beta$ such that $(\beta, \alpha) \in \mathcal{R}$ then $(\alpha, \beta) \notin \succ$ and if $\exists\ \gamma, \beta$ such that $(\gamma, \beta) \in \mathcal{R}$ then $(\beta, \gamma) \notin \succ$. This means that $\exists\ \alpha \in \mathcal{E}, \exists\ \beta \in \mathcal{A}$ such that (β, α) $\in$ Def and $\exists\ \gamma \in \mathcal{E}$ such that $(\gamma, \beta) \in$ Def (Def is built from $\mathcal{R}$ and $\succ$ according to Definition 4). This conclusion contradicts the fact that $\mathcal{E}$ does not defend its arguments in PAF.

Proof. of Property 4.

1. $\succ^V$ is irreflexive,
   Assume that $\succ^V$ is reflexive: $\forall\ \alpha \in \mathcal{A}$, $(\alpha, \alpha) \in \succ^V$. This means that (val($\alpha$), val($\alpha$)) $\in$ Pref, but this is not possible since Pref is irreflexive, showing a contradiction.

2. $\succ^V$ is asymmetric,
   Assume that $\alpha, \beta \in \mathcal{A}$ such that $(\alpha, \beta) \in \succ^V$ and $(\beta, \alpha) \in \succ^V$:
   - $(\alpha, \beta) \in \succ^V \Rightarrow$ (val(α), val(β)) $\in$ Pref,
   - $(\beta, \alpha) \in \succ^V \Rightarrow$ (val(β), val(α)) $\in$ Pref,

   By Definition 6, this is impossible since Pref is asymmetric, thus $\succ^V$ is also asymmetric.
3. $\succ^V$ is transitive,
   Let $\alpha, \beta, \gamma \in \mathcal{A}$, assume that $(\alpha, \beta) \in \succ^V$, $(\beta, \gamma) \in \succ^V$ and $(\alpha, \gamma) \notin \succ^V$:
   - $(\alpha, \beta) \in \succ^V \Rightarrow$ (val(α), val(β)) $\in$ Pref (1),
   - $(\beta, \gamma) \in \succ^V \Rightarrow$ (val(β), val(γ)) $\in$ Pref (2),

   Since $(\alpha, \gamma) \notin \succ^V \Rightarrow$ (val(α), val(γ)) $\notin$ Pref. However, since Pref is transitive, from (1) and (2) it follows that $\succ^V$ is transitive, showing a contradiction.

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