Using arguments for making and explaining decisions

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Abstract

Arguments play two different roles in day life decisions, as well as in the discussion of more crucial issues. Namely, they help to select one or several alternatives, or to explain and justify an already adopted choice.

This paper proposes the first general and abstract argument-based framework for decision making. This framework follows two main steps. At the first step, arguments for beliefs and arguments for options are built and evaluated using classical acceptability semantics. At the second step, pairs of options are compared using decision principles. Decision principles are based on the accepted arguments supporting the options. Three classes of decision principles are distinguished: *unipolar*, *bipolar* or *non-polar* principles depending on whether i) only arguments pro or only arguments con, or ii) both types, or iii) an aggregation of them into a meta-argument are used. The abstract model is then instantiated by expressing formally the mental states (beliefs and preferences) of a decision maker. In the proposed framework, information is given in the form of a stratified set of beliefs. The *bipolar* nature of preferences is emphasized by making an explicit distinction between prioritized *goals* to be pursued, and prioritized *rejections* that are stumbling blocks to be avoided. A typology that identifies four types of argument is also proposed. Indeed, each decision is *supported* by arguments emphasizing its positive consequences in terms of goals certainly satisfied and rejections certainly avoided. A decision can also be *attacked* by arguments emphasizing its negative consequences in terms of certainly missed goals, or rejections certainly led to by that decision. Finally, this paper articulates the optimistic and pessimistic decision criteria defined in qualitative decision making under uncertainty, in terms of an argumentation process. Similarly, different decision principles identified in multiple criteria decision making are restated in our argumentation-based framework.

Key words: Decision making, Argumentation.

1 Introduction

Decision making, often viewed as a form of reasoning toward action, has raised the interest of many scholars including philosophers, economists, psychologists, and computer scientists for a long time. Any decision problem amounts to selecting the "best" or sufficiently "good" action(s) that are feasible among different alternatives, given some available information about the current state of the world and the consequences of potential actions. Note that available information may be incomplete or pervaded with uncertainty. Besides, the goodness of an action is judged by estimating, maybe by means of several criteria, how much its possible consequences fit the preferences or the intentions of the decision maker. This agent is assumed to behave in a *rational* way [42,43,49], at least in the sense that his decisions should be as much as possible consistent with his preferences. However, we may have a more requiring view of rationality, such as demanding for the conformity of decision maker's behavior with postulates describing how a rational agent should behave [45].

Decision problems have been considered from different points of view. We may distinguish two main trends, which are currently influencing research in artificial intelligence (AI): *classical decision theory* on the one hand, and cognitively-oriented approaches such as *practical reasoning* or *beliefs-desires-intentions* (BDI) settings on the other hand.

1.1 Classical decision making vs. practical reasoning

Classical decision theory, as developed mainly by economists, has focused on making clear what is a rational decision maker. Thus, they have looked for principles for comparing different alternatives. A particular decision principle, such as the classical expected utility [45], should then be justified on the basis of a set of rationality postulates to which the preference relation between actions should obey. This means that in this approach, rationality is captured through a set of postulates that describe what is a rational decision behavior. Moreover, a minimal set of postulates is identified in such a way that it corresponds to a unique decision principle. The inputs of this approach are a set of candidate actions, and a function that assesses the value of their consequences when the actions are performed in a given state, together with complete or partial information about the current state of the world. In other words, such an

⋆ The present paper unifies and develops the content of several conference papers [3,8–10].

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approach distinguishes between knowledge and preferences, which are respectively encoded in practice by a distribution function assessing the plausibility of the different states of the world, and by a utility function encoding preferences by estimating how good a consequence is. The output is a preference relation between actions encoded by the associated principle. Note that such an approach aims at rank-ordering a group of candidate actions rather than focusing on a candidate action individually. Moreover, the candidate actions are supposed to be feasible. Roughly speaking, we may distinguish two groups of works in AI dealing with decision that follow the above type of approach. The first group is represented by researches using Bayesian networks [41], and works on planning under uncertainty (e.g. [22]). Besides, some AI works have aimed at developing more qualitative frameworks for decision, but still along the same line of thoughts (e.g. [23,28,47]).

Other researchers in AI, working on practical reasoning, starting with the generic question "what is the right thing to do for an agent in a given situation" [42,44], have proposed a two steps process to answer this question. The first step, often called deliberation [49], consists of identifying the goals of the agent. In the second step, they look for ways of achieving those goals, i.e. for plans, and thus for intermediary goals and sub-plans. Such an approach raises issues such as: how are goals generated ? are actions feasible ? do actions have undesirable consequences ? are sub-plans compatible ? are there alternative plans for achieving a given goal, ... In [17], it has been argued that this can be done by representing the cognitive states, namely agent's beliefs, desires and intentions (thus the so-called *BDI* architecture). This requires a rich knowledge/preference representation setting, which contrasts with the classical decision setting that directly uses an uncertainty distribution (a probability distribution in the case of expected utility), and a utility (value) function. Besides, the deliberation step is merely an inference problem since it amounts to finding a set of desires that are justified on the basis of the current state of the world and of conditional desires. Checking if a plan is feasible and does not lead to bad consequences is still a matter of inference. A decision problem only occurs when several plans or sub-plans are possible, and one of them has to be chosen. This latter issue may be viewed as a classical decision problem. What is worth noticing in most works on practical reasoning is the use of argument schemes for providing reasons for choosing or discarding an action (e.g. [31,36]). For instance, an action may be considered as potentially useful on the basis of the so-called *practical syllogism* [48]:

- G is a goal for agent X
- Doing action A is sufficient for agent X to carry out goal G
- Then, agent X ought to do action A

The above syllogism is in essence already an argument in favor of doing action A. However, this does not mean that the action is warranted, since other arguments (called counter-arguments) may be built or provided against the action. Those counter-arguments refer to critical questions identified in [48] for the above syllogism. In particular, relevant questions are "Are there alternative ways of realizing G?", "Is doing A feasible?", "Has agent X other goals than G?", "Are there other consequences of doing A which should be taken into account?". Recently in [11,12], the above syllogism has been extended to explicitly take into account the reference to ethical values in arguments. Anyway, the idea of using arguments for justifying or discarding candidate decisions is certainty very old, and its account in the literature at least dates back to Aristotle. See also Benjamin Franklin [34] for an early precise account on the way of balancing arguments pro and con a choice, more than two hundred years ago.

1.2 Argumentation and decision making

Generally speaking, argumentation is a reasoning model based on the construction and the evaluation of interacting arguments. Those arguments are intended to support / explain / attack statements that can be decisions, opinions, ... Argumentation has been used for different purposes [1], such as nonmonotonic reasoning (e.g. [29]). Indeed, several frameworks have been developed for handling inconsistency in knowledge bases (e.g. [2,4,14]). Moreover, it has been shown that such an approach is general enough to capture different existing approaches for nonmonotonic reasoning [29]. Argumentation has also been extensively used for modeling different kinds of dialogues, in particular persuasion (e.g. [6]), and negotiation (e.g. [38]). Indeed, an argumentation-based approach for negotiation has the advantage of exchanging in addition to offers, reasons that support these offers. These reasons may lead their receivers to change their preferences. Consequently, an agreement may be more easily reached with such approaches, when in other approaches (where agent's preferences are fixed) negotiation may fail. Adopting such an approach in a decision problem would have some obvious benefits. Indeed, not only would the user be provided with a "good" choice, but also with the reasons underlying this recommendation, in a format that is easy to grasp. Note that each potential choice usually has pros and cons of various strengths. Argumentation-based decision making is expected to be more akin with the way humans deliberate and finally make or also understand a choice. This has been pointed out for a long time (see e.g. [34]).

1.3 Contribution of the paper

In this paper we deal with an argumentative view of decision making, thus focusing on the issue of justifying the best decision to make in a given situation, and leaving aside the other related aspects of practical reasoning such as goal generation, feasibility, and planning. It is why we remain close to the classical view of decision, but now discussed in terms of arguments. The idea of articulating decisions on the basis of arguments is relevant for different decision problems or approaches such as decision making under uncertainty, multiple criteria decisions, or rule-based decisions. These problems are usually handled separately, and until recently without a close reference to argumentation. In practical applications, for instance in medical domain, the decision to be made has to be chosen under incomplete or uncertain information, the potential results of candidate decisions are evaluated from different criteria. Moreover, there may exist some expertise in the form of decision rules that associate possible decisions to given contexts. This makes the different decision problems somewhat related, and consequently a unified argumentation-based model is needed. This paper proposes such a model.

This paper proposes the first general, and abstract argument-based framework for decision making. This framework follows two main steps. At the first step, arguments for beliefs and arguments for options are built and evaluated using classical acceptability semantics. At the second step, pairs of options are compared using decision principles. Decision principles are based on the accepted arguments supporting the options. Three classes of decision principles are distinguished: *unipolar*, *bipolar* or *non-polar* principles depending on whether i) only arguments pro or only arguments con, or ii) both types, or iii) an aggregation of them into a meta-argument are used. The abstract model is then instantiated by expressing formally the mental states (beliefs and preferences) of a decision maker. In the proposed framework, information is given in the form of a stratified set of beliefs. The *bipolar* nature of preferences is emphasized by making an explicit distinction between prioritized *goals* to be pursued, and prioritized *rejections* that are stumbling blocks to be avoided. A typology that identifies four types of argument is also proposed. Indeed, each decision is *supported* by arguments emphasizing its positive consequences in terms of goals certainly satisfied and rejections certainly avoided. A decision can also be *attacked* by arguments emphasizing its negative consequences in terms of certainly missed goals, or rejections certainly led to by that decision. Another contribution of the paper consists of applying the general framework to decision making under uncertainty and to multiple criteria decision. Proper choices of decision principles are shown to be equivalent to known qualitative decision approaches.

The paper is organized as follows: Section 2 presents an abstract framework for decision making. Section 3 discusses a typology of arguments supporting or attacking candidate decisions. Section 4 applies the abstract framework to multiple criteria decision making, and section 5 applies the framework to decision making under uncertainty. Section 6 compares our approach to existing works on argumentation-based decision making, and section 7 is devoted to some concluding remarks and perspectives.

