A formal concept view of abstract argumentation

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Abstract. The paper presents a parallel between two important theories for the treatment of information which address questions that are apparently unrelated and that are studied by different research communities: an enriched view of formal concept analysis and abstract argumentation. Both theories exploit a binary relation (expressing object-property links, attacks between arguments). We show that when an argumentation framework rather considers the complementary relation does not attack, then its stable extensions can be seen as the exact counterparts of formal concepts. This leads to a cube of oppositions, a generalization of the well-known square of oppositions, between eight remarkable sets of arguments. This provides a richer view for argumentation in cases of bi-valued attack relations and fuzzy ones.

Keywords: argumentation, formal concept analysis, possibility theory, square of oppositions

1 Introduction

Formal concept analysis [34, 29] exploits a binary relation that links objects and properties. This relation, called 'formal context', is usually a classical 2-valued one (i.e., an object has, or not, a property), but may be also a fuzzy relation [5–7] when properties may be a matter of degree. From this relation, the notion of 'formal concept' is defined as maximal sets of pairs made of a subset of objects and a subset of properties, such that each object in a subset has all the properties in the associated subsets, and the objects in a subset are the only ones to have all these properties, in the considered context. Formal concepts are characterized by a fixed-point equation through a Galois connection. A recent parallel [18] with possibility theory [19] has shown the interest of introducing operators in this setting other than the one underlying the notion of formal concept, which leads to consider other connexions as well [15, 22].

In a fully independent way, an abstract theory of argumentation [24] has been developed on the basis of a binary attack relation between arguments. This relation, generally a classical one, may also become fuzzy when one tries to model the strength of arguments [25]. The objective is then to determine noticeable subsets of arguments that in particular constitute stable extensions in the sense they are without internal conflict, and where each argument outside the extension is attacked by an argument of the extension.

The exploitation in each setting of a classical binary relation, which may be more generally fuzzy, may lead to wonder about a possible parallel between the two theories, and about their possible mutual enrichment. In the next section we restate the formal elements of the abstract theory of argumentation and emphasize the different existing relational equations. Then in Section 3 in the same spirit, we recall the basis of formal concept analysis enriched by the operators induced by the parallel with possibility theory. In Section 4, we make a parallel between the abstract theory of argumentation and formal concept analysis, which especially sheds light on the parallel between stable extension and formal concept. Section 5 provides an analysis in terms of opposition structures that help to get an organized view of different subsets of remarkable arguments. The concluding remarks briefly considers the case of fuzzy relations, and in particular suggests lines of research for extending abstract theory of argumentation to situations where attacks are weighted.

2 Argumentation

P. M. Dung [24], in a famous article which has raised considerable interest, has proposed to define an *argumentation system* as a pair (A, R) where A is a set of arguments, and R ($\neq \emptyset$) a binary relation over A, i.e., $R \subseteq A \times A$. Given two arguments $a \in A$ and $b \in A$, $(a, b) \in R$, or equivalently aRb, then means that *a attacks b*. An *argumentation system* (A, R) can then be seen as an oriented graph, where arguments are its nodes, and where the elements of R are the vertices. As can be seen the notion of argument, which intuitively corresponds in the logical view (see, e.g., [31]) to a minimal consistent set of formulas that in a given logical setting enable us to deduce a formula of interest, is here "abstractized", as well as the notion of attack (which amounts in practice to challenge a deduced formula, either directly, or by challenging one of the formulas appearing in the argument for establishing its conclusion). Dung's framework has been often used as a reference setting and as a starting point in many artificial intelligence works in argumentation until now.

A subset $S \subseteq A$ of arguments attacks an argument a

$$\text{if } \exists s \in S \text{ and } sRa.$$

A subset $S \subseteq A$ of arguments attacks a subset $S' \subseteq A$

$$\text{if } \exists s \in S \text{ and } \exists s' \in S' \text{ and } sRs'.$$

A subset S of arguments is *conflict free*

$$\text{if } \nexists (a, b) \in S \times S \text{ such as } aRb.$$

Given an argumentation system (A, R), a key question that naturally arises is the definition of *acceptable* subsets of arguments; an acceptable subset of arguments is called *extension*. Different forms of acceptability exist. A well-known one is the notion of *stable extension.*

A subset S of arguments without conflict is a *stable* extension if and only if

$$\forall a \notin S, \exists s \in S \text{ and } sRa.$$

In other words, a stable extension attacks all the arguments outside. Other forms of acceptability use the notion of *defense*. An argument $a \in A$ is defended by a subset of arguments S if and only if for each argument $b \in A$ that attacks a, $\exists s \in S$ such that sRb. A conflict-free subset S of arguments is an *admissible extension* if and only if each argument of S is defended by S. A stable extension is admissible.

