On the quality of persuasion dialogs

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Abstract

Several systems have been proposed for generating *persuasion dialogs* in which agents try to persuade each others to change their mind on a state of affairs. In this paper, we focus on the evaluation of the *quality* of those dialogs. We particularly propose three families of *measures*: i) measures of the quality of exchanged arguments, ii) measures of the behavior of each participating agent in terms of *coherence*, *aggressiveness* and the *novelty* of her arguments, iii) measures of the quality of the dialog itself in terms of *relevance* and *usefulness* of its moves. A notion of *conciseness* of a dialog is also introduced. For each persuasion dialog, we compute its *ideal* dialog which is a concise sub-dialog. The closer a dialog to its ideal sub-dialog, the better it is.

1 Introduction

Persuasion is one of the main types of dialogs encountered in everyday life. It concerns two (or more) agents who disagree on a state of affairs, and each of them tries to persuade the others to change their minds. For that purpose, agents exchange arguments of different strengths. Several systems have been proposed in the literature for allowing agents to engage in persuasion dialogs (e.g. [6, 7, 9, 10, 11, 12, 14]). A dialog system is built around three main components: i) a *communication language* specifying the locutions that will be used by agents during a dialog for exchanging information, arguments, etc., ii) a *protocol* specifying the set of rules governing the well-definition of dialogs such as who is allowed to say what and when? and iii) agents' strategies which are the different tactics used by agents for selecting their moves at each step in a dialog. It is worth mentioning that in these systems, only properties that are related to the protocol can be proved. Those properties are related to the way a dialog is generated. For instance, one can show whether a dialog terminates, or whether turn shifts equally between agents (if such rule is specified by the protocol), etc. However, a protocol does not say anything about the *quality* of the generated dialogs. Moreover, it is well-known that under the same protocol, different dialogs on the same subject may be generated. It is important to be able to compare them w.r.t. their quality. Such a comparison may help to refine the protocols and to have more efficient ones. While there are a lot of works on dialog protocols, no work is done on defining criteria for evaluating the persuasion dialogs generated under those protocols.
Besides, judging the properties of a dialog may be seen as a subjective issue. Two people listening to the same political debate may disagree on the "winner" and may have different feelings about the dialog itself.

In this paper, we investigate objective criteria for analyzing already generated dialogs whatever the protocol and the strategies that are used. We place ourselves in the role of an external observer who tries to evaluate a dialog, and we propose three families of measures: 1) Measures that evaluate the quality of exchanged arguments, 2) Measures that analyze the behavior of each participating agent in terms of *coherence* and *aggressiveness* in the dialog, and finally in terms of *borrowing* (when an agent uses arguments coming from other participating agents), 3) Measures of the properties of the dialog itself in terms of *relevance* and *usefulness* of its moves. A move is relevant if it does not deviate from the subject of the dialog, and it is useful if it is important to determine the outcome of the dialog We propose also a criterion that evaluates the *conciseness* of a generated dialog. A dialog is concise if all its moves (i.e. the exchanged arguments) are both relevant to the subject and useful. Inspired from works on proof procedures that have been proposed in argumentation theory in order to check whether an argument is accepted or not [2], we compute and characterize a sub-dialog, called *ideal*, of the original one that is concise. The closer a dialog to its ideal sub-dialog, the better is its quality. All these measures are of great importance since they can be used as guidelines for generating the "best" dialogs. They can also serve as a basis for analyzing dialogs that hold between agents.

The paper is organized as follows: Section 2 recalls the basics of argumentation theory. Section 3 presents the basic concepts of a persuasion dialog. Section 4 describes the first family of measures, those evaluating arguments. Section 5 introduces measures that analyze the behavior of agents in a dialog. Section 6 presents the last family of measures, those devoted to the evaluation of a dialog. This paper unifies and develops the content of two previous works [3, 4].

2 Basics of argumentation systems

Argumentation is a reasoning model based on the construction and the comparison of arguments. Arguments are reasons for believing in statements, or for performing actions. In this paper, the origin of arguments is supposed to be unknown. In [8], an argumentation system is defined as follows:

Definition 1 (Argumentation system) *An* argumentation system *is a pair* $\mathsf{AS} = \langle \mathcal{A}, \mathcal{R} \rangle$*, where* $\mathcal{A}$ *is a set of arguments and* $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ *is an attack relation.* $(\alpha, \beta) \in \mathcal{R}$ *means that argument* α *attacks* β*.*

Note that to each argumentation system is associated a *directed graph* whose nodes are the different arguments, and the arcs represent the attack relation between them.

Since arguments are conflicting, it is important to know which arguments are acceptable. For that purpose different *acceptability semantics* have been proposed in [8]. In this paper, we only focus on *grounded* semantics. However, the work can be generalized to other semantics.

Definition 2 (Defense–Grounded extension) *Let* $\mathsf{AS} = \langle \mathcal{A}, \mathcal{R} \rangle$ *and* $\mathcal{E} \subseteq \mathcal{A}$*.*

- $\mathcal{E}$ defends *an argument* $\alpha \in \mathcal{A}$ *iff* $\forall\ \beta \in \mathcal{A}$*, if* $(\beta, \alpha) \in \mathcal{R}$*, then* $\exists \delta \in \mathcal{E}$ *s.t.* $(\delta, \beta) \in \mathcal{R}$*.*
- *The* grounded extension *of* AS *is the least fixed point of a function* $\mathcal{F}$ *where* $\mathcal{F}(\mathcal{E}) = \{\alpha \in \mathcal{A} \mid \mathcal{E}$ *defends* $\alpha\}$*.*

Each argumentation system has a unique grounded extension which may be empty. Moreover, when a system is finite (i.e. each argument is attacked by a finite number of arguments), its grounded extension is defined as follows: $\mathcal{E} = \bigcup_{i>0} \mathcal{F}^i(\emptyset)$. Depending on whether an argument belongs to this set or not, it is either accepted or rejected.

Definition 3 (Argument status) *Let* AS = $\langle \mathcal{A}, \mathcal{R} \rangle$ *be an argumentation system, and* $\mathcal{E}$ *its grounded extension. An argument* $\alpha \in \mathcal{A}$ *is* accepted *iff* $\alpha \in \mathcal{E}$*, it is* rejected *otherwise. We denote by* $\texttt{Status}(\alpha, \mathsf{AS})$ *the status of* α *in* AS*.*

Proposition 1 ([2]) *Let* AS = $\langle \mathcal{A}, \mathcal{R} \rangle$, $\mathcal{E}$ *its grounded extension, and* $\alpha \in \mathcal{A}$*. If* $\alpha \in \mathcal{E}$*, then* α *is* indirectly defended[1] *by non-attacked arguments against all its attackers.*

3 Persuasion dialogs

Throughout this section, $\mathcal{L}$ denotes a logical language. An *argument* is a reason for believing a statement. Thus, it has three main components: i) a *support* which is the set of premises on which the argument is grounded, it is thus a subset of $\mathcal{L}$, ii) a *conclusion* which is an element of $\mathcal{L}$ and iii) a *link* between the two.

Notations: `Support` is a function which returns for each argument α its support, thus $\texttt{Support}(\alpha) \subseteq \mathcal{L}$. $\arg$ is a function which returns all the arguments that can be built from a subset X of formulas ($X \subseteq \mathcal{L}$). `Formulas` is a function which returns the formulas included in the support of a set of arguments, hence if $A \subseteq \arg(\mathcal{L})$, $\texttt{Formulas}(A) = \bigcup_{\alpha \in A} \texttt{Support}(\alpha)$.

Conflicts among arguments of $\arg(\mathcal{L})$ are captured by a binary relation $\mathcal{R}_{\mathcal{L}}$ (i.e. $\mathcal{R}_{\mathcal{L}} \subseteq \arg(\mathcal{L}) \times \arg(\mathcal{L})$). We assume that each agent involved in a dialog recognizes any argument of $\arg(\mathcal{L})$ and any conflict in $\mathcal{R}_{\mathcal{L}}$. This assumption does not mean that each agent is aware of all the arguments. But, it means that agents use the same logical language and the same definitions of argument and attack relation.