2 A general framework for argumentative decision making

Solving a decision problem amounts to defining a pre-ordering, usually a complete one, on a set $\mathcal{D}$ of possible *options* (or candidate decisions), on the basis of the different consequences of each decision. Let us illustrate this problem through a simple example borrowed from [32].

Example 1 (Having or not a surgery) *The example is about having a surgery (sg) or not ($\neg sg$), knowing that the patient has colonic polyps. The knowledge base contains the following information:*

- *having a surgery has side-effects,*
- *not having surgery avoids having side-effects,*
- *when having a cancer, having a surgery avoids loss of life,*
- *if a patient has cancer and has no surgery, the patient would lose his life,*
- *the patient has colonic polyps,*
- *having colonic polyps may lead to cancer.*

In addition to the above knowledge, the patient has also some goals like: "no side effects" and "to not lose his life". Obviously it is more important for him to not lose his life than to not have side effects.

In what follows, $\mathcal{L}$ will denote a logical language. From $\mathcal{L}$, a finite set $\mathcal{D} = \{d_1, \ldots, d_n\}$ of n *options* is identified. Note that an option d_i may be a conjunction of other options in $\mathcal{D}$. Let us, for instance, assume that an agent wants a drink and has to choose between tea, milk or both. Thus, there are three options: d_1 : tea, d_2 : milk and d_3 : tea and milk. In Example 1, the set $\mathcal{D}$ contains only two options: $d_1 : sg$ and $d_2 : \neg sg$.

Argumentation is used in this paper for ordering the set $\mathcal{D}$. An argumentation-based decision process can be decomposed into the following steps:

(1) Constructing arguments in *favor/against* statements (pertaining to beliefs or decisions)
(2) Evaluating the strength of each argument

(3) Determining the different conflicts among arguments
(4) Evaluating the acceptability of arguments
(5) Comparing decisions on the basis of relevant "accepted" arguments

Note that the first four steps globally correspond to an "inference problem" in which one looks for accepted arguments, and consequently warranted beliefs. At this step, one only knows what is the quality of arguments in favor/against candidate decisions, but the "best" candidate decision is not determined yet. The last step answers this question once a *decision principle* is chosen.

2.1 Types of arguments

As shown in Example 1, decisions are made on the basis of available knowledge and the preferences of the decision maker. Thus, two categories of arguments are distinguished: i) *epistemic arguments* justifying beliefs and are themselves based only on beliefs, and ii) *practical arguments* justifying options and are built from both beliefs and preferences/goals. Note that a practical argument may highlight either a positive feature of a candidate decision, supporting thus that decision, or a negative one, attacking thus the decision.

Example 2 (Example 1 cont.) *In this example, α = ["the patient has colonic polyps", and "having colonic polyps may lead to cancer"] is considered as an argument for believing that the patient may have cancer. This epistemic argument involves only beliefs. While δ_1 = ["the patient may have a cancer", "when having a cancer, having a surgery avoids loss of life"] is an argument for having a surgery. This is a practical argument since it supports the option "having a surgery". Note that such argument involves both beliefs and preferences. Similarly, δ_2 = ["not having surgery avoids having side-effects"] is a practical argument in favor of "not having a surgery". However, the two practical arguments δ_3 = ["having a surgery has side-effects"] and δ_4 = ["the patient has colonic polyps", and "having colonic polyps may lead to cancer", "if a patient has cancer and has no surgery, the patient would lose his life"] are respectively against surgery and no surgery since they point out negative consequences of the two options.*

In what follows, $\mathcal{A}_e$ denotes a set of epistemic arguments, and $\mathcal{A}_p$ denotes a set of practical arguments such that $\mathcal{A}_e \cap \mathcal{A}_p = \emptyset$. Let $\mathcal{A} = \mathcal{A}_e \cup \mathcal{A}_p$ (i.e. $\mathcal{A}$ will contain all those arguments). The structure and origin of the arguments are assumed to be unknown. Epistemic arguments will be denoted by variables $\alpha_1, \alpha_2, \ldots$, while practical arguments will be referred to by variables $\delta_1, \delta_2, \ldots$ When no distinction is necessary between arguments, we will use the variables $a, b, c, \ldots$

Example 3 (Example 1 cont.) $\mathcal{A}_e = \{\alpha\}$ *while* $\mathcal{A}_p = \{\delta_1, \delta_2, \delta_3, \delta_4\}$.

Let us now define two functions that relate each option to the arguments supporting it and to the arguments against it.

- $\mathcal{F}_p : \mathcal{D} \rightarrow 2^{\mathcal{A}_p}$ is a function that returns the arguments *in favor* of a candidate decision. Such arguments are said pro the option.
- $\mathcal{F}_c : \mathcal{D} \rightarrow 2^{\mathcal{A}_p}$ is a function that returns the arguments *against* a candidate decision. Such arguments are said cons the option.

The two functions satisfy the following requirements:

- $\forall d \in \mathcal{D}$, $\nexists \delta \in \mathcal{A}_p$ s.t. $\delta \in \mathcal{F}_p(d)$ and $\delta \in \mathcal{F}_c(d)$. This means that an argument is either in favor of an option or against that option. It cannot be both.
- If $\delta \in \mathcal{F}_p(d)$ and $\delta \in \mathcal{F}_p(d')$ (resp. if $\delta \in \mathcal{F}_c(d)$ and $\delta \in \mathcal{F}_c(d')$), then $d = d'$. This means that an argument refers only to one option.
- Let $\mathcal{D} = \{d_1, \ldots, d_n\}$. $\mathcal{A}_p = (\bigcup \mathcal{F}_p(d_i)) \cup (\bigcup \mathcal{F}_c(d_i))$, with $i = 1, \ldots, n$. This means that the available practical arguments concern options of the set $\mathcal{D}$.

When $\delta \in \mathcal{F}_x(d)$ with $x \in \{p, c\}$, we say that d is the *conclusion* of δ, and we write $\texttt{Conc}(\delta) = d$.

Example 4 (Example 1 cont.) *The two options of the set* $\mathcal{D} = \{sg, \neg sg\}$ *are supported/attacked by the following arguments:* $\mathcal{F}_p(sg) = \{\delta_1\}$*,* $\mathcal{F}_c(sg) = \{\delta_3\}$*,* $\mathcal{F}_p(\neg sg) = \{\delta_2\}$*, and* $\mathcal{F}_c(\neg sg) = \{\delta_4\}$*.*

2.2 Comparing arguments

As pointed out by several researchers (e.g. [20,30]), arguments may have forces of various strengths. These forces play two key roles: i) they may be used in order to refine the notion of acceptability of epistemic or practical arguments, ii) they allow the comparison of practical arguments in order to rank-order candidate decisions. Generally, the strength of an epistemic argument reflects the quality, such as the certainty level, of the pieces of information involved in it. Whereas the strength of a practical argument reflects both the quality of knowledge used in the argument, as well as how important it is to fulfill the preferences to which the argument refers.

In our particular application, three preference relations between arguments are defined. The first one, denoted by $\geq_e$, is a (partial or total) *preorder* [1] on the set $\mathcal{A}_e$. The second relation, denoted by $\geq_p$, is a (partial or total) preorder on the set $\mathcal{A}_p$. Finally, a third relation, denoted by $\geq_m$ (m stands for mixed relation), captures the idea that any epistemic argument is stronger than any practical argument. The role of epistemic arguments in a decision problem

[1] A preorder is a binary relation that is *reflexive* and *transitive*

is to validate or to undermine the beliefs on which practical arguments are built. Indeed, decisions should be made under "certain" information. Thus, $\forall \alpha \in \mathcal{A}_e$, $\forall \delta \in \mathcal{A}_p$, $(\alpha, \delta) \in \geq_m$ and $(\delta, \alpha) \notin \geq_m$.
Note that $(a, b) \in \geq_x$, with $x \in \{e, p, m\}$, means that a is *at least as good as* b. At some places, we will also write $a \geq_x b$. In what follows, $>_x$ denotes the strict relation associated with $\geq_x$. It is defined as follows: $(a, b) \in >_x$ iff $(a, b) \in \geq_x$ and $(b, a) \notin \geq_x$. When $(a, b) \in \geq_x$ and $(b, a) \in \geq_x$, we say that a and b are *indifferent*, and we write $a \approx_x b$. When $(a, b) \notin \geq_x$ and $(b, a) \notin \geq_x$, the two arguments are said *incomparable*.

Example 5 (Example 1 cont.) $\geq_e = \{(\alpha, \alpha)\}$ *and* $\geq_m = \{(\alpha, \delta_1), (\alpha, \delta_2)\}$. *Now, regarding* $\geq_p$*, one may, for instance, assume that* δ_1 *is stronger than* δ_2 *since the goal satisfied by* δ_1 *(namely, not loss of life) is more important than the one satisfied by* δ_2 *(not having side effects). Thus,* $\geq_p = \{(\delta_1, \delta_1)$*,* (δ_2, δ_2)*,* $(\delta_1, \delta_2)\}$*. This example will be detailed in a next section.*

2.3 Attacks among arguments

Since knowledge may be inconsistent, the arguments may be conflicting too. Indeed, epistemic arguments may attack each others. Such conflicts are captured by the binary relation $\mathcal{R}_e \subseteq \mathcal{A}_e \times \mathcal{A}_e$. This relation is assumed abstract and its origin is not specified.

Epistemic arguments may also attack practical arguments when they challenge their knowledge part. The idea is that an epistemic argument may undermine the beliefs part of a practical argument. However, practical arguments are not allowed to attack epistemic ones. This avoids wishful thinking. This relation, denoted by $\mathcal{R}_m$, contains pairs (α, δ) where $\alpha \in \mathcal{A}_e$ and $\delta \in \mathcal{A}_p$.

We assume that practical arguments do not conflict. The idea is that each practical argument points out some advantage or some weakness of a candidate decision, and it is crucial in a decision problem to list all those arguments for each candidate decision, provided that they are accepted w.r.t. the current epistemic state, i.e built from warranted beliefs. According to the attitude of the decision maker in face of uncertain or inconsistent knowledge, these lists associated with the candidate decisions may be taken into account in different manners, thus leading to different orderings of the decisions. This is why all accepted arguments should be kept, whatever their strengths, for preserving all relevant information in the decision process. Otherwise, getting rid of some of those accepted arguments (w.r.t. knowledge), for instance because they would be weaker than others, may prevent us to have a complete view of the decision problem and then may even lead us to recommend decisions that would be wrong w.r.t. some decision principles (agreeing with the presumed decision maker's attitude). This point will be made more concrete in a next section. Thus, the relation $\mathcal{R}_p \subseteq \mathcal{A}_p \times \mathcal{A}_p$ is equal to the empty set ($\mathcal{R}_p = \emptyset$).

