Generating possible intentions with constrained argumentation systems [1]

Leila Amgoud⋆ Caroline Devred+ Marie-Christine Lagasquie-Schiex⋆
⋆ IRIT – CNRS, UPS, Toulouse, France
+ LERIA – UFR Sciences, Angers, France

Abstract

Practical reasoning (PR), which is concerned with the generic question of what to do, is generally seen as a two steps process: (1) *deliberation*, in which an agent decides what state of affairs it wants to reach –that is, its *desires*; and (2) *means-ends reasoning*, in which the agent looks for plans for achieving these desires. The agent's *intentions* are a consistent set of desires that are achievable together.

This paper proposes the first argumentation system for PR that computes in one step the possible intentions of an agent, avoiding thus the drawbacks of the existing systems. The proposed system is grounded on a recent work on constrained argumentation systems, and satisfies the rationality postulates identified in argumentation literature, namely the *consistency* and the *completeness* of the results.

Keywords: Practical reasoning, Argumentation theory.

1. Introduction

Practical reasoning (PR) [2] is concerned with the generic question of what to do for a rational agent in a given situation. In his seminal book [3], Wooldridge defines PR as a two step process. The first step, called *deliberation*, consists of identifying the states of affairs an agent wants to reach (*i.e.* the *desires*). This step is decomposed into two distinct components: i) an *option generation* component

[1]This paper extensively develops and extends the content of the conference paper [1]. The language of representation is refined and more results on the system are proposed.

in which the agent generates a set of possible desires, and ii) a *filtering* component in which the agent chooses between competing desires. In the second step of PR, called *means-end reasoning*, the agent looks for plans for reaching the chosen desires. If such plans exist, those desires will be called *intentions* and the agent commits to achieving them. Thus, an intention is a desire that is *justified* and *feasible*. In [4], it has been argued that generating options is an inference problem, while filtering those options is a decision making one. Regarding the means-end-reasoning step, the authors argue that it involves two problems: an inference problem in which an agent checks the feasibility of sets of plans, and a decision making problem in which the agent chooses among several feasible plans, the exact ones to carry out. The authors have then proposed another decomposition of a PR process as follows: i) option generation, ii) checking the feasibility of the options, *i.e.* to find sets of plans that are compatible in the sense that they are achievable together, and iii) filtering the options as well as the plans. The two decision problems are thus combined in a unique step. The new decomposition offers at least two advantages: First, it avoids that the filtering component selects an option for which no plan can be formed, and in so doing might exclude an option which could be carried out. The second advantage consists of the link that exists between the two decision problems. In [4], the authors have proposed different principles for choosing among competing and feasible options. For instance, an agent may choose a desire that has more plans for achieving it. It is clear that such a decision principle can only be applied after the means-end-principle. In this paper, we follow this decomposition of PR process.

Besides, what is worth noticing in most works on practical reasoning is the use of arguments for providing reasons for choosing or discarding a desire as an intention. These works can be divided into two groups: works that are interested in identifying argument schemes that are used in PR (e.g. [5, 6]), and works that propose concrete argumentation-based systems for PR (e.g. [7, 4, 8, 9, 10]) following the process proposed in [4]. Recall that an argumentation system consists mainly of a set of conflicting arguments, and the crucial issue is the selection of acceptable sets of arguments. Works of the second category suffer from three main drawbacks.

- The first problem is that the properties of the systems are not investigated; it is thus unclear whether the results of these systems are intuitive.
- The second one concerns the use of a skeptical acceptability semantics, namely grounded semantics, for evaluating arguments. However, skeptical

semantics are not suitable in practical reasoning as illustrated by the following example of an agent who has three equally preferred desires d_1, $\neg d_1$ and d_2. Assume that d_1 and $\neg d_1$ are not conditional while d_2 depends on d_1 (in [9] this is denoted by $d_1 \Rightarrow d_2$). According to the system proposed, for instance, in [9], there are three arguments: δ_1 (in favor of d_1), δ_2 (in favor of $\neg d_1$) and δ_3 (in favor of d_2) such that δ_1 and δ_2 attack each other and δ_2 attacks δ_3. It is clear that the grounded extension of this system is empty meaning that no desire will be pursued by this agent even if these desires are feasible. This is clearly counter-intuitive. Now, if a credulous semantics, like preferred semantics, is considered, then two preferred extensions are returned: $\{\delta_1, \delta_3\}$ and $\{\delta_2\}$ meaning that this agent can either pursue the two desires d_1 and d_2 together, or the desire $\neg d_1$ alone.

- The third drawback of existing approaches concerns the fact that the first and second steps of PR are modeled in terms of two separate systems. In such an approach, some desires that are not feasible may be accepted at the option generation step to the detriment of other justified and feasible desires, or may prevent some justified and feasible desires from being accepted. Let us consider again the previous example, and assume that the desire $\neg d_1$ is more important than the two others. However, this desire is not feasible since there is no plan for carrying it out while the agent has two plans: π_1 for achieving desire d_1 and π_2 for achieving d_2. According to the system proposed in [9], the argument δ_2 attacks both δ_1 and δ_3. The grounded semantics is empty in this case as well. Let us now consider preferred semantics. It can be checked that this system has a unique preferred extension which is the set $\{\delta_2\}$. The system concludes that the set of intentions is empty. This result is not desirable since the desire $\neg d_1$ prevents d_1 and d_2 from being accepted while it is itself not feasible.

This paper proposes the first argumentation system that computes the possible sets of intentions of an agent in one step. In other words, the paper presents a system that combines option generation and checking the feasibility of options. There are two motivations for this. The first one is optimization of resources: a unified process could be more effective, because it does not waste resources in the attempt to select desires among a large pool of desires, which may not all turn out to be feasible after all. The second one is completeness: a unified process would prevent selecting an unfeasible desire at the expense of a feasible one, in which case the agent may end up not realizing that there is after all a way to achieve at least some of its desires. Moreover, the use of argumentation theory presents another advantage: the choice of each set of intentions can be explained by the corresponding arguments.

The proposed system is grounded on a recent work on *constrained* argumentation systems [11]. These systems extend the general framework proposed by Dung [12] by adding a constraint on arguments. This constraint will serve to filter the results returned by Dung's acceptability semantics. Indeed, among all the extensions, only the ones that satisfy the constraint are kept.

Our system takes as input i) three categories of arguments: *epistemic* arguments that support beliefs, *explanatory* arguments that show that a desire holds in the current state of the world, and *instrumental* arguments that show that a desire is feasible, ii) different conflicts among those arguments, and iii) a particular constraint on arguments that captures the idea that for a desire to be pursued it should be both feasible and justified. This is translated by the fact that in a given extension each instrumental argument for a desire should be accompanied by at least one explanatory argument in favor of that desire and each explanatory argument for a desire should be accompanied by at least one instrumental argument for that desire. Two outputs are returned by the system: The first one is a set of extensions of arguments. Due to the constraint, only the "interesting" ones (*i.e.* the ones that support desires that are both justified and feasible) are kept. The second output is different sets of intentions. The agent should select one of them. In [4], it has been argued that this is a pure decision making problem, and several criteria have been proposed for rank-ordering sets of intentions. The output of our system can then be an input to those criteria. In this paper, we do not consider this step. The properties of this system are deeply investigated. In particular, we show that the results of such a system are safe, and satisfy the rationality postulates identified in [13], namely consistency and completeness.

The paper is organized as follows: Section 2 recalls the basics of an argumentation system. Section 3 introduces an example of practical reasoning. Section 4 presents the language used for representing the main notions (beliefs, desires and actions). Section 5 studies the different types of arguments involved in a practical reasoning problem, and Section 6 investigates the conflicts that may exist between them. Section 7 presents the constrained argumentation system for PR. The properties of the system are studied in Section 8. Section 9 compares our approach with existing systems of practical reasoning. All the proofs are given in an appendix at the end of the document.

2. Constrained argumentation systems: fundamentals

Argumentation is an established approach for reasoning with inconsistent knowledge (like clinical knowledge [14]), based on the construction and the comparison of arguments.

An argumentation formalism is built around an underlying logical language and an associated notion of logical consequence, defining the notion of argument. Argument construction is a monotonic process: new knowledge cannot rule out an argument but only gives rise to new arguments which may interact with the first argument. Since knowledge bases may give rise to inconsistent conclusions, the arguments may be conflicting too. Consequently, it is important to determine among all the available arguments the ones that are ultimately "acceptable".

In [12], an abstract argumentation system is defined as follows:

Definition 1. (Basic argumentation system [12]) *An* argumentation system *is a pair* $\mathsf{AS} = \langle \mathcal{A}, \mathcal{R} \rangle$ *with* $\mathcal{A}$ *is a set of arguments, and* $\mathcal{R}$ *is an attack relation (*$\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$*). For* $\alpha, \beta \in \mathcal{A}$*, writing* $\alpha \mathcal{R} \beta$ *means that the argument* α *attacks the argument* β*.*

In a recent study [15], this system was extended in such a way to take into account attacks on attacks. However, for the purpose of our paper, we focus on Dung's version of argumentation systems.

It is also worth noticing that in the previous definition, neither the origin nor the structure of arguments are specified. Indeed, the main purpose of Dung in [12] was to propose semantics for evaluating arguments whatever their structure is. The main semantics are based on two requirements: *conflict-freeness* and *defence*.

Definition 2. (Conflict-free, Defence [12]) *Let* $\mathsf{AS} = \langle \mathcal{A}, \mathcal{R} \rangle$ *and* $\mathcal{E} \subseteq \mathcal{A}$*.*

- $\mathcal{E}$ *is* conflict-free *iff* $\nexists\, \alpha, \beta \in \mathcal{E}$ *s.t.* $\alpha \,\mathcal{R}\, \beta$*.*
- $\mathcal{E}$ defends *an argument* α *iff* $\forall\, \beta \in \mathcal{A}$*, if* $\beta \,\mathcal{R}\, \alpha$*, then* $\exists\, \delta \in \mathcal{E}$ *s.t.* $\delta \,\mathcal{R}\, \beta$*.*

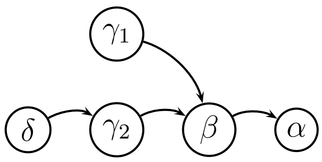
Different semantics were proposed in [12] and compared in [16]. For the purpose of our paper, we only need to recall two of them: stable and preferred semantics since they are the ones that are more suitable for practical reasoning as already explained in the introduction.

Definition 3. (Acceptability semantics [12]) *Let* $\mathsf{AS} = \langle \mathcal{A}, \mathcal{R} \rangle$ *and* $\mathcal{E} \subseteq \mathcal{A}$*.*

- $\mathcal{E}$ *is an* admissible *set iff it is conflict-free and defends every element in* $\mathcal{E}$*.*
- $\mathcal{E}$ *is a* preferred extension *iff it is a maximal (w.r.t. set-inclusion) admissible set.*
- $\mathcal{E}$ *is a* stable extension *iff it is a preferred extension that attacks all arguments in* $\mathcal{A} \backslash \mathcal{E}$*.*

Note that every stable extension is also a preferred one, but the converse is not always true.

Example 1. *Let* AS_1 *be an argumentation system such that* $\mathcal{A} = \{\alpha, \beta, \gamma_1, \gamma_2, \delta\}$ *and* $\mathcal{R} = \{(\delta, \gamma_2)$, (γ_1, β), (γ_2, β), $(\beta, \alpha)\}$*. The system* AS_1 *is depicted in the following figure:*



It can be checked that this argumentation system has six admissible sets: $\mathcal{E}_1 = \varnothing$, $\mathcal{E}_2 = \{\delta\}$, $\mathcal{E}_3 = \{\gamma_1\}$, $\mathcal{E}_4 = \{\delta, \gamma_1\}$, $\mathcal{E}_5 = \{\alpha, \gamma_1\}$ *and* $\mathcal{E}_6 = \{\delta, \gamma_1, \alpha\}$*. Among the six sets, only* $\mathcal{E}_6$ *is a preferred extension. In this example,* $\mathcal{E}_6$ *is also a stable extension.*

The basic argumentation system is extended in [11] by adding a *constraint* on arguments. This constraint should be satisfied by Dung's extensions (under a given semantics). In Example 1, one may imagine a constraint which requires that the two arguments α and γ_2 belong to the same stable extension. It is clear that this constraint can be satisfied neither by the stable extension $\mathcal{E}_6$, nor by any other admissible set of the system AS_1.

The constraint is a formula of a propositional language $\mathcal{L}_{\mathcal{A}}$ whose alphabet (*i.e.* propositional variables) is exactly the set $\mathcal{A}$ of arguments. Thus, each argument in $\mathcal{A}$ is a literal of $\mathcal{L}_{\mathcal{A}}$. Note that $\mathcal{L}_{\mathcal{A}}$ contains all the formulas that can be built using the usual logical operators ($\wedge$, $\vee$, $\rightarrow$, $\neg$, $\leftrightarrow$) and the constant symbols ($\top$ and $\bot$).

Definition 4. (Constraint, Completion [11]) *Let* $\mathcal{A}$ *be a set of arguments and* $\mathcal{L}_{\mathcal{A}}$ *its corresponding propositional language.*

- C *is a* constraint *on arguments of* $\mathcal{A}$ *iff* C *is a formula of* $\mathcal{L}_{\mathcal{A}}$*.*
- *The* completion *of a set* $\mathcal{E} \subseteq \mathcal{A}$ *is* $\widehat{\mathcal{E}} = \{\alpha \mid \alpha \in \mathcal{E}\} \cup \{\neg\alpha \mid \alpha \in \mathcal{A} \setminus \mathcal{E}\}$*.*
- *A set* $\mathcal{E} \subseteq \mathcal{A}$ satisfies C *iff* $\widehat{\mathcal{E}}$ *is a model of* C *(*$\widehat{\mathcal{E}} \vdash C$*).*

The completion of a set $\mathcal{E}$ of arguments is a set in which each argument of $\mathcal{A}$ appears either as a positive literal if the argument belongs to $\mathcal{E}$ or as a negative one otherwise. Thus, $|\widehat{\mathcal{E}}| = |\mathcal{A}|$.

Example 1 (Continued): *In the argumentation system* AS_1*, one may want to exclude the extensions that contain both arguments* α *and* δ*. This requirement is translated into the constraint:* $C = \delta \to \neg\alpha$*. In this case, the completion of the admissible extension* $\mathcal{E}_6 = \{\delta, \gamma_1, \alpha\}$ *is the set* $\widehat{\mathcal{E}_6} = \{\delta, \gamma_1, \alpha, \neg\beta, \neg\gamma_2\}$*. Note that* $\mathcal{E}_6$ *does not satisfy* C *since the set* $\widehat{\mathcal{E}_6}$ *does not infer* C*.*

A constrained argumentation system is defined as follows:

Definition 5. (Constrained argumentation system [11]) *A* constrained argumentation system *is a triple* $\mathsf{CAS} = \langle \mathcal{A}, \mathcal{R}, C \rangle$ *where* $\mathcal{A}$ *is a set of arguments,* $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ *is an attack relation and* C *is a constraint on arguments of the set* $\mathcal{A}$*.*

Note that, each argument may be a constraint. However, a constrained argumentation system has exactly one constraint. Thus, if this constraint is reduced to one argument, this means that all extensions of the system should contain this argument.

Let us now recall how Dung's extensions are extended to the case of constrained argumentation systems. As said before, the idea is to compute Dung's extensions, and to keep among those extensions only the ones that satisfy the constraint C.

Definition 6. (C-admissible set [11]) *Let* $\mathsf{CAS} = \langle \mathcal{A}, \mathcal{R}, C \rangle$ *and* $\mathcal{E} \subseteq \mathcal{A}$*. The set* $\mathcal{E}$ *is* C-admissible *in* CAS *iff*

1. $\mathcal{E}$ *is admissible,*
2. $\mathcal{E}$ *satisfies the constraint* C[2]*.*

[2] Note that the constraint on arguments corresponds to a constraint on extensions.

In [12], it has been shown that the empty set is always admissible; however, it is not always C-admissible since the set $\widehat{\varnothing}$ does not always imply C.

Definition 7. (C-preferred extension, C-stable extension [11]) *Let* $\mathsf{CAS} = \langle \mathcal{A}, \mathcal{R}, C \rangle$ *and* $\mathcal{E} \subseteq \mathcal{A}$.

- $\mathcal{E}$ *is a* C-preferred extension *of* CAS *iff* $\mathcal{E}$ *is maximal for set-inclusion among the* C*-admissible sets.*

- $\mathcal{E}$ *is a* C-stable extension *of* CAS *iff* $\mathcal{E}$ *is a* C*-preferred extension that attacks all arguments in* $\mathcal{A} \backslash \mathcal{E}$.

Example 1 (Continued): *The constrained version of* AS_1 *is* $\mathsf{CAS}_1 = \langle \mathcal{A}, \mathcal{R}, \delta \rightarrow \neg\alpha \rangle$. *The set* $\mathcal{E}_6 = \{\delta, \gamma_1, \alpha\}$ *is not a* C*-admissible extension since its completion* $\widehat{\mathcal{E}_6} = \{\delta, \gamma_1, \alpha, \neg\beta, \neg\gamma_2\}$ *does not infer the formula* $\delta \rightarrow \neg\alpha$. *However, the admissible extensions* $\mathcal{E}_4 = \{\delta, \gamma_1\}$ *and* $\mathcal{E}_5 = \{\alpha, \gamma_1\}$ *are both* C*-admissible and* C*-preferred extensions. Note that* CAS_1 *has no* C*-stable extensions.*

The following result summarizes the links between the extensions of a $\mathsf{CAS} = \langle \mathcal{A}, \mathcal{R}, C \rangle$ and those of its basic version $\mathsf{AS} = \langle \mathcal{A}, \mathcal{R} \rangle$.

Proposition 1. [11] *Let* $\mathsf{CAS} = \langle \mathcal{A}, \mathcal{R}, C \rangle$ *and* $\mathsf{AS} = \langle \mathcal{A}, \mathcal{R} \rangle$ *be its basic version.*

- *For each* C*-preferred extension* $\mathcal{E}$ *of* CAS*, there exists a preferred extension* $\mathcal{E}'$ *of* AS *such that* $\mathcal{E} \subseteq \mathcal{E}'$.

- *Every* C*-stable extension of* CAS *is a stable (hence preferred) extension of* AS. *The converse does not hold.*

Now that the acceptability semantics are defined, we are ready to define the status of any argument.

Definition 8. (Argument status) *Let* $\mathsf{CAS} = \langle \mathcal{A}, \mathcal{R}, C \rangle$, $\mathcal{E}_1, \ldots, \mathcal{E}_x$ *its* C*-extensions under a given semantics, and* $\alpha \in \mathcal{A}$.

1. α *is* sceptically accepted *(or accepted for short) iff* $\alpha \in \mathcal{E}_i$, $\forall \mathcal{E}_i$ *with* $i = 1, \ldots, x$.
2. α *is* rejected *iff* $\nexists \mathcal{E}_i$ *such that* $\alpha \in \mathcal{E}_i$.

3. α *is* credulously accepted *(or undecided) iff* α *is neither accepted nor rejected. This means that* α *is in some extensions and not in others.*

One can easily check that if an argument is rejected in a basic argumentation system AS under a given semantics, then it will also be rejected in the corresponding CAS under the same semantics.

Proposition 2. *Let* $\mathsf{CAS} = \langle \mathcal{A}, \mathcal{R}, C \rangle$ *and* $\mathsf{AS} = \langle \mathcal{A}, \mathcal{R} \rangle$ *be its basic version. For any* $\alpha \in \mathcal{A}$*, if* α *is rejected in* AS *under semantics* x *(where* x *is either preferred or stable), then* α *is also rejected in* CAS *under the same semantics* x*.*

Example 1 (Continued): *Under preferred semantics, the arguments* δ*,* γ_1 *and* α *are accepted while* γ_2 *and* β *are rejected in* AS_1*. Under the same semantics,* γ_1 *is accepted;* γ_2 *and* β *are rejected;* δ *and* α *are undecided in* CAS_1*. Finally, all the arguments are rejected under stable semantics in* CAS_1*, since there is no C-stable extension in* CAS_1*.*

3. Motivating example

Let us consider the case of Paula, a PhD student, who has four *desires* and would like to know whether she can reach them and with which plans. The four desires are:

- To be in central Africa for holidays (jca)
- To have her publication finished (fp)
- To be a lecturer (lec)
- To have visited her friend Carla (vc) if Carla is at home

What is worth noticing is that the three first desires are unconditional, whereas the fourth one depends on whether the friend is at home or not. Moreover, as argued in [17] a desire is a *state of the world* that an agent wants to reach in the future.

In addition to desires, Paula has beliefs on the way of moving from a state of the world to another one, namely:

- In order to have the paper finished, Paula should work (w)
- In order to be a lecturer, Paula should defend her thesis (dt) provided that her thesis is finished (ft)
- In order to visit her friend, Paula can go by car (gc) if it is in good state (gs)
- In order to have tickets (t), Paula can either go to an agency (ag) or ask a friend who may bring them (afr);
- In order to be vaccinated (vac), Paula can go either to a hospital (hop) or to a doctor (dr).

Paula has also some information about either the current state of the world:

- Actually, Paula's car is in good state (gs)
- Carla is not at home ($\neg ch$)
- Paula's thesis is not finished ($\neg ft$)
- Paula is not vaccinated ($\neg vac$) and does not have her tickets ($\neg t$)
- Paula's paper is not yet finished ($\neg fp$)

or the consequences of some actions or some states of the world over her desires:

- If Paula passes to an agency or goes to a doctor, then she cannot finish her paper
- If Paula has tickets and is vaccinated, then she can be in central africa for holidays.

Note that the term "belief" is a generic word representing informations believed by the agent about:

- the current state of the world,
- the way of moving from a state of the world to another one,
- the consequences of some actions or some states of the world over her desire.

The aim of this example is not to present a realistic situation, but to illustrate our ideas. Thus, it may be possible that more information can be added either as integrity constraints or even as conditional desires.

From the above information, it is clear that the desire of becoming a lecturer is not yet feasible. The desire of visiting Carla is feasible since there is a plan for reaching it; however, according to the current state of the world, this desire is not justified. Indeed, for Paula to consider this desire, she should be in a state where Carla is at home and this is not the case. Regarding the two first desires (*i.e.* jca and fp) things are different. Both desires are justified and feasible. However, in some cases, it is not possible to reach both desires as their plans conflict with each other. Of course, it would be ideal if all the desires can become intentions. As our example illustrates, this may not always be the case. In this paper we will answer the following questions: which desires will become the *intentions* of the agent and with which *plans*?

Next sections will give the formal material necessary for encoding this example of PR and computing the intentions of Paula.

4. Language of representation

The example discussed in the previous section shows that three notions are involved in a PR problem: *desires*, *actions* and *beliefs*. For encoding them, we will use a set $\mathcal{X}$ of propositional variables (atoms). From this set, two subsets are distinguished: $\mathcal{X}_{ac}$ and $\mathcal{X}_{nac}$ with $\mathcal{X}_{ac} \cup \mathcal{X}_{nac} = \mathcal{X}$ and $\mathcal{X}_{ac} \cap \mathcal{X}_{nac} = \varnothing$. The subset $\mathcal{X}_{ac}$ will be used for encoding actions while $\mathcal{X}_{nac}$ will be used for encoding non-actions (*i.e.* beliefs and desires).