One can then introduce remarkable sets associated with an argument a, or with a subset of arguments S in terms of attack or defense, which help to make the definitions more precise and to establish some properties :

- the set of arguments attacking a
  $Ra = \{s \in A | sRa\}$;
- the set of arguments attacked by a
  $aR = \{s \in A | aRs\}$;
- the set of arguments attacked by S
  $$R^+(S) = \{a \in A | S \text{ attacks } a\} = \{a \in A | \exists s \in S, sRa\} = \{a \in A | S \cap Ra \neq \emptyset\};$$
- the set of arguments attacking S
  $$R^-(S) = \{a \in A | a \text{ attacks} S\} = \{a \in A | \exists s \in S, aRs\} = \{a \in A | S \cap aR \neq \emptyset\};$$
- the set of arguments defended by S
  $$Def(S) = \{a \in A | S \text{ defends } a\} = \{a \in A | \forall b \in A \text{ t.q. } bRa, \exists s \in S \text{ s.t. } sRb\} = \{a \in A | Ra \subseteq R^+(S)\}.$$
  The set of arguments defended by S is indeed made of the arguments whose attackers are attacked by S.

It can be checked that

- S is *conflict-free* if and only if [1]
  $S \subseteq \overline{R^+(S)}$,
  where $\overline{T} = A \setminus T$. Indeed, the arguments that S attacks are then in $\overline{S}$ $(R^+(S) \subseteq \overline{S})$.

– S is a *stable extension* if and only if [24]

$$S = \overline{R^+(S)}. \tag{1}$$

This follows from above, and from the definition of stability that requires $\overline{S} \subseteq R^+(S)$. Note also that the set of arguments non attacked by S is equal to

$$\overline{R^+(S)} = \{a \in A | \forall s \in S, s\overline{R}a\}, \text{ i.e., we have}$$

$$\overline{R^+(S)} = \{a \in A | S \subseteq \overline{R}a\} \tag{2}$$

where $s\overline{R}a$ means that s does not attack a. One can then establish that:

– $Def(S) = \overline{R^+(\overline{R^+(S)})}$ [1].

Indeed, applying Equation 2 one gets $\overline{R^+(\overline{R^+(S)})} = \{a \in A | \overline{R^+(S)} \subseteq \overline{R}a\}$,

which provides the proof since $\overline{R}a = \overline{Ra}$, taking into account the definition of $Def(S)$.

Thus if S is a stable extension, Equation 1 holds, and then

$$Def(S) = S$$

In a stable extension, the arguments are thus defending themselves.

One can still establish that [8]

– S is an *admissible extension* if and only if

$$S \subseteq Def(S) \cap \overline{R^+(S)}.$$

Indeed, this is equivalent to $S \subseteq Def(S) \wedge S \subseteq \overline{R^+(S)}$, which indeed means that the arguments in S are both defended by S and non attacked by S (S is thus conflict-free). This condition can be still written

$$S \subseteq \{a \in A | \overline{R^+(S)} \subseteq \overline{R}a \wedge S \subseteq \overline{R}a\}$$
$$\Leftrightarrow S \subseteq \{a \in A | (\overline{R^+(S)} \cup S) \subseteq \overline{R}a\}$$
$$\Leftrightarrow S \subseteq \{a \in A | Ra \subseteq (R^+(S) \cap \overline{S})\}.$$

– S is an *admissible extension* if and only if

$$S \subseteq Def(S \ \cap \overline{R^-(S)}).$$

Indeed, this condition guarantees that S is conflict-free, since it expresses that each argument in S is defended by an argument in S that does not attack S (we have $\overline{R^-(S)} = \{a \in A | S \subseteq a\overline{R}\}$). Indeed, if aRb with $(a, b) \in S^2$, b cannot be defended by c (i.e. cRa and thus $c \in R^-(S)$) with $c \in \overline{R^-(S)}$.