Each preference relation $\geq_x$ (with $x \in \{e, p, m\}$) is combined with the conflict relation $\mathcal{R}_x$ into a unique relation between arguments, denoted by $\mathtt{Def}_x$ and called *defeat* relation, in the same way as in ([5], Definition 3.3, page 204).

Definition 1 (Defeat relation) *Let $\mathcal{A}$ be a set of arguments, and $a, b \in \mathcal{A}$. $(a, b) \in \mathtt{Def}_x$ iff:*

- $(a, b) \in \mathcal{R}_x$, *and*
- $(b, a) \notin >_x$

Let $\mathtt{Def}_e$, $\mathtt{Def}_p$ and $\mathtt{Def}_m$ denote the three defeat relations corresponding to the three attack relations. In case of $\mathtt{Def}_m$, the second bullet of Definition 1 is always true since epistemic arguments are strictly preferred (in the sense of $\geq_m$) to any practical arguments. Thus, $\mathtt{Def}_m = \mathcal{R}_m$ (i.e. the defeat relation is exactly the attack relation $\mathcal{R}_m$). The relation $\mathtt{Def}_p$ is the same as $\mathcal{R}_p$, thus it is empty. However, the relation $\mathtt{Def}_e$ coincides with its corresponding attack relation $\mathcal{R}_e$ in case all the arguments of the set $\mathcal{A}_e$ are incomparable.

2.4 Extensions of arguments

Now that the sets of arguments and the defeat relations are identified, we can define the decision system.

Definition 2 (Decision system) *Let $\mathcal{D}$ be a set of options. A* decision system *for ordering $\mathcal{D}$ is a triple $\mathtt{AF} = (\mathcal{D}, \mathcal{A}, \mathtt{Def})$ where $\mathcal{A} = \mathcal{A}_e \cup \mathcal{A}_p$* [2] *and $\mathtt{Def} = \mathtt{Def}_e \cup \mathtt{Def}_p \cup \mathtt{Def}_m$* [3] *.*

Note that a Dung style argumentation system is associated to a decision system $\mathtt{AF} = (\mathcal{D}, \mathcal{A}, \mathtt{Def})$, namely the system $(\mathcal{A}, \mathtt{Def})$. This latter can be seen as the union of two distinct argumentation systems: $\mathtt{AF}_e = (\mathcal{A}_e, \mathtt{Def}_e)$, called *epistemic system*, and $\mathtt{AF}_p = (\mathcal{A}_p, \mathtt{Def}_p)$, called *practical system*. The two systems are related to each other by the defeat relation $\mathtt{Def}_m$.

Due to Dung's acceptability semantics defined in [29], it is possible to identify among all the conflicting arguments, which ones will be kept for ordering the options. An acceptability semantics amounts to define sets of arguments that satisfy a consistency requirement and must defend all their elements.

[2] Recall that options are related to their supporting and attacking arguments by the functions $\mathcal{F}_p$ and $\mathcal{F}_c$ respectively.

[3] Since the relation $\mathtt{Def}_p$ is empty, then $\mathtt{Def} = \mathtt{Def}_e \cup \mathtt{Def}_m$.

Definition 3 (Conflict-free, Defence) *Let* $\mathtt{AF} = (\mathcal{D}, \mathcal{A}, \mathtt{Def})$ *be a decision system,* $\mathcal{B} \subseteq \mathcal{A}$*, and* $a \in \mathcal{A}$*.*

- $\mathcal{B}$ *is* conflict-free *iff* $\nexists\ a,\ b \in \mathcal{B}$ *s.t.* $(a, b) \in \mathtt{Def}$*.*
- $\mathcal{B}$ defends a *iff* $\forall\ b \in \mathcal{A}$*, if* $(b, a) \in \mathtt{Def}$*, then* $\exists\ c \in \mathcal{B}$ *s.t.* $(c, b) \in \mathtt{Def}$*.*

The main semantics introduced by Dung are recalled in the following definition. Note that other semantics have been defined in the literature (e.g. [13]). However, these will not be discussed in this paper.

Definition 4 (Acceptability semantics) *Let* $\mathtt{AF} = (\mathcal{D}, \mathcal{A}, \mathtt{Def})$ *be a decision system, and* $\mathcal{B}$ *be a conflict-free set of arguments.*

- $\mathcal{B}$ *is* admissible extension *iff it defends any element in* $\mathcal{B}$*.*
- $\mathcal{B}$ *is a* preferred extension *iff* $\mathcal{B}$ *is a maximal (w.r.t set* $\subseteq$*) admissible set.*
- $\mathcal{B}$ *is a* stable extension *iff it is a preferred extension that defeats any argument in* $\mathcal{A} \backslash \mathcal{B}$*.*

Using these acceptability semantics, a status is assigned to each argument of $\mathtt{AF}$ as follows.

Definition 5 (Argument status) *Let* $\mathtt{AF} = (\mathcal{D}, \mathcal{A}, \mathtt{Def})$ *be a decision system, and* $\mathcal{E}_1, \ldots, \mathcal{E}_x$ *its extensions under a given semantics. Let* $a \in \mathcal{A}$*.*

- a *is* skeptically accepted *iff* $a \in \mathcal{E}_i$*,* $\forall \mathcal{E}_i$ *with* $i = 1, \ldots, x$*.*
- a *is* credulously accepted *iff* $\exists \mathcal{E}_i$ *such that* $a \in \mathcal{E}_i$*.*
- a *is* rejected *iff* $\nexists \mathcal{E}_i$ *such that* $a \in \mathcal{E}_i$*.*

A direct consequence of the above definition is that an argument is skeptically accepted iff it belongs to the intersection of all extensions, and that it is rejected iff it does not belong to the union of all extensions. Formally:

Property 1 *Let* $\mathtt{AF} = (\mathcal{D}, \mathcal{A}, \mathtt{Def})$ *be a decision system, and* $\mathcal{E}_1, \ldots, \mathcal{E}_x$ *its extensions under a given semantics. Let* $a \in \mathcal{A}$*.*

- *a is skeptically accepted iff* $a \in \bigcap_{i=1}^{x} \mathcal{E}_i$
- *a is rejected iff* $a \notin \bigcup_{i=1}^{x} \mathcal{E}_i$

Let $\mathtt{Acc}(x, y)$ be a function that returns the skeptically accepted arguments of decision system x under semantics y ($y \in \{ad, st, pr\}$ with ad (resp. st and pr) stands for admissible (resp. stable and preferred) semantics). This set may contain both epistemic and practical arguments. Such arguments are very important in argumentation process since they support the conclusions to be inferred from a knowledge base or the options that will be chosen. Indeed, for ordering the different candidate decisions, only skeptically accepted practical arguments are used. The following property shows the links between the sets of accepted arguments under different semantics.

Property 2 *Let* $\mathtt{AF} = (\mathcal{D}, \mathcal{A}, \mathtt{Def})$ *be a decision system.*

- $\mathtt{Acc}(\mathtt{AF}, ad) = \emptyset$.
- *If* AF *has no stable extensions, then* $\mathtt{Acc}(\mathtt{AF}, st) = \emptyset$ *and* $\mathtt{Acc}(\mathtt{AF}, st) \subseteq \mathtt{Acc}(\mathtt{AF}, pr)$.
- *If* AF *has stable extensions, then* $\mathtt{Acc}(\mathtt{AF}, pr) \subseteq \mathtt{Acc}(\mathtt{AF}, st)$.

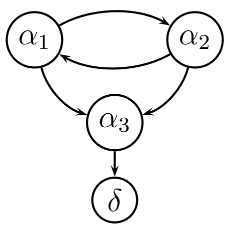
Proof *Let* $\mathtt{AF} = (\mathcal{D}, \mathcal{A}, \mathtt{Def})$ *be a decision system.*

- *In [29], it has been shown that the empty set is an admissible extension of any argumentation system. Thus,* $\cap\mathcal{E}_{i=1,\ldots,x} = \emptyset$ *where* $\mathcal{E}_1, \ldots, \mathcal{E}_x$ *are the admissible extensions of* AF. *Consequently,* $\mathtt{Acc}(\mathtt{AF}, ad) = \emptyset$.
- *Let us assume that the system* AF *has no stable extensions. Thus, according to Definition 5, all arguments of* $\mathcal{A}$ *are rejected. Thus,* $\mathtt{Acc}(\mathtt{AF}, st) = \emptyset$.
- *Let us now assume that the system* AF *has stable extensions, say* $\mathcal{E}_1, \ldots, \mathcal{E}_n$. *Dung has shown in [29] that any stable extension is a preferred one, but the converse is not true. Thus,* $\mathcal{E}_1, \ldots, \mathcal{E}_n$ *are also preferred extensions. Let us now assume that the system has other extensions that are preferred but not stable, say* $\mathcal{E}_{n+1}, \ldots, \mathcal{E}_x$ *with* $x \geq n+1$. *From set theory, it is clear that* $\bigcap_{i=1}^{x} \mathcal{E}_i \subseteq \bigcap_{i=1}^{n} \mathcal{E}_i$. *According to Property 1, it follows that* $\mathtt{Acc}(\mathtt{AF}, pr) \subseteq \mathtt{Acc}(\mathtt{AF}, st)$.

■

From the above property, one concludes that in a decision problem, it is not interesting to use admissible semantics. The reason is that no argument is accepted. Consequently, argumentation will not help at all for ordering the different candidate decisions. Let us illustrate this issue through the following simple example.

Example 6 *Let us consider the decision system* $\mathtt{AF} = (\mathcal{D}, \mathcal{A}_e \cup \mathcal{A}_p, \mathtt{Def})$ *where* $\mathcal{D} = \{d_1, d_2\}$, $\mathcal{A}_e = \{\alpha_1, \alpha_2, \alpha_3\}$, $\mathcal{A}_p = \{\delta\}$ *and* Def *is depicted in figure below. We assume that* $\mathcal{F}_p(d_1) = \delta$ *whereas* $\mathcal{F}_p(d_2) = \mathcal{F}_c(d_2) = \emptyset$.