Let $\mathcal{L}_{nac}$ be a *propositional language* built from $\mathcal{X}_{nac}$ using the classical logical operators $\wedge$, $\vee$, $\rightarrow$, $\neg$, $\leftrightarrow$ and the constant symbols $\top$, $\bot$. Note that $\mathcal{L}_{nac}$ is completely different from $\mathcal{L}_{\mathcal{A}}$ defined in Section 2[3]. $\mathcal{L}_{nac}$ will be used for encoding both desires and beliefs. As already said, a desire is a state of the world that an agent wants to reach. Thus, the main difference between a belief and a desire

[3]Their meaning and their use are different: $\mathcal{L}_{nac}$ will be used for representing beliefs and desires and *for building* arguments and interactions – see Sections 5 and 6 – and $\mathcal{L}_{\mathcal{A}}$ will be used for representing a constraint between arguments and restricting the set of the extensions – see Section 7 –.

is that the former is already true (or false) while the latter may only be true in the future (after the execution of an action). In [17], it has been argued that a desire can be encoded as a preference between two states of the world: the one in which the desire is satisfied and the one in which it is not satisfied. For instance, Paula prefers the state in which her publication is finished to that in which it is not yet finished. In our setting, desires are distinguished from beliefs by storing desires in a distinct set $\mathcal{D} \subseteq \mathcal{L}_{nac}$. Moreover, desires are *literals* and are denoted by $d_1, d_2, \ldots$[4] On their side, beliefs are propositional formulas of the whole language $\mathcal{L}_{nac}$.

Now regarding its source, a desire may be either unconditional or conditional. An unconditional desire does not depend on anything, it is expressed by an agent without justification. Some desires may depend on beliefs. This is, for instance, the case with the fourth desire of Paula. Indeed, visiting Carla depends on whether Carla is at home or not. Similarly, a desire may depend on other desires. For example, if there is a conference in India, and I have the desire to attend, then I desire also to attend the tutorials. In this example, the desire of attending the tutorials depends on my belief about the existence of a conference in India, and on my desire to attend that conference. These three sources of desires are captured by the notion of *desire rules*.

Definition 9. (Desire Rules) *A* desire rule *is an expression of the form* $\langle b, d_1, \ldots, d_{m-1}\rangle \hookrightarrow d_m$ *such that* b *is a propositional formula of* $\mathcal{L}_{nac}$ *and each* d_i *is an element of the set* $\mathcal{D}$.
$\langle b, d_1, \ldots, d_{m-1}\rangle$ *is called the* body *of the rule and* d_m *its* consequent. *Note that the body may be empty; in this case, the desire* d_m *is said* unconditional *and the desire rule is denoted by* $\langle\rangle \hookrightarrow d_m$.

The meaning of a rule $\langle b, d_1, \ldots, d_{m-1}\rangle \hookrightarrow d_m$ is "if the agent *believes* b and *desires* $d_1, \ldots, d_{m-1}$, then she will *desire* d_m as well". Note that the same desire d_i may appear in the consequent of several rules. This means that the same desire may depend on different beliefs or desires.

Example 2. ***(Paula's example)*** *In the motivating example,* $\mathcal{X}_{nac} = \{jca, fp, lec, vc, gs, ch, ft, vac, t\}$, *and the set of desires is* $\mathcal{D} = \{jca, \neg jca, fp, \neg fp, lec, \neg lec, vc, \neg vc\}$. *The desire rules of Paula are* $\langle\rangle \hookrightarrow jca$, $\langle\rangle \hookrightarrow fp$, $\langle\rangle \hookrightarrow lec$, $\langle ch\rangle \hookrightarrow vc$.

[4]Note that this notation will not be respected in the motivating example. We prefer to use more explicit strings of lowercase letters.

An agent is also equipped with a set of actions she can perform. These actions are provided by a correct and sound planning system (for instance [18, 19]) (not discussed in this paper). Note that the actions may not necessarily succeed since the environment is changing. In what follows, an action is defined as a triple: i) a set S of pre-conditions that should be satisfied before executing the action[5], ii) a set T of post-conditions that hold after executing the action, and iii) the name a of the action. Thus, an action allows to move from one state of the world to another. An action may either be atomic or a conjunction of atomic actions. Thus, each action is considered as a *plan* for reaching a state of the world[6]. Let $\mathcal{L}_{ac}$ be the propositional language built from $\mathcal{X}_{ac}$ using only the classical operator $\wedge$. Thus, formulas of $\mathcal{L}_{ac}$ are either atoms or conjunctions of atoms.

Definition 10. (Action) *An* action *(or a* plan*) is a triple* $\langle S, T, a \rangle$ *such that:*

- S *and* T *are two* consistent *sets of propositional formulas of* $\mathcal{L}_{nac}$
- $a \in \mathcal{L}_{ac}$

The set of pre-conditions may be empty ($S = \varnothing$), which means that the action can be carried out. It is also worth mentioning that there exists a link between S and T[7]. This link is not made explicit in this paper since we are not really interested by the exact definition of actions. We assume that they are given. Note also that a desire d may appear either in the pre-conditions or in the post-conditions of an action. When d is in the post-conditions of an action, this means that the action leads to the satisfaction of the desire. When d is in the pre-condition of an action, this means that in order to perform the action, we should be in a state of the world in which d is already reached. Let us illustrate this notion of action on the running example.

Example 2 (Continued): In this example, $\mathcal{X}_{ac} = \{dr, hop, ag, afr, gc, w, dt\}$ and the actions that are available for Paula are the following:

---

[5]The set S only describes the elements of the world which are mandatory for the execution of the action.

[6]For simplicity reasons, actions are encoded in a restricted propositional language.

[7]In the sense, that the formulas in T are obtained using the formulas in S.

$\langle\{\neg fp\}, \{fp\}, w\rangle$	$\langle\{\neg t\}, \{t, \neg fp\}, ag\rangle$
$\langle\{ft\}, \{lec\}, dt\rangle$	$\langle\{\neg vac\}, \{vac\}, hop\rangle$
$\langle\{\neg vac, \neg t\}, \{jca, vac, t, \neg fp\}, dr \wedge ag\rangle$	$\langle\{\neg vac\}, \{vac, \neg fp\}, dr\rangle$
$\langle\{\neg vac, \neg t\}, \{jca, vac, t, \neg fp\}, dr \wedge afr\rangle$	$\langle\{gs\}, \{vc\}, gc\rangle$
$\langle\{\neg vac, \neg t\}, \{jca, vac, t, \neg fp\}, hop \wedge ag\rangle$	$\langle\{\neg t\}, \{t\}, afr\rangle$
$\langle\{\neg vac, \neg t\}, \{jca, vac, t\}, hop \wedge afr\rangle$	

Note that the information "If Paula passes to an agency (ag) or goes to a doctor (dr), then she cannot finish her paper" is directly captured by the post-conditions of the two actions ag and dr. Similarly, the information "If Paula has tickets (t) and is vaccinated (vac), then she can be in central africa for holidays" is indirecly captured by the post-conditions of the compound actions $hop \wedge afr$, $hop \wedge ag$, $dr \wedge afr$, and $dr \wedge ag$.

In the remaining of the paper, we assume that an agent is equipped with the following three finite bases.

Definition 11. (Agent's bases) *An agent is equipped with three* finite bases*:*

1. $\mathcal{K}_b \subseteq \mathcal{L}_{nac}$ *containing its* basic beliefs *about the current state of the world,*
2. $\mathcal{K}_d$ *containing its desire rules,*
3. $\mathcal{K}_a$ *containing its actions.*

Example 2 (Continued): Paula is equipped with the following bases:

- $\mathcal{K}_b = \{gs, \neg ch, \neg ft, \neg vac, \neg t, \neg fp\}$,
- $\mathcal{K}_d = \{\langle\rangle \hookrightarrow jca, \langle\rangle \hookrightarrow fp, \langle\rangle \hookrightarrow lec, \langle ch\rangle \hookrightarrow vc\}$,
- $\mathcal{K}_a = \{$

$$\begin{array}{rl} \langle\{\neg fp\}, \{fp\}, w\rangle, & \langle\{ft\}, \{lec\}, dt\rangle, \\ \langle\{gs\}, \{vc\}, gc\rangle, & \langle\{\neg t\}, \{t, \neg fp\}, ag\rangle, \\ \langle\{\neg vac, \neg t\}, \{jca, vac, t, \neg fp\}, hop \wedge ag\rangle, & \langle\{\neg t\}, \{t\}, afr\rangle, \\ \langle\{\neg vac, \neg t\}, \{jca, vac, t, \neg fp\}, dr \wedge ag\rangle, & \langle\{\neg vac\}, \{vac, \neg fp\}, dr\rangle, \\ \langle\{\neg vac, \neg t\}, \{jca, vac, t, \neg fp\}, dr \wedge afr\rangle, & \langle\{\neg vac\}, \{vac\}, hop\rangle, \\ \langle\{\neg vac, \neg t\}, \{jca, vac, t\}, hop \wedge afr\rangle\}. & \end{array}$$

From $\mathcal{K}_d$, the set of *potential desires* of an agent can be identified as follows:

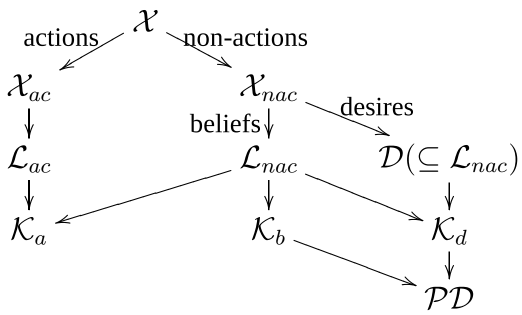
Definition 12. (Potential Desires) *Let $\mathcal{K}_b$ (resp. $\mathcal{K}_d$) be the belief base (resp. the set of desire rules) of an agent. The set of* potential desires *of this agent is* $\mathcal{PD} = \{d_m | \exists \langle b, d_1, \ldots, d_{m-1} \rangle \hookrightarrow d_m \in \mathcal{K}_d$ *and* $\mathcal{K}_b \nvdash d_m\}$.

These are "potential" desires because, when the body of the rule is not empty, the agent does not know yet whether the antecedents (*i.e.* bodies) of the corresponding rules are true or not. Moreover, throughout the paper, we assume that each potential desire of an agent is not yet reached in the current state of the world. This assumption is natural as a desire that is satisfied is no longer a desire.

Example 3. *Assume that Paula wants to be rich and, in the current state of the world, Peter is rich. In this case, the desire "to be rich" does not belong to Paula's set of potential desires.*

Example 2 (Continued): The set of potential desires of Paula is $\mathcal{PD} = \{jca$, fp, lec, $vc\}$.

The following schema gives a synthesis of the presented notions and their representation (from vocabulary to bases).



In the following sections, we propose different kinds of arguments (one for each notion introduced here: belief, desire and action/plan) and we study the conflicts between these arguments.

5. Typology of arguments

The aim of this section is to present the different kinds of arguments involved in a practical reasoning problem. Three categories of arguments are distinguished. The first category justifies/attacks beliefs of the knowledge base $\mathcal{K}_b$, while the two others justify the adoption of the potential desires of the $\mathcal{PD}$. Throughout the paper, arguments will be denoted with lowercase Greek letters.

5.1. Justifying beliefs

The first category of arguments is that studied in argumentation literature, especially for handling inconsistency in knowledge bases. Indeed, arguments are built from a knowledge base in order to support or to attack potential conclusions or inferences. These arguments are called *epistemic* in [20]. In our application, such arguments are built from the base $\mathcal{K}_b$. In what follows, we will use the definition proposed in [21].

Definition 13. (Epistemic Argument) *Let $\mathcal{K}_b$ be a beliefs base. An* epistemic argument α *is a pair* $\alpha = \langle H, h \rangle$ *such that:*

1. $H \subseteq \mathcal{K}_b$ *and* $h \in \mathcal{L}_{nac}$*,*
2. H *is consistent,*
3. $H \vdash h$ *and*
4. H *is minimal (for set $\subseteq$) among the sets satisfying conditions 1, 2, 3.*

The support *of the argument is given by the function* $\text{SUPP}(\alpha) = H$*, whereas its* conclusion *is returned by* $\text{CONC}(\alpha) = h$*.*

Definition 14. (Set of epistemic Arguments) $\mathcal{A}_b$ *stands for the set of all epistemic arguments that can be built from the base $\mathcal{K}_b$.*

> **Remark:** Due to the assumption that each potential desire is not yet true in the current state of the world, it is clear that the conclusion h of an epistemic argument cannot be a potential desire (*i.e.* an element of $\mathcal{PD}$). Thus, $\nexists \alpha \in \mathcal{A}_b$ such that $\text{CONC}(\alpha) \in \mathcal{PD}$.

Example 2 (Continued): Recall that the knowledge base of Paula is $\mathcal{K}_b = \{gs, \neg ch, \neg ft, \neg vac, \neg t, \neg fp\}$. The table below contains some epistemic arguments of the set $\mathcal{A}_b$. Other arguments, not presented here, can also be built from $\mathcal{K}_b$.

$\alpha_1 = \langle \{gs\}, gs \rangle$	$\alpha_5 = \langle \{\neg t\}, \neg t \rangle$
$\alpha_2 = \langle \{\neg ch\}, \neg ch \rangle$	$\alpha_6 = \langle \{\neg fp\}, \neg fp \rangle$
$\alpha_3 = \langle \{\neg ft\}, \neg ft \rangle$	$\alpha_7 = \langle \{\neg vac \wedge \neg t\}, \neg vac \wedge \neg t \rangle$
$\alpha_4 = \langle \{\neg vac\}, \neg vac \rangle$	$\alpha_8 = \langle \{gs, \neg ch\}, ch \rightarrow gs \rangle$
...	...

5.2. Justifying desires

A desire may be pursued by an agent only if this desire is *justified* and *feasible*. Thus, two kinds of *reasons* are needed for adopting a desire:

- the conditions underlying the desire hold[8] in the current state of world; such reasons will be called *explanatory arguments*;
- and there is a plan (an action) for reaching the desire; such reasons will be called *instrumental arguments*.

The definition of the first kind of arguments involves two bases: the belief base $\mathcal{K}_b$ and the base of desire rules $\mathcal{K}_d$. In what follows, we will use a tree-style definition of arguments [22]. This choice is not arbitrary but imposed by the logical language at hand. In particular, desire rules are not material implications, thus it is important to show how such rules are chained.

Before presenting that definition, let us first introduce some useful functions that will be used throughout the paper:

> **Notations:** The functions $\textsc{Beliefs}(\delta)$, $\textsc{Desires}(\delta)$, $\textsc{Conc}(\delta)$ and $\textsc{Sub}(\delta)$ return respectively, for a given explanatory argument δ, the beliefs used in δ, the desires supported by δ, the conclusion and the set of sub-arguments of the argument δ.

Definition 15. (Explanatory Argument) *Let $\mathcal{K}_b$, $\mathcal{K}_d$ be two bases. An* explanatory *argument is a pair $\delta = \langle S, d \rangle$ where $d \in \mathcal{PD}$ and S is defined recursively as follows:*

- *If $\exists \langle \rangle \hookrightarrow d \in \mathcal{K}_d$ then S is $\langle \rangle$ and*
  - $\textsc{Beliefs}(\delta) = \varnothing$,
  - $\textsc{Desires}(\delta) = \{d\}$,
  - $\textsc{Conc}(\delta) = d$,
  - $\textsc{Sub}(\delta) = \{\delta\}$.
- *If α is an epistemic argument, and $\delta_1, \ldots, \delta_m$ are explanatory arguments, and $\exists$ $\langle \textsc{Conc}(\alpha), \textsc{Conc}(\delta_1), \ldots, \textsc{Conc}(\delta_m) \rangle \hookrightarrow d \in \mathcal{K}_d$ then S is $\langle \alpha, \delta_1, \ldots, \delta_m \rangle$ and*

[8] In the sense that the conditions are inferred from the bases of the agent.

- $\text{BELIEFS}(\delta) = \text{SUPP}(\alpha) \cup \text{BELIEFS}(\delta_1) \cup \ldots \cup \text{BELIEFS}(\delta_m)$,
- $\text{DESIRES}(\delta) = \text{DESIRES}(\delta_1) \cup \ldots \cup \text{DESIRES}(\delta_m) \cup \{d\}$,
- $\text{CONC}(\delta) = d$,
- $\text{SUB}(\delta) = \{\alpha\} \cup \text{SUB}(\delta_1) \cup \ldots \cup \text{SUB}(\delta_m) \cup \{\delta\}$.

Definition 16. (Set of explanatory Arguments) *$\mathcal{A}_d$ stands for the set of all explanatory arguments δ that can be built from $\mathcal{K}_b$ and $\mathcal{K}_d$ such that the set* $\text{DESIRES}(\delta)$ *is consistent*[9].

Example 2 (Continued): Recall that $\mathcal{K}_b = \{gs, \neg ch, \neg ft, \neg vac, \neg t, \neg fp\}$ and $\mathcal{K}_d = \{\langle\rangle \hookrightarrow jca, \langle\rangle \hookrightarrow fp, \langle\rangle \hookrightarrow lec, \langle ch\rangle \hookrightarrow vc\}$. The set $\mathcal{A}_d = \{\delta_1, \delta_2, \delta_3\}$ where:

$\delta_1 = \langle\langle\rangle, jca\rangle$	$\delta_2 = \langle\langle\rangle, fp\rangle$	$\delta_3 = \langle\langle\rangle, lec\rangle$

Note that there is no explanatory argument in favor of desire vc since the pre-condition (ch) of the corresponding desire rule in not satisfied. Worse yet, $\neg ch \in \mathcal{K}_b$.

The same desire may be supported by several explanatory arguments since a desire may be the consequent of different desire rules. The set $\text{DESIRES}(\delta)$ of an explanatory argument δ contains the desire d (the conclusion of δ) and, in the case of a conditional desire, *all the desires* used for justifying d. The following trivial proposition follows from the previous definitions.

Proposition 3. *Let $\delta \in \mathcal{A}_d$.*

- *The set* DESIRES *of δ is a subset of $\mathcal{PD}$ (*$\text{DESIRES}(\delta) \subseteq \mathcal{PD}$*).*
- *The set* BELIEFS *of δ is a subset of the knowledge base $\mathcal{K}_b$ (*$\text{BELIEFS}(\delta) \subseteq \mathcal{K}_b$*).*

[9] The fact that the desires of a desire rule are not conflicting is not sufficient to ensure the consistency of the set $\text{DESIRES}(\delta)$ of an explanatory argument δ. Consider, for instance, the following example: $\mathcal{K}_d = \{\langle\rangle \hookrightarrow d_1;\ \langle\rangle \hookrightarrow \neg d_1;\ \langle d_1\rangle \hookrightarrow d_2;\ \langle\neg d_1\rangle \hookrightarrow d_3;\ \langle d_2, d_3\rangle \hookrightarrow d_4\}$. It is easy to check that only one explanatory argument, δ, can be built from $\mathcal{K}_d$ for the desire d_4, and that $\text{DESIRES}(\delta)$ contains both d_1 and $\neg d_1$. Such arguments are forbidden in our system.

The last category of arguments claims that "a desire may be pursued since it has a plan for achieving it". The definition of this kind of arguments involves the belief base $\mathcal{K}_b$, the base of actions/plans $\mathcal{K}_a$, and the set $\mathcal{PD}$.

Definition 17. (Instrumental Argument) *Let $\mathcal{K}_b, \mathcal{K}_a, \mathcal{PD}$ be three bases, and $d \in \mathcal{PD}$. An* instrumental *argument is a pair $\pi = \langle\langle S, T, x\rangle, d\rangle$ where:*

- $\langle S, T, x\rangle \in \mathcal{K}_a$,
- $d \in T$,
- $S \subseteq \mathcal{K}_b$.

The function CONC *will return for an argument π the desire d. Similarly, the functions* PLAN, PREC *and* POSTC *will return respectively the action $\langle S, T, d\rangle$ of the argument, the pre-conditions S of the action, its post-conditions T.*

Definition 18. (Set of instrumental Arguments) *$\mathcal{A}_p$ stands for the set of all instrumental arguments that can be built from $\langle \mathcal{K}_b, \mathcal{K}_a, \mathcal{PD}\rangle$.*

The second condition of the above definition ensures that the desire is reached when the action is executed. The third condition ensures that the pre-conditions of the action hold in the current state of the world. In other words, the action can be executed. Note that it may be the case that the base $\mathcal{K}_a$ contains actions whose pre-conditions are not true. Such actions cannot be executed and their corresponding instrumental arguments do not exist.

Example 2 (Continued): Let us recall here the three bases of Paula.

- $\mathcal{K}_b = \{gs, \neg ch, \neg ft, \neg vac, \neg t, \neg fp\}$,
- $\mathcal{K}_d = \{\langle\rangle \hookrightarrow jca, \langle\rangle \hookrightarrow fp, \langle\rangle \hookrightarrow lec, \langle ch\rangle \hookrightarrow vc\}$,
- $\mathcal{K}_a = \{$

$$\begin{array}{rl} \langle\{\neg fp\}, \{fp\}, w\rangle, & \langle\{ft\}, \{lec\}, dt\rangle, \\ \langle\{gs\}, \{vc\}, gc\rangle, & \langle\{\neg t\}, \{t, \neg fp\}, ag\rangle, \\ \langle\{\neg vac, \neg t\}, \{jca, vac, t, \neg fp\}, hop \wedge ag\rangle, & \langle\{\neg t\}, \{t\}, afr\rangle, \\ \langle\{\neg vac, \neg t\}, \{jca, vac, t, \neg fp\}, dr \wedge ag\rangle, & \langle\{\neg vac\}, \{vac, \neg fp\}, dr\rangle, \\ \langle\{\neg vac, \neg t\}, \{jca, vac, t, \neg fp\}, dr \wedge afr\rangle, & \langle\{\neg vac\}, \{vac\}, hop\rangle, \\ \langle\{\neg vac, \neg t\}, \{jca, vac, t\}, hop \wedge afr\rangle\}. & \end{array}$$

The only action that allows Paula to be a lecturer consists of defending her thesis (*i.e.* $\langle\{ft\}, \{lec\}, dt\rangle$). However, the pre-condition of this action (ft) is not satisfied in the current state of the world, namely the thesis is not finished yet ($\neg ft \in \mathcal{K}_b$). The other desires are all feasible. Their instrumental arguments are gathered in the set $\mathcal{A}_p = \{\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6\}$ where:

$\pi_1 : \langle\langle\{\neg vac, \neg t\}, \{jca, vac, t, \neg fp\}, dr \wedge ag\rangle, jca\rangle$
$\pi_2 : \langle\langle\{\neg vac, \neg t\}, \{jca, vac, t, \neg fp\}, dr \wedge af\rangle, jca\rangle$
$\pi_3 : \langle\langle\{\neg vac, \neg t\}, \{jca, vac, t, \neg fp\}, hop \wedge ag\rangle, jca\rangle$
$\pi_4 : \langle\langle\{\neg vac, \neg t\}, \{jca, vac, t\}, hop \wedge af\rangle, jca\rangle$
$\pi_5 : \langle\langle\{\neg fp\}, \{fp\}, w\rangle, fp\rangle$
$\pi_6 : \langle\langle\{gs\}, \{vc\}, gc\rangle, vc\rangle$

Remark: In what follows, $\mathcal{A} = \mathcal{A}_b \cup \mathcal{A}_d \cup \mathcal{A}_p$. Note that $\mathcal{A}$ is *finite* since the three initial bases ($\mathcal{K}_b$, $\mathcal{K}_d$ and $\mathcal{K}_a$) are finite.