In what follows, a persuasion dialog consists of an exchange of arguments between two or more agents. The *subject* of such a dialog is an argument and its *aim* is to determine the status of that argument. Note that in [6], other kinds of moves (like questions, assertions) may be exchanged in a persuasion dialog. For our purpose, we consider only arguments since they allow us to determine the output of a dialog.

Definition 4 (Move) *Let* `Ag` *be a set of symbols representing agents.*
A move m *is a triple* $\langle S, H, \alpha \rangle$ *such that:*

- $S \in \texttt{Ag}$ *is the agent that utters* m*, the function* `Speaker` *denotes this agent, i.e.,* $\texttt{Speaker}(m) = S$
- $H \subseteq \texttt{Ag}$ *is the set of agents to which the move is addressed, the function* `Hearer` *denotes this set of agents:* $\texttt{Hearer}(m) = H$
- $\alpha \in \arg(\mathcal{L})$ *is the content of the move, the function* `Content` *denotes the argument contained in the move:* $\texttt{Content}(m) = \alpha$.

[1]An argument α is *indirectly defended* by β iff there exists a finite sequence of distinct arguments $a_1, \ldots, a_{2n+1}$ such that $\alpha = a_1$, $\beta = a_{2n+1}$, and $\forall i \in [\![1, 2n]\!]$, $(a_{i+1}, a_i) \in \mathcal{R}, n \in \boldsymbol{N}^*$.

During a dialog several moves may be uttered. Those moves constitute a sequence denoted by $\langle m_1, \ldots, m_n \rangle$, where m_1 is the initial move whereas m_n is the final one. The empty sequence is denoted by $\langle\rangle$. These sequences are built under a given protocol like, for instance, the ones proposed in [6, 12]. For the purpose of our paper, we do not focus on particular protocols since we are not interested in generating dialogs but rather in analyzing a dialog which already took place.

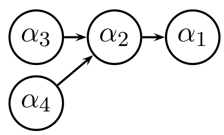
Definition 5 (Persuasion dialog) *A* persuasion dialog *D is a non-empty and finite sequence of moves $\langle m_1, \ldots, m_n \rangle$ s.t. the* subject *of D is $\texttt{Subject}(D) = \texttt{Content}(m_1)$, and the* length *of D, denoted $|D|$, is the number of moves: n. Each sub-sequence $\langle m_1, \ldots, m_i \rangle$ is a* sub-dialog *D^i of D, denoted by $D^i \sqsubseteq D$.*

An argumentation system is associated to each persuasion dialog in order to evaluate the status of its subject and that of each uttered argument.

Definition 6 (AS of a persuasion dialog) *Let $D = \langle m_1, \ldots, m_n \rangle$ be a persuasion dialog. The* argumentation system *of D is the pair $\mathsf{AS}_D = \langle \texttt{Args}(D), \texttt{Confs}(D) \rangle$ such that:*
- $\texttt{Args}(D) = \{\texttt{Content}(m_i) \mid i \in [\![1, n]\!]\}$
- $\texttt{Confs}(D) = \{(\alpha, \beta) \mid \alpha, \beta \in \texttt{Args}(D)$ *and* $(\alpha, \beta) \in \mathcal{R}_\mathcal{L}\}$

To put it differently, $\texttt{Args}(D)$ and $\texttt{Confs}(D)$ return respectively the set of arguments exchanged in a dialog and the different conflicts among them.

Example 1 *Let D_1 be a persuasion dialog between two agents a_1 and a_2 with $D_1 = \langle\langle a_1, \{a_2\}, \alpha_1\rangle, \langle a_2, \{a_1\}, \alpha_2\rangle, \langle a_1, \{a_2\}, \alpha_3\rangle, \langle a_1, \{a_2\}, \alpha_4\rangle, \langle a_2, \{a_1\}, \alpha_1\rangle\rangle$. The subject of D_1 is the argument α_1. Let us assume the following conflicts among some of these arguments.*



Thus, $\texttt{Args}(D_1) = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ *and* $\texttt{Confs}(D_1) = \{(\alpha_2, \alpha_1), (\alpha_3, \alpha_2), (\alpha_4, \alpha_2)\}$.

Remark 1 *For any sub-dialog $D' \sqsubseteq D$, $\texttt{Args}(D') \subseteq \texttt{Args}(D)$ and $\texttt{Confs}(D') \subseteq \texttt{Confs}(D)$.*

The *output* of a dialog is the status of the argument under discussion (i.e., the subject).

Definition 7 (Output of a persuasion dialog) *Let D be a persuasion dialog. The* output *of D, denoted by $\texttt{Output}(D)$, is $\texttt{Status}(\texttt{Subject}(D), \mathsf{AS}_D)$.*

Example 1 (Cont): The grounded extension of AS_{D_1} is the set $\{\alpha_1, \alpha_3, \alpha_4\}$. Thus, α_1 is accepted and consequently $\texttt{Output}(D_1)$ = Accepted.

In the rest of the paper, we evaluate the quality of a given persuasion dialog D according to three aspects:

1. the quality of the exchanged arguments
2. the behavior of each agent involved in the dialog
3. the conciseness of the dialog

We assume that the dialog D is finite. Note that this assumption is not too strong since a main property of any protocol is the termination of the dialogs it generates [13]. A consequence of this assumption is that the argumentation system AS_D associated to D is finite as well.

4 Measuring the quality of arguments

During a dialog, agents utter arguments that may have different *weights*. A weight may highlight the quality of information involved in the argument in terms, for instance, of certainty degree. It may also be related to the cost of revealing an information. In [1], several definitions of arguments' weights have been proposed, and their use for comparing arguments has been studied. It is worth noticing that the same argument may not have the same weight from one agent to another. In what follows, a weight in terms of a numerical value is associated to each argument. The greater this value is, the better the argument.

$$\texttt{weight} : \arg(\mathcal{L}) \longrightarrow \mathbb{N}^*$$

The function weight is given by *the agent who wants to analyze the dialog*. This agent may either be involved in the dialog or external. On the basis of arguments' weights, it is possible to compute the weight of a dialog as follows:

Definition 8 (Measure of dialog weight) *Let D be a persuasion dialog. The* weight *of D is* $\texttt{Weight}(D)$ = $\sum_{\alpha \in \texttt{Args}(D)} \texttt{weight}(\alpha)$

Property 1 *Let D be a persuasion dialog. $\forall D' \sqsubseteq D$,* $\texttt{Weight}(D') \leq \texttt{Weight}(D)$.

Proof *The result follows directly from Definition 8, the fact that* $\texttt{Args}(D') \subseteq \texttt{Args}(D)$, *and finally the fact that the function* weight *returns only positive values.* ■

This measure allows to compare pairs of persuasion dialogs only on the basis of the exchanged arguments. It is even more interesting when the two dialogs have the same subject and got the same output.

It is also possible to compute the weight of arguments uttered by each agent in a given dialog. For that purpose, one needs to know what has been said by each agent. This can be computed by a simple projection on the dialog given that agent. Note that this projection is not usually a sub-dialog of D (for instance, it may not contain m_1).

Definition 9 (Dialog projection) *Let $D = \langle m_1, \ldots, m_n \rangle$ be a persuasion dialog and $a_i \in$ Ag. The* projection *of D on agent a_i is $D^{a_i} = \langle m_{i_1}, \ldots, m_{i_k} \rangle$ such that $1 \leq i_1 \leq \ldots \leq i_k \leq n$ and $\forall l \in [1, k]$, $m_{i_l} \in D$ and* $\texttt{Speaker}(m_{i_l}) = a_i$.

The contribution of each agent is defined as follows:

Definition 10 (Measure of agent's contribution) *The* contribution *of an agent a_i in a dialog D is*

$$\texttt{Contr}(a_i, D) = \frac{\sum_{\alpha_i \in \texttt{Args}(D^{a_i})} \texttt{weight}(\alpha_i)}{\texttt{Weight}(D)}$$

Example 1 (Cont): $D_1^{a_1} = \{\alpha_1, \alpha_3, \alpha_4\}$ and $D_1^{a_2} = \{\alpha_1, \alpha_2\}$. Suppose that an external agent who wants to analyze this dialog assigns the following weights to arguments: $\texttt{weight}(\alpha_1)$ = 1, $\texttt{weight}(\alpha_2)$ = 4, $\texttt{weight}(\alpha_3)$ = 2 and $\texttt{weight}(\alpha_4)$ = 3. Note that $\texttt{Weight}(D_1)$ = 10. The contributions of the two agents are respectively $\texttt{Contr}(a_1, D_1)$ = 6/10 and $\texttt{Contr}(a_2, D_1)$ = 5/10.