The admissible extensions of this system are: $\mathcal{E}_1 = \{\}$, $\mathcal{E}_2 = \{\alpha_1\}$, $\mathcal{E}_3 = \{\alpha_2\}$, $\mathcal{E}_4 = \{\alpha_1, \delta\}$ *and* $\mathcal{E}_5 = \{\alpha_2, \delta\}$. *Under admissible semantics, the practical argument* δ *is not skeptically accepted. Thus, the two options* d_1 *and* d_2 *may* *be equally preferred since the first one has an argument but not an accepted one, and the second has no argument at all. However, the same decision system has two preferred extensions: $\mathcal{E}_4$ and $\mathcal{E}_5$. Under preferred semantics, the set $\texttt{Acc}(\texttt{AF}, pr)$ contains the argument δ (i.e. $\texttt{Acc}(\texttt{AF}, pr) = \{\delta\}$). Thus, it is natural to prefer the option d_1 to d_2.*

Consequently, in the following, we will use stable semantics if the system has stable extensions, otherwise preferred semantics will be considered for computing the set $\texttt{Acc}(\texttt{AF}, y)$.

Since the defeat relation $\texttt{Def}_p$ is empty, it is trivial that the practical system $\texttt{AF}_p$ has exactly one preferred/stable extension which is the set $\mathcal{A}_p$ itself.

Property 3 *The practical system $\texttt{AF}_p = (\mathcal{A}_p, \texttt{Def}_p)$ has a unique preferred/stable extension, which is the set $\mathcal{A}_p$.*

Proof *This follows directly from the fact that the set $\mathcal{A}_p$ is conflict-free since $\texttt{Def}_p = \emptyset$.* ■

It is important to notice that the epistemic system $\texttt{AF}_e$ in its side is very general and does not necessarily present particular properties like for instance the existence of stable/preferred extensions.

In what follows, we will show that the result of the decision system depends broadly on the outcome of its epistemic system. The first result states that the epistemic arguments of each admissible extension of AF constitute an admissible extension of the epistemic system $\texttt{AF}_e$.

Theorem 1 *Let $\texttt{AF} = (\mathcal{D}, \mathcal{A}_e \cup \mathcal{A}_p, \texttt{Def}_e \cup \texttt{Def}_p \cup \texttt{Def}_m)$ be a decision system, $\mathcal{E}_1, \ldots, \mathcal{E}_n$ its admissible extensions, and $\texttt{AF}_e = (\mathcal{A}_e, \texttt{Def}_e)$ its associated epistemic system.*

- *$\forall \mathcal{E}_i$, the set $\mathcal{E}_i \cap \mathcal{A}_e$ is an admissible extension of $\texttt{AF}_e$.*
- *$\forall \mathcal{E}'$ such that $\mathcal{E}'$ is an admissible extension of $\texttt{AF}_e$, $\exists \mathcal{E}_i$ such that $\mathcal{E}' \subseteq \mathcal{E}_i \cap \mathcal{A}_e$.*

Proof

- *Let $\mathcal{E}_i$ be an admissible extension of AF. Let $\mathcal{E} = \mathcal{E}_i \cap \mathcal{A}_e$. Let us assume that $\mathcal{E}$ is not an admissible extension of $\texttt{AF}_e$. There are two cases:*
  Case 1: *$\mathcal{E}$ is not conflict-free. This means that $\exists \alpha_1, \alpha_2 \in \mathcal{E}$ such that $(\alpha_1, \alpha_2) \in \texttt{Def}_e$. Thus, $\exists \alpha_1, \alpha_2 \in \mathcal{E}_i$ such that $(\alpha_1, \alpha_2) \in \texttt{Def}$. This is impossible since $\mathcal{E}_i$ is an admissible extension, thus conflict-free.*
  Case 2: *$\mathcal{E}$ does not defend its elements. This means that $\exists \alpha \in \mathcal{E}$, such that $\exists \alpha' \in \mathcal{A}_e$, $(\alpha', \alpha) \in \texttt{Def}_e$ and $\nexists \alpha'' \in \mathcal{E}$ such that $(\alpha'', \alpha') \in \texttt{Def}_e$. Since $(\alpha', \alpha) \in \texttt{Def}_e$, this means that $(\alpha', \alpha) \in \texttt{Def}$ with $\alpha \in \mathcal{E}_i$. However, $\mathcal{E}_i$ is admissible, then $\exists a \in \mathcal{E}_i$ such that $(a, \alpha') \in \texttt{Def}$. Assume that $a \in \mathcal{A}_p$. This is impossible since practical arguments are not allowed to defeat*

epistemic ones. Thus, $a \in \mathcal{A}_e$. Hence, $a \in \mathcal{E}$. Contradiction.

- *Let $\mathcal{E}'$ be an admissible extension of $\mathtt{AF}_e$. Let us prove that $\mathcal{E}'$ is an admissible extension of $\mathtt{AF}$. Assume that $\mathcal{E}'$ is not an admissible extension of $\mathtt{AF}$. There are two possibilities: i) $\mathcal{E}'$ is not conflict-free in $\mathtt{AF}$. This is not possible since $\mathcal{E}'$ an admissible extension of $\mathtt{AF}_e$, thus conflict-free.*

  ii) $\mathcal{E}'$ does not defend all its elements in the system $\mathtt{AF}$. This means that $\exists a \in \mathcal{E}'$ such that $\mathcal{E}'$ does not defend a. This means also that $\exists b \notin \mathcal{E}'$ such that $(b, a) \in \mathtt{Def}$ and $\nexists c \in \mathcal{E}'$ such that $(c, b) \in \mathtt{Def}$. There are two cases: either $b \in \mathcal{A}_e$ or $b \in \mathcal{A}_p$. b cannot be in $\mathcal{A}_e$ since $\mathcal{E}'$ is an admissible extension thus defends its arguments against any attack, consequently it defends also a against b. Assume now that $b \in \mathcal{A}_p$, this is also impossible since practical arguments are not allowed to attack epistemic ones. Thus, $\mathcal{E}'$ is an admissible extension of the system $\mathtt{AF}$.

■

Note that the above theorem holds as well for stable and preferred extensions since each stable (resp. preferred) extension is an admissible one.

It is easy to show that when $\mathtt{Def}_m$ is empty, i.e. no epistemic argument defeats a practical one, then the extensions of $\mathtt{AF}$ (under a given semantics) are exactly the different extensions of $\mathtt{AF}_e$ (under the same semantics) augmented by the set $\mathtt{AF}_p$.

Theorem 2 *Let $\mathtt{AF} = (\mathcal{D}, \mathcal{A}_e \cup \mathcal{A}_p, \mathtt{Def}_e \cup \mathtt{Def}_p \cup \mathtt{Def}_m)$ be a decision system. Let $\mathcal{E}_1, \ldots, \mathcal{E}_n$ be the extensions of $\mathtt{AF}_e$ under a given semantics. If $\mathtt{Def}_m = \emptyset$ then $\forall \mathcal{E}_i$ with $i = 1, \ldots, n$, then the set $\mathcal{E}_i \cup \mathcal{A}_p$ is an extension of $\mathtt{AF}$.*

Proof *Let $\mathcal{E}$ be an admissible extension of $\mathtt{AF}_e$. Let us assume that $\mathcal{E} \cup \mathcal{A}_p$ is not an admissible extension of $\mathtt{AF}$. There are two cases:*

Case 1: *$\mathcal{E} \cup \mathcal{A}_p$ is not conflict-free. Since $\mathcal{E}$ and $\mathcal{A}_p$ are conflict-free, then $\exists \alpha \in \mathcal{E}$ and $\exists \delta \in \mathcal{A}_p$ such that $(\alpha, \delta) \in \mathtt{Def}$. Contradiction with the fact that $\mathtt{Def}_m = \emptyset$.*

Case 2: *$\mathcal{E} \cup \mathcal{A}_p$ does not defend its elements. This means that: i) $\exists \alpha \in \mathcal{E}$ such that $\exists \alpha' \in \mathcal{A}_e$, $(\alpha', \alpha) \in \mathtt{Def}_e$ and $\mathcal{E} \cup \mathcal{A}_p$ does not defend it. Impossible since $\mathcal{E}$ is an admissible extension then it defends its arguments. ii) $\exists \delta \in \mathcal{A}_p$ such that $\exists a \in \mathcal{A}$, and $(a, \delta) \in \mathtt{Def}$ and δ is not defended by $\mathcal{E} \cup \mathcal{A}_p$. Since $\mathtt{Def}_m = \emptyset$ then $a \in \mathcal{A}_p$. This is impossible since $\mathcal{R}_p = \emptyset$. Contradiction.*

■

Finally, it can be shown that if the empty set is the only admissible extension of the decision system $\mathtt{AF}$, then the empty set is also the only admissible extension of the corresponding epistemic system $\mathtt{AF}_e$. Moreover, each practical argument is attacked by at least one epistemic argument.

Theorem 3 *Let* $\mathtt{AF} = (\mathcal{D}, \mathcal{A}_e \cup \mathcal{A}_p, \mathtt{Def}_e \cup \mathtt{Def}_p \cup \mathtt{Def}_m)$ *be a decision system. The only admissible extension of* $\mathtt{AF}$ *is the empty set iff:*

(1) The only admissible extension of $\mathtt{AF}_e$ *is the empty set, and*
(2) $\forall \delta \in \mathcal{A}_p$, $\exists \alpha \in \mathcal{A}_e$ *such that* $(\alpha, \delta) \in \mathtt{Def}_m$.