5.3. Summary

The following table summarizes the different arguments involved in a PR problem.

Type of argument	Type of its conclusion	Set	Bases involved
Epistemic	belief	$\mathcal{A}_b$	$\mathcal{K}_b$
Explanatory	desire	$\mathcal{A}_d$	$\mathcal{K}_b, \mathcal{K}_d$
Instrumental	desire	$\mathcal{A}_p$	$\mathcal{K}_b, \mathcal{K}_a, \mathcal{PD}$

The next section presents the different conflicts between all these arguments.

6. Interactions between arguments

Arguments built from a knowledge base cannot generally be considered separately in an inference problem. Indeed, an argument constitutes a reason for believing, or adopting a desire. However, it is not a proof that the belief is true, or in our case that the desire should be adopted. The reason is that an argument can be attacked by other arguments. In this section, we will investigate the different kinds of conflicts among the arguments identified in the previous section.

6.1. *Conflicts among epistemic arguments*

An argument can be attacked by another argument for three main reasons: i) they have contradictory conclusions (this is known as *rebuttal*) [23], ii) the conclusion of an argument contradicts a premise of another argument (*assumption attack*) [23], iii) the conclusion of an argument contradicts an inference rule used in order to build the other argument (*undercutting*) [24].

Since the base $\mathcal{K}_b$ contains propositional formulas, it has been shown in [25] that the notion of assumption attack is sufficient to capture conflicts between epistemic arguments.

Definition 19. *Let α_1, $\alpha_2 \in \mathcal{A}_b$. The conflict relation $\mathcal{R}_b$ on $\mathcal{A}_b$ is defined as follows:*
$\alpha_1 \; \mathcal{R}_b \; \alpha_2$ iff $\exists h \in \textsc{Supp}(\alpha_2)$ such that $\textsc{Conc}(\alpha_1) \equiv \neg h$.

Example 2 (Continued): In our running example, the base $\mathcal{K}_b = \{gs, \neg ch, \neg ft, \neg vac, \neg t, \neg fp\}$ is clearly consistent. Thus, epistemic arguments are not conflicting and $\mathcal{R}_b = \varnothing$.

Let us now consider another knowledge base.

Example 4. *Let $\mathcal{K}_b = \{a, \neg b, a \to b\}$ be a propositional knowledge base. The argument $\langle \{a, \neg b\}, a \wedge \neg b\rangle$ attacks in the sense of $\mathcal{R}_b$ the argument $\langle \{a, a \to b\}, b\rangle$.*

Note that the assumption attack is a binary relation that *is not symmetric*. Moreover, it can be shown that there are no self-attacking arguments.

Proposition 4. *Let $\mathcal{A}_b$ be the set of all epistemic arguments that can be built from a beliefs base $\mathcal{K}_b$. It holds that $\nexists \alpha \in \mathcal{A}_b$ such that $\alpha \; \mathcal{R}_b \; \alpha$.*

In [26], the argumentation system $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$ has been applied for handling inconsistency in a knowledge base, say $\mathcal{K}_b$. In this particular case, a full correspondence has been established between the stable extensions of the system and the maximal consistent subsets of the base $\mathcal{K}_b$. Before presenting formally the result, let us introduce two useful functions:

Notations:

- Let $\mathcal{E} \subseteq \mathcal{A}_b$, $\textsc{Base}(\mathcal{E}) = \bigcup H_i$ such that $\langle H_i, h_i \rangle \in \mathcal{E}$.

- Let $T \subseteq \mathcal{K}_b$, $\textsc{Arg}(T) = \{\langle H_i, h_i \rangle$ is an epistemic argument $| H_i \subseteq T\}$.

Proposition 5. **[26]** *Let $\mathcal{E}$ be a stable extension of $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$.*

- $\textsc{Base}(\mathcal{E})$ *is a maximal (for set inclusion) consistent subset of $\mathcal{K}_b$.*
- $\textsc{Arg}(\textsc{Base}(\mathcal{E})) = \mathcal{E}$*.*

Proposition 6. **[26]** *Let T be a maximal (for set inclusion) consistent subset of $\mathcal{K}_b$.*

- $\textsc{Arg}(T)$ *is a stable extension of $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$.*
- $\textsc{Base}(\textsc{Arg}(T)) = T$*.*

A direct consequence of the above result is that if the base $\mathcal{K}_b$ is not reduced to $\perp$, then the system $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$ has at least one non-empty stable extension.

Proposition 7. *If $\mathcal{K}_b \neq \varnothing$ and $\mathcal{K}_b \neq \{\perp\}$, then the argumentation system $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$ has non-empty stable extensions.*

In addition, it has been shown in [27] that each preferred extension of $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$ returns a consistent subset of $\mathcal{K}_b$.

Proposition 8. **[27]** *Let $\mathcal{E}$ be preferred extension of $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$. It holds that* $\textsc{Base}(\mathcal{E})$ *is a consistent subset of $\mathcal{K}_b$.*

6.2. Conflicts among explanatory arguments

Two explanatory arguments may also be conflicting, in particular, when they are based on contradictory desires. This kind of conflict is captured by the following relation:

Definition 20. *Let δ_1, $\delta_2 \in \mathcal{A}_d$. The conflict relation $\mathcal{R}_d$ on $\mathcal{A}_d$ is defined as follows:*
$\delta_1 \; \mathcal{R}_d \; \delta_2$ *iff* $\exists d_1 \in \textsc{Desires}(\delta_1)$, $d_2 \in \textsc{Desires}(\delta_2)$ *such that* $d_1 \equiv \neg d_2$*.*

Proposition 9. *The relation $\mathcal{R}_d$ is symmetric and irreflexive.*

Example 2 (Continued): The three explanatory arguments $\delta_1 = \langle\langle\rangle, jca\rangle$, $\delta_2 = \langle\langle\rangle, fp\rangle$ and $\delta_3 = \langle\langle\rangle, lec\rangle$ are not conflicting. Thus, $\mathcal{R}_d = \varnothing$.

Let us consider another example in which two explanatory arguments are conflicting.

Example 5. *Let $\mathcal{K}_d = \{\langle\rangle \hookrightarrow d_1, \langle\rangle \hookrightarrow \neg d_1, \langle d_1\rangle \hookrightarrow d_2\}$. The following three explanatory arguments are built from this base:*

- $\delta_1 = \langle\langle\rangle, d_1\rangle$
- $\delta_2 = \langle\langle\rangle, \neg d_1\rangle$
- $\delta_3 = \langle\langle\delta_1\rangle, d_2\rangle$

It is clear that $\delta_2 \mathcal{R}_d \delta_3$ and $\delta_3 \mathcal{R}_d \delta_2$ since $\text{DESIRES}(\delta_2) = \{\neg d_1\}$ *and* $\text{DESIRES}(\delta_3) = \{d_1, d_2\}$*. Similarly, $\delta_1 \mathcal{R}_d \delta_2$ and $\delta_2 \mathcal{R}_d \delta_1$ since* $\text{DESIRES}(\delta_1) = \{d_1\}$

It can also be checked that any two explanatory arguments having conflicting desires are conflicting in the sense of the relation $\mathcal{R}_d$. Formally:

Proposition 10. *Let $d_1, d_2 \in \mathcal{PD}$. If $d_1 \equiv \neg d_2$, then $\forall \delta_1, \delta_2 \in \mathcal{A}_d$ such that:*

1. $\exists \delta'_1 \in \text{SUB}(\delta_1)$ *with* $\text{CONC}(\delta'_1) = d_1$*, and*
2. $\exists \delta'_2 \in \text{SUB}(\delta_2)$ *with* $\text{CONC}(\delta'_2) = d_2$*,*

then $\delta_1 \; \mathcal{R}_d \; \delta_2$.

Note that, from the definition of an explanatory argument δ, the set $\text{DESIRES}(\delta)$ cannot be inconsistent. However, the set $\text{BELIEFS}(\delta)$ may be inconsistent. The union of the beliefs of two explanatory arguments may also be inconsistent. Later in the paper we will show that it is unnecessary to consider these kinds of conflict, since they are captured by conflicts between explanatory and epistemic arguments (see Propositions 13 and 14).

6.3. Conflicts among instrumental arguments

Two actions (or plans) may be conflicting for three main reasons:

1. incompatibility of their pre-conditions (indeed, both plans cannot be executed at the same time).
2. incompatibility of their post-conditions (the execution of both plans will lead to contradictory states of the world). This captures also the case of two plans leading to contradictory desires.
3. incompatibility between the post-conditions of a plan and the pre-conditions of the other (this means that the execution of a plan will prevent the execution of the second plan in the future).

The above reasons are captured in the following definition of attack among instrumental arguments.

Definition 21. *Let π_1, $\pi_2 \in \mathcal{A}_p$ and $\pi_1 \neq \pi_2$. The conflict relation $\mathcal{R}_p$ on $\mathcal{A}_p$ is defined as follows: $\pi_1 \; \mathcal{R}_p \; \pi_2$ iff*

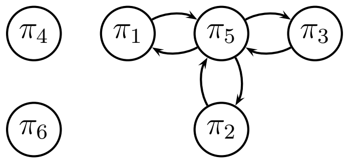
- $\text{PREC}(\pi_1) \wedge \text{PREC}(\pi_2) \models \bot$, *or*
- $\text{POSTC}(\pi_1) \wedge \text{POSTC}(\pi_2) \models \bot$, *or*
- $\text{POSTC}(\pi_1) \wedge \text{PREC}(\pi_2) \models \bot$ *or* $\text{PREC}(\pi_1) \wedge \text{POSTC}(\pi_2) \models \bot$

It is clear from the above definition that $\mathcal{R}_p$ is symmetric and irreflexive[10]

Proposition 11. *The relation $\mathcal{R}_p$ is symmetric and irreflexive.*

Example 2 (Continued): Some instrumental arguments are conflicting. These conflicts are summarized in the figure below.

[10]The fact that the post-conditions of a plan are inconsistent with its pre-conditions is not considered as a conflict. In this case, after the execution of the plan, we must have an update mechanism which will modify the beliefs. It is also for this reason that there is no conflict between epistemic arguments and instrumental arguments on the post-conditions of a plan (see Definition 22).
Note also that the order in which plans are executed is not considered in this paper. This order may be very important, for instance when we must manage resources consumed by plans. So, this will be the subject of future work.



From the above definition, it can be shown that if two plans realize conflicting desires, then their corresponding instrumental arguments are conflicting too.

Proposition 12. *Let* d_1*,* $d_2 \in \mathcal{PD}$*. If* $d_1 \equiv \neg d_2$*, then* $\forall \pi_1, \pi_2 \in \mathcal{A}_p$ *s.t.* $\text{CONC}(\pi_1)$ = d_1 *and* $\text{CONC}(\pi_2)$ = d_2*, then* π_1 $\mathcal{R}_p$ π_2*.*

> **Assumption 1.** In this section, we have considered only *binary conflicts* between plans, and consequently between their corresponding instrumental arguments. However, in every-day life, one may have for instance three plans such that any pair of them is not conflicting, but the three together are incompatible. For simplicity reasons, in this paper we suppose that we do not have such conflicts.

6.4. Conflicts among mixed arguments

In the previous sections we have shown how arguments of the same category can interact with each other. In this section, we will show that arguments of different categories can also interact. Namely, epistemic arguments play a key role in defining the status of explanatory and instrumental arguments. An epistemic argument can attack both types of arguments. The basic idea is to invalidate any belief used in an explanatory argument and any belief used in the pre-conditions of an instrumental one. The end goal is to ensure that only "warranted" beliefs are used in explanatory and instrumental arguments.

It is worth mentioning that an epistemic argument cannot invalidate a state of the world that is not yet reached like for instance desires and post-conditions of actions. Indeed, epistemic arguments support beliefs that hold in the current state of the world. Thus, if they attack a state of the world which is true in the future, they will forbid desires to be reached. Let us consider the case of Paula who thinks that she is not rich and would like to be rich. Thus, $\mathcal{K}_b = \{\neg rich\}$ and $\mathcal{K}_d = \{\langle\rangle \hookrightarrow rich\}$. If the epistemic argument $\alpha = \langle\{\neg rich\}, \neg rich\rangle$ attacks the explanatory argument $\delta = \langle\langle\rangle, rich\rangle$, then this latter will never be pursued by Paula even if we can imagine that she has a good plan for it.

Similarly, let us assume that Paula has the following action/plan for reaching her desire: $\langle \{\}, \{rich\}, x \rangle$. Thus, she has an instrumental argument $\pi = \langle \langle \{\}, \{rich\}, x \rangle, rich \rangle$. If α attacks π, then the plan can never be executed. Consequently, Paula will not consider her desire as an intention.

Finally, let us note that explanatory arguments and instrumental arguments are not allowed to attack epistemic arguments. In fact, a desire cannot invalidate a belief. Let us illustrate this issue by an example borrowed from [28]. An agent thinks that it will be raining, and that when it is raining, she gets wet. It is clear that this agent does not desire to be wet when it is raining. Intuitively, we should get one extension $\{rain, wet\}$. The idea is that if the agent believes that it is raining, and she will get wet if it rains, then she should believe that she will get wet, regardless what she wants. To do otherwise would be to indulge in *wishful thinking*.

Definition 22 summarizes all these remarks and gives the exhaustive list of allowed mixed conflicts in our setting[11].

Definition 22. *Let $\alpha \in \mathcal{A}_b$, $\delta \in \mathcal{A}_d$, $\pi \in \mathcal{A}_p$. The conflict relations between mixed arguments are defined as follows:*

- $\alpha \, \mathcal{R}_{bd} \, \delta$ *iff* $\exists h \in \textsc{Beliefs}(\delta)$ *s.t.* $h \equiv \neg\textsc{Conc}(\alpha)$.
- $\alpha \, \mathcal{R}_{bp} \, \pi$ *iff* $\exists h \in \textsc{PreC}(\pi)$*, s.t.* $h \equiv \neg\textsc{Conc}(\alpha)$.
- $\delta \, \mathcal{R}_{pdp} \, \pi$ *and* $\pi \, \mathcal{R}_{pdp} \, \delta$ *iff* $\textsc{Conc}(\pi) \equiv \neg d$ *with* $d \in \textsc{Desires}(\delta)$[12].

Example 2 (Continued): In this example, the relations $\mathcal{R}_{bd}$, $\mathcal{R}_{bp}$ and $\mathcal{R}_{pdp}$ are *empty* since the beliefs base $\mathcal{K}_b$ is consistent and there is no contradictory desires. The absence of conflict between $\alpha_6 = \langle \{\neg fp\}, \neg fp \rangle$ and $\delta_2 = \langle \langle \rangle, fp \rangle$ illustrates the previous remarks about the temporal difference between the current state of the world (α_6) and the future state of the world (δ_2).

A trivial consequence of this definition is the following link between $\mathcal{R}_b$ and $\mathcal{R}_{bd}$:

Consequence 1. *Let $\alpha_1, \alpha_2 \in \mathcal{A}_b$ and $\delta \in \mathcal{A}_d$ such that $\alpha_1 \in \textsc{Sub}(\delta)$. If $\alpha_2 \mathcal{R}_b \alpha_1$ then $\alpha_2 \mathcal{R}_{bd} \delta$.*

[11] $\mathcal{R}_{xy}$ (resp. $\mathcal{R}_{xyx}$) denotes conflicts (resp. symmetric conflicts) emanating from arguments of $\mathcal{A}_x$ towards arguments of $\mathcal{A}_y$.

[12] Note that if $\delta_1 \mathcal{R}_{pdp} \pi_2$ and there exists δ_2 such that $\textsc{Conc}(\delta_2) = \textsc{Conc}(\pi_2)$ then $\delta_1 \mathcal{R}_d \delta_2$.

Moreover, as already said, the set of beliefs of an explanatory argument may be inconsistent. In such a case, the explanatory argument is certainly attacked (in the sense of $\mathcal{R}_{bd}$) by an epistemic argument. Formally:

Proposition 13. *Let* $\delta \in \mathcal{A}_d$. *If* $\textsc{Beliefs}(\delta) \vdash \bot$, *then* $\exists \alpha \in \mathcal{A}_b$ *s.t.* $\alpha \ \mathcal{R}_{bd} \ \delta$.

Similarly, when the beliefs of two explanatory arguments are inconsistent, it can be checked that there exist epistemic arguments that attack the two explanatory arguments.

Proposition 14. *Let* $\delta_1, \delta_2 \in \mathcal{A}_d$ *with* $\textsc{Beliefs}(\delta_1) \nvdash \bot$ *and* $\textsc{Beliefs}(\delta_2) \nvdash \bot$. *If* $\textsc{Beliefs}(\delta_1) \cup \textsc{Beliefs}(\delta_2) \vdash \bot$, *then* $\exists \alpha_1, \alpha_2 \in \mathcal{A}_b$ *s.t.* $\alpha_1 \ \mathcal{R}_{bd} \ \delta_1$ *and* $\alpha_2 \ \mathcal{R}_{bd} \ \delta_2$.

Conflicts may also exist between an instrumental argument and an explanatory one since the beliefs of the explanatory argument may be conflicting with the pre-conditions of the instrumental one. Here again, we will show that there exist epistemic arguments that attack the two arguments. Note that in this case, the set of pre-conditions of the instrumental argument is not empty.

Proposition 15. *Let* $\delta \in \mathcal{A}_d$ *and* $\pi \in \mathcal{A}_p$ *with* $\textsc{Beliefs}(\delta) \nvdash \bot$. *If* $\textsc{Beliefs}(\delta) \cup \textsc{Prec}(\pi) \vdash \bot$ *then* $\exists \alpha_1, \alpha_2 \in \mathcal{A}_b$ *s.t.* $\alpha_1 \ \mathcal{R}_{bd} \ \delta$ *and* $\alpha_2 \ \mathcal{R}_{bp} \ \pi$.

Later in the paper, it will be shown that the three above propositions are sufficient for ignoring these conflicts (between two explanatory arguments, and between an explanatory argument and an instrumental one).

6.5. Summary of conflict relations between arguments

The following table summarizes the possible conflicts between arguments.

Conflict relation	From	To	Symmetric
$\mathcal{R}_b$	epistemic arg. ($\mathcal{A}_b$)	epistemic arg.	no
$\mathcal{R}_d$	explanatory arg. ($\mathcal{A}_d$)	explanatory arg.	yes
$\mathcal{R}_p$	instrumental arg. ($\mathcal{A}_p$)	instrumental arg.	yes
$\mathcal{R}_{bd}$	epistemic arg.	explanatory arg.	no
$\mathcal{R}_{bp}$	epistemic arg.	instrumental arg.	no
$\mathcal{R}_{pdp}$	instrumental arg. explanatory arg.	explanatory arg. instrumental arg.	yes

Now, all the mandatory pieces are ready for the definition of an argumentation system for practical reasoning.

7. Argumentation system for PR

The notion of constraint forms the backbone of constrained argumentation systems. In a practical reasoning context, it encodes two important points:

- First, it gives the link between the justification of a desire and the plan for achieving it. The basic idea is the following: as already said, for a desire to be pursued, it should be both justified (*i.e.* supported by an explanatory argument) and feasible (*i.e.* supported by an instrumental argument). Thus, explanatory arguments that are not accompanied by instrumental arguments for their conclusions will not be considered (see Part 2 of Definition 23). Similarly, instrumental arguments that cannot be accompanied by explanatory arguments in favor of their desires will also be discarded (see Part 1 of Definition 23).

- Secondly, it takes into account the recursive form of the explanatory arguments. Indeed, because this particular form, each explanatory argument must be accompanied by all its subarguments (see Part 3 of Definition 23).

So, the constraint is formalized as follows:

Definition 23. (Constraint for PR) *Let $\mathcal{A}_d$ and $\mathcal{A}_p$ be two sets of arguments and $\mathcal{L}_{\mathcal{A}_d \cup \mathcal{A}_p}$ be the propositional language defined using $\mathcal{A}_d \cup \mathcal{A}_p$ as the set of propositional variables. A* constraint for PR *is a constraint C on arguments of $\mathcal{A}_d \cup \mathcal{A}_p$ such that:*

$$
\begin{aligned}
C \;=\; & \Big(\bigwedge_{\pi_i \in \mathcal{A}_p} (\pi_i \rightarrow (\bigvee_{\delta_j \in \{\delta \in \mathcal{A}_d | \mathrm{CONC}(\pi_i) \equiv \mathrm{CONC}(\delta)\}} \delta_j)))\\
& \wedge \\
& \Big(\bigwedge_{\delta_k \in \mathcal{A}_d} (\delta_k \rightarrow (\bigvee_{\pi_l \in \{\pi \in \mathcal{A}_p | \mathrm{CONC}(\delta_k) \equiv \mathrm{CONC}(\pi)\}} \pi_l)))\\
& \wedge \\
& \Big(\bigwedge_{\delta_k \in \mathcal{A}_d} (\bigwedge_{\beta \in \mathrm{SUB}(\delta_k)} (\delta_k \rightarrow \beta)))
\end{aligned}
$$

with the convention: $(\bigvee_{x \in X} x) = \bot$ *if* $X = \varnothing$.

Example 2 (Continued): In the example on Paula, the constraint C is on arguments of $\mathcal{A}_b \cup \mathcal{A}_d \cup \mathcal{A}_p$. It is defined as follows:

$$
\begin{array}{rll}
C = & (& (\pi_1 \to \delta_1) \\
& & \wedge(\pi_2 \to \delta_1) \\
& & \wedge(\pi_3 \to \delta_1) \\
& & \wedge(\pi_4 \to \delta_1) \\
& & \wedge(\pi_5 \to \delta_2) \\
& & \wedge(\pi_6 \to \bot)) \\
& \wedge(& (\delta_1 \to (\pi_1 \vee \pi_2 \vee \pi_3 \vee \pi_4) \\
& & \wedge(\delta_2 \to \pi_5) \\
& & \wedge(\delta_3 \to \bot)) \\
& \wedge(& (\delta_1 \to \delta_1) \\
& & \wedge(\delta_2 \to \delta_2) \\
& & \wedge(\delta_3 \to \delta_3))
\end{array}
$$

Note the particular cases of δ_3 and π_6: for δ_3 (resp. π_6) there is no corresponding instrumental (resp. explanatory) argument.

Example 5 (Continued): In this example, there are three explanatory arguments $\delta_1 = \langle\langle\rangle, d_1\rangle$, $\delta_2 = \langle\langle\rangle, \neg d_1\rangle$ and $\delta_3 = \langle\langle\delta_1\rangle, d_2\rangle$. Suppose that there exists only one instrumental argument $\pi = \langle\langle S, T, x\rangle, d_2\rangle$. The constraint is thus:

$$
\begin{array}{rll}
C = & (& (\pi \to \delta_3)) \\
& \wedge(& (\delta_1 \to \bot) \\
& & \wedge(\delta_2 \to \bot) \\
& & \wedge(\delta_3 \to \pi) \\
& \wedge(& (\delta_1 \to \delta_1) \\
& & \wedge(\delta_2 \to \delta_2) \\
& & \wedge((\delta_3 \to \delta_1) \wedge (\delta_3 \to \delta_3)))
\end{array}
$$

Let $\mathcal{A} = \{\delta_1, \delta_2, \delta_3, \pi\}$. The constrained argumentation system of this example has only one C-preferred extension which is the empty set.