– An admissible extension S is said *complete* if and only if each argument which is defended by S is in S [24]. Thus S is complete if and only if S is admissible and $Def(S) \subseteq S$. Thus, we have

S is a *complete extension* if and only if

$$S = Def(S) \cap \overline{R^{+}(S)}.$$

Moreover, if S is a complete extension, then

$$S = Def(S).$$

3 Formal concept analysis

Formal concept analysis (FCA) [4, 34] provides a theoretical setting for the learning of hierarchies of concepts (from which association rules can be extracted). It starts with a *formal context* $\mathcal{K} = (\mathcal{O}, \mathcal{P}, \mathcal{R})$ where $\mathcal{R}$ is a binary relation completely defined between a set of objects $\mathcal{O}$ and a set of Boolean properties $\mathcal{P}$. Namely, $\mathcal{R} \subseteq \mathcal{O} \times \mathcal{P}$. A formal context is often visualized under the form of a table such that the presence of a cross ($\times$) (resp. its absence) in a cell indicates if an objet satisfies (resp. does not satisfy) the corresponding property.

Given an object x and a property y, let $R(x) = \{y \in \mathcal{P} \mid x\mathcal{R}y\}$ be the set of properties satisfied by object x ($x\mathcal{R}y$ means that x has property y) and let $R(y) = \{x \in \mathcal{O} \mid x\mathcal{R}y\}$ be the set of objects having property y. In FCA, one defines correspondences between the sets $2^{\mathcal{O}}$ and $2^{\mathcal{P}}$. These correspondences are called Galois derivation operators. The Galois operator, which is at the basis of FCA, here denoted $(.)^{\Delta}$ (for reasons made clear later), enables us to express the set of properties satisfied by *all* the objects in $X \subseteq \mathcal{O}$ as :

$$\begin{aligned} X^{\Delta} &= \{y \in \mathcal{P} \mid \forall x \in \mathcal{O}\ (x \in X \Rightarrow x\mathcal{R}y)\} \\ &= \{y \in \mathcal{P} \mid X \subseteq R(y)\} = \textstyle\bigcap_{x \in X} R(x) \end{aligned}$$

We can also express, in a dual manner, the set of objects satisfying all the properties in Y as :

$$\begin{aligned} Y^{\Delta} &= \{x \in \mathcal{O} \mid \forall y \in \mathcal{P}\ (y \in Y \Rightarrow x\mathcal{R}y)\} \\ &= \{x \in \mathcal{O} \mid Y \subseteq R(x)\} = \textstyle\bigcap_{y \in Y} R(y) \end{aligned}$$

The dual pair of operators $((.)^{\Delta}, (.)^{\Delta})$ applied respectively to $2^{\mathcal{O}}$ and to $2^{\mathcal{P}}$ constitutes a Galois connexion that enables the definition of formal concepts. A*formal concept* is a pair (X, Y) such as

$$X^{\Delta} = Y \text{ and } Y^{\Delta} = X.$$

In other words, X is the maximal set of objects satisfying all the properties already satisfied by all the objects in X. The set X (resp. Y) is called *extension* (resp. *intension*) of the concept. It can be shown that in an equivalent way, (X, Y) is a formal concept if and only if it is a maximal pair in the sense of set inclusion such as

$$X \times Y \subseteq \mathcal{R}.$$

The set of all the formal concepts is naturally equipped with an order relation (denoted $\preccurlyeq$) and defined as : $(X_1, Y_1) \preceq (X_2, Y_2)$ iff $X_1 \subseteq X_2$ (or $Y_2 \subseteq Y_1$). This set equipped with the order relation $\preccurlyeq$ forms a complete lattice $\mathfrak{B}(\mathcal{K})$. The operators *meet* and *join* in the lattice are described by the fundamental result due to Ganter and Wille [29] :

$$\bigwedge_{j\in J}(X_j, Y_j) = \left(\bigcap_{j\in J} X_j, \left(\left(\bigcup_{j\in J} Y_j\right)^{\Delta}\right)^{\Delta}\right)$$

$$\bigvee_{j\in J}(X_j, Y_j) = \left(\left(\left(\bigcup_{j\in J} X_j\right)^{\Delta}\right)^{\Delta}, \bigcap_{j\in J} Y_j\right)$$

In [18], on the basis of a parallel with *possibility theory* (indeed $X^{\Delta} = \bigcap_{x\in X} R(x)$ may be seen as the counterpart of the definition of a guaranteed possibility measure $\Delta(F) = \min_{x\in F} \pi(x)$ where π is a possibility distribution), other operators have been introduced: namely the possibility operator (denoted $(.)^{\Pi}$) and its dual, the necessity operator (denoted $(.)^{N}$), as well as the operator $(.)^{\nabla}$, dual of the operator $(.)^{\triangle}$ at the basis of FCA, defined as follows:

- X^{Π} is the set of properties satisfied by at least one object in X:

$$\begin{aligned} X^{\Pi} &= \{y \in \mathcal{P} \mid \exists x \in X,\, x\mathcal{R}y\} \\ &= \{y \in \mathcal{P} \mid X \cap R(y) \neq \emptyset\} \\ &= \textstyle\bigcup_{x\in X} R(x) \end{aligned}$$

- X^{N} is the set of properties that only the objects in X have:

$$\begin{aligned} X^{N} &= \{y \in \mathcal{P} \mid \forall x \in \mathcal{O}\, (x\mathcal{R}y \Rightarrow x \in X)\} \\ &= \{y \in \mathcal{P} \mid R(y) \subseteq X\} \\ &= \textstyle\bigcap_{x\notin X} \overline{R}(x) \end{aligned}$$

  (where $\overline{R}(x)$ is the set of properties that x does not have)

- X^{∇} is the set of properties that are not satisfied by at least one object outside X (X^{∇} should not be confused with the notion of weak opposition in FCA, often denoted in a similar way):

$$\begin{aligned} X^{\nabla} &= \{y \in \mathcal{P} \mid \exists x \in \overline{X},\, x\overline{\mathcal{R}}y\} \\ &= \{y \in \mathcal{P} | R(y) \cup X \neq \mathcal{O}\} \\ &= \textstyle\bigcup_{x\notin X} \overline{R}(x) \end{aligned}$$

The operators Y^{Π}, Y^{N}, Y^{∇} are obtained in a dual manner. As established in [15, 22], the pairs (X, Y) such as $X^N = Y$ and $Y^N = X$ (or in an equivalent way $X^{\Pi} = Y$ and $Y^{\Pi} = X$) characterize independent sub-contexts (i.e. which have not any objects or properties in common) inside the initial context. The pairs (X, Y) such as $X^N = Y$ and $Y^N = X$ are such that:

$$(X \times Y) \cup (\overline{X} \times \overline{Y}) \supseteq \mathcal{R}.$$

Regarding $X^{\nabla} = Y$ and $Y^{\nabla} = X$, it constitutes another characterization of formal concepts.

It has been shown [18, 22] that the four sets X^{Π}, X^N, X^{Δ}, X^{∇} represent complementary pieces of information, which are all necessary for a complete analysis of the situation of a set X in the formal context $\mathcal{K} = (\mathcal{O}, \mathcal{P}, \mathcal{R})$.

4 Stable extensions in argumentation and formal concepts

There is a striking parallel between Equation 2 in Section 2

$$\overline{R^+(S)} = \{a \in A | S \subseteq \overline{R}a\}$$

and the expression

$$X^{\Delta} = \{y \in \mathcal{P} \mid X \subseteq R(y)\} = \bigcap_{x \in X} R(x)$$

as well as between the definition 1 of a stable extension S

$$S = \overline{R^+(S)}$$

and the one of a formal concept (X, Y)

$$X^{\Delta} = Y \text{ and } Y^{\Delta} = X,$$

taking into account the similarity of the definitions of $\overline{R^+(S)}$ and X^{Δ}.

However, there is an obvious difference: in argumentation one is in the particular case $\mathcal{O} = \mathcal{P} = A$. What plays the role of the formal context is thus the relation $\overline{R}$ ("does not attack") defined on $A \times A = \mathcal{O} \times \mathcal{P}$.

It is well-known that stable extensions do not always exist. For instance, $(A = \{a, b, c, d\}, R = \{(a, b), (b, c), (c, a)\})$ has no stable extension. While formal concepts always exist when $R \neq \emptyset$, here it is no longer the case, when we work on $A \times A$, rather than with $\mathcal{O} \times \mathcal{P}$ where $\mathcal{O} \neq \mathcal{P}$. Since here the only acceptable formal concepts (X, Y) should be such that $X = Y$ ($= S$ in the above notation).