Proof *Let* $\mathtt{AF} = (\mathcal{D}, \mathcal{A}_e \cup \mathcal{A}_p, \mathtt{Def}_e \cup \mathtt{Def}_p \cup \mathtt{Def}_m)$ *be a decision system.*

Case 1: *Assume that the empty set is the only admissible extension of* $\mathtt{AF}$. *Assume also that the epistemic system* $\mathtt{AF}_e$ *has a non-empty admissible extension,say* E. *This means that* E *is not an admissible extension of* $\mathtt{AF}$. *There are two cases:*
a) E *is not conflict-free. This is impossible since* E *is an admissible extension of* $\mathtt{AF}_e$.
b) E *does not defend its elements. This means that* $\exists a \in \mathcal{A}_e \cup \mathcal{A}_p$ *such that* $\exists b \in E$ *and* $(a, b) \in \mathtt{Def}$ *and* $\nexists c \in E$ *such that* $(c, a) \in \mathtt{Def}$. *There are two possibilities: i)* $a \in \mathcal{A}_p$. *This is impossible since practical arguments are not allowed to attack epistemic arguments. ii)* $a \in \mathcal{A}_e$. *Since* E *is an admissible extension of* $\mathtt{AF}_e$, *then* $\exists c \in E$ *such that* $(c, a) \in \mathtt{Def}_e$. *Thus,* $(c, a) \in \mathtt{Def}$. *Contradiction.*

Case 2: *Let us now assume that the empty set is the only admissible extension of* $\mathtt{AF}_e$ *and that* $\forall \delta \in \mathcal{A}_p$, $\exists \alpha \in \mathcal{A}_e$ *such that* $(\alpha, \delta) \in \mathtt{Def}_m$. *Assume also that* $\exists E \neq \emptyset$ *such that* E *is an admissible extension of the decision system* $\mathtt{AF}$.

From Theorem 1, $E \cap \mathcal{A}_e$ *is an admissible extension of* $\mathtt{AF}_e$. *Since the only admissible extension of* $\mathtt{AF}_e$ *is the empty set, then* $E \cap \mathcal{A}_e = \emptyset$. *Thus,* $E \subseteq \mathcal{A}_p$.

Let $\delta \in E$. *By assumption,* $\exists \alpha \in \mathcal{A}_e$ *such that* $(\alpha, \delta) \in \mathtt{Def}_m$. *Since* E *is an admissible extension, thus it defends all its elements. Consequently,* $\exists \delta' \in E$ *such that* $(\delta', \alpha) \in \mathtt{Def}$. *Since* $E \subseteq \mathcal{A}_p$, *then* $\delta' \in \mathcal{A}_p$. *It is impossible to have* $(\delta', \alpha) \in \mathtt{Def}$ *since practical arguments are not allowed to attack epistemic ones.*

■

At this step, we have only defined the accepted arguments among all the existing ones. However, nothing is yet said about which option to prefer. In the next section, we will study different ways of comparing pairs of options on the basis of skeptically accepted practical arguments.

2.5 Ordering options

Comparing candidate decisions, i.e. defining a preference relation $\succeq$ on the set $\mathcal{D}$ of options, is a key step in a decision process. In an argumentation-based approach, the definition of this relation is based on the sets of "accepted" arguments pro or cons associated with candidate decisions. Thus, the input of this relation is no longer $\mathcal{A}_p$, but the set $\texttt{Acc}(\texttt{AF}, y) \cap \mathcal{A}_p$, where $\texttt{Acc}(\texttt{AF}, y)$ is the set of skeptically accepted arguments of the decision system $(\mathcal{D}, \mathcal{A}, \texttt{Def})$ under stable or preferred semantics. In what follows, we will use the notation $\texttt{Acc}(\texttt{AF})$ for short.

Note that in a decision system, when the defeat relation $\texttt{Def}_m$ is empty, the epistemic arguments become useless for the decision problem, i.e. for ordering options. Thus, only the practical system $\texttt{AF}_p$ is needed.

Depending on what sets are considered and how they are handled, one can roughly distinguish between three categories of principles:

Unipolar principles: are those that only refer to either the arguments pro or the arguments con.

Bipolar principles: are those that take into account both types of arguments at the same time.

Non-polar principles: are those where arguments pro and arguments con a given choice are aggregated into a unique *meta-argument*. It results that the negative and positive polarities disappear in the aggregation.

Whatever the category is, a relation $\succeq$ should suitably satisfy the following minimal requirements:

(1) **Transitivity**: The relation should be *transitive* (as usually required in decision theory).
(2) **Completeness**: Since one looks for the "best" candidate decision, it should then be possible to compare any pair of choices. Thus, the relation should be *complete*.

2.5.1 Unipolar principles

In this section we present basic principles for comparing decisions on the basis of only arguments pro. Similar ideas apply to arguments con. We start by presenting those principles that do not involve the strength of arguments, then their respective refinements when strength is taken into account.

A first natural criterion consists of preferring the decision that has more arguments pro.

Definition 6 (Counting arguments pro) *Let* $\texttt{AF} = (\mathcal{D}, \mathcal{A}, \texttt{Def})$ *be a decision system and* $\texttt{Acc}(\texttt{AF})$ *its accepted arguments. Let* d_1*,* $d_2 \in \mathcal{D}$*.*

$$d_1 \succeq d_2 \text{ iff } |\mathcal{F}_p(d_1) \cap \texttt{Acc}(\texttt{AF})| \geq |\mathcal{F}_p(d_2) \cap \texttt{Acc}(\texttt{AF})|.$$

Property 4 *This relation is a* complete *preorder.*

Note that when the decision system has no accepted arguments (i.e. $\texttt{Acc(AF)} = \emptyset$), all the options in $\mathcal{D}$ are equally preferred w.r.t. the relation $\succeq$. It can be checked that if a practical argument is defined as done later in Definition 18, then with such a principle, one may prefer a decision d, which has three arguments pointing all to the same goal, to decision d', which is supported by two arguments pointing to different goals.

When the strength of arguments is taken into account in the decision process, one may think of preferring a choice that has a dominant argument, i.e. an argument pro that is preferred w.r.t. the relation $\geq_p \subseteq \mathcal{A}_p \times \mathcal{A}_p$ to any argument pro the other choices. This principle is called *promotion focus* principle in [3].

Definition 7 *Let* $\texttt{AF} = (\mathcal{D}, \mathcal{A}, \texttt{Def})$ *be a decision system and* $\texttt{Acc(AF)}$ *its accepted arguments. Let* d_1*,* $d_2 \in \mathcal{D}$*.*

$$d_1 \succeq d_2 \textit{ iff } \exists \delta \in \mathcal{F}_p(d_1) \cap \texttt{Acc(AF)} \textit{ such that } \forall \delta' \in \mathcal{F}_p(d_2) \cap \texttt{Acc(AF)}, \delta \geq_p \delta'.$$

With this criterion, if the decision system has no accepted arguments, then all the options in $\mathcal{D}$ are equally preferred. The above definition relies heavily on the relation $\geq_p$ that compares practical arguments. Thus, the properties of this criterion depend on those of $\geq_p$. Namely, it can be checked that the above criterion works properly if $\geq_p$ is a complete preorder.

Property 5 *If the relation* $\geq_p$ *is a complete preorder, then* $\succeq$ *is also a complete preorder.*

Note that the above relation may be found to be too restrictive, since when the strongest arguments in favor of d_1 and d_2 have equivalent strengths (i.e. are indifferent), d_1 and d_2 are also seen as equivalent. However, we can refine the above definition by ignoring the strongest arguments with equal strengths, by means of the following *strict preorder*.

Definition 8 *Let* $\texttt{AF} = (\mathcal{D}, \mathcal{A}, \texttt{Def})$ *be a decision system and* $\texttt{Acc(AF)}$ *its accepted arguments. Let* d_1*,* $d_2 \in \mathcal{D}$*, and* $\geq_p$ *be a complete preorder. Let* $(\delta_1, \ldots, \delta_r)$*,* $(\delta'_1, \ldots, \delta'_s)$ *such that* $\forall \delta_{i=1,\ldots,r}$*,* $\delta_i \in \mathcal{F}_p(d_1) \cap \texttt{Acc(AF)}$*, and* $\forall \delta'_{j=1,\ldots,s}$*,* $\delta'_j \in \mathcal{F}_p(d_2) \cap \texttt{Acc(AF)}$*.*

Each of these vectors is assumed to be decreasingly ordered w.r.t $\geq_p$ *(e.g.* $\delta_1 \geq_p \ldots \geq_p \delta_r$*). Let* $v = min(r, s)$*.*

$d_1 \succeq d_2$ *iff:*

- $\delta_1 >_p \delta'_1$*, or*

- $\exists\ k \leq v$ *such that* $\delta_k >_p \delta'_k$ *and* $\forall\ j < k$, $\delta_j \approx_p \delta'_j$, *or*
- $r > v$ *and* $\forall\ j \leq v$, $\delta_j \approx_p \delta'_j$.

Till now, we have only discussed decision principles based on arguments pro. However, the counterpart principles when arguments con are considered can also be defined. Thus, the counterpart principle of the one defined in Definition 6 is the following complete preorder:

Definition 9 (Counting arguments con) *Let* $\mathtt{AF} = (\mathcal{D}, \mathcal{A}, \mathtt{Def})$ *be a decision system and* $\mathtt{Acc(AF)}$ *its accepted arguments. Let* d_1, $d_2 \in \mathcal{D}$.

$$d_1 \succeq d_2 \textit{ iff } |\mathcal{F}_c(d_1) \cap \mathtt{Acc(AF)}| \leq |\mathcal{F}_c(d_2) \cap \mathtt{Acc(AF)}|.$$

The principles that take into account the strengths of arguments have also their counterparts when handling arguments con. The *prevention focus* principle prefers a decision when all its cons are weaker than at least one argument against the other decision. Formally:

Definition 10 *Let* $\mathtt{AF} = (\mathcal{D}, \mathcal{A}, \mathtt{Def})$ *be a decision system and* $\mathtt{Acc(AF)}$ *its accepted arguments. Let* d_1, $d_2 \in \mathcal{D}$.

$$d_1 \succeq d_2 \textit{ iff } \exists \delta \in \mathcal{F}_c(d_2) \cap \mathtt{Acc(AF)} \textit{ such that } \forall \delta' \in \mathcal{F}_c(d_1) \cap \mathtt{Acc(AF)}, \delta \geq_p \delta'.$$

As in the case of arguments pro, when the relation $\geq_p$ is a complete preorder, the above relation is also a complete preorder, and can be refined into the following strict one.

Definition 11 *Let* $\mathtt{AF} = (\mathcal{D}, \mathcal{A}, \mathtt{Def})$ *be a decision system and* $\mathtt{Acc(AF)}$ *its accepted arguments. Let* d_1, $d_2 \in \mathcal{D}$.