A constrained argumentation system for PR is defined as follows:

Definition 24. (Constrained argumentation system for PR) *A* constrained argumentation system *for practical reasoning is the triple* $\mathsf{CAS_{PR}} = \langle \mathcal{A}, \mathcal{R}, C \rangle$ *with:*

- $\mathcal{A} = \mathcal{A}_b \cup \mathcal{A}_d \cup \mathcal{A}_p$,
- $\mathcal{R} = \mathcal{R}_b \cup \mathcal{R}_d \cup \mathcal{R}_p \cup \mathcal{R}_{bd} \cup \mathcal{R}_{bp} \cup \mathcal{R}_{pdp}$,
- C *a constraint on arguments defined on* $\mathcal{A}_d \cup \mathcal{A}_p$ *as in Definition 23.*

Remember that the aim of this paper is to compute the intentions to be pursued by an agent, *i.e.* the desires that are both justified and feasible together (this is one of the purposes of a practical reasoning problem). These intentions are defined as follows:

Definition 25. (Set of intentions) *Let* $\mathcal{K}_b, \mathcal{K}_d, \mathcal{K}_a$ *be three bases and* $\mathsf{CAS_{PR}}$ *be the corresponding constrained system. Let* $\mathcal{E}_1, \ldots, \mathcal{E}_n$ *be the* C*-extensions of* $\mathsf{CAS_{PR}}$ *under a given semantics.*

A set $\mathcal{I} \subseteq \mathcal{PD}$ *is a* set of intentions *of* $\mathsf{CAS_{PR}}$ *under the given semantics iff there exists a* C*-extension* $\mathcal{E}_i$ *such that for each* $d \in \mathcal{I}$*, there exists* $\pi \in \mathcal{A}_p \cap \mathcal{E}_i$ *such that* $d = \textsc{Conc}(\pi)$.

Different intention sets may be returned by our $\mathsf{CAS_{PR}}$. Indeed, each extension gives birth to a set of intentions, the state of the world which justifies these intentions and the plans which can realize them. The exact set that an agent decides to pursue is merely a decision problem as argued in [4]. This choice is beyond the scope of this paper. Recall that the aim of this paper is only to identify the different possibilities for an agent.

Example 2 (Continued): The constrained argumentation system that will help Paula to define her intentions is thus $\mathsf{CAS_{PR}}$ = $\langle \mathcal{A}_b \cup \mathcal{A}_d \cup \mathcal{A}_p, \mathcal{R}_p$[13]$, C \rangle$ where C is the constraint defined above.

The system $\mathsf{AS_{PR}}$ has two stable and preferred extensions[14]:

[13]Recall that $\mathcal{R}_b$, $\mathcal{R}_d$, $\mathcal{R}_{bd}$, $\mathcal{R}_{bp}$, and $\mathcal{R}_{pdp}$ are all empty.

[14]Note that the notion of defence has two different semantics in PR context. When we consider only epistemic or explanatory arguments, the defence corresponds exactly to the notion defined in Dung's argumentation systems and in its constrained extension: an attacked argument must be "reinstated" by a defender. Things are different with instrumental arguments because of the symmetry of the conflict relation. In this case, it would be sufficient to take into account the notion of conflict-free in order to identify the plans which belong to an admissible set.

- $\mathcal{E}_1 = \mathcal{A}_b \cup \{\delta_1, \delta_2, \delta_3, \pi_1, \pi_2, \pi_3, \pi_4, \pi_6\}$ and
- $\mathcal{E}_2 = \mathcal{A}_b \cup \{\delta_1, \delta_2, \delta_3, \pi_4, \pi_5, \pi_6\}$

Note that the above extensions contain the explanatory argument δ_3 in favor of the desire lec even if this desire is not feasible. Similarly, they contain the instrumental argument π_6 while the desire vc is not justified. If now, we apply the system $\mathsf{CAS}_{\mathsf{PR}}$, then we will get two C-preferred extensions (there is no C-stable extensions in this example):

- $\mathcal{E}'_1 = \mathcal{A}_b \cup \{\delta_1, \pi_1, \pi_2, \pi_3, \pi_4\}$ and
- $\mathcal{E}'_2 = \mathcal{A}_b \cup \{\delta_1, \delta_2, \pi_4, \pi_5\}$.

It is worth mentioning that the C-preferred extensions contain only useful information. Thus, the use of the constraint C makes it possible to remove uninteresting information from the extensions (like δ_3 and π_6).

Now that the C-extensions are defined, we are able to define Paula's sets of intentions. She has two sets of intentions under the preferred semantics:

- $\mathcal{I}_1 = \{jca\}$
- $\mathcal{I}_2 = \{jca, fp\}$

Our framework does not make choice between these two sets. The choice of the exact set is a decision problem and is beyond the scope of this paper. For instance, one may think that since the two desires may be satisfied, it is natural to assume that Paula will choose the second set. Consequently, she should choose the plans π_4 and π_5. Assume now that Paula is very cautious, and she does not want to miss her journey to central Africa. In this case, we can easily imagine that she chooses the set I_1 since she has four plans for reaching this desire, and if for any reason one of them fails, she can still satisfy her desire by another plan.

Note: In this example, $\mathsf{CAS}_{\mathsf{PR}}$ does not have C-stable extensions. This means that at least one of the potential desires of the agent cannot be both justified and feasible, whereas its justification or its feasibility are not attacked in this state of the world. Here, it is the case for the desire *lec* (to be a lecturer) and for the desire *vc* (to have visited her friend Carla); the first one is justified (argument δ_3) but not feasible and the second one is feasible (argument π_6) but not justified. However both δ_3 and π_6 are not attacked in $\mathsf{CAS}_{\mathsf{PR}}$,

so the justification of *lec* and the feasibility of *vc* are “compatible” with the current state of the world. Stable semantics emphasizes this kind of “compatibility” to the detriment of the constraint C (desires must be both justified and feasible). So, in these cases, it is natural to consider that there is no C-stable extensions and the set of intentions remains empty.
With preferred semantics, things are different because the use of set-inclusion maximality allows more flexibility: even if an argument is not attacked, it can be rejected in order to satisfy the constraint C of the system.

Example 5 (Continued): In this example, we have shown that if $\mathcal{A} = \{\delta_1, \delta_2, \delta_3, \pi\}$, then the only C-preferred extension of the corresponding constrained system is the empty set. Consequently, the empty set is also the unique set of intentions.

Let us now consider a more elaborate version of this example, in particular the one discussed in [9]. Recall that this version is not handled correctly by existing systems for PR, namely the one proposed in [9].

In the elaborate version, the agent has three potential desires d_1, $\neg d_1$ and d_2 such that $\langle d_1 \rangle \hookrightarrow d_2$. The explanatory arguments are gathered in $\mathcal{A}_d = \{\delta_1, \delta_2, \delta_3\}$ with $\textsc{Conc}(\delta_1) = d_1$, $\textsc{Conc}(\delta_2) = \neg d_1$, and $\textsc{Conc}(\delta_3) = d_2$. The relation $\mathcal{R}_d$ is defined as follows: $\mathcal{R}_d = \{(\delta_1, \delta_2), (\delta_2, \delta_1), (\delta_2, \delta_3), (\delta_3, \delta_2)\}$. Assume that there are two instrumental arguments, thus $\mathcal{A}_p = \{\pi_1, \pi_2\}$ with $\textsc{Conc}(\pi_1) = d_1$ and $\textsc{Conc}(\pi_2) = d_2$. Let us assume that $\mathcal{R}_p = \varnothing$, $\mathcal{R}_b = \varnothing$, $\mathcal{R}_{xy} = \varnothing$ with $x \neq y$, and $\mathcal{R}_{pdp} = \{(\delta_2, \pi_1), (\pi_1, \delta_2)\}$. The constraint of the corresponding $\mathsf{CAS_{PR}}$ is:

$$
\begin{aligned}
C = \quad & (\quad (\pi_1 \rightarrow \delta_1) \wedge (\pi_2 \rightarrow \delta_3)) \\
& \wedge(\quad (\delta_1 \rightarrow \pi_1) \wedge (\delta_2 \rightarrow \bot) \wedge (\delta_3 \rightarrow \pi_2)) \\
& \wedge(\quad (\delta_1 \rightarrow \delta_1) \wedge (\delta_2 \rightarrow \delta_2) \wedge ((\delta_3 \rightarrow \delta_1) \wedge (\delta_3 \rightarrow \delta_3)))
\end{aligned}
$$

It can be checked that the corresponding CAS has one C-preferred extension $\mathcal{E} = \mathcal{A}_b \cup \{\delta_1, \delta_3, \pi_1, \pi_2\}$. This agent has thus one intention set which is $\mathcal{I} = \{d_1, d_2\}$. Remind that according to the system proposed in [9], this agent has no intentions meaning that she will abandon her three desires.

8. Properties of the system

The aim of this section is to study the properties of the proposed argumentation system for PR ($\mathsf{CAS_{PR}}$ = $\langle \mathcal{A}, \mathcal{R}, C \rangle$). At some places, we will refer by $\mathsf{AS_{PR}}$

to the corresponding basic argumentation system $\langle \mathcal{A}, \mathcal{R} \rangle$ (*i.e.* the system without the constraint C).

The first results concern the extensions of the system, and are mainly direct consequences of results obtained in [11]. The first proposition establishes a link between C-admissible sets and C-preferred extensions, and shows the impact of applying the constraint on the notion of admissibility.

Proposition 16. *Let* $\mathsf{CAS}_{\mathsf{PR}} = \langle \mathcal{A}, \mathcal{R}, C \rangle$*. Let* Ω *be the set of* C*-admissible sets of* $\mathsf{CAS}_{\mathsf{PR}}$.

1. *Let* $\mathcal{E} \in \Omega$*. There exists a* C*-preferred extension* $\mathcal{E}'$ *of* $\mathsf{CAS}_{\mathsf{PR}}$ *s.t.* $\mathcal{E} \subseteq \mathcal{E}'$.
2. *Let* ${\mathsf{CAS}_{\mathsf{PR}}}' = \langle \mathcal{A}, \mathcal{R}, C' \rangle$ *s.t.* $C' \models C$*. Let* Ω' *be the set of* C'*-admissible sets of* ${\mathsf{CAS}_{\mathsf{PR}}}'$*. The inclusion* $\Omega' \subseteq \Omega$ *holds.*

The two following properties show that the constrained argumentation system is more general than its basic version. However, the two systems may coincide in some circumstances.

Proposition 17. *Let* $\mathsf{CAS}_{\mathsf{PR}} = \langle \mathcal{A}, \mathcal{R}, C \rangle$*. For each* C*-preferred extension* $\mathcal{E}$ *of* $\mathsf{CAS}_{\mathsf{PR}}$*, there exists a preferred extension* $\mathcal{E}'$ *of* $\mathsf{AS}_{\mathsf{PR}}$ *such that* $\mathcal{E} \subseteq \mathcal{E}'$.

This proposition is illustrated in the running example. Indeed, $\mathcal{E}'_1 \subseteq \mathcal{E}_1$ and $\mathcal{E}'_2 \subseteq \mathcal{E}_2$.

Proposition 18. *Let* $\mathsf{CAS}_{\mathsf{PR}} = \langle \mathcal{A}, \mathcal{R}, C \rangle$ *be such that* C *is a valid formula on* $\mathcal{A}$*. Then the preferred extensions of* $\mathsf{AS}_{\mathsf{PR}}$ *are the* C*-preferred extensions of* $\mathsf{CAS}_{\mathsf{PR}}$.

As already said, due to the constraint C, each C-extension $\mathcal{E}$ of $\mathsf{CAS}_{\mathsf{PR}}$ contains, among the instrumental arguments, only the ones for which there exists at least one explanatory argument in the same set for their conclusions. Similarly, it contains, among the explanatory arguments, only the ones for which we can find at least one instrumental argument in favor of their conclusions. This means that the constraint makes it possible to filter the content of the extensions and to keep only useful information. Formally

Consequence 2. *Let* $\mathsf{CAS}_{\mathsf{PR}} = \langle \mathcal{A}, \mathcal{R}, C \rangle$ *and* $\mathcal{E}$ *be its* C*-extension under preferred or stable semantics.*

- *For all* $\delta \in \mathcal{E} \cap \mathcal{A}_d$*,* $\exists \pi \in \mathcal{E} \cap \mathcal{A}_p$ *such that* $\textsc{Conc}(\delta) = \textsc{Conc}(\pi)$.

- *For all $\pi \in \mathcal{E} \cap \mathcal{A}_p$, $\exists \delta \in \mathcal{E} \cap \mathcal{A}_d$ such that* $\textsc{Conc}(\delta) = \textsc{Conc}(\pi)$.

Due to the particular constraint used in our system, the empty set is always C-admissible and the system has at least one C-preferred extension.

Proposition 19.
- *The empty set is a C-admissible of the practical system* $\mathsf{CAS_{PR}}$.
- *The practical system* $\mathsf{CAS_{PR}}$ *has at least one C-preferred extension.*

Recall that $\mathsf{AS_{PR}} = \langle \mathcal{A}_b \cup \mathcal{A}_d \cup \mathcal{A}_p, \mathcal{R}_b \cup \mathcal{R}_d \cup \mathcal{R}_p \cup \mathcal{R}_{bd} \cup \mathcal{R}_{bp} \cup \mathcal{R}_{pdp} \rangle$. An important proposition shows that the set of epistemic arguments in a given stable extension of $\mathsf{AS_{PR}}$ is itself a stable extension of the system $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$. Knowing that the argumentation system $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$ is intended to handle inconsistency in the knowledge base $\mathcal{K}_b$, the following result shows that stable extensions of $\mathsf{AS_{PR}}$ are "complete" w.r.t. epistemic arguments. This means also that explanatory and instrumental arguments have no impact on the status of beliefs, and that wishful thinking is avoided.

Proposition 20. *If $\mathcal{E}$ is a stable extension of* $\mathsf{AS_{PR}}$, *then the set $\mathcal{E} \cap \mathcal{A}_b$ is a stable extension of $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$.*

We also show that the basic argumentation system $\mathsf{AS_{PR}}$ for PR has always stable extensions.

Proposition 21. *If $\mathcal{K}_b \neq \varnothing$ and $\mathcal{K}_b \neq \{\bot\}$, then the system* $\mathsf{AS_{PR}}$ *has at least one non-empty stable extension.*

It can be shown that if an explanatory argument belongs to a stable extension of $\mathsf{AS_{PR}}$, then all its sub-arguments belong to that extension.

Proposition 22. *Let $\mathcal{E}$ be a stable extension of* $\mathsf{AS_{PR}}$. *If $\delta \in \mathcal{E} \cap \mathcal{A}_d$, then* $\textsc{Sub}(\delta) \subseteq \mathcal{E}$.

This means that the beliefs on which this explanatory argument is built are "warranted" and the desires on which depend its conclusion are justified[15].

Similarly, we can show that if an instrumental argument belongs to a stable extension then all its pre-conditions are supported by this extension.

[15] Note that, in this case, Part 3 of Definition 23 is trivially satisfied. However, it could be not the case under other semantics.

Proposition 23. *Let $\mathcal{E}$ be a stable extension of* $\mathsf{AS}_{\mathsf{PR}}$*. If $\pi \in \mathcal{E} \cap \mathcal{A}_p$, then* $\textsc{Prec}(\pi) \subseteq \bigcup_{\alpha_j \in \mathcal{E} \cap \mathcal{A}_b} \textsc{Supp}(\alpha_j)$.

In a previous section, we have shown that an explanatory argument may be based on contradictory beliefs. We have also shown that such an argument is attacked by an epistemic argument. In what follows, we will show that the situation is worse since such an argument is attacked by each stable extension of the system $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$. That's why these arguments will be discarded.

Proposition 24. *Let $\delta \in \mathcal{A}_d$. If* $\textsc{Beliefs}(\delta) \vdash \bot$*, then $\forall \mathcal{E}$ with $\mathcal{E}$ is a stable extension of $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$, $\exists \alpha \in \mathcal{E}$ such that $\alpha \mathcal{R}_{bd} \delta$.*

A direct consequence of the above result is that such explanatory argument (with contradictory beliefs) will never belong to a stable extension of the system $\mathsf{AS}_{\mathsf{PR}}$.

Proposition 25. *Let $\delta \in \mathcal{A}_d$ with* $\textsc{Beliefs}(\delta) \vdash \bot$*. Under the stable semantics, the argument δ is rejected in* $\mathsf{AS}_{\mathsf{PR}}$.

Since an explanatory argument with contradictory beliefs is rejected in $\mathsf{AS}_{\mathsf{PR}}$, then it will also be rejected in $\mathsf{CAS}_{\mathsf{PR}}$.

Proposition 26. *Let $\delta \in \mathcal{A}_d$ with* $\textsc{Beliefs}(\delta) \vdash \bot$*. Under the stable semantics, δ is a rejected argument in* $\mathsf{CAS}_{\mathsf{PR}}$.

Besides in Proposition 14, we have shown that when two explanatory arguments are based on contradictory beliefs, then the two arguments are attacked by epistemic arguments. We will show that they are even attacked by each stable extension of the system $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$.

Proposition 27. *Let $\delta_1, \delta_2 \in \mathcal{A}_d$ with* $\textsc{Beliefs}(\delta_1) \nvdash \bot$ *and* $\textsc{Beliefs}(\delta_2) \nvdash \bot$*. If* $\textsc{Beliefs}(\delta_1) \cup \textsc{Beliefs}(\delta_2) \vdash \bot$*, then $\forall \mathcal{E}$ with $\mathcal{E}$ is a stable extension of $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$, $\exists \alpha \in \mathcal{E}$ such that $\alpha \mathcal{R}_{bd} \delta_1$ or $\alpha \mathcal{R}_{bd} \delta_2$.*

We go further, and we show that the two arguments cannot be accepted at the same time, *i.e.* they cannot belong to the same stable extension simultaneously. This guarantees that the system proposed here returns safe results (there is no pairs of explanatory arguments with contradictory beliefs in a C-stable extension).

Proposition 28. *Let* $\delta_1, \delta_2 \in \mathcal{A}_d$ *with* $\text{BELIEFS}(\delta_1) \nvdash \bot$ *and* $\text{BELIEFS}(\delta_2) \nvdash \bot$. *If* $\text{BELIEFS}(\delta_1) \cup \text{BELIEFS}(\delta_2) \vdash \bot$, *then* $\nexists \mathcal{E}$ *with* $\mathcal{E}$ *a* C*-stable extension of* $\mathsf{CAS}_{\mathsf{PR}}$ *such that* $\delta_1 \in \mathcal{E}$ *and* $\delta_2 \in \mathcal{E}$.

Similarly, some conflicts between explanatory and instrumental arguments were discarded. We have shown in Proposition 15 that in such a case, the two arguments are attacked by epistemic arguments. Here we will show that the explanatory argument cannot be accepted at the same time with the instrumental one. One of them will be for sure rejected in the system.

Proposition 29. *Let* $\delta \in \mathcal{A}_d$ *and* $\pi \in \mathcal{A}_p$ *with* $\text{BELIEFS}(\delta) \nvdash \bot$. *If* $\text{BELIEFS}(\delta) \cup \text{PREC}(\pi) \vdash \bot$, *then* $\forall \mathcal{E}$ *with* $\mathcal{E}$ *is a stable extension of* $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$, $\exists \alpha \in \mathcal{E}$ *such that* $\alpha \mathcal{R}_{bd} \delta$, *or* $\alpha \mathcal{R}_{bp} \pi$.

Proposition 30. *Let* $\delta \in \mathcal{A}_d$ *and* $\pi \in \mathcal{A}_p$ *with* $\text{BELIEFS}(\delta) \nvdash \bot$. *If* $\text{BELIEFS}(\delta) \cup \text{PREC}(\pi) \vdash \bot$ *then* $\nexists \mathcal{E}$ *with* $\mathcal{E}$ *a* C*-stable extension of* $\mathsf{CAS}_{\mathsf{PR}}$ *such that* $\delta \in \mathcal{E}$ *and* $\pi \in \mathcal{E}$.

The next results are of great importance. They show that the proposed argumentation system for PR satisfies the "consistency" rationality postulate proposed in [13]. Indeed, each C-stable (C-preferred) extension of our system supports a consistent set of beliefs about the current state of the world. Moreover, the consequences of the plans of each extension are consistent. In particular, the set of desires is consistent. Thus, each C-stable (C-preferred) extension represents a consistent state of the world before and after the execution of the corresponding actions.

Notations: The following notations will be used: Let $\mathcal{E} \subseteq \mathcal{A}$.

$$\text{BEL}(\mathcal{E}) = \Big(\bigcup_{\alpha_i \in \mathcal{E} \cap \mathcal{A}_b} \text{SUPP}(\alpha_i)\Big) \cup \Big(\bigcup_{\delta_j \in \mathcal{E} \cap \mathcal{A}_d} \text{BELIEFS}(\delta_j)\Big) \cup \Big(\bigcup_{\pi_k \in \mathcal{E} \cap \mathcal{A}_p} \text{PREC}(\pi_k)\Big)$$

$$\text{DES}(\mathcal{E}) = \Big(\bigcup_{\delta_j \in \mathcal{E} \cap \mathcal{A}_d} \text{DESIRES}(\delta_j)\Big) \cup \Big(\bigcup_{\pi_k \in \mathcal{E} \cap \mathcal{A}_p} \text{CONC}(\pi_k)\Big)$$

Theorem 1. (Consistency) *Let* $\mathsf{CAS}_{\mathsf{PR}}$ *be a constrained argumentation system for PR, and* $\mathcal{E}_1, \ldots, \mathcal{E}_n$ *its* C*-stable extensions.* $\forall \mathcal{E}_i$, $i = 1, \ldots, n$, *it holds that:*

1. *The set* $\text{BEL}(\mathcal{E}_i) = \text{BEL}(\mathcal{E}_i \cap \mathcal{A}_b)$.

2. *The set* $\text{BEL}(\mathcal{E}_i)$ *is a maximal (for set inclusion) consistent subset of* $\mathcal{K}_b$.
3. *The set* $\bigcup_{\pi_k \in \mathcal{E}_i \cap \mathcal{A}_p} \text{POST}(\pi_k)$ *is consistent.*
4. *The set* $\text{DES}(\mathcal{E}_i)$ *is consistent.*

Consistency is also ensured with preferred semantics.