Consider now an example in which an agent sends several times the same argument.

Example 2 *Consider a persuasion dialog D_2 between two agents a_1 and a_2 with* $\texttt{Args}(D_2) = \{\alpha, \beta\}$, $D_2^{a_1} = \{\alpha\}$ *and* $D_2^{a_2} = \{\beta\}$. *Assume that there are 50 moves in D_2 of which 49 moves are uttered by agent a_1 and one move uttered by a_2. Assume also that an external agent assigns the following weights to arguments:* $\texttt{weight}(\alpha)$ *= 1 and* $\texttt{weight}(\beta)$ *= 30. The overall weight of the dialog is* $\texttt{Weight}(D_2)$ *= 31. The contributions of the two agents are respectively* $\texttt{Contr}(a_1, D_2)$ *= 1/31 and* $\texttt{Contr}(a_2, D_2)$ *= 30/31.*

It is easy to check that when the protocol under which a dialog is generated does not allow an agent to repeat an argument already given by another agent, then the sum of the contributions of the different agents is equal to 1.

Property 2 *Let $D = \langle m_1, \ldots, m_n\rangle$ be a persuasion dialog and $a_1, \ldots, a_m$ the agents involved in D.* $\sum_{i=1,\ldots,m} \texttt{Contr}(a_i, D)$ *= 1 iff $\nexists m_i, m_j$, $1 \leq i, j \leq n$, such that* $\texttt{Speaker}(m_i) \neq \texttt{Speaker}(m_j)$ *and* $\texttt{Content}(m_i) = \texttt{Content}(m_j)$.

Proof *The proof follows directly from the definition.* ■

As we will see in the next section, a more specific measure of contribution maybe defined if we focus on formulas that are involved in arguments. Indeed, contribution may be defined on the basis of formulas revealed by each agent. This requires to assign weights to formulas instead of arguments.
It is worth noticing that measure Contr is *not monotonic* since the contribution of an agent may change during a dialog. However, at a given step of a dialog, the contribution of the agent who will present the next move will never decrease, whereas the contributions of the other agents may decrease.

Proposition 2 *Let $D = \langle m_1, \ldots, m_n\rangle$ be a persuasion dialog, $a_i \in \texttt{Ag}$ and m be a move such that* $\texttt{Speaker}(m) = a_i$. *It holds that* $\texttt{Contr}(a_i, D \oplus m) \geq \texttt{Contr}(a_i, D)$ *and $\forall a_j \in \texttt{Ag}$ with $a_j \neq a_i$,* $\texttt{Contr}(a_j, D \oplus m) \leq \texttt{Contr}(a_j, D)$, *with $D \oplus m = \langle m_0, \ldots, m_n, m\rangle$.*

5 Analyzing the behavior of agents

The behavior of an agent in a given persuasion dialog may be analyzed on the basis of three main criteria: i) her degree of *aggressiveness* in the dialog, ii) the source of her arguments, i.e. whether she builds arguments using her own formulas, or rather the ones revealed by other agents, and finally iii) her degree of *coherence* in the dialog.

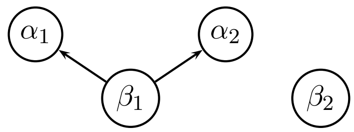
The first criterion, i.e. the aggressiveness of an agent in a dialog, amounts to computing to what extent an agent was attacking arguments sent by other agents. An aggressive agent prefers to destroy arguments presented by other parties rather than presenting arguments supporting her own point of view. Formally, the *aggressiveness degree* of an agent a_i towards an agent a_j during a persuasion dialog is equal to the number of its arguments that attack the other agent's arguments over the number of arguments it has uttered in that dialog.

Definition 11 (Measure of aggressiveness) *Let D be a persuasion dialog and $a_i, a_j \in \texttt{Ag}$. The* aggressiveness degree *of agent a_i towards a_j in D is*

$$\texttt{Agr}(a_i, a_j, D) = \frac{|\{\alpha \in \texttt{Args}(D^{a_i}) \text{ such that } \exists \beta \in \texttt{Args}(D^{a_j}) \text{ and } (\alpha,\beta) \in \texttt{Confs}(D)\}|^2}{|\texttt{Args}(D^{a_i})|}.$$

[2]The expression $|E|$ denotes the cardinal of the set E.

Example 3 *Let D_3 be a persuasion dialog between two agents a_1 and a_2. Assume that* $\texttt{Args}(D_3)$ = $\{\alpha_1, \alpha_2, \beta_1, \beta_2\}$*,* $D_3^{a_1}$ = $\{\alpha_1, \alpha_2\}$*,* $D_3^{a_2}$ = $\{\beta_1, \beta_2\}$ *and the conflicts are depicted in the figure below.*



The aggressiveness degrees of the two agents are $\texttt{Agr}(a_1, a_2, D_3)$ *= 0 and* $\texttt{Agr}(a_2, a_1, D_3)$ *= 1/2.*

The aggressiveness degree of an agent changes as soon as a new argument is uttered by that agent. It decreases when that argument does not attack any argument of the other agent, and increases otherwise.

Proposition 3 *Let D = $\langle m_1, \ldots, m_n \rangle$ be a persuasion dialog and $a_i, a_j \in$* $\texttt{Ag}$*. Let m be a move such that* $\texttt{Speaker}(m) = a_i$ *and $D \oplus m$ = $\langle m_1, \ldots, m_n, m \rangle$,*

$$\texttt{Agr}(a_i, a_j, D \oplus m) \geq \texttt{Agr}(a_i, a_j, D) \quad \textit{iff} \quad \exists \alpha \in \texttt{Args}(D^{a_j}) \textit{ such that } (\texttt{Content}(m), \alpha) \in \mathcal{R}_\mathcal{L}$$

The second criterion concerns the source of arguments. An agent can build her arguments either from her own knowledge base using her own formulas, or using formulas revealed by other agents in the dialog. In [5], this idea of borrowing formulas from other agents has been presented as one of the tactics used by agents for selecting the argument to utter at a given step of a dialog. The authors argue that by doing so, an agent minimizes the risk of being attacked subsequently. Let us now check to what extent an agent borrows information from other agents. Before that, let us first determine which formulas are owned by each agent according to what has been said in a dialog. Informally, a formula is owned by an agent if it is revealed for the first time by that agent. Note that a formula revealed for the first time by agent a_i may also pertain to the base of another agent a_j but, here, we are interested by *who reveals first* that formula.

Definition 12 (Agent's formulas) *Let D = $\langle m_1, \ldots, m_n \rangle$ be a persuasion dialog and $a_i \in$* $\texttt{Ag}$*. The* formulas owned *by agent a_i are:* $\texttt{OwnF}(a_i, D) =$

$$\left\{x \in \mathcal{L} | \exists m_j \textit{ with } j \leq n \textit{ and } \left| \begin{array}{l} \texttt{Speaker}(m_j) = a_i \textit{ and } x \in \texttt{Support}(\texttt{Content}(m_j)) \\ \textit{and } \nexists m_k \textit{ with } k < j \textit{ and } \left| \begin{array}{l} \texttt{Speaker}(m_k) \neq a_i \\ \textit{and } x \in \texttt{Support}(\texttt{Content}(m_k)) \end{array} \right. \end{array} \right. \right\}$$

Now that we know which formulas are owned by each agent, we can compute the *degree of loan* of each agent. Note that from a strategical point of view, it is interesting to turn out an agent's argument against her in order to weaken her position. The borrowing degree can thus help for evaluating the strategical behavior of an agent.

Definition 13 (Measure of loan) *Let D be a persuasion dialog and $a_i, a_j \in$* $\texttt{Ag}$*. The* loan degree *of agent a_i from agent a_j in D is:* $\texttt{Loan}(a_i, a_j, D) = \frac{|\texttt{Formulas}(\texttt{Args}(D^{a_i})) \cap \texttt{OwnF}(a_j, D)|}{|\texttt{Formulas}(\texttt{Args}(D^{a_i}))|}$.

