Gradual Semantics for Weighted Graphs: An Unifying Approach

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Abstract

The paper bridges the gap between two general settings of gradual semantics for weighted argumentation graphs: the *evaluation method* setting (EMS) and the *principle-based* one (PBS). The former defines a semantics by three aggregation functions, each of which satisfies specific *properties*. The latter considers a semantics as any function that follows some high-level *principles*. The paper shows that (EMS) is one way of defining semantics that satisfy principles. Indeed, some principles follow from properties of aggregation functions.

Introduction

Gradual semantics are methods that evaluate overall strengths of individual arguments in graphs. Two general settings for such semantics are proposed in the literature:
Evaluation method setting (EMS): Initiated by Cayrol and Lagasquie in 2005 for flat attack graphs, it was later extended by Leite and Martins in 2011 for weighted attack graphs (graphs where basic weights are ascribed to arguments). The idea is to define a semantics by an *evaluation method*, i.e. a pair of aggregation functions, each of which should satisfy some *properties* (like continuity). This approach specifies the elements of a (flat, weighted) graph that are taken into account in the evaluation of a single argument.
Principle-based setting (PBS): Initiated by Amgoud and Ben-Naim in 2013 and further developed by Amgoud et al. in 2017, it defines a semantics as any function that follows some high-level *principles* (like considering the number of attackers). This approach does not provide any methodology for implementing semantics that satisfy some/all principles.

In this paper, we compare for the first time the two settings. For that purpose, we start by simplifying EMS. We present its main ideas using two novel concepts: rational and well-behaved evaluation methods. We also extend this setting by releasing constraints on functions, and integrating the uniqueness condition on overall strengths. The latter is required in the original setting, however it is not part of its definition. Then, we show that EMS is one way of implementing gradual semantics that satisfy principles of PBS. Indeed, some properties of aggregation functions lead to the satisfaction of principles from (Amgoud et al. 2017).

Principle-based Setting

Let us introduce *weighted argumentation graphs*. Their nodes are *arguments*, each of which has a *basic weight* representing different issues (eg. votes given by users). Edges represent *attacks* (i.e., conflicts) between arguments.

Definition 1 *A* weighted argumentation graph *is a tuple* $\mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R} \rangle$*, where* $\mathcal{A}$ *is a non-empty finite set of arguments,* $\sigma : \mathcal{A} \to [0,1]$*,* $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$*. Let* $\mathtt{AG}$ *be the set of all weighted graphs.*

Notations For $\mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R} \rangle \in \mathtt{AG}$, $a \in \mathcal{A}$, $\mathtt{Att}(a)$ denotes the set $\{b \in \mathcal{A} \mid (b,a) \in \mathcal{R}\}$. Let $\mathbf{G}' = \langle \mathcal{A}', \sigma', \mathcal{R}' \rangle \in \mathtt{AG}$ s.t $\mathcal{A} \cap \mathcal{A}' = \emptyset$. $\mathbf{G} \oplus \mathbf{G}' = \langle \mathcal{A} \cup \mathcal{A}', \sigma'', \mathcal{R} \cup \mathcal{R}' \rangle \in \mathtt{AG}$ s.t $\forall x \in \mathcal{A}$ (resp. $x \in \mathcal{A}'$), $\sigma''(x) = \sigma(x)$ (resp. $\sigma''(x) = \sigma'(x)$).

A semantics is a function assigning a value from an ordered scale (generally $[0,1]$) to each argument. The greater the value, the stronger the argument.

Definition 2 *A* semantics *is a function* $\mathbf{S}$ *assigning to any* $\mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R} \rangle \in \mathtt{AG}$ *a weighting* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}$ *on* $\mathcal{A}$*, i.e.,* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}} : \mathcal{A} \to [0,1]$*. For* $a \in \mathcal{A}$*,* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a)$ *is the* overall strength *of* a*.*