Theorem 2. (Consistency) *Let* $\mathsf{CAS}_{\mathsf{PR}}$ *be a constrained argumentation system for PR, and* $\mathcal{E}_1, \ldots, \mathcal{E}_n$ *its* C*-preferred extensions.* $\forall \mathcal{E}_i$, $i = 1, \ldots, n$, *it holds that:*

1. *The set* $\text{BEL}(\mathcal{E}_i)$ *is consistent.*
2. *The set* $\bigcup_{\pi_k \in \mathcal{E}_i \cap \mathcal{A}_p} \text{POST}(\pi_k)$ *is consistent.*
3. *The set* $\text{DES}(\mathcal{E}_i)$ *is consistent.*

As direct consequence of the above results, a set of intentions is consistent. Formally:

Theorem 3. *Under stable and preferred semantics, each set of intentions of* $\mathsf{CAS}_{\mathsf{PR}}$ *is consistent.*

We have also shown that our system satisfies the rationality postulate concerning the closure of the extensions [13]. Namely, we have shown that the set of arguments that can be built from the beliefs, desires, and plans involved in a given stable extension, is that extension itself. Before giving this result, let us first introduce some notations:

> **Notations:** Let $\mathcal{E}$ be a C-stable extension of $\mathsf{CAS}_{\mathsf{PR}}$.
> $\mathcal{A}$s will denote the set of all (epistemic, explanatory and instrumental) arguments that can be built from $\text{BEL}(\mathcal{E})$, $\text{DES}(\mathcal{E})$, the plans involved in building arguments of $\mathcal{E}$ and the base $\mathcal{K}_d$.

Theorem 4. (closure) *Let* $\mathsf{CAS}_{\mathsf{PR}}$ *be a constrained argumentation system for PR, and* $\mathcal{E}_1, \ldots, \mathcal{E}_n$ *its* C*-stable extensions.* $\forall \mathcal{E}_i$, $i = 1, \ldots, n$, *it holds that:*

- $\text{ARG}(\text{BEL}(\mathcal{E}_i))$ = $\mathcal{E}_i \cap \mathcal{A}_b$.
- $\mathcal{A}s$ = $\mathcal{E}_i$.

In fact, this shows that every “good” argument is included in a C-stable extension. Thus, each desire that deserves to be pursued will be returned in an intention set.

Note that the property of closure is not satisfied under preferred semantics as shown in the following example:

Example 6. *Consider* $\mathsf{CAS_{PR}}$ *such that* $\mathcal{R}$ *is empty and there exist two explanatory arguments* $\delta_1 = \langle\langle\rangle, d_1\rangle$ *and* $\delta_2 = \langle\langle\delta_1\rangle, d_2\rangle$ *and only one instrumental argument* $\pi_1 = \langle\langle\{\}, \{d_1\}, a\rangle, d_1\rangle$. $\mathcal{E} = \mathcal{A}_b \cup \{\delta_1, \pi_1\}$ *is the only one C-preferred extension of this system.*

However, $\text{DES}(\mathcal{E}) = \{d_1\}$. *So, using* $\text{DES}(\mathcal{E})$*, one can create the argument* δ_2 *(i.e.* $\delta_2 \in \mathcal{A}s$*). In this case,* $\mathcal{A}s \neq \mathcal{E}$.

However, it is clear that when $\mathsf{CAS_{PR}}$ is coherent (*i.e.* its stable extensions coincide with the preferred ones), then it satisfies closure even under C-preferred semantics.

Property 1. *Let* $\mathsf{CAS_{PR}}$ *be a constrained argumentation system for PR, and* $\mathcal{E}_1$*, ...,* $\mathcal{E}_n$ *its* C*-preferred extensions. If* $\mathsf{CAS_{PR}}$ *is coherent, then* $\forall \mathcal{E}_i$, $i = 1, \ldots, n$*, it holds that:*

- $\text{ARG}(\text{BEL}(\mathcal{E}_i)) = \mathcal{E}_i \cap \mathcal{A}_b$.
- $\mathcal{A}s = \mathcal{E}_i$.

9. Related Work

As already mentioned in the introduction, a number of attempts have been made to use argumentation as a basis for practical reasoning. These attemps can be divided into two groups of works: works that are interested in identifying argument schemes that one may encounter in practical reasoning (e.g. [5, 6]), and works that propose concrete argumentation-based systems for PR (e.g. [7, 4, 8, 9, 10]).

The starting point of Atkinson and Bench Capon in [5] was the following practical syllogism advocated by the philosopher Walton in [6].

- G is a goal/desire for agent X
- Doing action A is sufficient for agent X to carry out G
- Then, agent X ought to do action A

The above syllogism, which would apply to the means-end reasoning step, is in essence already an argument in favor of doing action A. However, this does not mean that the action is warranted, since other arguments (called counter-arguments) may be built or provided against the action. The authors have defined an extended version of this syllogism as well as different ways of attacking it. However, it is not clear how all these arguments can be put together in order to answer the critical question of PR "what is the right thing to do in a given situation?". It is neither clear how these arguments are evaluated, nor which decision principle is followed in order to choose between competing desires or between competing plans. It is worth mentioning that most of the schemes and attacks suggested in [5] are already captured in our constrained system. For instance, to the above syllogism the following critical questions are associated:

1. Are there alternative ways of realizing G?
2. Is it possible to do A?
3. Does the agent have other goals that can be taken into account?
4. Are there other consequences of doing A which should be taken into account?

The first question amounts to find the different instrumental arguments for the desire G and to take all of them into account in the reasoning, *i.e.* when computing the set of intentions. The second question amounts to verify whether we are in a state of the world where A can be executed. In our approach this is captured by the pre-conditions of the plans. The third question is also captured in our approach. Indeed, we start with the set of all potential desires of the agent, and then we select the ones that will become its intentions. The last question is captured in our system by the post-conditions of the plans and with the beliefs in the base $\mathcal{K}_b$. Nevertheless, in [5], agent's preferences (her *values*) are taken into account while in our system these are left for investigation.

Regarding the second category of works, it can itself be partitioned into two sub-groups of models: models that are instantiations of the *abstract* argumentation framework of Dung [12] (e.g. [29, 7, 30]), and models that are based on an encoding of argumentative reasoning in logic programs (e.g. [31]). Our framework builds on the former.

In [29], Amgoud was only interested by the second step of PR process (*i.e.* generating and checking the feasibility of plans). She has developed an argumentation framework for generating consistent plans from a given set of desires and planning rules. Later in [32], she has proposed together with Cayrol an ATMS-based proof theory for that framework. The framework was then extended with another argumentation framework that generates the desires themselves in [7], taking thus into account the first step of PR process. For that purpose, a notion of "desire generation rules" has been introduced. These rules are meant to generate desires from beliefs. Thus, our desire generation rules are more general since we allow the generation of desires not only from beliefs, but also from other desires. Another problem with the work proposed in [7] arises because desires and beliefs are not correctly distinguished in the antecedent and consequent of the desire generation rules. This may lead to incorrect inferences where an agent may conclude beliefs on the basis of yet-unachieved desires, hence exhibiting a form of wishful thinking. Our approach resolves this by distinguishing between beliefs and desires in the rules, and refining the notion of attack among explanatory arguments accordingly. The problem of the logical language has been fixed in [9]. In that work, the authors considered three separate systems: one for reasoning about beliefs, one for generating justified desires, and finally one for generating feasible desires. The three systems are related with each others by attacks. Indeed, arguments supporting beliefs may attack both explanatory arguments and instrumental ones. However, explanatory arguments do not conflict with the instrumental ones. Once the results of the three systems are known, the intentions of an agent are computed. The main drawback of this approach is the following: it may be the case that two desires, say d_1 and d_2, are supported by two conflicting explanatory arguments, however d_1 is not feasible since there is no plan for reaching it. What happens is that the system may discard the desire d_2 since its explanatory argument is stronger than the one in favor of d_1. However, when computing the set of intentions, d_1 will neither be considered since it is not feasible. Thus, we lose both desires even if it was possible to achieve d_2 since it is both justified and feasible. In summary, handling separately the three types of arguments may lead to undesirable situations.

Hulstijn and van der Torre [30], on the other hand, have a notion of "desire rule," which contains only desires in the consequent. But their approach is still problematic. It requires that the selected goals are supported by goal trees[16] which contain both desire rules and belief rules that are deductively consistent. This consistent deductive closure again does not distinguish between desire literals and belief literals (see Proposition 2 in [30]). This means that one cannot both believe $\neg p$ and desire p. In our framework, on the other hand, the distinction enables us to have an acceptable belief argument for believing $\neg p$ and, at the same time, an

[16]Similar respectively to our justified desires and our explanatory arguments.

acceptable explanatory argument for desiring p.

In [31], Simari et al. were interested by the first step of a PR process, and have developed an argumentation system for generating desires. This makes our system more general since it tackles also the second step. Like us, they separate in the language rules for reasoning about beliefs and rules for reasoning about desires.

In [33], a defeasible logic based on modal logic is used to reason about motivational attitudes (such as obligations, intentions and desires). In that work, the authors focused on the links between the different attitudes. They show how to infer information from different (nested) rules describing either the beliefs of an agent, or her obligations, desires and intentions. However, they do not take into account the feasibility of desires. In this sense, our work is more general.

A last work which is less related to ours is that developed in [34, 35]. In these two papers, the authors are interested in argumentative dialogues/negotiations. Each agent has final goal and a plan for reaching it. The actions of the plan are arguments that should be uttered. In our paper, we are more interested in generating the final goal(s) of an agent.

10. Conclusion

The paper tackles an important aspect of the practical reasoning problem using argumentation theory. It computes the set of intentions that an agent mat pursue.

The contribution is twofold. To the best of our knowledge, this paper proposes the first argumentation system that computes the possible intentions in one step, *i.e.* by combining desire generation and planning. This avoids undesirable results encountered by previous proposals in the literature. The second contribution consists of studying deeply the properties of argumentation-based PR.

This work can be extended in different ways:

- To improve the language in such a way to take into account temporal aspects.
- To relax the assumption that the attack relation among instrumental arguments is binary. Indeed, it may be the case that more than two plans may be conflicting while each pair of them is compatible.

- Another urgent extension would be to introduce preferences to the system. The idea is that beliefs may be pervaded with uncertainty, desires may not have equal priorities, and plans may have different costs. Thus, taking into account these preferences will help to refine the intention sets.

- In [36, 37], it has been shown that an argument may not only be attacked by other arguments, but may also be supported by arguments. It would be interesting to study the impact of such a relation between arguments in the context of PR.

- Another interesting area of future work is investigating the proof theories of this system. The idea is to answer the question "is a given potential desire a possible intention of the agent ?" without computing the whole preferred extensions.

- Finally, we are planning to implement the system. For that purpose, we may take advantage of existing algorithms developed recently in [38] for generating arguments and counter-arguments.

References

[1] L. Amgoud, C. Devred, M. Lagasquie-Schiex, A constrained argumentation system for practical reasoning, in: Proceedings of the 7th International Joint Conference on Autonomous Agents and Multiagent Systems (AAMAS'08), ACM, 2008, pp. 429–436.

[2] J. Raz, Practical reasoning, Oxford, Oxford University Press.

[3] M. J. Wooldridge, Reasoning about Rational Agents, MIT Press, 2000.

[4] L. Amgoud, H. Prade, Formalizing practical reasoning under uncertainty: An argumentation-based approach, in: Proc. of IAT'07, IEEE, 2007, pp. 189–195.

[5] K. Atkinson, T. Bench-Capon, Practical reasoning as presumptive argumentation using action based alternating transition systems, Artificial Intelligence Journal 171 (2007) 855–874.

[6] D. Walton, Argument schemes for presumptive reasoning, Vol. 29, Lawrence Erlbaum Associates, Mahwah, NJ, USA, 1996.

[7] L. Amgoud, S. Kaci, On the generation of bipolar goals in argumentation-based negotiation, in: I. Rahwan, P. Moraitis, C. Reed (Eds.), Argumentation in Multi-Agent Systems: (Proceedings of the First International Workshop (ArgMAS'04): Expanded and Invited Contributions), Vol. 3366 of Lecture Notes in Computer Science, Springer Verlag, Berlin, Germany, 2005.

[8] J. Hulstijn, L. van der Torre, Combining goal generation and planning in an argumentation framework, in: T. Heskes, P. Lucas, L. Vuurpijl, W. Wiegerinck (Eds.), Proceedings of the 15th Belgium-Netherlands Conference on Artificial Intelligence (BNAIC 2003), Katholieke Universiteit Nijmegen, 2003, pp. 155–162.

[9] I. Rahwan, L. Amgoud, An Argumentation-based Approach for Practical Reasoning , in: G. Weiss, P. Stone (Eds.), 5th International Joint Conference on Autonomous Agents & Multi Agent Systems, AAMAS'2006, Hakodate, Japan, ACM Press, New York, USA, 2006, pp. 347–354.

[10] N. Rotstein, A. García, G. R. Simari, Defeasible argumentation support for an extended BDI architecture, in: ArgMAS, 2007, pp. 145–163.

[11] S. Coste-Marquis, C. Devred, P. Marquis, Constrained argumentation frameworks, in: Proceedings of the 10th International Conference on Principles of Knowledge Representation and Reasoning (KR' 06), 2006, pp. 112–122.

[12] P. M. Dung, On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games, Artificial Intelligence 77 (2) (1995) 321–358.

[13] M. Caminada, L. Amgoud, On the evaluation of argumentation formalisms, Artificial Intelligence 171 (5–6) (2007) 286–310.

[14] N. Gorogiannis, A. Hunter, M. Williams, An argument-based approach to reasoning with clinical knowledge, International Journal of Approximate Reasoning 51 (1) (2009) 16–22.

[15] P. Baroni, F. Cerutti, M. Giacomin, G. Guida, AFRA: Argumentation framework with recursive attacks 52 (1) (2011) 19–37.

[16] P. Baroni, M. Giacomin, Skepticism relations for comparing argumentation semantic 50 (6) (2009) 854–866.

[17] J. Lang, L. van der Torre, E. Weydert, Utilitarian desires, Autonomous Agents and Multi-Agent Systems 5 (3) (2002) 329–363.

[18] M. Ghallab, D. Nau, P. Traverso, Automated planning, theory and practice, Elsevier, Morgan Kaufmann, 2004.

[19] S. Russel, P. Norvig, Artificial Intelligence. A modern approach, Prentice hall, 1995.

[20] G. Harman, Practical aspects of theoretical reasoning, The Oxford Handbook of Rationality (2004) 45–56.

[21] G. R. Simari, R. P. Loui, A mathematical treatment of defeasible reasoning and its implementation, Artificial Intelligence 53 (1992) 125–157.

[22] G. Vreeswijk, Abstract argumentation systems, Artificial Intelligence 90 (1–2) (1997) 225–279.

[23] M. Elvang-Gransson, P. Krause, J. Fox, Dialectic reasoning with inconsistent information, in: D. Heckerman, A. Mamdani (Eds.), Proceedings of the 9th Conference on Uncertainty in Artificial Intelligence, Morgan Kaufmann, Washington D. C., USA, 1993, pp. 114–121.

[24] J. L. Pollock, How to reason defeasibly, Artificial Intelligence 57 (1992) 1–42.

[25] L. Amgoud, C. Cayrol, Inferring from inconsistency in preference-based argumentation frameworks, International Journal of Automated Reasoning 29 (2) (2002) 125–169.

[26] C. Cayrol, On the relation between argumentation and non-monotonic coherence-based entailment, in: Proceedings of the International Joint Conference on Artifitial Intelligence, IJCAI'95, 1995, pp. 1443 – 1448.

[27] L. Amgoud, P. Besnard, A formal analysis of logic-based argumentation systems, in: Proceedings of the 4th International Conference on Scalable Uncertainty Management (SUM 2010), 2010, pp. 42–55.

[28] R. H. Thomason, Desires and defaults: A framework for planning with inferred goals, in: A. G. Cohn, F. Giunchiglia, B. Selman (Eds.), KR 2000: Principles of Knowledge Representation and Reasoning, Proceedings of the

Seventh International Conference, Breckenridge, Colorado, USA, Morgan Kaufmann Publishers, 2000, pp. 702–713.

[29] L. Amgoud, A formal framework for handling conflicting desires, in: T. D. Nielsen, N. L. Zhang (Eds.), Symbolic and Quantitative Approaches to Reasoning with Uncertainty, 7th European Conference (ECSQARU 2003), Vol. 2711 of Lecture Notes in Computer Science, Springer Verlag, Berlin, Germany, 2003, pp. 552–563.

[30] J. Hulstijn, L. van der Torre, Combining goal generation and planning in an argumentation framework, in: A. Hunter, J. Lang (Eds.), Proceedings of the Workshop on Argument, Dialogue and Decision at the International Workshop on Non-monotonic Reasoning (NMR 2004), Whistler, Canada, 2004.

[31] G. R. Simari, A. J. Garcia, M. Capobianco, Actions, planning and defeasible reasoning, in: Proceedings of the 10th International Workshop on Non-Monotonic Reasoning, Whistler BC, Canada, 2004, pp. 377–384.

[32] L. Amgoud, C. Cayrol, On the use of an ATMS for handling conflicting desires, in: D. Dubois, C. A. Welty, M.-A. Williams (Eds.), Principles of Knowledge Representation and Reasoning: Proceedings of the Ninth International Conference (KR2004), Whistler, Canada, June 2-5, 2004, 2004, pp. 194–201.

[33] M. Dastani, G. Governatori, A. Rotolo, I. Song, L. van der Torre, Contextual deliberation of cognitive agents in defeasible logic, in: Proc. of AAMAS"07, ACM Press, 2007, pp. 148–150.

[34] M. Mbarki, J. Bentahar, B. Moulin, A. Moazin, Constraints-based negotiation using argumentation, in: Proceedings of the 13th International Workshop on Non-Monotonic Reasoning (NMR'10), 2010.

[35] M. Mbarki, J. Bentahar, B. Moulin, Specification and complexity of strategic-based reasoning using argumentation, in: Third International Workshop on Argumentation in Multi-Agent Systems (ArgMAS'06), 2006, pp. 142–160.

[36] C. Cayrol, M.-C. Lagasquie-Schiex, On the acceptability of arguments in bipolar argumentation frameworks, in: Proc. of ECSQARU'05 - LNAI 3571, Springer-Verlag, 2005, pp. 378–389.

[37] N. Karacapilidis, D. Papadias, Computer supported argumentation and collaborative decision making: the HERMES system, Information systems 26 (4) (2001) 259–277.

[38] V. Efstathiou, A. Hunter, Algorithms for generating arguments and counterarguments in propositional logic 52 (6) (2011) 672–704.

[39] S. Coste-Marquis, C. Devred, P. Marquis, Symmetric argumentation frameworks, in: Proc. of ECSQARU'05 - LNAI 3571, Springer-Verlag, 2005, pp. 317–328.

Appendix A. Proofs of propositions and theorems

Proposition 2. Let $\mathsf{CAS} = \langle \mathcal{A}, \mathcal{R}, C \rangle$ and $\mathsf{AS} = \langle \mathcal{A}, \mathcal{R} \rangle$ be its basic version. For any $\alpha \in \mathcal{A}$, if α is rejected in AS under semantics x (where x is either preferred or stable), then α is also rejected in CAS under the same semantics x.

Proof . *Assume that $\alpha \in \mathcal{A}$ is rejected in* AS *under semantics x and not rejected in* CAS.

Case of stable semantics: Since α is not rejected in CAS*, then there exists $\mathcal{E}$ such that $\mathcal{E}$ is a C-stable extension of* CAS *and $\alpha \in \mathcal{E}$. According to Proposition 2.9, $\mathcal{E}$ is also a stable extension. Since α is rejected in* AS*, then $\alpha \notin \mathcal{E}$, contradiction.*

Case of preferred semantics: Since α is not rejected in CAS*, then there exists $\mathcal{E}$ such that $\mathcal{E}$ is a C-preferred extension of* CAS *and $\alpha \in \mathcal{E}$. According to Proposition 2.9, each C-preferred extension is a subset of a preferred extension. This means that $\exists \mathcal{E}'$ such $\mathcal{E}'$ is a preferred extension of* AS *and $\mathcal{E} \subseteq \mathcal{E}'$. However, since α is rejected in* AS*, then $\alpha \notin \mathcal{E}'$, contradiction with the fact that $\alpha \in \mathcal{E}$.*

■

Proposition 3. Let $\delta \in \mathcal{A}_d$.

- The set DESIRES of δ is a subset of $\mathcal{PD}$ ($\text{DESIRES}(\delta) \subseteq \mathcal{PD}$).
- The set BELIEFS of δ is a subset of the knowledge base $\mathcal{K}_b$ ($\text{BELIEFS}(\delta) \subseteq \mathcal{K}_b$).

Proof . *Let $\delta \in \mathcal{A}_d$.*

- *Let us show that* $\textsc{Desires}(\delta) \subseteq \mathcal{PD}$. *This is a direct consequence from the definition of an explanatory argument and the definition of the set* $\mathcal{PD}$.
- *Let us show that* $\textsc{Beliefs}(\delta) \subseteq \mathcal{K}_b$. $\textsc{Beliefs}(\delta) = \bigcup \textsc{Supp}(\alpha_i)$ *with* $\alpha_i \in \mathcal{A}_b \cap \textsc{Sub}(\delta)$. *According to the definition of an epistemic argument* α_i, $\textsc{Supp}(\alpha_i) \subseteq \mathcal{K}_b$, *thus* $\textsc{Beliefs}(\delta) \subseteq \mathcal{K}_b$.

■

Proposition 4. Let $\mathcal{A}_b$ be the set of all epistemic arguments that can be built from a beliefs base $\mathcal{K}_b$. It holds that $\nexists \alpha \in \mathcal{A}_b$ such that $\alpha\ \mathcal{R}_b\ \alpha$.

Proof . *Let $\alpha \in \mathcal{A}_b$. Let us suppose that $\alpha\ \mathcal{R}_b\ \alpha$. According to Definition 19,* $\exists h \in \textsc{Supp}(\alpha)$ *such that* $\textsc{Conc}(\alpha) \equiv \neg h$. *Moreover, according to the definition of an epistemic argument, it holds that* $\textsc{Supp}(\alpha) \vdash \textsc{Conc}(\alpha)$, *thus,* $\textsc{Supp}(\alpha) \vdash \neg h$. *Since* $h \in \textsc{Supp}(\alpha)$, *this means that* $\textsc{Supp}(\alpha) \vdash h, \neg h$, *thus* $\textsc{Supp}(\alpha) \vdash \bot$. *This contradicts the fact that the support of an epistemic argument (α in our case) should be consistent.* ■

Proposition 7. If $\mathcal{K}_b \neq \{\bot\}$ and $\mathcal{K}_b \neq \varnothing$, then the argumentation system $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$ has non-empty stable extensions.

Proof . *Since $\mathcal{K}_b \neq \{\bot\}$ and $\mathcal{K}_b \neq \varnothing$ then the base $\mathcal{K}_b$ has at least one maximal (for set inclusion) consistent subset, say T. According to Proposition 6,* $\textsc{Arg}(T)$ *is a stable extension of* $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$. ■

Proposition 9. The relation $\mathcal{R}_d$ is symmetric and irreflexive.

Proof . *This is a direct consequence of Definition 20.* ■

Proposition 10. Let d_1, $d_2 \in \mathcal{PD}$. If $d_1 \equiv \neg d_2$, then $\forall \delta_1, \delta_2 \in \mathcal{A}_d$ such that: (1) $\exists \delta'_1 \in \textsc{Sub}(\delta_1)$ with $\textsc{Conc}(\delta'_1) = d_1$, and (2) $\exists \delta'_2 \in \textsc{Sub}(\delta_2)$ with $\textsc{Conc}(\delta'_2) = d_2$, then $\delta_1\ \mathcal{R}_d\ \delta_2$.