Let $(\delta_1, \ldots, \delta_r)$, $(\delta'_1, \ldots, \delta'_s)$ *such that* $\forall \delta_{i=1,\ldots,r}$, $\delta_i \in \mathcal{F}_c(d_1) \cap \mathtt{Acc(AF)}$, *and* $\forall \delta'_{j=1,\ldots,s}$, $\delta'_j \in \mathcal{F}_c(d_2) \cap \mathtt{Acc(AF)}$.

Each of these vectors is assumed to be decreasingly ordered w.r.t $\geq_p$ *(e.g.* $\delta_1 \geq_p \ldots \geq_p \delta_r$*). Let* $v = min(r, s)$.
$d_1 \succ d_2$ *iff:*

- $\delta'_1 >_p \delta_1$, *or*
- $\exists\ k \leq v$ *such that* $\delta'_k >_p \delta_k$ *and* $\forall\ j < k$, $\delta_j \approx_p \delta'_j$, *or*
- $v < s$ *and* $\forall\ j \leq v$, $\delta_j \approx_p \delta'_j$.

2.5.2 Bipolar principles

Let's now define some principles where both types of arguments (pros and cons) are taken into account when comparing decisions. Generally speaking, we can conjunctively combine the principles dealing with arguments pro with their counterpart handling arguments con. For instance, the principles given in Definition 6 and Definition 9 can be combined as follows:

Definition 12 *Let* $\texttt{AF} = (\mathcal{D}, \mathcal{A}, \texttt{Def})$ *be a decision system and* $\texttt{Acc(AF)}$ *its accepted arguments. Let* d_1, $d_2 \in \mathcal{D}$. $d_1 \succeq d_2$ *iff*

- $|\mathcal{F}_p(d_1) \cap \texttt{Acc(AF)}| \geq |\mathcal{F}_p(d_2) \cap \texttt{Acc(AF)}|$, *and*
- $|\mathcal{F}_c(d_1) \cap \texttt{Acc(AF)}| \leq |\mathcal{F}_c(d_2) \cap \texttt{Acc(AF)}|$.

However, note that unfortunately this is no longer a complete preorder. Similarly, the principles given respectively in Definition 7 and Definition 10 can be combined into the following one:

Definition 13 *Let* $\texttt{AF} = (\mathcal{D}, \mathcal{A}, \texttt{Def})$ *be a decision system and* $\texttt{Acc(AF)}$ *its accepted arguments. Let* d_1, $d_2 \in \mathcal{D}$. $d_1 \succeq d_2$ *iff:*

- $\exists\ \delta \in \mathcal{F}_p(d_1) \cap \texttt{Acc(AF)}$ *such that* $\forall\ \delta' \in \mathcal{F}_p(d_2) \cap \texttt{Acc(AF)}$, $\delta \geq_p \delta'$, *and*
- $\nexists\ \delta \in \mathcal{F}_c(d_1) \cap \texttt{Acc(AF)}$ *such that* $\forall\ \delta' \in \mathcal{F}_c(d_2) \cap \texttt{Acc(AF)}$, $\delta \geq_p \delta'$.

This means that one prefers a decision that has at least one supporting argument which is better than any supporting argument of the other decision, and also has not a very strong argument against it. Note that the above definition can be also refined in the same spirit as Definitions 8 and 11.

Another family of bipolar decision principles applies the *Franklin principle* which is a natural extension to the bipolar case of the idea underlying Definition 8. This principle consists, when comparing pros and cons a decision, of ignoring pairs of arguments pro and cons which have the same strength. After such a simplification, one can apply any of the above bipolar principles. In what follows, we will define formally the Franklin simplification.

Definition 14 (Franklin simplification) *Let* $\texttt{AF} = (\mathcal{D}, \mathcal{A}, \texttt{Def})$ *be a decision system and* $\texttt{Acc(AF)}$ *its accepted arguments. Let* $d \in \mathcal{D}$.

Let $P = (\delta_1, \ldots, \delta_r)$, $C = (\delta'_1, \ldots, \delta'_m)$ *such that* $\forall \delta_i, \delta_i \in \mathcal{F}_p(d) \cap \texttt{Acc(AF)}$ *and* $\forall \delta'_j, \delta'_j \in \mathcal{F}_c(d) \cap \texttt{Acc(AF)}$.

Each of these vectors is assumed to be decreasingly ordered w.r.t $\geq_p$ *(e.g.* $\delta_1 \geq_p \ldots \geq_p \delta_r$*). The* result of the simplification *is* $P' = (\delta_{j+1}, \ldots, \delta_r)$, $C' = (\delta'_{j+1}, \ldots, \delta'_m)$ *s.t.*

- $\forall\ 1 \leq i \leq j$, $\delta_i \approx_p \delta'_i$ *and* $(\delta_{j+1} >_p \delta'_{j+1}$ *or* $\delta'_{j+1} >_p \delta_{j+1})$
- *If* $j = r$ *(resp.* $j = m$*), then* $P' = \emptyset$ *(resp.* $C' = \emptyset$*).*

2.5.3 Non-polar principles

In some applications, the arguments in favor of and against a decision are aggregated into a unique *meta-argument* having a unique strength. Thus, comparing two decisions amounts to compare the resulting meta-arguments. Such a view is well in agreement with current practice in multiple criteria decision making, where each decision is evaluated according to different criteria using the same scale (with a positive and a negative part), and an aggregation function is used to obtain a global evaluation of each decision.

Definition 15 (Aggregation criterion) *Let* $\mathtt{AF} = (\mathcal{D}, \mathcal{A}, \mathtt{Def})$ *be a decision system and* $\mathtt{Acc}(\mathtt{AF})$ *its accepted arguments. Let* $d_1, d_2 \in \mathcal{D}$. *Let* $(\delta_1, \ldots, \delta_n)$ [4] *and* $(\delta'_1, \ldots, \delta'_m)$ [5] *(resp.* $(\gamma_1, \ldots, \gamma_l)$ [6] *and* $(\gamma'_1, \ldots, \gamma'_k)$ [7] *) the vectors of the arguments pro and cons the decision* d_1 *(resp.* d_2*).*
$d_1 \succeq d_2$ *iff* $h(\delta_1, \ldots, \delta_n, \delta'_1, \ldots, \delta'_m) \geq_p h(\gamma_1, \ldots, \gamma_l, \gamma'_1, \ldots, \gamma'_k)$*, where* h *is an* aggregation function.

A simple example of this aggregation attitude is computing the difference of the number of arguments pros and cons.

Definition 16 *Let* $\mathtt{AF} = (\mathcal{D}, \mathcal{A}, \mathtt{Def})$ *be a decision system and* $\mathtt{Acc}(\mathtt{AF})$ *its accepted arguments. Let* $d_1, d_2 \in \mathcal{D}$. $d_1 \succeq d_2$ *iff* $|\mathcal{F}_p(d_1) \cap \mathtt{Acc}(\mathtt{AF})| - |\mathcal{F}_c(d_1) \cap \mathtt{Acc}(\mathtt{AF})| \geq |\mathcal{F}_p(d_2) \cap \mathtt{Acc}(\mathtt{AF})| - |\mathcal{F}_c(d_2) \cap \mathtt{Acc}(\mathtt{AF})|$.

This has the advantage to be again a complete preorder, while taking into account both pros and cons arguments.

3 A typology of formal practical arguments

This section aims at presenting a systematic study of practical arguments. Epistemic arguments will not be discussed here because they have been much studied in the literature (eg. [4,14,46]), and their handling does not make new problems in the general setting of Section 2, even in the decision process perspective of this paper. Moreover, they only play a role when the knowledge base is inconsistent. Before presenting the different types of practical arguments, we start first by introducing the logical language as well as the different bases needed in a decision making problem.

[4] Each $\delta_i \in \mathcal{F}_p(d_1) \cap \mathtt{Acc}(\mathtt{AF})$.
[5] Each $\delta'_i \in \mathcal{F}_c(d_1) \cap \mathtt{Acc}(\mathtt{AF})$.
[6] Each $\gamma_i \in \mathcal{F}_p(d_2) \cap \mathtt{Acc}(\mathtt{AF})$.
[7] Each $\gamma'_i \in \mathcal{F}_c(d_2) \cap \mathtt{Acc}(\mathtt{AF})$.

This section introduces the representation setting of knowledge and preference which are here distinct, as it is in classical decision theory. Moreover, preferences are supposed to be handled in a *bipolar* way, which means that what the decision maker is really looking for may be more restrictive than what it is just willing to avoid.

In what follows, a vocabulary $\mathcal{P}$ of propositional variables contains two kinds of variables: *decision variables*, denoted by $v_1, \ldots, v_n$, and *state variables*. Decision variable are controllable, that is their value can be fixed by the decision maker. Making a decision then amounts to fixing the truth value of every decision variable. On the contrary, state variables are fixed by nature, and their value is a matter of knowledge by the decision maker. He has no control on them (although he may express preferences about their values).

(1) $\mathcal{D}$ is a set of formulas built from the decision variables. Elements of $\mathcal{D}$ represent the different *alternatives*, or candidate *decisions*. Let us consider the following example of an agent who wants to know whether she should take her umbrella, her raincoat or both. In this case, there are two decision variables: *umb* (for umbrella) and *rac* (for raincoat). Assume that this agent hesitates between the three following options: i) $d_1 : umb$ (i.e. to take her umbrella), ii) $d_2 : rac$ (i.e. to take her raincoat), or iii) $d_3 : umb \wedge rac$ (i.e. to take both). Thus, $\mathcal{D} = \{d_1, d_2, d_3\}$. Note that elements of $\mathcal{D}$ are not necessarily mutually exclusive. In the example, if the agent chooses the option d_3 then the two other options are satisfied.

(2) $\mathcal{G}$ is a set of propositional formulas built from state variables. It gathers the *goals* of an agent (the decision maker). A goal represents what the agent wants to achieve, and has thus a positive flavor. This means that if $g \in \mathcal{G}$, the decision maker wants that the chosen decision leads to a state of affairs where g is true. This base may be *inconsistent*. In this case it would be for sure impossible to satisfy all the goals, which would induce the simultaneous existence of practical arguments pro and cons. In general $\mathcal{G}$ contains several goals. Clearly, an agent should try to satisfy all goals in its goal base $\mathcal{G}$ if possible. This means that $\mathcal{G}$ may be thought as a conjunction. However, the two goal bases $\mathcal{G} = \{g_1, g_2\}$ and $\mathcal{G}' = \{g_1 \wedge g_2\}$ although they are logically equivalent, will not be handled in the same way in an argumentative perspective, since in the second case there is no way to consider intermediary objectives such as here satisfying g_1, or satisfying g_2 only, in case it turns out that it is impossible to satisfy $g_1 \wedge g_2$. This means that our approach is syntax-dependent.