It is worth mentioning that if agents do not borrow any formula to each others, then their contributions are independent. Hence, due to proposition 2, the sum of these contributions is equal to 1.

Proposition 4 *Let $a_1, \ldots, a_m \in$* $\texttt{Ag}$ *be the agents involved in a persuasion dialog D. If $\forall i \neq j$,* $\texttt{Loan}(a_i, a_j, D) = 0$*, then* $\sum_{i=1,\ldots,m} \texttt{Contr}(a_i, D) = 1$.

The third criterion concerns the coherence of an agent. Indeed, in a persuasion dialog where an agent a_i defends her point of view, it is important to detect when this agent contradicts herself. There are two kinds of self contradiction:

1. an *explicit* contradiction in which an agent presents an argument and a counter-argument in the same dialog. Such conflicts appear in the argumentation system $\mathsf{AS}_{D^{a_i}} = \langle \texttt{Args}(D^{a_i}), \texttt{Confs}(D^{a_i}) \rangle$ associated to the moves uttered by agent a_i. Thus, the set $\texttt{Confs}(D^{a_i})$ is not empty.

2. an *implicit* contradiction appearing in a "complete" version of the agent's argumentation system.

The complete version of an argumentation system takes into account not only the set of arguments which are explicitly expressed in a dialog by an agent, i.e. $\texttt{Args}(D^{a_i})$, but also all the arguments that may be built from the set of formulas involved in the arguments of $\texttt{Args}(D^{a_i})$. Due to the monotonic construction of arguments, for any set A of arguments, $A \subseteq \arg(\texttt{Formulas}(A))$ but the reverse is not necessarily true. As a consequence, new conflicts may appear. This shows clearly that the argumentation system associated with a dialog is not necessarily "complete".

Definition 14 (Complete AS) *The* complete AS *of a persuasion dialog D is*

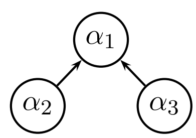
$$\mathsf{CAS}_D = \langle \arg(\texttt{Formulas}(\texttt{Args}(D))), \mathcal{R}_c \rangle$$

where $\mathcal{R}_c = \{(\alpha, \beta)$ such that $\alpha, \beta \in \arg(\texttt{Formulas}(\texttt{Args}(D)))$ and $(\alpha, \beta) \in \mathcal{R}_\mathcal{L}\}$.

This definition is valid for any dialog projection D^{a_i}. Recall that $\texttt{Args}(D) \subseteq \arg(\texttt{Formulas}(\texttt{Args}(D)))$ $\subseteq \arg(\mathcal{L})$ and $\texttt{Confs}(D) \subseteq \mathcal{R}_c \subseteq \mathcal{R}_\mathcal{L}$. Note also that the status of an argument α in a system AS_D is not necessarily the same in the complete system CAS_D. The next definition evaluates to what extent an agent is incoherent in a dialog.

Definition 15 (Measure of incoherence) *Let D be a persuasion dialog, $a_i \in \texttt{Ag}$ and $\mathsf{CAS}_{D^{a_i}} = \langle \mathcal{A}_c^{a_i}, \mathcal{R}_c^{a_i} \rangle$. The* incoherence degree *of agent a_i in D is $\texttt{Inc}(a_i, D) = \frac{|\mathcal{R}_c^{a_i}|}{|\mathcal{A}_c^{a_i} \times \mathcal{A}_c^{a_i}|}$.*

Example 4 *Let D_4 be a persuasion dialog in which agent a_1 has uttered two arguments α_1 and α_2. Let us assume that from the formulas of those arguments a third argument, say α_3, is built. The figure below depicts the conflicts among the three arguments. The incoherence degree of agent a_1 is equal to 2/9.*



Note that, the above definition is general enough to capture both explicit and implicit contradictions. Moreover, this measure is more precise than the one defined on the basis of attacked arguments, i.e. $\texttt{Inc\_bis}(a_i, D) = \frac{|\{\beta \in \mathcal{A}_c^{a_i} \text{ such that } \exists (\alpha,\beta) \in \mathcal{R}_c^{a_i}\}|}{|\mathcal{A}_c^{a_i}|}$. Using this measure, the incoherence degree of agent a_1 is 1/3. Even if the argument α_1 is attacked by two arguments, only one conflict is considered.

It is easy to check that if an agent is aggressive towards herself, then she is incoherent.

Property 3 *Let D be a persuasion dialog and $a_i \in \texttt{Ag}$. If $\texttt{Agr}(a_i, a_i, D) > 0$, then $\texttt{Inc}(a_i, D) > 0$.*

Proof *Let D be a persuasion dialog and $a_i \in$ `Ag`. Assume that $\texttt{Agr}(a_i, a_i, D) > 0$. This means that $\exists(\alpha, \beta) \in \texttt{Confs}(D^{a_i})$. Consequently, $|\mathcal{R}_c^{a_i}| > 0$. This is due to the fact that $\texttt{Confs}(D^{a_i}) \subseteq \mathcal{R}_c^{a_i}$.* ■

The following example shows that the reverse is not always true.

Example 5 *Let D_5 be a persuasion dialog and $a_i \in$ `Ag`. Assume that $\texttt{Args}(D_5^{a_i}) = \{\alpha_1, \alpha_2\}$, and $\texttt{Confs}(D_5^{a_i}) = \emptyset$. It means that $\texttt{Agr}(a_i, a_i, D_5) = 0$. Suppose that $\mathsf{CAS}_{D_5^{a_i}} = \langle \{\alpha_1, \alpha_2, \alpha_3\}, \{(\alpha_3, \alpha_1), (\alpha_3, \alpha_2)\}\rangle$ is its associated complete argumentation system. It is clear that $\texttt{Inc}(a_i, D_5)$ = 2/9.*

Similarly, it can be shown that if agent a_i is aggressive towards agent a_j and if all the formulas of a_i are borrowed from a_j, then a_j is for sure incoherent. Note that a_i might be coherent if she has not used conflicting arguments.

Proposition 5 *Let D be a persuasion dialog and $a_i, a_j \in$ `Ag`. If $\texttt{Loan}(a_i, a_j, D) = 1$ and $\texttt{Agr}(a_i, a_j, D) > 0$, then $\texttt{Inc}(a_j, D) > 0$.*

Proof *Let $\mathsf{CAS}_{D^{a_i}} = \langle \mathcal{A}_c^{a_i}, \mathcal{R}_c^{a_i}\rangle$ and $\mathsf{CAS}_{D^{a_j}} = \langle \mathcal{A}_c^{a_j}, \mathcal{R}_c^{a_j}\rangle$. It is clear that $\texttt{Loan}(a_i, a_j, D) = 1$ means that every formula used by a_i has been first revealed by a_j, it implies that $\mathcal{A}_c^{a_i} \subseteq \mathcal{A}_c^{a_j}$ (1). Now if $\texttt{Agr}(a_i, a_j, D) > 0$ then it means that $\exists \alpha \in \texttt{Args}(D^{a_i})$ that is attacked by an argument of $\texttt{Args}(D^{a_j})$. From (1), we get that $\alpha \in \mathcal{A}_c^{a_j}$ hence a_j is self-contradicting.* ■

Note that incoherence is not necessarily a bad behavior, it depends on the aim of the participants: the goal may either be to win the debate whatever the other says or to discuss and take into account new information. In the last case, changing its opinion is a self-contradiction but may be a constructive attitude.

6 Measuring the conciseness of a dialog

It is very common that a dialog contains redundancies or useless moves. Thus, only some arguments may be useful for computing the output of the dialog. In this section, we are interested in characterizing the useful moves in a dialog and identifying the *ideal* version of a dialog. We start by presenting different criteria for evaluating each move in a dialog, then we provide a procedure for computing the ideal version of a given dialog.

6.1 Quality of moves

In everyday life, it is very common that agents deviate from the subject of the dialog. We first define a criterion that evaluates to what extent the moves uttered are in relation with the subject of the dialog. This amounts to check whether there exists a path from the argument presented by the agent towards the argument representing the subject in the graph of the argumentation system associated to the dialog.