Proof . *Let d_1, $d_2 \in \mathcal{PD}$. Suppose that $d_1 \equiv \neg d_2$. Let $\delta_1, \delta_2 \in \mathcal{A}_d$ such that: (1)* $\exists \delta'_1 \in \textsc{Sub}(\delta_1)$ *with* $\textsc{Conc}(\delta'_1) = d_1$, *and (2)* $\exists \delta'_2 \in \textsc{Sub}(\delta_2)$ *with* $\textsc{Conc}(\delta'_2) = d_2$. *According to the definition of an explanatory argument, it is clear that* $d_1 \in \textsc{Desires}(\delta_1)$ *and* $d_2 \in \textsc{Desires}(\delta_2)$. *Since* $d_1 \equiv \neg d_2$ *then* $\delta_1\ \mathcal{R}_d\ \delta_2$. ■

Proposition 11. The relation $\mathcal{R}_p$ is symmetric and irreflexive.

Proof . *This is a direct consequence of Definition 21.* ■

Proposition 12. Let d_1, $d_2 \in \mathcal{PD}$. If $d_1 \equiv \neg d_2$, then $\forall \pi_1, \pi_2 \in \mathcal{A}_p$ s.t. $\text{CONC}(\pi_1)$ = d_1 and $\text{CONC}(\pi_2)$ = d_2, then $\pi_1\ \mathcal{R}_p\ \pi_2$.

Proof . *Let* d_1, $d_2 \in \mathcal{PD}$. *Suppose that* $d_1 \equiv \neg d_2$. *Let us also suppose that* $\exists\ \pi_1, \pi_2 \in \mathcal{A}_p$ *with* $\text{CONC}(\pi_1)$ = d_1, *and* $\text{CONC}(\pi_2)$ = d_2. *According to Definition 17, it holds that* $d_1 \in \text{POSTC}(\pi_1)$ *and* $d_2 \in \text{POSTC}(\pi_2)$. *Since* $d_1 \equiv \neg d_2$, *then* $\text{POSTC}(\pi_2) \vdash \neg d_1$. *However, the two sets* $\text{POSTC}(\pi_1)$ *and* $\text{POSTC}(\pi_2)$ *are both consistent (according to Definition 10), thus* $\text{POSTC}(\pi_1) \cup \text{POSTC}(\pi_2) \vdash \bot$. *Consequently,* $\pi_1\ \mathcal{R}_p\ \pi_2$. ■

Consequence 1. Let $\alpha_1, \alpha_2 \in \mathcal{A}_b$ and $\delta \in \mathcal{A}_d$ such that $\alpha_1 \in \text{SUB}(\delta)$. If $\alpha_2 \mathcal{R}_b \alpha_1$ then $\alpha_2 \mathcal{R}_{bd} \delta$.

Proof . *By definition, if* $\alpha_1 \in \text{SUB}(\delta)$ *then* $\text{SUPP}(\alpha_1) \subseteq \text{BELIEFS}(\delta)$. *Moreover, also by definition, if* $\alpha_2 \mathcal{R}_b \alpha_1$ *then* $\exists h \in \text{SUPP}(\alpha_1)$ *such that* $\text{CONC}(\alpha_2) \equiv \neg h$. *Thus,* $\exists h \in \text{BELIEFS}(\delta)$ *such that* $\text{CONC}(\alpha_2) \equiv \neg h$. *Consequently,* $\alpha_2 \mathcal{R}_{bd} \delta$. ■

Proposition 13. Let $\delta \in \mathcal{A}_d$. If $\text{BELIEFS}(\delta) \vdash \bot$, then $\exists \alpha \in \mathcal{A}_b$ such that $\alpha\ \mathcal{R}_{bd}\ \delta$.

Proof . *Let* $\delta \in \mathcal{A}_d$. *Suppose that* $\text{BELIEFS}(\delta) \vdash \bot$. *This means that* $\exists T$ *that is minimal for set inclusion among subsets of* $\text{BELIEFS}(\delta)$ *with* $T \vdash \bot$. *Thus*[17], $\exists h \in T$ *such that* $T \backslash \{h\} \vdash \neg h$ *with* $T \backslash \{h\}$ *is consistent. Since* $\text{BELIEFS}(\delta) \subseteq \mathcal{K}_b$ *(according to Prop. 3), then* $T \backslash \{h\} \subseteq \mathcal{K}_b$. *Consequently,* $\exists\ \langle T \backslash \{h\}, \neg h \rangle \in \mathcal{A}_b$ *with* $h \in \text{BELIEFS}(\delta)$. *Thus,* $\langle T \backslash \{h\}, \neg h \rangle\ \mathcal{R}_{bd}\ \delta$. ■

Proposition 14. Let δ_1, $\delta_2 \in \mathcal{A}_d$ with $\text{BELIEFS}(\delta_1) \nvdash \bot$ and $\text{BELIEFS}(\delta_2) \nvdash \bot$. If $\text{BELIEFS}(\delta_1) \cup \text{BELIEFS}(\delta_2) \vdash \bot$, then $\exists \alpha_1, \alpha_2 \in \mathcal{A}_b$ s.t. $\alpha_1\ \mathcal{R}_{bd}\ \delta_1$ and $\alpha_2\ \mathcal{R}_{bd}\ \delta_2$.

[17] Since T is $\subseteq$-minimal among inconsistent subsets of $\text{BELIEFS}(\delta)$, each subset of T is consistent; so, $\exists T' = T \setminus \{h\}$ strictly included in T s.t. $T' \nvdash \bot$; so $T' \vdash \neg h$ (otherwise, $T' \cup \{h\} = T$ would be consistent).

Proof . *Let* $\delta_1, \delta_2 \in \mathcal{A}_d$ *with* $\textsc{Beliefs}(\delta_1) \nvdash \bot$ *and* $\textsc{Beliefs}(\delta_2) \nvdash \bot$. *Assume that* $\textsc{Beliefs}(\delta_1) \cup \textsc{Beliefs}(\delta_2) \vdash \bot$. *So,* $\exists T_1 \subseteq \textsc{Beliefs}(\delta_1)$ *and* $\exists T_2 \subseteq \textsc{Beliefs}(\delta_2)$ *with* $T_1 \cup T_2 \vdash \bot$ *and* $T_1 \cup T_2$ *is minimal for set inclusion, i.e.* $T_1 \cup T_2$ *is a minimal conflict. Since* $\textsc{Beliefs}(\delta_1) \nvdash \bot$ *and* $\textsc{Beliefs}(\delta_2) \nvdash \bot$, *then* $T_1 \neq \varnothing$ *and* $T_2 \neq \varnothing$. *Thus,* $\exists h_1 \in T_1$ *such that* $(T_1 \cup T_2) \setminus \{h_1\} \vdash \neg h_1$. *Since* $T_1 \cup T_2$ *is a minimal conflict, then each subset of* $T_1 \cup T_2$ *is consistent, thus the set* $(T_1 \cup T_2) \setminus \{h_1\}$ *is consistent. Moreover, according to Proposition 3,* $\textsc{Beliefs}(\delta_1) \subseteq \mathcal{K}_b$ *and* $\textsc{Beliefs}(\delta_2) \subseteq \mathcal{K}_b$. *Thus,* $T_1 \subseteq \mathcal{K}_b$ *and* $T_2 \subseteq \mathcal{K}_b$. *It is then clear that* $(T_1 \cup T_2) \setminus \{h_1\} \subseteq \mathcal{K}_b$. *Consequently* $\langle (T_1 \cup T_2) \setminus \{h_1\}, \neg h_1 \rangle$ *is an argument of* $\mathcal{A}_b$. *Thus,* $\langle (T_1 \cup T_2) \setminus \{h_1\}, \neg h_1 \rangle \ \mathcal{R}_{bd}\ \delta_1$. *Similar reasoning applies for* $h_2 \in T_2$ *(since* $T_2 \neq \varnothing$*). Thus,* $\langle (T_1 \cup T_2) \setminus \{h_2\}, \neg h_2 \rangle \ \mathcal{R}_{bd}\ \delta_2$. ■

Proposition 15. Let $\delta \in \mathcal{A}_d$ and $\pi \in \mathcal{A}_p$ with $\textsc{Beliefs}(\delta) \nvdash \bot$. If $\textsc{Beliefs}(\delta) \cup \textsc{Prec}(\pi) \vdash \bot$ then $\exists \alpha_1, \alpha_2 \in \mathcal{A}_b$ s.t. $\alpha_1 \ \mathcal{R}_{bd}\ \delta$ and $\alpha_2 \ \mathcal{R}_{bp}\ \pi$.

Proof . *Let* $\delta \in \mathcal{A}_d$ *and* $\pi \in \mathcal{A}_p$. *Suppose that* $\textsc{Beliefs}(\delta) \nvdash \bot$. *Since* $\textsc{Beliefs}(\delta) \nvdash \bot$ *and* $\textsc{Prec}(\pi) \nvdash \bot$, *then* $\exists T_1 \subseteq \textsc{Beliefs}(\delta)$ *and* $\exists T_2 \subseteq \textsc{Prec}(\pi)$ *with* $T_1 \neq \varnothing$, $T_2 \neq \varnothing$ *and* $T_1 \cup T_2$ *is the smallest inconsistent subset of* $\textsc{Beliefs}(\delta) \cup \textsc{Prec}(\pi)$.

Since $T_1 \neq \varnothing$, *then* $\exists h_1 \in T_1$ *such that* $T_1 \cup T_2 \setminus \{h_1\} \vdash \neg h_1$ *with* $T_1 \cup T_2 \setminus \{h_1\}$ *is consistent. Since* $\textsc{Beliefs}(\delta) \subseteq \mathcal{K}_b$ *and since* $\textsc{Prec}(\pi) \subseteq \mathcal{K}_b$, *then* $T_1 \cup T_2 \subseteq \mathcal{K}_b$. *Consequently,* $T_1 \cup T_2 \setminus \{h_1\} \subseteq \mathcal{K}_b$. *Thus,* $\langle T_1 \cup T_2 \setminus \{h_1\}, \neg h_1 \rangle \in \mathcal{A}_b$. *Moreover,* $\langle T_1 \cup T_2 \setminus \{h_1\}, \neg h_1 \rangle \ \mathcal{R}_{bd}\ \delta$. *Similar reasoning applies for* $h_2 \in T_2$. *We build an argument* $\langle T_1 \cup T_2 \setminus \{h_2\}, \neg h_2 \rangle \ \mathcal{R}_{bp}\ \pi$. ■

Proposition 16. Let $\mathsf{CAS}_{\mathsf{PR}} = \langle \mathcal{A}, \mathcal{R}, C \rangle$. Let Ω be the set of C-admissible sets of $\mathsf{CAS}_{\mathsf{PR}}$.

1. Let $\mathcal{E} \in \Omega$. There exists a C-preferred extension $\mathcal{E}'$ of $\mathsf{CAS}_{\mathsf{PR}}$ such that $\mathcal{E} \subseteq \mathcal{E}'$.
2. Let $\mathsf{CAS}_{\mathsf{PR}}{}' = \langle \mathcal{A}, \mathcal{R}, C' \rangle$ such that $C' \models C$. Let Ω' be the set of C'-admissible sets of $\mathsf{CAS}_{\mathsf{PR}}{}'$. We have $\Omega' \subseteq \Omega$.

Proof . *This is a direct consequence of Proposition in [11].* ■

Proposition 17. Let $\mathsf{CAS}_{\mathsf{PR}} = \langle \mathcal{A}, \mathcal{R}, C \rangle$. For each C-preferred extension $\mathcal{E}$ of $\mathsf{CAS}_{\mathsf{PR}}$, there exists a preferred extension $\mathcal{E}'$ of $\mathsf{AS}_{\mathsf{PR}}$ such that $\mathcal{E} \subseteq \mathcal{E}'$.

Proof . *This is a direct consequence of Proposition in [11].* ■

Proposition 18. Let $\mathsf{CAS}_{\mathsf{PR}} = \langle \mathcal{A}, \mathcal{R}, C \rangle$ such that C is a valid formula on $\mathcal{A}$. Then the preferred extensions of $\mathsf{AS}_{\mathsf{PR}}$ are the C-preferred extensions of $\mathsf{CAS}_{\mathsf{PR}}$.

Proof . *This is a direct consequence of Proposition in [11].* ■

Consequence 2 Let $\mathsf{CAS}_{\mathsf{PR}} = \langle \mathcal{A}, \mathcal{R}, C \rangle$ and $\mathcal{E}$ be its C-extension under preferred or stable semantics.

- For all $\delta \in \mathcal{E} \cap \mathcal{A}_d$, $\exists \pi \in \mathcal{E} \cap \mathcal{A}_p$ such that $\textsc{Conc}(\delta) = \textsc{Conc}(\pi)$.
- For all $\pi \in \mathcal{E} \cap \mathcal{A}_p$, $\exists \delta \in \mathcal{E} \cap \mathcal{A}_d$ such that $\textsc{Conc}(\delta) = \textsc{Conc}(\pi)$.

Proof . *These are direct consequences of the constraint C.* ■

Proposition 19

- The empty set is a C-admissible of the practical system $\mathsf{CAS}_{\mathsf{PR}}$.
- The practical system $\mathsf{CAS}_{\mathsf{PR}}$ has at least one C-preferred extension.

Proof . *$\varnothing$ is admissible (as shown by Dung in [12]) and all π_i and δ_k variables are false in $\widehat{\varnothing}$, so $\widehat{\varnothing} \vdash C$) (this is due to the particular form of the constraint for practical reasoning). Thus, the empty set is C-admissible, consequently, the argumentation system $\mathsf{CAS}_{\mathsf{PR}}$ has a C-preferred extension.* ■

Proposition 20. If $\mathcal{E}$ is a stable extension of $\mathsf{AS}_{\mathsf{PR}}$, then the set $\mathcal{E} \cap \mathcal{A}_b$ is a stable extension of $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$.

Proof . *Let $\mathcal{E}$ be a stable extension of $\mathsf{AS}_{\mathsf{PR}}$. Let us suppose that $\mathcal{E}' = \mathcal{E} \cap \mathcal{A}_b$ is not a stable extension of $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$. Two cases exist:*

***Case 1**: $\mathcal{E}'$ is not conflict-free. This means that there exist $\alpha, \alpha' \in \mathcal{E}'$ such that $\alpha \mathcal{R}_b \alpha'$. Since $\mathcal{E}' = \mathcal{E} \cap \mathcal{A}_b$, then $\alpha, \alpha' \in \mathcal{E}$. This means that $\mathcal{E}$ is not conflict-free. This contradicts the fact that $\mathcal{E}$ is a stable extension.*

***Case 2**: $\mathcal{E}'$ does not attack every argument that is not in $\mathcal{E}'$. This means that $\exists \alpha \in \mathcal{A}_b$ and $\alpha \notin \mathcal{E}'$ and $\mathcal{E}'$ does not attack (w.r.t. $\mathcal{R}_b$) α. This means that $\mathcal{E}' \cup \{\alpha\}$ is conflict-free, thus $\mathcal{E} \cup \{\alpha\}$ is also conflict-free, and does not attack an argument that is not in it (because only an epistemic argument can attack another epistemic argument and all epistemic arguments of $\mathcal{E}$ belong to $\mathcal{E}'$). This contradicts the fact that $\mathcal{E}$ is a stable extension.* ■

Proposition 21. If $\mathcal{K}_b \neq \varnothing$ and $\mathcal{K}_b \neq \{\bot\}$, then the system $\mathsf{AS}_{\mathsf{PR}}$ has at least one non-empty stable extension.

Proof . *$\mathsf{AS}_{\mathsf{PR}}$ can be viewed as the union of 2 argumentation systems: $\mathsf{AS}_b = \langle \mathcal{A}_b, \mathcal{R}_b \rangle$ and $\mathsf{AS}_{dp} = \langle \mathcal{A}_d \cup \mathcal{A}_p, \mathcal{R}_d \cup \mathcal{R}_p \cup \mathcal{R}_{pdp} \rangle$ plus the $\mathcal{R}_{bd} \cup \mathcal{R}_{bp}$ relation.*

Since $\mathcal{K}_b \neq \varnothing$ and $\mathcal{K}_b \neq \{\bot\}$, then the system AS_b has stable extensions (according to Proposition 7). Let $\mathcal{E}_1, \ldots, \mathcal{E}_n$ be those extensions. The system AS_{dp} is symmetric in the sense of [39] since the relation $\mathcal{R}_d \cup \mathcal{R}_p \cup \mathcal{R}_{pdp}$ is symmetric. In [39], it has been shown that such a system has stable extensions which correspond to maximal (for $\subseteq$) sets of arguments that are conflict-free. Let $\mathcal{E}'_1, \ldots, \mathcal{E}'_m$ be those extensions.

The two systems are linked with $\mathcal{R}_{bd} \cup \mathcal{R}_{bp}$. Two cases can be distinguished:

- **case1:** *$\mathcal{R}_{bd} \cup \mathcal{R}_{bp} = \varnothing$. $\forall \mathcal{E}_i, \mathcal{E}'_j$, the set $\mathcal{E}_i \cup \mathcal{E}'_j$ is a stable extension of $\mathsf{AS}_{\mathsf{PR}}$. Indeed, $\mathcal{E}_i \cup \mathcal{E}'_j$ is conflict-free since $\mathcal{E}_i$, $\mathcal{E}'_j$ are both conflict-free, and the relation $\mathcal{R}_{bd} \cup \mathcal{R}_{bp} = \varnothing$. Moreover, $\mathcal{E}_i \cup \mathcal{E}'_j$ attacks every argument that is not in $\mathcal{E}_i \cup \mathcal{E}'_j$, since if $\alpha \notin \mathcal{E}_i \cup \mathcal{E}'_j$, then: i) if $\alpha \in \mathcal{A}_b$, then $\mathcal{E}_i$ attacks w.r.t. $\mathcal{R}_b$ α since $\mathcal{E}_i$ is a stable extension. Now, assume that $\alpha \in \mathcal{A}_d \cup \mathcal{A}_p$. Then, $\mathcal{E}'_j \cup \{\alpha\}$ is conflicting since $\mathcal{E}'_j$ is a maximal (for $\subseteq$) set that is conflict-free. Thus, $\mathcal{E}'_j$ attacks α.*

- **case2:** *$\mathcal{R}_{bd} \cup \mathcal{R}_{bp} \neq \varnothing$. Let $\mathcal{E}$ be a maximal (for set inclusion) set of arguments that is built with the following algorithm:*

  1. $\mathcal{E} = \mathcal{E}_i$
  2. *while ($\exists \beta \in \mathcal{A}_p \cup \mathcal{A}_d$ such that $\mathcal{E} \cup \{\beta\}$ is conflict-free) do $\mathcal{E} = \mathcal{E} \cup \{\beta\}$*

  This algorithm stops after a finite number of steps (because $\mathcal{A}_p \cup \mathcal{A}_d$ is a finite set) and gives a set of arguments which is $\subseteq$-maximal among the conflict-free sets which include $\mathcal{E}_i$. It is easy to see that $\mathcal{E}$ is stable because, by construction, $\forall \gamma \in (\mathcal{A}_p \cup \mathcal{A}_d) \setminus \mathcal{E}$, $\exists \gamma' \in \mathcal{E}$ such that $\gamma' \mathcal{R} \gamma$, (because if $\gamma' \in \mathcal{A}_b \cap \mathcal{E}$ it is impossible that $\gamma \mathcal{R} \gamma'$ and because if $\gamma' \in (\mathcal{A}_d \cup \mathcal{A}_p) \cap \mathcal{E}$ if we have $\gamma \mathcal{R} \gamma'$ we also have $\gamma' \mathcal{R} \gamma$) and, because $\mathcal{E}_i \subseteq \mathcal{E}$, we also have $\forall \alpha \in \mathcal{A}_b \setminus \mathcal{E}$, $\exists \alpha' \in \mathcal{E}$ such that $\alpha' \mathcal{R} \alpha$ (because $\mathcal{E}_i$ is stable in AS_b).

So there is always a stable extension of $\mathsf{AS}_{\mathsf{PR}}$. ■

Proposition 22. Let $\mathcal{E}$ be a stable extension of $\mathsf{AS}_{\mathsf{PR}}$. If $\delta \in \mathcal{E} \cap \mathcal{A}_d$, then $\textsc{Sub}(\delta) \subseteq \mathcal{E}$.

Proof . *Let $\mathcal{E}$ be a stable extension of $\mathsf{AS}_{\mathsf{PR}}$. Let $\delta \in \mathcal{A}_d$. Let us suppose that $\delta \in \mathcal{E}$ and $\exists \delta' \in \textsc{Sub}(\delta)$ such that $\delta' \notin \mathcal{E}$. Since $\delta' \notin \mathcal{E}$, then $\exists x \in \mathcal{E}$ such that $x\mathcal{R}\delta'$. There are three possible cases:*

1. *$x \in \mathcal{A}_b$, thus $x\mathcal{R}_{bd}\delta'$. This means that $\exists h \in \textsc{Beliefs}(\delta')$ such that $\textsc{Conc}(x) \equiv \neg h$. However, $\delta' \in \textsc{Sub}(\delta)$, thus $\textsc{Beliefs}(\delta') \subseteq \textsc{Beliefs}(\delta)$. Thus, $x\mathcal{R}_{bd}\delta$ and consequently, $x\mathcal{R}\delta$. This contradicts the fact that $\mathcal{E}$ is conflict-free.*
2. *$x \in \mathcal{A}_d$, thus $x\mathcal{R}_d\delta'$. Thus, $\exists d_1 \in \textsc{Desires}(x)$ and $\exists d_2 \in \textsc{Desires}(\delta')$ such that $d_1 \equiv \neg d_2$. However, $\textsc{Desires}(\delta') \subseteq \textsc{Desires}(\delta)$, thus $x\mathcal{R}_d\delta$ and consequently, $x\mathcal{R}\delta$. This contradicts the fact that $\mathcal{E}$ is conflict-free.*
3. *$x \in \mathcal{A}_p$, thus $x\mathcal{R}_{pdp}\delta'$. This means that $\textsc{Conc}(x) \equiv \neg d$ with $d \in \textsc{Desires}(\delta')$. However, $\textsc{Desires}(\delta') \subseteq \textsc{Desires}(\delta)$, thus $x\mathcal{R}_{pdp}\delta$ and consequently, $x\mathcal{R}\delta$. This contradicts the fact that $\mathcal{E}$ is conflict-free.*

■

Proposition 23. Let $\mathcal{E}$ be a stable extension of $\mathsf{AS}_{\mathsf{PR}}$. If $\pi \in \mathcal{E} \cap \mathcal{A}_p$, then $\textsc{PreC}(\pi) \subseteq \bigcup_{\alpha_j \in \mathcal{E} \cap \mathcal{A}_b} \textsc{Supp}(\alpha_j)$.