Then, one can look at the argumentative counterparts of X^{Π}, X^N, or X^{∇}. They are respectively:

– $\overline{R}^{+}(S) = \{a \in A | S \cap \overline{R}a \neq \emptyset\}$

the set of arguments not attacked by *all* the arguments in S. It means that for each argument in $\overline{R}^{+}(S)$ there exists at least one argument in S that does not attack it. It should not be confused with the set of arguments not attacked by *some* arguments in S: $\overline{R^{+}(S)} = \{a \in A | S \cap Ra = \emptyset\}$; Thus we have $\overline{R^{+}(S)} \subseteq \overline{R}^{+}(S)$, just as $\Delta \leq \Pi$ in possibility theory.

– $\overline{\overline{R}^{+}(\overline{S})} = \{a \in A | \overline{R}a \subseteq S\} = \{a \in A | \overline{S} \subseteq Ra\}$

the set of arguments that are attacked by all the arguments outside S ;

– $R^{+}(\overline{S}) = \{a \in A | S \cup \overline{R}a \neq A\}$
$= \{a \in A | \overline{S} \cap Ra \neq \emptyset\}$

the set of arguments that are attacked by arguments outside S. We have $\overline{\overline{R}^{+}(\overline{S})} \subseteq R^{+}(\overline{S})$, as well as $N \leq \nabla$ holds in possibility theory. Moreover, if $R \neq \emptyset$ and $\overline{R} \neq \emptyset$, we have $\overline{\overline{R}^{+}(\overline{S})} \subseteq \overline{R}^{+}(S)$ and $\overline{R^{+}(S)} \subseteq R^{+}(\overline{S})$, counterparts of $N \leq \Pi$ and $\Delta \leq \nabla$ respectively. Thus, finally it holds that

$$\overline{R^{+}(S)} \cup \overline{\overline{R}^{+}(\overline{S})} \subseteq R^{+}(\overline{S}) \cap \overline{R}^{+}(S).$$

If one leaves aside complementations, it can thus be seen that given S, there are four basic sets of arguments:

$$R^{+}(S), \overline{R}^{+}(S), R^{+}(\overline{S}), \overline{R}^{+}(\overline{S}).$$

They are i) the arguments attacked by S, ii) the arguments not attacked by S, iii) the arguments attacked by non S, iv)the arguments not attacked by non S. Considering these four sets is necessary for a complete characterization of the relative position of the set of attackers of an argument a with respect to a set S of arguments (see [22] for the detailed possibilistic counterpart of this fact). It is clear that in a dual manner, there are four other noticeable sets in terms of R^{-} rather than of R^{+}.

We are thus led to consider the counterparts of the four conditions $X^{\Delta} = Y$ and $Y^{\Delta} = X$, $X^{\nabla} = Y$ and $Y^{\nabla} = X$, $X^{\Pi} = Y$ and $Y^{\Pi} = X$, and $X^{N} = Y$ and $Y^{N} = X$. They are respectively $S = \overline{R^{+}(S)}$, $S = R^{+}(\overline{S})$, which equivalently characterizes a stable extension on the one hand, and the equivalent constraints $S = \overline{\overline{R}^{+}(S)}$ and $S = \overline{R}^{+}(\overline{S})$ on the other hand, which correspond to extensions S and $\overline{S}$ that present a form of independence. Indeed $S = \overline{R}^{+}(\overline{S}) \Leftrightarrow \overline{S} = \overline{\overline{R}^{+}(\overline{S})}$ expresses that the set of arguments that are attacked by all the arguments outside S are precisely the arguments outside S.

5 Structures of opposition and abstract argumentation

Structures of opposition have been studied in logic for a long time. In particular, the square of oppositions invented by Aristotle and its modern generalization to an hexagon of oppositions after the works of Robert Blanché [11] and Béziau [10] are encountered each time an internal negation and an external negation are at work on formal expressions.

Taking advantage of results presented in [23] regarding the structures of oppositions in formal concept analysis and in possibility theory, one may study in a similar manner the structures of oppositions at work in the theory of abstract argumentation, and in particular obtain the cube of oppositions pictured in Figure 1, where the four sets of arguments and their complements appear (a set and its complement are at the two extremities of diagonals). The vertical arrows express inclusions. For example $\overline{R^+(S)} \subseteq R^+(\overline{S})$ (provided that $R \neq \emptyset$).