Definition 16 (Relevant and useful move) *Let $D = \langle m_1, \ldots, m_n\rangle$ be a persuasion dialog. A move m_i, with $i \in [\![1, n]\!]$, is* relevant *to D iff there exists a path (not necessarily directed) from $\texttt{Content}(m_i)$ to $\texttt{Subject}(D)$ in the directed graph associated with AS_D. A move m_i is* useful *iff there exists a directed path from $\texttt{Content}(m_i)$ to $\texttt{Subject}(D)$ in this graph.*

Example 3 (Cont): Assume that $\texttt{Subject}(D_3) = \alpha_1$. It is clear that α_3, β_1 are relevant while β_2 is not and that β_1 is useful while α_3 is not.

Property 4 *If a move m is useful in a dialog D, then m is relevant to D.*

Proof *If a move m is useful then there exists a directed path from $\texttt{Content}(m)$ to $\texttt{Subject}(D)$, thus m is relevant to D.* ■

One can define a measure, called $\texttt{Relevance}(D)$, that computes the percentage of moves that are relevant in a dialog D[3]. In Example 3, $\texttt{Relevance}(D)$ = 3/4. It is clear that the greater this degree is, the better the dialog. When the relevance degree of a dialog is equal to 1, this means that agents did not deviate from the subject of the dialog. Useful moves are those that have a more direct influence on the status of the subject. However, this does not mean that their presence has an impact on the output of the dialog. Moves that have a real impact on the status of the subject are said *decisive*.

Definition 17 (Decisive move) *Let $D = \langle m_1, \ldots, m_n\rangle$ be a persuasion dialog and AS_D its argumentation system. A move m_i, with $i \in [\![1, n]\!]$, is* decisive *in D iff*

$$\texttt{Status}(\texttt{Subject}(D), \mathsf{AS}_D) \neq \texttt{Status}(\texttt{Subject}(D), \mathsf{AS}_D \ominus \texttt{Content}(m_i))$$

where $\mathsf{AS}_D \ominus \texttt{Content}(m_i) = \langle A', R'\rangle$ such that $A' = \texttt{Args}(D) \setminus \{\texttt{Content}(m_i)\}$ and $R' = \texttt{Confs}(D) \setminus \{(x, \texttt{Content}(m_i)), (\texttt{Content}(m_i), x) \mid x \in \texttt{Args}(D)\}$.

It can be checked that if a move is decisive, then it is useful. This means that there exists a directed path from the content of this move to the subject of the dialog in the graph of the argumentation system associated to the dialog.

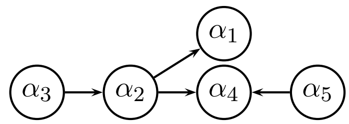
Proposition 6 *If a move m is decisive in a persuasion dialog D, then m is useful in D.*

Proof *Assume that m is a decisive move in D and that $\texttt{Subject}(D)$ is accepted in AS_D. According to Proposition 1, for any attacker of $\texttt{Subject}(D)$, $\texttt{Subject}(D)$ is indirectly defended by a non-attacked argument. Since m is decisive, $\texttt{Subject}(D)$ is rejected in $\mathsf{AS}_D \ominus \texttt{Content}(m)$. This means that at least one attacker is no more indirectly defended by a non-attacked argument. Hence, removing $\texttt{Content}(m)$ eliminates a path from a non-attacked argument to this attacker. Hence $\texttt{Content}(m)$ is useful.*
If $\texttt{Subject}(D)$ is rejected in AS_D and accepted in $\mathsf{AS}_D \ominus \texttt{Content}(m)$. This means that every attacker is defended by a non-attacked argument in $\mathsf{AS}_D \ominus \texttt{Content}(m)$. Hence the deletion of $\texttt{Content}(m)$ has eliminated every direct or indirect attacker of the subject. This means that $\texttt{Content}(m)$ was on a path from an attacker to the subject hence it was useful in D. ■

From Property 4, it follows that each decisive move is also relevant. Note that the converse is not true as shown in the following example.

Example 6 *Let D_6 be a dialog whose subject is α_1 and whose graph is the following:*

[3] $\texttt{Relevance}(D) = \frac{|\{m_{i=1,\ldots,n} \text{ such that } m_i \text{ is relevant to D}\}|}{|D|}$



The grounded extension of AS_{D_6} is $\{\alpha_1, \alpha_3, \alpha_5\}$. It is clear that the argument α_4 is relevant to α_1, but it is not decisive for D_6. Indeed, the removal of α_4 will not change the status of α_1 which is accepted.

The converse of Proposition 6 is not true since useful moves may not be decisive:

Example 7 *Let D_7 be a dialog whose argumentation system is the one given in Example 4 and whose subject is α_1. Note that neither α_2 nor α_3 is decisive in D_7. However, this does not mean that the two arguments should be removed since the status of α_1 depends on at least one of them (they are both useful).*

On the basis of the above notion of decisiveness of moves, we can define *the degree of decisiveness* of the entire dialog as the percentage of moves that are decisive.

6.2 Canonical dialogs

As shown in the previous sub-section, some moves may not be important in a dialog and removing them does not have any impact on the output of the dialog. In this section, we characterize sub-dialogs, called *canonical*, which return the same output as an original dialog. In [2], a proof procedure that tests the membership of an argument to a grounded extension has been proposed. The basic notions of this procedure are revisited and adapted for the purpose of characterizing canonical dialogs.

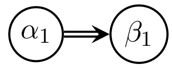
Definition 18 (Dialog branch) *Let D be a persuasion dialog and $\mathsf{AS}_D = \langle \mathtt{Args}(D), \mathtt{Confs}(D)\rangle$ its argumentation system. A* dialog branch *for D is a sequence $\langle \alpha_0, \ldots, \alpha_p\rangle$ of arguments such that $\forall i, j \in [\![0, p]\!]$*

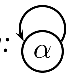
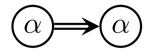
1. $\alpha_i \in \mathtt{Args}(D)$
2. $\alpha_0 = \mathtt{Subject}(D)$
3. *if $i \neq 0$ then $(\alpha_i, \alpha_{i-1}) \in \mathtt{Confs}(D)$*
4. *if i and j are even and $i \neq j$ then $\alpha_i \neq \alpha_j$*
5. *if i is even and $i \neq 0$ then $(\alpha_{i-1}, \alpha_i) \notin \mathtt{Confs}(D)$*
6. *$\forall \beta \in \mathtt{Args}(D)$, $\langle \alpha_0, \ldots, \alpha_p, \beta\rangle$ is not a dialog branch for D.*

Intuitively, a dialog branch is a kind of partial sub-graph of AS_D in which the nodes contains arguments and the arcs represents inverted conflicts. Note that arguments that appear at even levels are not allowed to be repeated. Moreover, these arguments should strictly attack[4] the preceeding argument. The last point requires that a branch is maximal. Let us illustrate this notion on examples.

Example 3 (Cont): The only dialog branch that can be built from dialog D_3 is:

[4]An argument α strictly attacks an argument β in a argumentation system $\langle \mathcal{A}, \mathcal{R}\rangle$ iff $(\alpha, \beta) \in \mathcal{R}$ and $(\beta, \alpha) \notin \mathcal{R}$.



Example 8 *Let D_8 be a persuasion dialog whose subject is α and whose graph is the following:* 
The only possible dialog branch associated to this dialog is the following: 

Proposition 7 *A dialog branch is non-empty and finite.*

Proof
- A dialog branch is non-empty since the subject of the original persuasion dialog belongs to the branch.
- Let us assume that there exists an infinite dialog branch for a given persuasion dialog D. This means that there is an infinite sequence $\langle \alpha_0, \alpha_1, \ldots \rangle$ that forms a dialog branch. In this sequence, the number of arguments of even index and of odd index are infinite. According to Definition 5, the persuasion dialog D is finite, thus both sets $\texttt{Args}(D)$ and $\texttt{Confs}(D)$ are finite. Consequently, the set of arguments that belong to the sequence $\langle \alpha_0, \alpha_1, \ldots \rangle$ is finite. Hence, there is at least one argument that is repeated at an even index. This is impossible. ■

Moreover, it is easy to check the following result:

Proposition 8 *For each dialog branch $\langle \alpha_0, ..., \alpha_k \rangle$ of a persuasion dialog D there exists a unique directed path $(\alpha_k, \alpha_{k-1}, ..., \alpha_0)$ of same length*[5] *(k) in the directed graph associated to AS_D.*

Proof *Let $\langle \alpha_0, ..., \alpha_k \rangle$ be a dialog branch for D, from Definition 18.3, it follows that $\forall i \in [\![1, k]\!]$, $(\alpha_i, \alpha_{i-1}) \in Confs(D)$. Hence there is a path of length k in AS_D from α_k to α_0. From Definition 18.2, $\alpha_0 = Subject(D)$.* ■

In what follows, we show that when a dialog branch is of even-length, then its leaf is not attacked in the original dialog.