Proof . *Let $\mathcal{E}$ be a stable extension of $\mathsf{AS}_{\mathsf{PR}}$, and let $\pi \in \mathcal{A}_p$ such that $\pi \in \mathcal{E}$. Let us assume that $\exists x \in \textsc{PreC}(\pi)$ and $x \notin \bigcup_{\alpha_j \in \mathcal{E} \cap \mathcal{A}_b} \textsc{Supp}(\alpha_j)$. Let $\mathcal{E}' = \mathcal{E} \cap \mathcal{A}_b$.*

According to Proposition 20, the set $\mathcal{E}'$ is a stable extension of the system $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$. Moreover, according to Proposition 5, $\textsc{Base}(\mathcal{E}') = \bigcup_{\alpha_j \in \mathcal{E} \cap \mathcal{A}_b} \textsc{Supp}(\alpha_j)$ is a maximal (for set inclusion) consistent subbase of the knowledge base $\mathcal{K}_b$. Thus, $\textsc{Base}(\mathcal{E}') \cup \{x\}$ is inconsistent. It follows that $\textsc{Base}(\mathcal{E}') \vdash \neg x$.

According to Proposition 7, $\textsc{Base}(\mathcal{E}') \neq \varnothing$. Thus, $\exists H \subseteq \textsc{Base}(\mathcal{E}')$ such that $H \neq \varnothing$, H is consistent and $H \vdash \neg x$. Consequently, $\langle H, \neg x \rangle$ is an argument of the set $\mathcal{A}_b$, and $\langle H, \neg x \rangle \in \textsc{Arg}(\textsc{Base}(\mathcal{E}'))$.

According to Proposition 5, $\textsc{Arg}(\textsc{Base}(\mathcal{E}')) = \mathcal{E}'$. Thus, $\langle H, \neg x \rangle \in \mathcal{E}'$. Consequently, $\langle H, \neg x \rangle \in \mathcal{E}$. From Definition 22, $\langle H, \neg x \rangle \mathcal{R}_{bp}\pi$. This means that $\mathcal{E}$ is not conflict free. This contradicts the fact that $\mathcal{E}$ is a stable extension. ■

Proposition 24. Let $\delta \in \mathcal{A}_d$. If $\textsc{Beliefs}(\delta) \vdash \perp$, then $\forall \mathcal{E}$ with $\mathcal{E}$ is a stable extension of $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$, $\exists \alpha \in \mathcal{E}$ such that $\alpha\mathcal{R}_{bd}\delta$.

Proof . *Let $\delta \in \mathcal{A}_d$ with $\textsc{Beliefs}(\delta) \vdash \perp$. Let $\mathcal{E}_1, \ldots \mathcal{E}_n$ be the stable extensions of the system $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$. Suppose that $\exists \mathcal{E}_i$ such that $\mathcal{E}_i$ does not attack δ, i.e. $\nexists \alpha \in \mathcal{E}_i$ such that $\alpha\mathcal{R}_{bd}\delta$.*

According to Prop. 5, $\textsc{Base}(\mathcal{E}_i)$ *is a maximal (for set inclusion) consistent subset of* $\mathcal{K}_b$*. Since* $\textsc{Beliefs}(\delta) \vdash \bot$*, then* $\exists T \subseteq \textsc{Beliefs}(\delta)$ *with* T *is the smallest inconsistent subset of* $\textsc{Beliefs}(\delta)$ *(i.e.* $T \vdash \bot$*). Moreover, according to Prop. 3,* $\textsc{Beliefs}(\delta) \subseteq \mathcal{K}_b$*, thus* $T \subseteq \mathcal{K}_b$*.*

Since $\textsc{Base}(\mathcal{E}_i)$ *is a maximal (for set inclusion) consistent subset of* $\mathcal{K}_b$*, and* T *a minimal conflict of* $\mathcal{K}_b$*, then we have two cases:*

- *Case 1:* $\textsc{Base}(\mathcal{E}_i) \cap T = \varnothing$*. This means that* $\forall h \in T$*,* $\textsc{Base}(\mathcal{E}_i) \cup \{h\} \vdash \bot$*. Thus,* $\textsc{Base}(\mathcal{E}_i) \vdash \neg h$*. Consequently,* $\exists H \subseteq \textsc{Base}(\mathcal{E}_i)$ *with* H *is minimal for set-inclusion among subsets of* $\textsc{Base}(\mathcal{E}_i)$ *that satisfy* $H \vdash \neg h$*. The pair* $\langle H, \neg h\rangle$ *is then an argument of* $\mathcal{A}_b$*. However, according to Prop. 5,* $\textsc{Arg}(\textsc{Base}(\mathcal{E}_i))$ = $\mathcal{E}_i$*, this means that* $\langle H, \neg h\rangle \in \mathcal{E}_i$ *and* $\langle H, \neg h\rangle$ $\mathcal{R}_{bd}$ δ*.*

- *Case 2:* $\textsc{Base}(\mathcal{E}_i) \cap T \neq \varnothing$*. Since* $\textsc{Base}(\mathcal{E}_i) \nvdash \bot$ *and* $T \vdash \bot$*, then* $\exists h \in T$ *and* $h \notin \textsc{Base}(\mathcal{E}_i)$ *such that* $\textsc{Base}(\mathcal{E}_i) \vdash \neg h$ *(this is due to the fact that* $\textsc{Base}(\mathcal{E}_i)$ *is a maximal consistent subset of* $\mathcal{K}_b$*). Consequently,* $\exists H \subseteq \textsc{Base}(\mathcal{E}_i)$ *with* H *is minimal for set-inclusion among subsets of* $\textsc{Base}(\mathcal{E}_i)$ *that satisfy* $H \vdash \neg h$*. The pair* $\langle H, \neg h\rangle$ *is then an argument of* $\mathcal{A}_b$*. According to Prop. 5,* $\textsc{Arg}(\textsc{Base}(\mathcal{E}_i))$ = $\mathcal{E}_i$*, this means that* $\langle H, \neg h\rangle$ $\in \mathcal{E}_i$ *and* $\langle H, \neg h\rangle$ $\mathcal{R}_{bd}$ δ*.*

■

Proposition 25. Let $\delta \in \mathcal{A}_d$ with $\textsc{Beliefs}(\delta) \vdash \bot$. Under the stable semantics, the argument δ is rejected in $\mathsf{AS}_{\mathsf{PR}}$.

Proof . *Let* $\delta \in \mathcal{A}_d$ *with* $\textsc{Beliefs}(\delta) \vdash \bot$*.*

According to Proposition 21, the system $\mathsf{AS}_{\mathsf{PR}}$ *has at least one stable extension. Let* $\mathcal{E}$ *be one of these stable extensions. Suppose that* $\delta \in \mathcal{E}$*.*

According to Proposition 20, the set $\mathcal{E} \cap \mathcal{A}_b$ *is a stable extension of* $\langle \mathcal{A}_b, \mathcal{R}_b\rangle$*. Moreover, according to Proposition 24,* $\exists \alpha \in \mathcal{E} \cap \mathcal{A}_b$ *such that* $\alpha \mathcal{R}_{bd} \delta$*. This contradicts the fact that a stable extension is conflict-free.* ■

Proposition 26. Let $\delta \in \mathcal{A}_d$ with $\textsc{Beliefs}(\delta) \vdash \bot$. Under the stable semantics, δ is a rejected argument in $\mathsf{CAS}_{\mathsf{PR}}$.

Proof . *Let* $\delta \in \mathcal{A}_d$ *with* $\textsc{Beliefs}(\delta) \vdash \bot$*. According to Prop. 25,* δ *is rejected in* $\mathsf{AS}_{\mathsf{PR}}$*. Moreover, according to Prop. 2; we know that each argument that is rejected in* $\mathsf{AS}_{\mathsf{PR}}$ *is also rejected in* $\mathsf{CAS}_{\mathsf{PR}}$*.* ■

Proposition 27. Let δ_1, $\delta_2 \in \mathcal{A}_d$ with $\textsc{Beliefs}(\delta_1) \nvdash \bot$ and $\textsc{Beliefs}(\delta_2) \nvdash \bot$. If $\textsc{Beliefs}(\delta_1) \cup \textsc{Beliefs}(\delta_2) \vdash \bot$, then $\forall \mathcal{E}$ with $\mathcal{E}$ is a stable extension of $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$, $\exists \alpha \in \mathcal{E}$ such that $\alpha \mathcal{R}_{bd} \delta_1$, or $\alpha \mathcal{R}_{bd} \delta_2$.

Proof . *Let $\delta_1, \delta_2 \in \mathcal{A}_d$ with* $\textsc{Beliefs}(\delta_1) \nvdash \bot$, $\textsc{Beliefs}(\delta_2) \nvdash \bot$, $\textsc{Beliefs}(\delta_1) \cup \textsc{Beliefs}(\delta_2) \vdash \bot$.

Let $\mathcal{E}_1, \ldots \mathcal{E}_n$ be the stable extensions of the system $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$. Suppose that $\exists \mathcal{E}_i$ such that $\mathcal{E}_i$ does not attack δ_1 and $\mathcal{E}_i$ does not attack δ_2, i.e. $\nexists \alpha \in \mathcal{E}_i$ such that $\alpha \mathcal{R}_{bd} \delta_1$, or $\alpha \mathcal{R}_{bd} \delta_2$.

$\textsc{Beliefs}(\delta_1) \cup \textsc{Beliefs}(\delta_2) \vdash \bot$, *so* $\exists T \subseteq \textsc{Beliefs}(\delta_1) \cup \textsc{Beliefs}(\delta_2)$ *with T is the smallest inconsistent subset of* $\textsc{Beliefs}(\delta_1) \cup \textsc{Beliefs}(\delta_2)$ *(i.e. $T \vdash \bot$).*

Moreover, according to Proposition 3, $\textsc{Beliefs}(\delta_1) \subseteq \mathcal{K}_b$ *and* $\textsc{Beliefs}(\delta_2) \subseteq \mathcal{K}_b$*, thus $T \subseteq \mathcal{K}_b$.*

According to Proposition 5, $\textsc{Base}(\mathcal{E}_i)$ *is a maximal (for set inclusion) consistent subset of $\mathcal{K}_b$. Since* $\textsc{Base}(\mathcal{E}_i)$ *is a maximal (for set inclusion) consistent subset of $\mathcal{K}_b$, and T a minimal conflict of $\mathcal{K}_b$, then we have two cases:*

- *Case 1:* $\textsc{Base}(\mathcal{E}_i) \cap T = \varnothing$. *This means that* $\forall h \in T$, $\textsc{Base}(\mathcal{E}_i) \cup \{h\} \vdash \bot$. *Thus,* $\textsc{Base}(\mathcal{E}_i) \vdash \neg h$. *Consequently,* $\exists H \subseteq \textsc{Base}(\mathcal{E}_i)$ *with H is minimal for set-inclusion among subsets of* $\textsc{Base}(\mathcal{E}_i)$ *that satisfy $H \vdash \neg h$. The pair $\langle H, \neg h \rangle$ is then an argument of $\mathcal{A}_b$. However, according to Proposition 5,* $\textsc{Arg}(\textsc{Base}(\mathcal{E}_i)) = \mathcal{E}_i$*, this means that $\langle H, \neg h \rangle \in \mathcal{E}_i$.*

  If $h \in T \cap \textsc{Beliefs}(\delta_1)$*, then* $\langle H, \neg h \rangle \; \mathcal{R}_{bd} \; \delta_1$.

  If $h \in T \cap \textsc{Beliefs}(\delta_2)$*, then* $\langle H, \neg h \rangle \; \mathcal{R}_{bd} \; \delta_2$.

- *Case 2:* $\textsc{Base}(\mathcal{E}_i) \cap T \neq \varnothing$. *Since* $\textsc{Base}(\mathcal{E}_i) \nvdash \bot$ *and* $T \vdash \bot$*, then* $\exists h \in T$ *and* $h \notin \textsc{Base}(\mathcal{E}_i)$ *such that* $\textsc{Base}(\mathcal{E}_i) \vdash \neg h$ *(this is due to the fact that* $\textsc{Base}(\mathcal{E}_i)$ *is a maximal consistent subset of $\mathcal{K}_b$). Consequently,* $\exists H \subseteq \textsc{Base}(\mathcal{E}_i)$ *with H is minimal for set-inclusion among subsets of* $\textsc{Base}(\mathcal{E}_i)$ *that satisfy $H \vdash \neg h$. The pair $\langle H, \neg h \rangle$ is then an argument of $\mathcal{A}_b$. According to Prop. 5,* $\textsc{Arg}(\textsc{Base}(\mathcal{E}_i)) = \mathcal{E}_i$*, this means that $\langle H, \neg h \rangle \in \mathcal{E}_i$. If* $h \in T \cap \textsc{Beliefs}(\delta_1)$*, then and* $\langle H, \neg h \rangle \; \mathcal{R}_{bd} \; \delta_1$*. If* $h \in T \cap \textsc{Beliefs}(\delta_2)$*, then and* $\langle H, \neg h \rangle \; \mathcal{R}_{bd} \; \delta_2$.

■

Proposition 28. Let δ_1, $\delta_2 \in \mathcal{A}_d$ with $\textsc{Beliefs}(\delta_1) \nvdash \bot$ and $\textsc{Beliefs}(\delta_2) \nvdash \bot$. If $\textsc{Beliefs}(\delta_1) \cup \textsc{Beliefs}(\delta_2) \vdash \bot$, then $\nexists \mathcal{E}$ with $\mathcal{E}$ a C-stable extension of $\mathsf{CAS}_{\mathsf{PR}}$ such that $\delta_1 \in \mathcal{E}$ and $\delta_2 \in \mathcal{E}$.

Proof . *Let $\delta_1, \delta_2 \in \mathcal{A}_d$ with* $\text{BELIEFS}(\delta_1) \nvdash \bot$, $\text{BELIEFS}(\delta_2) \nvdash \bot$, $\text{BELIEFS}(\delta_1) \cup \text{BELIEFS}(\delta_2) \vdash \bot$.

Assume that $\exists\mathcal{E}$ with $\mathcal{E}$ a C-stable extension of $\mathsf{CAS}_{\mathsf{PR}}$. *According to [11], $\mathcal{E}$ is also a stable extension of* $\mathsf{AS}_{\mathsf{PR}}$. *Suppose that $\delta_1 \in \mathcal{E}$ and $\delta_2 \in \mathcal{E}$.*

According to Proposition 20, the set $\mathcal{E} \cap \mathcal{A}_b$ is a stable extension of $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$. Moreover, according to Proposition 27, $\exists\alpha \in \mathcal{E}\cap\mathcal{A}_b$ such that $\alpha\mathcal{R}_{bd}\delta_1$, or $\alpha\mathcal{R}_{bd}\delta_2$. Thus, there is a contradiction, and we can conclude that $\nexists\mathcal{E}$ with $\mathcal{E}$ a stable extension of $\mathsf{AS}_{\mathsf{PR}}$ *such that $\delta_1 \in \mathcal{E}$ and $\delta_2 \in \mathcal{E}$. Thus, we have a contradiction.* ■

Proposition 29. Let $\delta \in \mathcal{A}_d$ and $\pi \in \mathcal{A}_p$ with $\text{BELIEFS}(\delta) \nvdash \bot$. If $\text{BELIEFS}(\delta) \cup \text{PREC}(\pi) \vdash \bot$ then $\forall\mathcal{E}$ with $\mathcal{E}$ is a stable extension of $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$, $\exists\alpha \in \mathcal{E}$ such that $\alpha\mathcal{R}_{bd}\delta$, or $\alpha\mathcal{R}_{bp}\pi$.

Proof . *Let $\delta \in \mathcal{A}_d$, $\pi \in \mathcal{A}_p$ with* $\text{BELIEFS}(\delta) \nvdash \bot$ *and* $\text{BELIEFS}(\delta) \cup \text{PREC}(\pi) \vdash \bot$. *Let us suppose that $\mathcal{E}$ is a stable extension of $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$, and that $\delta \in \mathcal{E}$ and $\pi \in \mathcal{E}$.*

Since $\text{BELIEFS}(\delta) \cup \text{PREC}(\pi) \vdash \bot$, $\text{BELIEFS}(\delta) \nvdash \bot$, *and* $\text{PREC}(\pi) \nvdash \bot$, *then* $\exists T_1 \subseteq \text{BELIEFS}(\delta)$ *and* $\exists T_2 \subseteq \text{PREC}(\pi)$ *such that $T_1 \cup T_2 \vdash \bot$ and $T_1 \cup T_2$ is the minimal inconsistent subset of* $\text{BELIEFS}(\delta) \cup \text{PREC}(\pi)$. *We know also that $T_1 \subseteq \mathcal{K}_b$ (since according to Proposition 3,* $\text{BELIEFS}(\delta) \subseteq \mathcal{K}_b$*) and $T_2 \subseteq \mathcal{K}_b$ (since* $\text{PREC}(\pi) \subseteq \mathcal{K}_b$*). Let $T = T_1 \cup T_2$.*

According to Proposition 5, $\text{BASE}(\mathcal{E})$ *is a maximal (for set inclusion) consistent subset of $\mathcal{K}_b$. Then, two cases are distinguished:*

- *Case 1:* $\text{BASE}(\mathcal{E}) \cap T = \varnothing$. *This means that $\forall h \in T$,* $\text{BASE}(\mathcal{E}) \cup \{h\} \vdash \bot$. *Thus,* $\text{BASE}(\mathcal{E}) \vdash \neg h$. *Consequently,* $\exists H \subseteq \text{BASE}(\mathcal{E})$ *with H is minimal for set-inclusion among subsets of* $\text{BASE}(\mathcal{E})$ *that satisfy $H \vdash \neg h$. The pair $\langle H, \neg h \rangle$ is then an argument of $\mathcal{A}_b$. However, according to Proposition 5,* $\text{ARG}(\text{BASE}(\mathcal{E})) = \mathcal{E}$, *this means that $\langle H, \neg h \rangle \in \mathcal{E}$.*

  If $h \in T_1$, then $\langle H, \neg h \rangle\ \mathcal{R}_{bd}\ \delta$.

  If $h \in T_2$, then $\langle H, \neg h \rangle\ \mathcal{R}_{bp}\ \pi$.

- *Case 2:* $\text{BASE}(\mathcal{E}) \cap T \neq \varnothing$. *Since* $\text{BASE}(\mathcal{E}) \nvdash \bot$ *and $T \vdash \bot$, then $\exists h \in T$ and* $h \notin \text{BASE}(\mathcal{E})$ *such that* $\text{BASE}(\mathcal{E}) \vdash \neg h$ *(this is due to the fact that* $\text{BASE}(\mathcal{E})$ *is a maximal consistent subset of $\mathcal{K}_b$). Consequently,* $\exists H \subseteq \text{BASE}(\mathcal{E})$ *with H is minimal for set-inclusion among subsets of* $\text{BASE}(\mathcal{E})$ *that satisfy $H \vdash \neg h$. The pair $\langle H, \neg h \rangle$ is then an argument of $\mathcal{A}_b$.*

According to Prop. 5, $\textsc{Arg}(\textsc{Base}(\mathcal{E})) = \mathcal{E}$*, this means that* $\langle H, \neg h\rangle \in \mathcal{E}$*. If* $h \in T \cap \textsc{Beliefs}(\delta_1)$*, then* $\langle H, \neg h\rangle \ \mathcal{R}_{bd}\ \delta_1$*. If* $h \in T \cap \textsc{Beliefs}(\delta_2)$*, then* $\langle H, \neg h\rangle \ \mathcal{R}_{bd}\ \delta_2$*.*

■

Proposition 30. Let $\delta \in \mathcal{A}_d$ and $\pi \in \mathcal{A}_p$ with $\textsc{Beliefs}(\delta) \nvdash \bot$. If $\textsc{Beliefs}(\delta) \cup \textsc{PreC}(\pi) \vdash \bot$ then $\nexists \mathcal{E}$ with $\mathcal{E}$ a C-stable extension of $\mathsf{CAS}_{\mathsf{PR}}$ such that $\delta \in \mathcal{E}$ and $\pi \in \mathcal{E}$.

Proof . *Let* $\delta \in \mathcal{A}_d$ *and* $\pi \in \mathcal{A}_p$ *with* $\textsc{Beliefs}(\delta) \nvdash \bot$ *and* $\textsc{Beliefs}(\delta) \cup \textsc{PreC}(\pi) \vdash \bot$*. Let* $\mathcal{E}$ *be a* C*-stable extension of* $\mathsf{CAS}_{\mathsf{PR}}$*. So, according to [11],* $\mathcal{E}$ *is also a stable extension of* $\mathsf{AS}_{\mathsf{PR}}$*. Let us suppose that* $\delta \in \mathcal{E}$ *and* $\pi \in \mathcal{E}$*. Since* $\mathcal{E}$ *is a stable extension of* $\mathsf{AS}_{\mathsf{PR}}$*, then* $\mathcal{E}' = \mathcal{E} \cap \mathcal{A}_b$ *is a stable extension of* $\langle \mathcal{A}_b, \mathcal{R}_b\rangle$ *(according to Proposition 20). Moreover, according to Proposition 29, since* $\textsc{Beliefs}(\delta) \cup \textsc{PreC}(\pi) \vdash \bot$ *then* $\exists \alpha \in \mathcal{E}'$ *such that* $\alpha \mathcal{R}_{bd} \delta$ *or* $\alpha \mathcal{R}_{bp} \pi$*. This means that* $\mathcal{E}$ *attacks* δ *or* $\mathcal{E}$ *attacks* π*. However,* $\delta \in \mathcal{E}$ *and* $\pi \in \mathcal{E}$*. This contradicts the fact that* $\mathcal{E}$ *is conflict-free.* ■

Theorem 1. Let $\mathsf{CAS}_{\mathsf{PR}}$ be a constrained argumentation system for PR, and $\mathcal{E}_1$, $\ldots$, $\mathcal{E}_n$ its C-stable extensions. $\forall \mathcal{E}_i$, $i = 1, \ldots, n$, it holds that:

1. The set $\textsc{Bel}(\mathcal{E}_i) = \textsc{Bel}(\mathcal{E}_i \cap \mathcal{A}_b)$.
2. The set $\textsc{Bel}(\mathcal{E}_i)$ is a maximal (for set inclusion) consistent subset of $\mathcal{K}_b$.
3. The set $\bigcup_{\pi_k \in \mathcal{E}_i \cap \mathcal{A}_p} \textsc{PosT}(\pi_k)$ is consistent.
4. The set $\textsc{Des}(\mathcal{E}_i)$ is consistent.