Definition 2 is too coarse and does not tell much on what is considered in the evaluation of arguments. For instance, it does not exclude crude semantics like ones that ignore basic weights of arguments or even attacks. *Principles* are proposed for restricting the set of candidate functions. They describe high-level properties that a semantics may satisfy. In what follows, we consider some principles proposed in (Amgoud et al. 2017). Let $\mathbf{S}$ be an arbitrary semantics.
Anonymity: $\forall \mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R} \rangle, \mathbf{G}' = \langle \mathcal{A}', \sigma', \mathcal{R}' \rangle \in \mathtt{AG}$, for any isomorphism f from $\mathbf{G}$ to $\mathbf{G}'$, it holds: $\forall\, a \in \mathcal{A}, \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}'}(f(a))$.
Independence: $\forall \mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R} \rangle, \mathbf{G}' = \langle \mathcal{A}', \sigma', \mathcal{R}' \rangle \in \mathtt{AG}$ s.t $\mathcal{A} \cap \mathcal{A}' = \emptyset$, it holds: $\forall a \in \mathcal{A}$, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G} \oplus \mathbf{G}'}(a)$.
Directionality: $\forall \mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R} \rangle \in \mathtt{AG}$, $\forall a, b \in \mathcal{A}$, $\forall \mathbf{G}' = \langle \mathcal{A}', \sigma', \mathcal{R}' \rangle \in \mathtt{AG}$ s.t. $\mathcal{A}' = \mathcal{A}$, $\sigma' = \sigma$, $\mathcal{R}' = \mathcal{R} \cup \{(a,b)\}$, it holds: $\forall x \in \mathcal{A}$, if there is no path from b to x, then $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(x) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}'}(x)$.
Equivalence: $\forall \mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R} \rangle \in \mathtt{AG}$, $\forall a, b \in \mathcal{A}$, if i) $\sigma(a) = \sigma(b)$, ii) there exists a bijective function f from $\mathtt{Att}(a)$ to $\mathtt{Att}(b)$ s.t. $\forall x \in \mathtt{Att}(a)$, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(x) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(f(x))$, then $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(b)$.
Maximality: $\forall \mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R} \rangle \in \mathtt{AG}, \forall a \in \mathcal{A}$, if $\mathtt{Att}(a) = \emptyset$, then $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) = \sigma(a)$.

Neutrality: $\forall \mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R} \rangle \in \mathtt{AG}$, $\forall a, b \in \mathcal{A}$, if i) $\sigma(a) = \sigma(b)$, ii) $\mathtt{Att}(b) = \mathtt{Att}(a) \cup \{x\}$ s.t. $x \in \mathcal{A} \setminus \mathtt{Att}(a)$ and $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(x) = 0$, then $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(b)$.
Weakening: $\forall \mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R} \rangle \in \mathtt{AG}$, $\forall a \in \mathcal{A}$, if i) $\sigma(a) > 0$, ii) $\exists b \in \mathtt{Att}(a)$ s.t. $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(b) > 0$, then $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) < \sigma(a)$.
Proportionality: $\forall \mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R} \rangle \in \mathtt{AG}$, $\forall a, b \in \mathcal{A}$, if i) $\mathtt{Att}(a) = \mathtt{Att}(b)$, ii) $\sigma(a) > \sigma(b)$, iii) $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) > 0$, then $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) > \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(b)$.
Resilience: $\forall \mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R} \rangle \in \mathtt{AG}$, $\forall a \in \mathcal{A}$, if $\sigma(a) > 0$, then $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) > 0$.
Reinforcement: $\forall \mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R} \rangle \in \mathtt{AG}$, $\forall a, b \in \mathcal{A}$, if i) $\sigma(a) = \sigma(b)$, ii) $\mathtt{Att}(a) \setminus \mathtt{Att}(b) = \{x\}$, $\mathtt{Att}(b) \setminus \mathtt{Att}(a) = \{y\}$, iii) $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(y) > \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(x)$, iv) $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) > 0$, then $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) > \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(b)$.
Counting: $\forall \mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R} \rangle \in \mathtt{AG}$, $\forall a, b \in \mathcal{A}$, if i) $\sigma(a) = \sigma(b)$, ii) $\mathtt{Att}(b) = \mathtt{Att}(a) \cup \{x\}$ s.t. $x \in \mathcal{A} \setminus \mathtt{Att}(a)$ and $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(x) > 0$, iii) $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) > 0$, then $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) > \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(b)$.

Simplifying the Evaluation Method Setting

The backbone of the setting is what we call *evaluation method* (EM), which is a pair of aggregation functions.

Definition 3 *An* evaluation method *is a pair* $\mathbf{M} = \langle f, g \rangle$ *s.t.*

- $g : \bigcup_{n=0}^{+\infty} [0,1]^n \to [0, +\infty)$ *such that g is symmetric*
- $f : [0,1] \times \mathtt{Range}(g) \to [0,1]$[1]

The function g evaluates how strongly an argument is attacked. It aggregates the overall strengths of all attackers of the argument. Since the ordering of attackers should not be important, we posed the symmetry condition, i.e., $g(x_1, \ldots, x_n) = g(x_{\rho(1)}, \ldots, x_{\rho(n)})$, for any permutation ρ of the set $\{1, \ldots, n\}$. The function f returns the overall strength of an argument by combining its basic weight with the value returned by g. In the evaluation method setting, a semantics **should** be based on an evaluation method.