Theorem 1 *$\langle \alpha_0, .., \alpha_p \rangle$ being a dialog branch for D, if p is even then $\nexists \beta \in \texttt{Args}(D)$ such that $(\beta, \alpha_p) \in \texttt{Confs}(D)$*

Proof *If $\exists \beta \in \texttt{Args}(D)$ such that $(\beta, \alpha_p) \in \texttt{Confs}(D)$ then a new sequence beginning by $\langle \alpha_0, \ldots \alpha_p, \beta \rangle$ would be a dialog branch, which is forbidden by Definition 18.6.* ■

Let us now introduce the notion of a dialog tree.

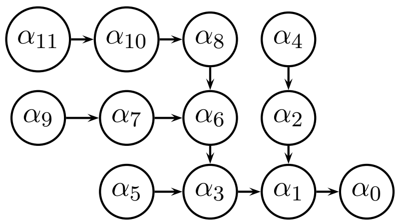
Definition 19 (Dialog tree) *A* dialog tree *of D, denoted by D^t, is a finite tree whose branches are all the possible dialog branches that can be built from D.*
We denote by AS_{D^t} the argumentation system associated to D^t, $\mathsf{AS}_{D^t} = \langle A^t, C^t \rangle$ such that $A^t = \{\alpha \in \texttt{Args}(D)$ such that α appears in a node of $D^t\}$ and $C^t = \{(\alpha, \beta) \in \texttt{Confs}(D)$ such that (β, α) is an arc of $D^t\}$.

Hence, a dialog tree is a tree whose root is the subject of the persuasion dialog.

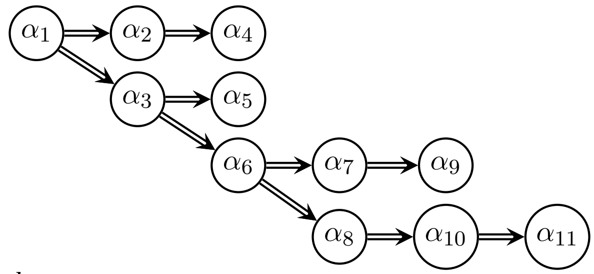
[5]The length of a path is defined by its number of arcs.

Example 9

Let us consider D_9 whose subject is α_1 and whose graph is the following:



The dialog tree associated to this dialog is:



Note that the argument α_0 does not belong to the dialog tree.

Proposition 9 *Each persuasion dialog has exactly one corresponding dialog tree.*

Proof *This follows directly from the definition of the dialog tree. Indeed, the root of the tree is the subject of the persuasion dialog. Moreover, all the possible branches are considered.* ■

An important result states that the status of the subject of the original persuasion dialog D is exactly the same in both argumentation systems AS_D and AS_{D^t} (where AS_{D^t} is the argumentation system whose arguments are all the arguments that appear in the dialog tree D^t and whose attacks are obtained by inverting the arcs between those arguments in D^t).

Theorem 2 $\texttt{Status}(\texttt{Subject}(D), \mathsf{AS}_D) = \texttt{Status}(\texttt{Subject}(D), \mathsf{AS}_{D^t})$.

Proof *The proof of this theorem is based on two theorems given farther that are referring to the notion of canonical tree.*

- *If $\texttt{Subject}(D)$ is accepted in AS_D. then using Theorem 4 we get that there exists a canonical tree D_i^c such that $\texttt{Subject}(D)$ is accepted in $\mathsf{AS}_{D_i^c}$. Moreover, the way D_i^c has been constructed (by an AND/OR process) imposes that D_i^c contains every direct child of the subject in D^t. Furthermore, Theorem 3 shows that every branch of D_i^c is of even length. Every leaf of this canonic tree, by definition, is non-attacked in D_i^c and by definition in AS_{D^t}. Using Definition 18.4 we get that in each branch of AS_{D^t}, each even node strictly attacks the previous node. Hence, by construction, for each direct attacker of the subject in AS_{D^t}, there exists at least one defender non-attacked in AS_{D^t} (leaf of D_i^c), the defense being strict, the subject belongs to the basic extension of AS_{D^t}.*

- *If $\texttt{Subject}(D)$ is accepted in AS_{D^t} then there exists a non-attacked defender against every direct attacker of the subject in AS_{D^t}. This means that there exists a canonical tree based on AS_{D^t} having only even length branches. The subject is accepted in this canonical tree using Theorem 3, which implies that the subject is accepted in D using Theorem 4.* ■

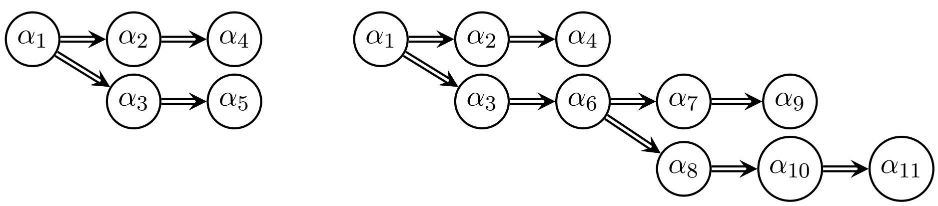
In order to compute the status of the subject of a dialog, we can consider the dialog tree as an And/Or tree. A node of an even level is an And node, whereas a node of odd level is an Or one. This distinction between nodes is due to the fact that an argument is accepted if it can be defended against all its attackers. A dialog tree can be decomposed into one or several trees called canonical trees. A *canonical tree* is a subtree of D^t whose root is $\texttt{Subject}(D)$ and which contains all the arcs starting from an even node and exactly one arc starting from an odd node.

Definition 20 (Canonical tree) *Let D be a persuasion dialog, and let D^t its dialog tree. D^c is a* canonical tree *of D^t if it is a subtree of D^t built by levels as follows:*

- `Subject`(D) *is its root (of level 0)*
- *and inductively:*
  - *if α is a node of even level in D^c then for **every** $\beta \in D^t$ such that $(\alpha, \beta) \in D^t$, the node β and the arc (α, β) is added to D^c.*
  - *if α is a node of odd level in D^c and if α has at least one attacker in D^t then for **exactly one** $\beta \in D^t$ such that $(\alpha, \beta) \in D^t$, the node β and the arc (α, β) is added to D^c.*

It is worth noticing that from a dialog tree one may extract at least one canonical tree. Let $D_1^c, \ldots, D_m^c$ denote those canonical trees. We will denote by $\mathsf{AS}_1^c, \ldots, \mathsf{AS}_m^c$ their corresponding argumentation systems. It can be checked that the status of `Subject`(D) is not necessarily the same in these different systems.

Example 10 *From the dialog tree of D_9, two canonical trees can be extracted:*



It can be checked that the argument α_1 is accepted in the argumentation system of the canonical tree on the left while it is rejected in the one of the right.

The following result characterizes the status of `Subject`(D) in the argumentation system AS_i^c associated to a canonical tree D_i^c.

Theorem 3 *Let D be a persuasion dialog, D_i^c a canonical tree and AS_i^c its corresponding argumentation system.* `Subject`(D) *is accepted in AS_i^c iff all the branches of D_i^c are of even-length.*

Proof *Let D be a persuasion dialog, D_i^c a canonical tree and AS_i^c its corresponding argumentation system.*

- *Assume that* `Subject`(D) *is accepted in AS_i^c, and that there is a branch of D_i^c whose length is odd. This means that the leaf of this branch, say α, indirectly attacks* `Subject`(D) *(the root of the branch).*
  - *Either α is not attacked in AS_i^c it means that α is accepted hence the second node of the branch is a direct attacker of* `Subject`(D) *that is not defended by a non attacked argument, i.e.,* `Subject`(D) *would not be accepted in AS_i^c.*
  - *Either α is attacked in AS_i^c then it can only be attacked by an argument already present in the branch (hence itself attacked), else the branch would not satisfied Definition18.6. This also means that the second node of the branch is a direct attacker of* `Subject`(D) *that is not defended by a non attacked argument.*

- *Assume now that all the branches of D_i^c are of even length, then for each branch the leaf is accepted since it is not attacked in AS_i^c (using Theorem 1). Then iteratively considering each even node from the leaf to the root, they can all be added to the grounded extension since the leaf defends the penultimate even node against the attack of the last odd node and so on and by construction for each odd node attacking an even node there is a deeper even node that strictly defends it (due to Definition 18.5). Hence each even node is in the grounded extension, so* $\texttt{Subject}(D)$ *is accepted in* AS_i^c ■

The following result follows immediately from this Theorem and Theorem 1.