Proof . *Let* $\mathcal{E}_i$ *be a stable extension of the system* $\mathsf{CAS}_{\mathsf{PR}}$*.*

1. *Let us show that the set* $\textsc{Bel}(\mathcal{E}_i) = \textsc{Bel}(\mathcal{E}_i \cap \mathcal{A}_b)$*.*
   In order to prove this, one should handle two cases:
   - $\textsc{Bel}(\mathcal{E}_i \cap \mathcal{A}_b) \subseteq \textsc{Bel}(\mathcal{E}_i)$*. This is a direct consequence from the fact that* $\textsc{Bel}(\mathcal{E}_i \cap \mathcal{A}_b) = \bigcup \textsc{Supp}(\alpha_i)$ *with* $\alpha_i \in \mathcal{E}_i \cap \mathcal{A}_b$ *(cf. definition of* $\textsc{Bel}(\mathcal{E})$*).*
   - $\textsc{Bel}(\mathcal{E}_i) \subseteq \textsc{Bel}(\mathcal{E}_i \cap \mathcal{A}_b)$*. Let us suppose that* $\exists h \in \textsc{Bel}(\mathcal{E}_i)$ *and* $h \notin \textsc{Bel}(\mathcal{E}_i \cap \mathcal{A}_b)$*. According to Proposition 20,* $\mathcal{E}_i \cap \mathcal{A}_b$ *is a stable extension of* $\langle \mathcal{A}_b, \mathcal{R}_b\rangle$*. Moreover, according to Proposition 5,* $\textsc{Bel}(\mathcal{E}_i \cap \mathcal{A}_b)$

is a maximal (for set-$\subseteq$) consistent subset of $\mathcal{K}_b$[18]*. However, $\text{BEL}(\mathcal{E}_i) \subseteq \mathcal{K}_b$, then $h \in \mathcal{K}_b$. Since $h \notin \text{BEL}(\mathcal{E}_i \cap \mathcal{A}_b)$, then $\text{BEL}(\mathcal{E}_i \cap \mathcal{A}_b) \cup \{h\} \vdash \perp$ (this is due to the fact that $\text{BEL}(\mathcal{E}_i \cap \mathcal{A}_b)$ is a maximal (for set-$\subseteq$) consistent subset of $\mathcal{K}_b$). Thus, $\text{BEL}(\mathcal{E}_i \cap \mathcal{A}_b) \vdash \neg h$. This means that $\exists H \subseteq \text{BEL}(\mathcal{E}_i \cap \mathcal{A}_b)$ such that H is the minimal consistent subset of $\text{BEL}(\mathcal{E}_i \cap \mathcal{A}_b)$, thus $H \vdash \neg h$. Since $H \subseteq \mathcal{K}_b$ (since $\text{BEL}(\mathcal{E}_i \cap \mathcal{A}_b) \subseteq \mathcal{K}_b$), then $\langle H, \neg h\rangle \in \mathcal{A}_b$. However, according to Proposition 5, $\text{ARG}(\text{BEL}(\mathcal{E}_i \cap \mathcal{A}_b)) = \mathcal{E}_i \cap \mathcal{A}_b$. Besides, $h \in \text{BEL}(\mathcal{E}_i)$, there are three possibilities:*

(a) *$h \in \text{BELIEFS}(\delta)$ with $\delta \in \mathcal{E}_i$. In this case, $\langle H, \neg h\rangle\ \mathcal{R}_{bd}\ \delta$. This contradicts the fact that $\mathcal{E}_i$ is a stable extension that is conflict-free.*

(b) *$h \in \text{PREC}(\pi)$ with $\pi \in \mathcal{E}_i$. In this case, $\langle H, \neg h\rangle\ \mathcal{R}_{bp}\ \pi$. This contradicts the fact that $\mathcal{E}_i$ is a stable extension that is conflict-free.*

(c) *$h \in \text{SUPP}(\alpha)$ with $\alpha \in \mathcal{E}_i$. This is impossible since the set $\mathcal{E}_i \cap \mathcal{A}_b$ is a stable extension, thus it is conflict free.*

2. *Let us show that the set $\text{BEL}(\mathcal{E}_i)$ is a maximal (for set inclusion) consistent subset of $\mathcal{K}_b$.*
   Since $\mathcal{E}_i$ is a C-stable extension of $\mathsf{CAS}_{\mathsf{PR}}$, then $\mathcal{E}_i$ is also a stable extension of $\mathsf{AS}_{\mathsf{PR}}$ (according to [11]). Moreover, according to the first item of Theorem 1, $\text{BEL}(\mathcal{E}_i) = \text{BEL}(\mathcal{E}_i \cap \mathcal{A}_b)$. However, according to Proposition 20, $\mathcal{E}_i \cap \mathcal{A}_b$ is a stable extension of $\langle \mathcal{A}_b, \mathcal{R}_b\rangle$, and according to Proposition 5, $\text{BEL}(\mathcal{E}_i \cap \mathcal{A}_b)$ is a maximal (for set-$\subseteq$) consistent subset of $\mathcal{K}_b$. Thus, $\text{BEL}(\mathcal{E}_i)$ is a maximal (for set inclusion) consistent subset of $\mathcal{K}_b$.
3. *Let us show that the set $\bigcup_{\pi_k \in \mathcal{E}_i \cap \mathcal{A}_p} \text{POST}(\pi_k)$ is consistent. Assume that $\bigcup_{\pi_k \in \mathcal{E}_i \cap \mathcal{A}_p} \text{POST}(\pi_k)$ is inconsistent. This means that $\exists \pi_1, \ldots, \pi_n \in \mathcal{E}_i$ such that $\text{POST}(\pi_1) \cup \ldots \cup \text{POST}(\pi_n)$ is inconsistent. According to Assumption 1 given in the end of Section 6.3, $\mathcal{R}_p$ is binary, and thus, by definition of the relation $\mathcal{R}_p$, it holds that $\pi_i \mathcal{R}_p \pi_j$, for all $i, j \in 1 \ldots n$ and $i \neq j$. This contradicts the fact that $\mathcal{E}_i$ is a C-stable extension, thus conflict-free.*
4. *Let us show that the set $\text{DES}(\mathcal{E}_i)$ is consistent.*
   Since $\mathcal{E}_i$ is a C-stable extension of $\mathsf{CAS}_{\mathsf{PR}}$, then $\mathcal{E}_i$ is also a stable extension of $\mathsf{AS}_{\mathsf{PR}}$ (according to [11]). Let us suppose that $\text{DES}(\mathcal{E}_i)$ is inconsistent, this means that $\bigcup \text{DESIRES}(\delta_k) \cup \bigcup \text{CONC}(\pi_j) \vdash \perp$ with $\delta_k \in \mathcal{E}_i$ and

[18] Because $\text{BEL}(\mathcal{E}_i \cap \mathcal{A}_b) = \bigcup \text{SUPP}(\alpha_i)$ with $\alpha_i \in \mathcal{E}_i \cap \mathcal{A}_b$; so, $\text{BEL}(\mathcal{E}_i \cap \mathcal{A}_b) = \text{BASE}(\mathcal{E}_i \cap \mathcal{A}_b)$.

$\pi_j \in \mathcal{E}_i$. *Since* $\text{DES}(\mathcal{E}_i) \subseteq \mathcal{PD}$ *(according to Proposition 3), then* $\exists d_1, d_2 \in \text{DES}(\mathcal{E}_i)$ *such that* $d_1 \equiv \neg d_2$. *Three possible situations may occur:*

(a) $\exists \pi_1, \pi_2 \in \mathcal{E}_i \cap \mathcal{A}_p$ *such that* $\text{CONC}(\pi_1) = d_1$, *and* $\text{CONC}(\pi_2) = d_2$. *This means that* $\pi_1 \mathcal{R}_p \pi_2$, *thus* $\pi_1 \mathcal{R} \pi_2$. *This is impossible since* $\mathcal{E}_i$ *is a stable extension, thus it is supposed to be conflict-free.*

(b) $\exists \delta_1, \delta_2 \in \mathcal{E}_i \cap \mathcal{A}_d$ *such that* $d_1 \in \text{DESIRES}(\delta_1)$ *and* $d_2 \in \text{DESIRES}(\delta_2)$. *This means that* $\delta_1 \mathcal{R}_d \delta_2$, *thus* $\delta_1 \mathcal{R} \delta_2$. *This is impossible since* $\mathcal{E}_i$ *is a stable extension, thus it is supposed to be conflict-free.*

(c) $\exists \delta \in \mathcal{E}_i \cap \mathcal{A}_d$, $\exists \pi \in \mathcal{E}_i \cap \mathcal{A}_p$ *such that* $d_1 \in \text{DESIRES}(\delta)$ *and* $d_2 = \text{CONC}(\pi)$.
Since $d_1 \in \text{DESIRES}(\delta)$, *thus* $\exists \delta' \in \text{SUB}(\delta)$ *such that* $\text{CONC}(\delta') = d_1$. *This means that* $\delta' \mathcal{R}_{pdp} \pi$, *thus* $\delta' \mathcal{R} \pi$. *However, since* $\delta \in \mathcal{E}_i$, *thus according to Proposition 22* $\delta' \in \mathcal{E}_i$. *This is impossible since* $\mathcal{E}_i$ *is a stable extension, thus it is supposed to be conflict-free.*

■

Theorem 2. Let $\mathsf{CAS_{PR}}$ be a constrained argumentation system for PR, and $\mathcal{E}_1, \ldots, \mathcal{E}_n$ its C-preferred extensions. $\forall \mathcal{E}_i$, $i = 1, \ldots, n$, it holds that:

1. The set $\text{BEL}(\mathcal{E}_i)$ is consistent.
2. The set $\bigcup_{\pi_k \in \mathcal{E}_i \cap \mathcal{A}_p} \text{POST}(\pi_k)$ is consistent.
3. The set $\text{DES}(\mathcal{E}_i)$ is consistent.

Proof . *Let* $\mathsf{CAS_{PR}}$ *be a constrained argumentation system for PR.*

1. *Let* $\mathcal{E}$ *be a preferred extension of* $\mathsf{AS_{PR}}$. *Assume that* $\text{BEL}(\mathcal{E})$ *is inconsistent. Thus, there exists* $C \subseteq \text{BEL}(\mathcal{E})$ *s.t.* C *is a minimal (for set inclusion) subset of* $\text{BEL}(\mathcal{E})$ *that is inconsistent. Since* $C \vdash \bot$, *there exists* $h \in C$ *s.t.* $C \setminus \{h\} \vdash \neg h$. *Since* C *is minimal, thus* $\nexists H \subset C \setminus \{h\}$ *s.t.* $H \vdash \neg h$. *Moreover,* $\text{BEL}(\mathcal{E}) \subseteq \mathcal{K}_b$, *thus* $C \setminus \{h\} \subseteq \mathcal{K}_b$. *Consequently,* $\langle C \setminus \{h\}, \neg h \rangle \in \mathcal{A}_b$ *and there exists* $y \in \mathcal{E}$ *such that:*
   (a) *either* $y = \delta \subseteq \mathcal{E} \cap \mathcal{A}_d$ *and* $h \in \text{BELIEFS}(\delta)$. *Thus,* $\langle C \setminus \{h\}, \neg h \rangle \; \mathcal{R}_{bd} \; \delta$.
   (b) *or* $y = \pi \subseteq \mathcal{E} \cap \mathcal{A}_p$ *and* $h \in \text{PREC}(\pi)$. *Thus,* $\langle C \setminus \{h\}, \neg h \rangle \; \mathcal{R}_{bp} \; \pi$.
   (c) *or* $y = \alpha \subseteq \mathcal{E} \cap \mathcal{A}_p$ *and* $h \in \text{SUPP}(\alpha)$. *Thus,* $\langle C \setminus \{h\}, \neg h \rangle \; \mathcal{R}_b \; \alpha$.

   In each situation ($y = \delta$, $y = \pi$, $y = \alpha$*), since* $y \in \mathcal{E}$, *then* $\exists \alpha' \in \mathcal{E} \cap \mathcal{A}_b$ *which attacks the attacker of* y, *so s.t.* $\alpha' \; \mathcal{R}_b \; \langle C \setminus \{h\}, \neg h \rangle$. *This means that* $\exists h' \in C \setminus \{h\}$ *s.t.* $\text{CONC}(\alpha') = \neg h'$. *However, since* $h' \in C \setminus \{h\}$ *which is included in* $\text{BEL}(\mathcal{E})$, *then* $\exists x \in \mathcal{E}$ *s.t:*

(a) $x \in \mathcal{A}_b$ *and* $h' \in \text{SUPP}(x)$. *Thus,* $\alpha' \mathcal{R}_b x$. *This contradicts the fact that* $\mathcal{E}$ *is conflict-free.*
(b) $x \in \mathcal{A}_d$ *and* $h' \in \text{BELIEFS}(x)$. *Thus,* $\alpha' \mathcal{R}_{bd} x$. *This contradicts the fact that* $\mathcal{E}$ *is conflict-free.*
(c) $x \in \mathcal{A}_p$ *and* $h' \in \text{PREC}(x)$. *Thus,* $\alpha' \mathcal{R}_{bp} x$. *This contradicts the fact that* $\mathcal{E}$ *is conflict-free.*

Since for each preferred extension $\mathcal{E}$ *of* $\mathsf{AS}_{\mathsf{PR}}$, $\text{BEL}(\mathcal{E})$ *is consistent, then each* C*-preferred extension* $\mathcal{E}'$, $\text{BEL}(\mathcal{E}')$ *is consistent as well since* $\mathcal{E}'$ *is a subset of a preferred extension* $\mathcal{E}$. *Thus,* $\text{BEL}(\mathcal{E}') \subseteq \text{BEL}(\mathcal{E})$.

2. *Let* $\mathcal{E}$ *be a* C*-preferred extension of* $\mathsf{CAS}_{\mathsf{PR}}$. *Assume that* $\bigcup_{\pi_k \in \mathcal{E} \cap \mathcal{A}_p} \text{POST}(\pi_k)$ *is inconsistent. Thus, there exists* $C \subseteq \bigcup_{\pi_k \in \mathcal{E} \cap \mathcal{A}_p} \text{POST}(\pi_k)$ *s.t.* C *is minimal (for set inclusion) and inconsistent. According to Assumption 1 given in Section 6.3 and Definition 10,* $C = C_1 \cup C_2$ *with* $C_1, C_2 \neq \varnothing$ *and* $\exists \pi_1, \pi_2 \in \mathcal{E} \cap \mathcal{A}_p$ *s.t.* $C_1 \subseteq \text{POST}(\pi_1)$ *and* $C_2 \subseteq \text{POST}(\pi_2)$. *Thus,* $\pi_1 \mathcal{R}_p \pi_2$ *(and* $\pi_2 \mathcal{R}_p \pi_1$*). This contradicts the fact that* $\mathcal{E}$ *is conflict-free.*
3. *Let* $\mathcal{E}$ *be a* C*-preferred extension of* $\mathsf{CAS}_{\mathsf{PR}}$. *Assume that* $\text{DES}(\mathcal{E})$ *is inconsistent. Thus,* $\exists d_1, d_2 \in \text{DES}(\mathcal{E})$ *s.t.* $d_1 \equiv \neg d_2$. *There are three cases:*
   (a) $d_1 \in \text{DESIRES}(\delta_1)$ *and* $d_2 \in \text{DESIRES}(\delta_2)$ *with* $\delta_1, \delta_2 \in \mathcal{A}_d \cap \mathcal{E}$. *This means that* $\delta_1 \mathcal{R}_d \delta_2$ *and* $\delta_2 \mathcal{R}_d \delta_1$. *This contradicts the fact that* $\mathcal{E}$ *is conflict-free.*
   (b) $d_1 \in \text{DESIRES}(\delta)$ *and* $d_2 \in \text{CONC}(\pi)$ *with* $\delta \in \mathcal{A}_d \cap \mathcal{E}$ *and* $\pi \in \mathcal{A}_p \cap \mathcal{E}$. *This means that* $\delta \mathcal{R}_{dp} \pi$. *This contradicts the fact that* $\mathcal{E}$ *is conflict-free.*
   (c) $d_1 \in \text{CONC}(\pi_1)$ *and* $d_2 \in \text{CONC}(\pi_2)$ *with* $\pi_1, \pi_2 \in \mathcal{A}_p \cap \mathcal{E}$. *This means that* $\pi_1 \mathcal{R}_p \pi_2$. *This contradicts the fact that* $\mathcal{E}$ *is conflict-free.*

■

Theorem 3. Under the stable and preferred semantics, each set of intentions of $\mathsf{CAS}_{\mathsf{PR}}$ is consistent.

Proof . *Let* $\mathcal{I}$ *be a set of intentions of* $\mathsf{CAS}_{\mathsf{PR}}$. *Let us suppose that* $\mathcal{I}$ *is inconsistent. From the definition of an intention set, it is clear that* $\mathcal{I} \subseteq \text{DES}(\mathcal{E}_i)$ *with* $\mathcal{E}_i$ *is an extension of* $\mathsf{CAS}_{\mathsf{PR}}$. *However, according to Theorem 1 and Theorem 2 the set* $\text{DES}(\mathcal{E}_i)$ *is consistent.* ■

Theorem 4. Let $\mathsf{CAS}_{\mathsf{PR}}$ be a constrained argumentation system for PR, and $\mathcal{E}_1$, $\ldots$, $\mathcal{E}_n$ its C-stable extensions. $\forall \mathcal{E}_i$, $i = 1, \ldots, n$, it holds that:

1. The set $\text{ARG}(\text{BEL}(\mathcal{E}_i))$ = $\mathcal{E}_i \cap \mathcal{A}_b$.

2. $\mathcal{A}$s = $\mathcal{E}_i$.

Proof . *Let $\mathcal{E}_i$ be a C-stable extension of the system* $\mathsf{CAS}_{\mathsf{PR}}$*. $\mathcal{E}_i$ is also a stable extension of* $\mathsf{AS}_{\mathsf{PR}}$ *(according to [11]).*

1. *Let us show that* $\textsc{Arg}(\textsc{Bel}(\mathcal{E}_i))$ = $\mathcal{E}_i \cap \mathcal{A}_b$.
   According to Theorem 1, it is clear that $\textsc{Bel}(\mathcal{E}_i)$ = $\textsc{Bel}(\mathcal{E}_i \cap \mathcal{A}_b)$*. Moreover, according to Proposition 20, $\mathcal{E}_i \cap \mathcal{A}_b$ is a stable extension of $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$. Besides, according to Proposition 5,* $\textsc{Arg}(\textsc{Bel}(\mathcal{E}_i \cap \mathcal{A}_b))$ = $\mathcal{E}_i \cap \mathcal{A}_b$*, thus* $\textsc{Arg}(\textsc{Bel}(\mathcal{E}_i))$ = $\mathcal{E}_i \cap \mathcal{A}_b$.
2. *Let us show that $\mathcal{A}$s = $\mathcal{E}_i$.*
   - $\mathcal{E}_i \subseteq \mathcal{A}$*s: This is trivial.*
   - $\mathcal{A}$*s* $\subseteq \mathcal{E}_i$*: Let us suppose that $\exists y \in \mathcal{A}$s and $y \notin \mathcal{E}_i$. There are three possible situations:*
     (a) $y \in \mathcal{A}$*s* $\cap \mathcal{A}_b$*: Since $y \notin \mathcal{E}_i$, this means that $\exists \alpha \in \mathcal{E}_i \cap \mathcal{A}_b$ such that $\alpha \mathcal{R}_b y$. Thus,* $\textsc{Supp}(\alpha) \cup \textsc{Supp}(y) \vdash \perp$*. However,* $\textsc{Supp}(\alpha) \subseteq \textsc{Bel}(\mathcal{E}_i)$ *and* $\textsc{Supp}(y) \subseteq \textsc{Bel}(\mathcal{E}_i)$*, thus* $\textsc{Supp}(\alpha) \cup \textsc{Supp}(y) \subseteq \textsc{Bel}(\mathcal{E}_i)$*. This means that* $\textsc{Bel}(\mathcal{E}_i)$ *is inconsistent. According to Theorem 1 this is impossible.*
     (b) $y \in \mathcal{A}$*s* $\cap \mathcal{A}_d$*: Since $y \notin \mathcal{E}_i$, this means that $\exists x \in \mathcal{E}_i$ such that $x \mathcal{R} y$. There are three situations:*
     Case 1: $x \in \mathcal{A}_b$ This means that $\textsc{Beliefs}(y) \cup \textsc{Supp}(x) \vdash \perp$*. However,* $\textsc{Beliefs}(y) \cup \textsc{Supp}(x) \subseteq \textsc{Bel}(\mathcal{E}_i)$*. Thus,* $\textsc{Bel}(\mathcal{E}_i)$ *is inconsistent. This contradicts Theorem 1.*
     Case 2: $x \in \mathcal{A}_d$ This means that $\textsc{Desires}(y) \cup \textsc{Desires}(x) \vdash \perp$*. However,* $\textsc{Desires}(y) \cup \textsc{Desires}(x) \subseteq \textsc{Des}(\mathcal{E}_i)$*. So,* $\textsc{Des}(\mathcal{E}_i)$ *is inconsistent. This contradicts Theorem 1.*
     Case 3: $x \in \mathcal{A}_p$ This means that $\textsc{Desires}(y) \cup \textsc{Conc}(x) \vdash \perp$*. However,* $\textsc{Desires}(y) \cup \textsc{Conc}(x) \subseteq \textsc{Des}(\mathcal{E}_i)$*. Thus,* $\textsc{Des}(\mathcal{E}_i)$ *is inconsistent. This contradicts Theorem 1.*
     (c) $y \in \mathcal{A}$*s* $\cap \mathcal{A}_p$*: Since $y \notin \mathcal{E}_i$, this means that $\exists x \in \mathcal{E}_i$ such that $x \mathcal{R} y$. There are three situations:*
     Case 1: $x \in \mathcal{A}_b$ This means that $x \mathcal{R}_{bp} y$, thus $\textsc{Supp}(x) \cup \textsc{PreC}(y) \vdash \perp$*. However,* $\textsc{Supp}(x) \cup \textsc{PreC}(y) \subseteq \textsc{Bel}(\mathcal{E}_i)$*. Thus,* $\textsc{Bel}(\mathcal{E}_i)$ *is inconsistent. This contradicts Theorem 1.*
     Case 2: $x \in \mathcal{A}_d$ This means that $x \mathcal{R}_{pdp} y$, so we have $\textsc{Desires}(x) \cup \textsc{Conc}(y) \vdash \perp$*. However,* $\textsc{Desires}(x) \cup \textsc{Conc}(y) \subseteq \textsc{Des}(\mathcal{E}_i)$*. Thus,* $\textsc{Des}(\mathcal{E}_i)$ *is inconsistent. This contradicts Theorem 1.*

Case 3: $x \in \mathcal{A}_p$ *This means that* $x\mathcal{R}_p y$. *There are three different cases:*

- $\textsc{PreC}(x) \cup \textsc{PreC}(y) \vdash \bot$.
  However, $\textsc{PreC}(x) \cup \textsc{PreC}(y) \subseteq \textsc{Bel}(\mathcal{E}_i)$. *Thus,* $\textsc{Bel}(\mathcal{E}_i)$ *is inconsistent. This contradicts Theorem 1.*
- $\textsc{PostC}(x) \cup \textsc{PreC}(y) \vdash \bot$. *We know that* y *is built using one of the plans of* $\mathcal{E}_i$*, say* $p = \langle S, T, a \rangle$. *Thus,* $\exists \pi \in \mathcal{E}_i$ *such that* $\pi = \langle p, d \rangle$. *Thus,* $\textsc{PostC}(x) \cup \textsc{PreC}(\pi) \vdash \bot$, *consequently,* $x\mathcal{R}\pi$. *This is impossible since* $\mathcal{E}_i$ *is a stable extension, thus it is supposed to be conflict-free.*
- $\textsc{PostC}(x) \cup \textsc{PostC}(y) \vdash \bot$. *Since* $y \in \mathcal{A}s$*, thus* y *is built using one of the plans of* $\mathcal{E}_i$*, say* $p = \langle S, T, a \rangle$. *Thus,* $\exists \pi \in \mathcal{E}_i$ *such that* $\pi = \langle p, d \rangle$. *Thus,* $\textsc{PostC}(x) \cup \textsc{PostC}(\pi) \vdash \bot$, *consequently,* $x\mathcal{R}\pi$. *This is impossible since* $\mathcal{E}_i$ *is a stable extension, thus it is supposed to be conflict-free.*

■