(3) The set $\mathcal{R}$ is a set of propositional formulas built from state variables. It

gathers the *rejections* of an agent. A rejection represents what the agent wants to avoid. Clearly rejections express negative preferences. The set $\{\neg r | r \in \mathcal{R}\}$ describing what is acceptable for the agent is assumed to be *consistent*, since acceptable alternatives should satisfy $\neg r$ due to the rejection of r, and at least there should remain some possible worlds that are not rejected. There are at least two reasons for separately considering a set of goals and a set of rejections. First, since agents naturally express themselves in terms of what they are looking for (i.e. their goals), and in terms of what they want to avoid (i.e. their rejections), it is better to consider goals and rejections separately in order to articulate arguments referring to them in a way easily understandable for the agents. Moreover, recent cognitive psychology studies [18] have confirmed the cognitive validity of this distinction between goals and rejections. Second, if r is a rejection, this does not necessarily mean that $\neg r$ is a goal, and thus rejections cannot be equivalently restated as goals. For instance, in case of choosing a medical drug, one may have as a goal the immediate availability of the drug, and as a rejection its availability only after at least two days. In such a case, if the candidate decision guarantees the availability only after one day, this decision will for sure avoid the rejection without satisfying the goal. Another simple example is the case of an agent who wants to get a cup of either coffee or tea, and wants to avoid getting no drink. If the agent obtains a glass of water, again he would avoid its rejection, without being completely satisfied.
We can imagine different forms of consistency between the goals and the rejections. A minimal requirement is to have $\mathcal{G} \cap \mathcal{R} = \emptyset$, otherwise it will mean that an agent both wants to have p true and to avoid it.

(4) The set $\mathcal{K}$ represents the *background knowledge* that is not necessarily assumed to be *consistent*. The argumentation framework for inference presented in Section 2 will handle such inconsistency, namely with the epistemic system. Elements of $\mathcal{K}$ are propositional formulas built from the alphabet $\mathcal{P}$, and assumed to be put in a clausal form. The base $\mathcal{K}$ contains basically two kinds of clauses: i) those not involving any element from $\mathcal{D}$, which encode pieces of knowledge or factual information (possibly involving goals) about how the world is; ii) those involving one negation of a formula d of the set $\mathcal{D}$, and which states what follows when decision d is applied.

Thus, the decision problem we consider will always be encoded with the four above sets of formulas (with the restrictions stated above). Moreover, we suppose that each of the three bases $\mathcal{K}$, $\mathcal{G}$, and $\mathcal{R}$ are stratified. Having $\mathcal{K}$ stratified would mean that we consider that some pieces of knowledge are fully certain, while others are less certain (maybe distinguishing between several levels of partial certainty such as “almost certain”, “rather certain”, ...). Clearly, formulas that are not certain at all cannot be in $\mathcal{K}$. Similarly, having $\mathcal{G}$ (resp. $\mathcal{R}$)

stratified means that some goals (resp. rejections) are imperative, while some others are less important (one may have more than two levels of importance). Completely unimportant goals (resp. rejections) do not appear in any stratum of $\mathcal{G}$ (resp. $\mathcal{R}$).

It is worth pointing out that we assume that candidate decisions are all considered as *a priori* equally potentially suitable, and thus there is no need to have $\mathcal{D}$ stratified.

For encoding the stratifications, we use the set $\{0, 1, \ldots, n\}$ of integers as a linearly ordered scale, where n stands for the highest level of certainty if dealing with $\mathcal{K}$ (resp. level of importance if dealing with $\mathcal{G}$ or $\mathcal{R}$) and '0' corresponds to the complete lack of certainty (resp. importance). Other encodings (e.g. using levels inside the unit interval or using the integer scale in a reversed way) would be equivalent.

Definition 17 (Decision theory) *A* decision theory *(or a* theory *for short) is a tuple $\mathcal{T} = \langle \mathcal{D}, \mathcal{K}, \mathcal{G}, \mathcal{R} \rangle$.*

- *The base $\mathcal{K}$ is partitioned and stratified into $\mathcal{K}_1, \ldots, \mathcal{K}_n$ ($\mathcal{K} = \mathcal{K}_1 \cup \ldots \cup \mathcal{K}_n$) such that formulas in $\mathcal{K}_i$ have the same certainty level and are more certain than formulas in $\mathcal{K}_j$ where $j < i$. Moreover, $\mathcal{K}_0$ is not considered since it gathers formulas which are completely uncertain.*
- *The base $\mathcal{G}$ is partitioned and stratified into $\mathcal{G}_1, \ldots, \mathcal{G}_n$ ($\mathcal{G} = \mathcal{G}_1 \cup \ldots \cup \mathcal{G}_n$) such that goals in $\mathcal{G}_i$ have the same importance and are more important than goals in $\mathcal{G}_j$ where $j < i$. Moreover, $\mathcal{G}_0$ is not considered since it gathers goals which are completely unimportant.*
- *The base $\mathcal{R}$ is partitioned and stratified into $\mathcal{R}_1, \ldots, \mathcal{R}_n$ ($\mathcal{R} = \mathcal{R}_1 \cup \ldots \cup \mathcal{R}_n$) such that rejections in $\mathcal{R}_i$ have the same importance and are more important than rejections in $\mathcal{R}_j$ where $j < i$. Moreover, $\mathcal{R}_0$ is not considered since it gathers rejections which are completely unimportant.*

3.2 A typology of formal practical arguments

Each candidate decision may have arguments in its favor (called pros), and arguments against it (called cons). In the following, an argument is associated with an alternative, and always either refers to a goal or to a rejection.

Arguments pros point out the "existence of good consequences" or the "absence of bad consequences" for a candidate decision. A good consequence means that applying decision d will lead to the satisfaction of a goal, or to the avoidance of a rejection. Similarly, a bad consequence means that the application of d leads for sure to miss a goal, or to reach a rejected situation.

We can distinguish between practical arguments referring to a goal, and those arguments referring to rejections. When focusing on the base $\mathcal{G}$, an argument pro corresponds to the guaranteed satisfaction of a goal when there exists a consistent subset S of $\mathcal{K}$ such that $S \cup \{d\} \vdash g$.

Definition 18 (Positive arguments pro) *Let $\mathcal{T}$ be a theory. A* positively expressed argument in favor of *an option d is a tuple $\delta = \langle S, d, g \rangle$ s.t:*

(1) $S \subseteq \mathcal{K}$, $d \in \mathcal{D}$, $g \in \mathcal{G}$, $S \cup \{d\}$ is consistent
(2) $S \cup \{d\} \vdash g$, and S is minimal for set inclusion among subsets of $\mathcal{K}$ satisfying the above criteria (arguments of Type `PP`*).*

S is called the support *of the argument, and d is its* conclusion*. Let $\mathcal{A}_{\mathtt{PP}}$ be the set of all arguments of type* `PP` *that can be built from a decision theory $\mathcal{T}$.*

In what follows, `Supp` denotes a function that returns the support S of an argument, `Conc` denotes a function that returns the conclusion d of the argument, and `Result` denotes a function that returns the consequence of the decision. The consequence may be either a goal as in the previous definition, or a rejection as we can see in the next definitions of argument types.

The above definition deserves several comments.

- The consistency of $S \cup \{d\}$ means that d is applicable in the context S, in other words that we cannot prove from S that d is impossible. This means that impossible alternatives w.r.t. $\mathcal{K}$ have been already taken out when defining the set $\mathcal{D}$. In the particular case where the base $\mathcal{K}$ would be consistent, then condition 1, namely $S \cup \{d\}$ is consistent, is equivalent to $\mathcal{K} \cup \{d\}$ is consistent. But, in the case where $\mathcal{K}$ is inconsistent, independently from the existence of a `PP` argument, it may happen that for another consistent subset S' of $\mathcal{K}$, $S' \vdash \neg d$. This would mean that there is some doubt about the feasibility of d, and then constitute an epistemic argument against d. In the general framework proposed in section 2, such an argument will overrule decision d since epistemic arguments take precedence over any practical argument (provided that this epistemic argument is not itself killed by another epistemic argument).
- Note that argument of type `PP` are reminiscent of the practical syllogism recalled in the introduction. Indeed, it emphasizes that a candidate decision might be chosen if it leads to the satisfaction of a goal. However, this is only a clue for choosing the decision since this last may have arguments against, which would weaken it, or there may exist other candidate decisions with stronger arguments. Moreover, due to the nature of the practical syllogism, it is worth noticing that practical arguments have an *abductive* form, contrarily to epistemic arguments that are defined in a deductive way, as revealed by their formal respective definitions.

Another type of arguments pro refers to rejections. It amounts to avoid a rejection for sure, i.e. $S \cup \{d\} \vdash \neg r$ (where S is a consistent subset of $\mathcal{K}$).

Definition 19 (Negative arguments pro) *Let $\mathcal{T}$ be a theory. A* negatively expressed argument in favor of *an option is a tuple $\delta = \langle S, d, r \rangle$ s.t:*

(1) $S \subseteq \mathcal{K}$, $d \in \mathcal{D}$, $r \in \mathcal{R}$, $S \cup \{d\}$ is consistent
(2) $S \cup \{d\} \vdash \neg r$ and S is minimal for set inclusion among subsets of $\mathcal{K}$ satisfying the above criteria (arguments of Type `NP`*).*

Let $\mathcal{A}_{\texttt{NP}}$ be the set of all arguments of type `NP` *that can be built from a decision theory $\mathcal{T}$.*

Arguments cons highlight the existence of bad consequences for a given candidate decision. Negatively expressed arguments con are defined by exhibiting a rejection that is necessarily satisfied. Formally:

Definition 20 (Negative arguments con) *Let $\mathcal{T}$ be a theory. A* negatively expressed argument against *an option d is a tuple $\delta = \langle S, d, r \rangle$ s.t:*

(1) $S \subseteq \mathcal{K}$, $d \in \mathcal{D}$, $r \in \mathcal{R}$, $S \cup \{d\}$ is consistent,
(2) $S \cup \{d\} \vdash r$ and S is minimal for set inclusion among subsets of $\mathcal{K}$ satisfying the above criteria (arguments of Type `NC`*).*

Let $\mathcal{A}_{\texttt{NC}}$ be the set of all arguments of type `NC` *that can be built from a decision theory $\mathcal{T}$.*

Lastly, the absence of positive consequences can also be seen as an argument against (cons) an alternative.