Corollary 1 *Let D be a persuasion dialog, D_i^c a canonical tree and AS_i^c its corresponding argumentation system. If* $\texttt{Subject}(D)$ *is accepted in AS_i^c, then all the leaves of D_i^c are not attacked in D.*

Proof *According to Theorem 3, since* $\texttt{Subject}(D)$ *is accepted in AS_i^c, then all its branches are of even-length. According to Theorem 1, the leaf of each branch of even-length is an argument that is not attacked in D. Thus, all the leaves of D_i^c are not attacked in D.* ■

An important result shows the link between the outcome of a dialog D and the outcomes of the different canonical trees.

Theorem 4 *Let D be a persuasion dialog, D_1^c, ..., D_m^c its different canonical trees and $\mathsf{AS}_1^c, \ldots, \mathsf{AS}_m^c$ their corresponding argumentation systems.*
$\texttt{Output}(D)$[6] *is accepted iff $\exists\, i \in [\![1, m]\!]$ such that* $\texttt{Status}(\texttt{Subject}(D), \mathsf{AS}_i^c)$ *is accepted.*

Proof *Let D be a persuasion dialog, D_1^c, ..., D_m^c its different canonical trees and $\mathsf{AS}_1^c, \ldots, \mathsf{AS}_m^c$ their corresponding argumentation systems.*

- *Let us assume that there exists D_j^c with $1 \leq j \leq m$ and* $\texttt{Status}(\texttt{Subject}(D), \mathsf{AS}_j^c)$ *is accepted. According to Theorem 3, this means that all the branches of D_j^c are of even length. From Corollary 1, it follows that the leaves of D_j^c are all not attacked in the graph of the original dialog D.*

  Let $2i$ be the depth of D_j^c (i.e. the maximum number of moves of all dialog branches of D_j^c).

  We define the height of a node N in a tree as the depth of the sub-tree of root N.

  We show by induction on p that $\forall$ p such that $0 \leq p \leq i$, the set $\{y|y$ is an argument of even indice and in a node of height $\leq 2p$ belonging to $D_j^c\}$ is included in the grounded extension of AS_D).

  - *Case $p = 0$. The leaves of D_j^c are not attacked in D (according to Corollary 1). Thus, they belong to the grounded extension of AS_D.*
  - *Assume that the property is true to an order p and show that it is also true to the order $p + 1$. It is sufficient to consider the arguments that appear at even levels and in a node of height $2p + 2$ of D_j^c. Let y be such an argument. Since y appears at an even level, then all the arguments y' attacking y in AS_D appear in D_j^c as children of y (otherwise the branch would not be maximal or D_j^c would not be canonic), and each y' is itself strictly attacked in AS_D by*

[6] Recall that $\texttt{Output}(D)$ = $\texttt{Status}(\texttt{Subject}(D), \mathsf{AS}_D)$.

exactly one argument z appearing in D_j^c as a child of y'. Thus, each z is at an even level in D_j^c and appears as a node of height $2p$ of D_j^c. By induction hypothesis, each argument z is in the grounded extension of AS_D. Since all attackers of y have been considered, thus the grounded extension of AS_D defends y. Consequently y is also in this grounded extension.

- *Let us assume that $\texttt{Status}(\texttt{Subject}(D), \mathsf{AS}_D)$ is accepted. Let i_0 be the smallest index ≥ 0 such that $\texttt{Subject}(D) \in \mathcal{F}^{i_0}(\mathcal{C}^7)$. Let us show by induction on i that if an argument $\alpha \in \texttt{Args}(D)$ is in $\mathcal{F}^i(\mathcal{C})$ then there exists a canonical tree of root α for D^8 having a depth $\leq 2i$ and having only branches of even length.*
  - *Case $i = 0$: if $\alpha \in \mathcal{C}$, then α itself is a canonical tree of root α and depth 0.*
  - *Assume that the property is true at order i and consider the order $i+1$. Hence, let us consider $\alpha \in \mathcal{F}^{i+1}(\mathcal{C})$ and $\alpha \notin \mathcal{F}^k(\mathcal{C})$ with $k < i+1$.*
    Let $x_1, \ldots, x_n$ be the attackers of α. Consider an attacker x_j. x_j attacks α, and $\alpha \in \mathcal{F}^{i+1}(\mathcal{C})$ = $\mathcal{F}(\mathcal{F}^i(\mathcal{C}))$. According to Proposition 4.1 in [2], it exists y in the grounded extension of AS_D such that y attacks strictly x_j. Since y defends α (definition of $\mathcal{F}$) then $y \in \mathcal{F}^i(\mathcal{C})$. By induction hypothesis applied to y, there exists a canonical tree whose root is y and the depth is $\leq 2i$. The same construction is done for each x_j. So we get a canonical tree whose root is α and its depth is $\leq 2(i+1)$ and in which each branch has still an even length.

  Now, from the fact that $\texttt{Subject}(D) \in \mathcal{F}^{i_0}(\mathcal{C})$ we conclude that it exists a canonical tree of root $\texttt{Subject}(D)$ having each branch of even length. Using Theorem 3, we get that $\texttt{Subject}(D)$ is accepted in this canonical tree. ■

This result is of great importance since it shows that a canonical tree whose branches are all of even-length is sufficient to reach the same outcome as the original dialog in case the subject is accepted. When the subject is rejected, the whole dialog tree is necessary to ensure the outcome.

Example 9 (Cont): The subject α_1 of dialog D_9 is accepted since there is a canonical tree whose branches are of even length (it is the canonical tree on the left in Example 10). It can also be checked that α_1 is in the grounded extension $\{\alpha_1, \alpha_4, \alpha_5, \alpha_8, \alpha_9, \alpha_{11}\}$ of AS_{D_9}.

So far, we have shown how to extract from a graph associated with a dialog its canonical trees. These canonical trees contain only useful (hence relevant) moves:

Theorem 5 *Let D_i^c be a canonical tree of a persuasion dialog D. Any move built on an argument of D_i^c is* useful *in the dialog D.*

Proof *By construction of D_i^c, there is a path in this tree from the root to each argument α of the canonical tree. According to Proposition 8, we get that there exists a corresponding directed path in AS_D from α to $\texttt{Subject}(D)$, hence a move containing the argument α is useful in D.* ■

[7] The set $\mathcal{C}$ contains all the arguments that are not attacked in D.

[8] Here, we consider a "canonical tree of root α for a dialog D". Its definition is more general than canonical tree for a dialog D since it does not requires that all the branches start from the subject of the dialog (modifying item 2 of Definition 18) but requires that all the branches start from the node α.

The previous theorem gives an upper bound of the set of moves that can be used to build a canonical tree, a lower bound is the set of decisive moves.