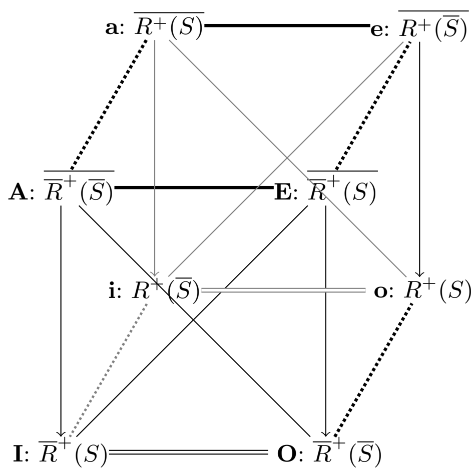

Fig. 1. Cube of oppositions between 8 remarkable sets of arguments

It is worth noticing that the different meaningful sets of arguments can be organized in such a structure that has played an important role through the whole history of logic. Moreover, the hexagonal structure of oppositions obtained in the logic of argumentation proposed in [3] should also be compared to the one obtained here.

6 Concluding remarks: The gradual case

The idea to extend FCA to a fuzzy formal context, which enables us to express that an object satisfies a property to an intermediary degree, has been initially proposed by Burusco and Fuentes-Gonzalez [12] before being considerably developed by Belohlavek [5–7], and by a number of other authors, as in particular [30, 26, 32, 33]. For a discussion of different meaningful gradual extensions of FCA, the reader is referred to [14, 16].

We only give here the basic operator of fuzzy FCA [6]:

$$X^{\Delta}(y) = \bigwedge_{x \in O} (X(x) \to R(x, y))$$

where now R is a fuzzy relation, $R(x, y)$ is the degree to which x is in relation R with y, and X and X^{Δ} are fuzzy sets of objects and properties respectively, and $\bigwedge$ is the conjunction operator min and $\to$ an implication operator. An appropriate choice of this connective (such as Gödel residuated implication: $a \to b = 1$ if $a \leq b$, and $a \to b = b$ if $a > b$) enables us to see a fuzzy formal concept in terms of its level cuts X_α, Y_α in such a way that

$$(X_\alpha \times Y_\alpha) \subseteq R_\alpha$$

where $X_\alpha \times Y_\alpha$ is maximal, with $R_\alpha = \{(x, y) | R(x, y) \geq \alpha\}$, $X_\alpha = \{x \in \mathcal{O} | X(x) \geq \alpha\}$, $Y_\alpha = \{y \in \mathcal{P} | Y(y) \geq \alpha\}$.

The idea of an abstract argumentation theory allowing for a *graded attack relation* has been recently advocated by some authors, in particularly in [25]. Following the parallel presented here, we are thus led to characterize a *fuzzy stable extension* by the equation

$$S(s) = \bigwedge_{a \in A} (S(s) \to \overline{R}(s, a))$$

where $S(s)$ is the degree to which the argument s belongs to the fuzzy stable extension S, $\overline{R}(s, a) = 1 - R(s, a)$, $R(s, a)$ being the degree with which s attacks a, which generalizes $S = \overline{R^{+}(S)} = \{a \in A | S \subseteq \overline{R}a\}$.

By exploiting the counterpart of$(X_\alpha \times Y_\alpha) \subseteq R_\alpha$, in the argumentative setting, one sees that we are back to the study of the level cuts of the relation of "non-attack" $\overline{R}$.

In the same spirit, one could define fuzzy admissible extensions, or define fuzzy extensions of $\overline{R}^{+}(S)$, $R^{+}(\overline{S})$, and $\overline{R}^{+}(\overline{S})$.

The association of degrees to arguments may have different meanings: They may in particular reflect the strength of the argument, or the uncertainty associated to its components. The nature of the degrees is as much important in FCA, since uncertainty and satisfaction level of a gradual property should not be handled in the same way [16]. Different treatments should as well be considered in argumentation according to the meaning of the degrees. What is suggested above rather applies to the strength of the arguments rather than to their uncertainty.

The computation of extensions in argumentation can be expressed in the setting of propositional logic in terms of algebraic equations as shown in [9] (see also [2]). This idea has been recently reused by Gabbay [28], thus putting abstract argumentation in the framework of the equational semantics of propositional logic, first developed one century ago by Louis Couturat [13]. The exploitation of this idea can be extended to fuzzy logic [28]. One can thus also reconsider what is proposed above in this paper in that perspective.

This paper is a preliminary attempt at bridging four noticeable areas in the formal treatment of information, namely abstract argumentation, formal concept analysis, but also possibility theory and squares of opposition, which have remained completely related until recently. Such parallels should contribute to enrich each domain: for instance, in argumentation by considering new sets of arguments and understanding better how they are related. It may also provide useful guidelines for introducing grades in argumentation.

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