Definition 4 *A semantics* $\mathbf{S}$ is based on *an evaluation method* $\mathbf{M} = \langle f, g \rangle$ *iff* $\forall \mathbf{G} = \langle \mathcal{A}, \sigma, \mathcal{R} \rangle \in \mathtt{AG}$, $\forall a \in \mathcal{A}$,

$$\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) = f(\sigma(a), g(\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(b_1), \ldots, \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(b_n))), \quad (1)$$

where $\{b_1, \ldots, b_n\} = \mathtt{Att}(a)$.

Evaluating arguments with a semantics amounts thus to solving a system of equations (one equation per argument). Such system may have *one* or *several* solutions. Uniqueness of overall strengths is however desirable for several reasons: a unique strength is more informative than multiple ones, it can be used in applications like nonmonotonic reasoning, and as argued in (Leite and Martins 2011), users of online debate platforms would prefer a system that informs them which is the overall strength of an argument. Next, we extend the existing setting by integrating the uniqueness condition. For that purpose, we define the concept of *rational evaluation methods*, i.e methods that characterize semantics.

Definition 5 *An evaluation method* $\mathbf{M} = \langle f, g \rangle$ *is* rational *iff there is a unique semantics* $\mathbf{S}$ *which is based on* $\mathbf{M}$. $\mathbf{S}(\mathbf{M})$ *denotes the semantics characterized by* $\mathbf{M}$.

[1] $\mathtt{Range}(g)$ denotes the co-domain of g. In the literature, it is usually either $[0, 1]$ or $[0, +\infty)$.

In (Cayrol and Lagasquie 2005; Leite and Martins 2011), an evaluation method should be well-behaved.

Definition 6 *An evaluation method* $\mathbf{M} = \langle f, g \rangle$ *is* well-behaved *iff the following conditions hold:*

1. *f is increasing in the first variable, decreasing in the second variable whenever the first variable is not equal to* 0, *$f(x, 0) = x$, and $f(0, x) = 0$.*
2. *$g() = 0$, $g(x) = x$, $g(x_1, \ldots, x_n) = g(x_1, \ldots, x_n, 0)$, and $g(x_1, \ldots, x_n, y) \leq g(x_1, \ldots, x_n, z)$ if $y \leq z$.*

Linking the Two Settings

The first result shows that the equivalence principle follows already from Equation (1). In other words, any semantics that is based on an evaluation method satisfies the principle.

Proposition 1 *If a semantics* $\mathbf{S}$ *is based on an evaluation method, then* $\mathbf{S}$ *satisfies equivalence.*

Rational evaluation methods ensure the three important principles: anonymity, independence, directionality. However, the converse is not true.

Proposition 2 *If a semantics* $\mathbf{S}$ *is based on a rational evaluation method, then* $\mathbf{S}$ *satisfies anonymity, independence and directionality.*

The two previous results do not require any constraint on the two aggregation functions of an evaluation method. The following result states that well-behaved evaluation methods define semantics that satisfy additional principles.

Proposition 3 *If a semantics* $\mathbf{S}$ *is based on a well-behaved evaluation method, then* $\mathbf{S}$ *satisfies maximality, neutrality, weakening, and proportionality.*

Well behaved methods that satisfy the positivity constraint on f, described in the following proposition, define semantics that satisfy resilience.

Proposition 4 *If a semantics* $\mathbf{S}$ *is based on a well-behaved evaluation method* $\mathbf{M} = \langle f, g \rangle$ *such that* $f(x_1, x_2) > 0$ *whenever* $x_1 > 0$, *then* $\mathbf{S}$ *satisfies resilience.*

The function g of a well-behaved evaluation method is *monotonic*. When it is *strictly monotonic*, the method defines semantics that satisfy reinforcement and counting.

Proposition 5 *If a semantics* $\mathbf{S}$ *is based on a well-behaved evaluation method* $\mathbf{M} = \langle f, g \rangle$ *such that*

$$g(x_1, \ldots, x_n, y) < g(x_1, \ldots, x_n, z) \text{ whenever } y < z,$$

then $\mathbf{S}$ *satisfies reinforcement and counting.*

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