Theorem 6 *Every argument of a decisive move belongs to the dialog tree and to each canonical tree.*

Proof *If a move m is decisive then, as seen in the proof of proposition 6,*

- *if the subject is accepted in AS_D then it exists at least a direct attacker of the subject that is no more inderectly defended by a non attacked argument in $\mathsf{AS}_D \ominus \texttt{Content}(m)$. The subject being accepted in AD_D, this means that there is a canonical tree having only branches of even length (according to Theorem 3). By construction, this canonic tree contains every direct attacker of the subject. If $\texttt{Content}(m)$ does not belong to this canonic tree then there is a defender of the subject on a path that does not contains $\texttt{Content}(m)$ in AS_D, if it is the case for every direct attacker of the subject then the subject should have been accepted in $\mathsf{AS}_D \ominus \texttt{Content}(m)$. This is not possible, hence $\texttt{Content}(m)$ belongs to the canonical tree that accepts the subject.*

- *if the subject is rejected in AS_D but accepted in $\mathsf{AS}_D \ominus \texttt{Content}(m)$ then there exists a canonical tree hwhere all the branches are of even length in $\mathsf{AS}_D \ominus \texttt{Content}(m)$. Since the adding of $/content(m)$ leads to reject the subject, it means that $\texttt{Content}(m)$ attacks at least one direct or indirect defender of the subject belonging to each canonical tree that accepts the subject in $\mathsf{AS}_D \ominus \texttt{Content}(m)$. The sequence containing the branch from the subject to that defender can be prolongated with $\texttt{Content}(m)$ in order to form a new branch of odd length in $\mathcal{D}^t$. Hence for every canonical tree that rejects the subject, $\texttt{Content}(m)$ has to belong one of their branch.* ∎

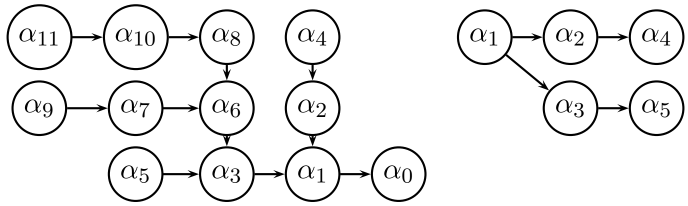
The converse is false since many arguments are not decisive. It is illustrated in Example 7, there are two attackers that are not decisive but the dialog tree contains both of them (as does the only canonical dialog for this example).

6.3 The ideal dialog

In the previous section, we have shown that from each dialog, a dialog tree can be built. This dialog tree contains direct and indirect attackers and defenders of the subject. From this dialog tree, interesting subtrees can be extracted and are called canonical trees. A canonical tree is a subtree containing only particular entire branches of the dialog tree (only one argument in favor of the subject is chosen for attacking an attacker while each argument against a defender is selected). In case the subject of the dialog is accepted it has been proved that there exists at least one canonical tree such that the subject is accepted in its argumentation system. This canonical tree is a candidate for being an ideal tree since it is sufficient to justify the acceptance of the subject against any attack available in the initial dialog. Among all these candidate, we define the ideal tree as the smallest one. In the case the subject is rejected in the initial dialog, then the dialog tree contains all the reasons to reject it, hence we propose to consider the dialog tree itself as the only ideal tree.

Definition 21 (ideal trees and dialogs) *If a dialog D has an accepted output*
- then an ideal tree *associated to D is a canonical tree of D in which $\texttt{Subject}(D)$ is accepted and having a minimal number of nodes among all the canonical graphs that also accept $\texttt{Subject}(D)$*
- else the ideal tree *is the dialog tree of D.*
A dialog using once each argument of an ideal graph is called an ideal dialog.

Example 9 (Cont): An ideal Dialog for Dialog D_9 (on the left) has the following graph (on the right):

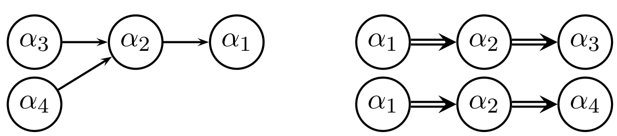


Given the above definition, an ideal dialog contains exactly the same number of moves that the number of nodes of the ideal graph.

Proposition 10 *Given a dialog D whose subject is accepted. An ideal dialog ID for D is the shortest dialog with the same output, and such that every argument in favor of the subject in ID (including $Subject(D)$ itself) is defended against any attack (existing in D).*

Proof *If the subject is accepted in D then, by construction, a canonical graph of D contains every argument existing in D that directly attacks the subject since they belongs to all the possible dialog branches that can be built from D. But for any of them it contains only one attacker that is in favor of the subject (this attacker is a son of an "OR" node in the dialog tree), for each chosen argument in favor of the subject, all the attackers are present in the canonical tree (they are the sons of an "AND" node in the dialog tree). Moreover, if the subject is accepted then every branch of the canonical graph is of even length. It means that the leafs are in favor of the subject and not attacked in the initial dialog D. This property is true for any canonical graph. Then since the ideal dialog correspond to the smallest canonical graph it means that it is the shortest dialog that satisfy this property.* ■

This property ensures that, when the subject is accepted in the initial dialog D, an ideal dialog ID is the more concise dialog that entails an acceptation. In other words, we require that the ideal dialog should contain a set of arguments that sumarize D. Note that the ideal dialog exists but is not always unique. Here is an example of an argumentation system of a dialog which leads to two ideal trees (hence it will lead to at least two ideal dialogs).



So far, we have formally defined the notion of ideal dialog, and have shown how it is extracted from a persuasion dialog. It is clear that the closer (in terms of set-inclusion of the exchanged arguments) the dialog from its ideal version, the better the dialog.

7 Conclusion

Several systems have been proposed in the literature for allowing agents to engage in persuasion dialogs. Different dialog protocols have then been discussed. These latter are the high level rules that govern a dialog. Examples of such rules are 'how the turn shifts between agents', and 'how moves are chained in a dialog'. All these rules should ensure 'correct' dialogs, i.e. dialogs that terminate and reach their goals. However, they do not say anything on the quality of the dialogs. One even wonders whether there are criteria for measuring the quality of a dialog. In this paper, we argue that the answer to this question is yes. Indeed, under the same protocol, different dialogs on the same subject may be generated, and some of them may be judged better than others. There are three kinds of reasons, each of them is translated into quality measures: i) the exchanged arguments are stronger, ii) the behavior of agents was 'ideal'. iii) the generated dialogs are more concise (i.e. all the uttered arguments have an impact on the result of the dialog). In this paper, the behavior of an agent is analyzed on the basis of three main criteria: its degree of aggressiveness, its degree of loan, and its degree of coherence.

We have also proposed three criteria for evaluating the moves of a persuasion dialog with respect to its subject: relevance, usefulness and decisiveness. Relevance only expresses that the argument of the move has a link with the subject (this link is based on the attack relation of the argumentation system). Usefulness is a more stronger relevance since it requires a directed link from the argument of the move to the subject. Decisive moves have a heavier impact on the dialog, since their omission changes the output of the dialog.

Inspired from works on proof theories for grounded semantics in argumentation, we have defined a notion of "ideal dialog". More precisely, we have first defined a dialog tree associated to a given dialog as the graph that contains every possible direct and indirect attackers and defenders of the subject. From this dialog tree, it is then possible to extract sub-trees called "ideal trees" that are sufficient to prove that the subject is accepted or rejected in the original dialog and this, against any possible argument taken from the initial dialog. A dialog is good if it is close to that ideal tree. Ideal dialogs have nice properties with respect to conciseness, namely they contain only useful and relevant arguments for the subject of the dialog. Moreover for every decisive move its argument belongs to all ideal trees.

From the results of this paper, it seems natural that a protocol generates dialogs of good quality if (1) irrelevant and not useful moves are penalized until there is a set of arguments that relate them to the subject (2) adding arguments in favor of the subject that are attacked by already present arguments has no interest (since they do not belong to any ideal tree). By doing so, the generated dialogs are more *concise* (*i.e.*, all the uttered arguments have an impact on the result of the dialog), and more *efficient* (*i.e.*, they are the minimal dialogs that can be built from the information exchanged and that reach the goal of the persuasion).

Note that in our proposal, the order of the arguments has not to be constrained since the generated graph does not take it into account. The only thing that matters in order to obtain a conclusion is the final set of interactions between the exchanged arguments. But the criteria of being relevant to the previous move or at least to a move not too far in the dialog sequence could be taken into account for analyzing dialog quality. Moreover, all the measures already defined in the literature and cited in the introduction could also be used to refine the proposed preference relation on dialogs and finally could help to formalize general properties of protocols in order to generate good dialogs.

Furthermore, it may be the case that from the set of formulas involved in a set of arguments, new arguments may be built. This give birth to a new set of arguments and to a new set of attack relations called complete argumentation system associated to a dialog. Hence, it could be interesting to define dialog trees on the basis of the complete argumentation system then more efficient dialogs could be obtained (but this is not guaranteed). However, some arguments of the complete argumentation system may require the cooperation of the agents. It would mean that in an ideal but practicable dialog, the order of the utterance of the arguments would be constrained by the fact that each agent should be able to build each argument at each step.

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