Definition 21 (Positive arguments con) *Let $\mathcal{T}$ be a theory. A* positively expressed argument against *an option d is a tuple $\delta = \langle S, d, g \rangle$ s.t:*

(1) $S \subseteq \mathcal{K}$, $d \in \mathcal{D}$, $g \in \mathcal{G}$, $S \cup \{d\}$ is consistent,
(2) $S \cup \{d\} \vdash \neg g$ and S is minimal for set inclusion among subsets of $\mathcal{K}$ satisfying the above criteria (arguments of Type `PC`*).*

Let $\mathcal{A}_{\texttt{PC}}$ be the set of all arguments of type `PC` *that can be built from a decision theory $\mathcal{T}$.*

Let us illustrate the previous definitions on an example.

Example 7 *Two decisions are possible, organizing a show (d), or not ($\neg d$). Thus $\mathcal{D} = \{d, \neg d\}$. The knowledge base $\mathcal{K}$ contains the following pieces of knowledge: if a show is organized and it rains then small money loss ($\neg d \vee \neg r \vee sml$); if a show is organized and it does not rain then benefit ($\neg d \vee r \vee b$); small money loss entails money loss ($\neg sml \vee ml$); if benefit there is no money* *loss ($\neg b \vee \neg ml$); small money loss is not large money loss ($\neg sml \vee \neg lml$); large money loss is money loss ($\neg lml \vee ml$); there are clouds (c); if there are clouds then it may rain ($\neg c \vee r$). All these pieces of knowledge are in the stratum of level n, except the last one which is in a stratum with a lower level due to uncertainty. Consider now the cases of two organizers (O_1 and O_2) having different preferences. O_1 does not want any loss $\mathcal{R} = \{ml\}$, and would like benefit $\mathcal{G} = \{b\}$. O_2 does not want large money loss $\mathcal{R} = \{lml\}$, and would like benefit $\mathcal{G} = \{b\}$. In such case, it is expected that O_1 prefers $\neg d$ to d, since there is a* `NC` *argument against d and no argument for $\neg d$. For O_2, there is no longer any* `NC` *argument against d. He might even prefer d to $\neg d$, if he is optimistic and he considers that there is a possibility that it does not rain (leading to a potential* `PP` *argument under the hypothesis to have $\neg r$ in $\mathcal{K}$.*

Due to the asymmetry in human mind between what is rejected and what is desired, the former being usually considered as stronger than the latter, one may assume that `NC` arguments are stronger than `PC` arguments, and conversely `PP` arguments are stronger than `NP` arguments.

In classical decision frameworks, bipolarity is not considered. Indeed, we are in the particular case where rejections mirror goals in the sense that g is a goal iff $\neg g$ is a rejection. Consequently, in our argumentation setting the two types `NC` and `PC` coincide. Similarly, the two types `PP` and `NP` are the same.

4 Application to multiple criteria decision making

4.1 Introduction to multiple criteria decision making

In multiple criteria decision making, each candidate decision d in $\mathcal{D}$ is evaluated from a set $\mathcal{C}$ of m different points of view ($i = 1, m$), called *criteria*. The evaluation can be done in an absolute manner or in a relative way. This means that for each i, d can be either evaluated by an absolute estimate $C_i(d)$ belonging to the evaluation scale used for i, or there exists a valued preference relation $R_i(d, d')$ associated with each i that is applicable to any pair (d, d') of elements of $\mathcal{D}$. Then one can distinguish between two families of approaches: i) the ones based on a global aggregation of value criteria-based functions where the obtained global absolute evaluations are of the form $g(f_1(C_1(d), \ldots, f_m(C_m(d)))$ where the mappings f_i map the original evaluations on a unique scale, which assumes commensurability, and ii) the ones that aggregate the preference indices $R_i(d, d')$ into a global preference $R(d, d')$ from which a ranking of the elements in $\mathcal{D}$ can be obtained. In the following, only the first type of approach is considered.

The decision maker uses a set $\mathcal{C}$ of different *criteria*. For each criterion c_i, one assumes that we have a *bipolar univariate* ordered scale T_i which enables us to distinguish between positive and negative values. Such a scale has a neutral point, or more generally a neutral area that separates positive and negative values. The lower bound of the scale stands for total *dissatisfaction* and the upper bound for total *satisfaction*, while neutral value(s) stand for indifference. The closer to the upper bound the value of criterion c_i for choice d, denoted $c_i(d)$ is, the more satisfactory choice d is w.r.t c_i; the closer to the lower bound the value of criterion c_i for choice d is, the more dissatisfactory choice d is w.r.t c_i. As in multiple criteria aggregation, we assume that the different scales T_i can be mapped on a unique bipolar scale T, i.e. for any i, $f_i(c_i(d)) \in T$. Moreover, we assume here that T is discrete and will be denoted $T = \{-k, \ldots, -1, 0, +1, \ldots, +k\}$ with the classical ordering convention of relative integers.

Example 8 (Choosing an apartment) *Imagine we have a set C of three criteria for choosing an apartment: Price (c_1), Size (c_2), and Location w.r.t. downtown (c_3). The criteria are valued on the same bipolar univariate scale $\{-2, -1, 0, +1, +2\}$ (this means that all the f_i mappings are the identity). Prices of apartments may be judged 'very expensive', 'expensive', 'reasonably priced', 'cheap', 'very cheap'. Size may be 'very small', 'small', 'normal sized', 'large', 'very large'. Distance may be 'very far', 'far', 'medium', 'close', 'very close'. In each case, the five linguistic expressions would be valued by $-2, -1, 0, +1, +2$ respectively. Thus an apartment d that is expensive, medium-sized, and very close to downtown will be evaluated as $c_1(d) = -1$, $c_2(d) = 0$, and $c_3(d) = +2$. It is clear that this scale implicitly encodes that the best apartments are those that are very cheap, very large, and very close to both downtown and transportation.*

From this setting, it is possible to express goals and rejections in terms of criteria values. A bipolar-valued criterion can be straightforwardly translated into a set of stratified goals, and a stratified set of rejections. The idea is the following. The criteria may be satisfied either in a positive way (if the satisfaction degree is higher than the neutral point 0 of T) or in a negative way (if the satisfaction degree is lower than the neutral point of T). Formally speaking, the two bases $\mathcal{G}$ and $\mathcal{R}$ are defined as follows: having the condition $f_i(c_i(d)) \geq +j$ satisfied, where $+j$ belongs to the positive part of T, is a goal g_j for the agent that uses c_i as a criterion. This goal is all the more important as j is small (but positive), since as suggested by the above example, the less restrictive conditions are the most imperative ones. The importance of g_j can be taken as equal to $k - j + 1$ for $j \geq 1$ (using the standard order-reversing map on $\{1, ..., k\}$). Indeed the most important condition $f_i(c_i(d))$

$\geq +1$ will have the maximal value in T, while the condition $f_i(c_i(d)) \geq +k$ will have the minimal positive level in T, i.e. $+1$. We can proceed similarly with rejections. The rejection r_j corresponding to the condition $f_i(c_i(d)) \leq -j$ will have importance j (importance uses only the positive part of the scale). This corresponds to the view of a fuzzy set as a nested family of level cuts, which translates in possibilistic logic into a collection of propositions whose extensions are all the larger as the proposition is more imperative. In the above example, consider for instance the price criterion. We will have two goals: $g_1 =$ very cheap and $g_2 =$ cheap with respective weights 1 and 2. Thus, being cheap is more imperative than being very cheap as expected. Similarly, $r_1 =$ very expensive and $r_2 =$ expensive are rejections with respective weights 2 and 1. Note that if an apartment is normally sized, then there will be no argument in favor or against it w.r.t. its size.

In multiple criteria aggregation, criteria may have different levels of importance. Let $w_i \in \{0, +1, \ldots, +k\}$ be the importance of criterion c_i. Then, we can apply the above translation procedure where, now the importance $k-j+1$ of condition $f_i(c_i(d)) \geq +j$ is changed into $min(w_i, k-j+1)$. Indeed, if w_i is maximal, i.e. $w_i = +k$, the importance is unchanged; in case the importance w_i of criterion c_i would be minimal, i.e. $w_i = 0$, then the resulting importance of the associated goal (the condition $f_i(c_i(d)) \geq +j$) is indeed also 0 expressing its complete lack of importance.

In addition to the bases $\mathcal{D}$, $\mathcal{C}$, $\mathcal{G}$ and $\mathcal{R}$, the decision maker is also equipped with a stratified knowledge base $\mathcal{K}$ encoding what he knows. In particular, $\mathcal{K}$ contains factual information about the values of the $f_i(c_i(x))$'s for the different criteria and the different candidate decisions. $\mathcal{K}$ also contains rules expressing that values in T are linearly ordered, i.e. rules of the form if $c_i(x) \geq j$, then $c_i(x) \geq j'$ if $j \geq j' \in T$. More generally, $\mathcal{K}$ can also contain pieces of knowledge that enable the decision maker to evaluate criteria from more elementary evaluation of facts. This may be useful in practice for describing complex notions, e.g. comfort of a house in our example, which indeed may depend on many parameters. A goal is assumed to be associated with a unique criterion are no longer allowed). Then, a goal g_i^j is associated to a criterion c_i by a propositional formula of the form $g_i^j \rightarrow c_i$ meaning just that the goal g_i^j refers to the evaluation of criterion c_i. Such formulas will be added to $\mathcal{K}_n$. Note that in classical multiple criteria problems, complete information is usually assumed w.r.t. the precise evaluation of criteria. Clearly, our setting is more general since it leaves room to incomplete information, and facilitates the expression of goals and rejections.

Now that the different bases are introduced, we can apply our general decision system, and build the arguments pro and cons for any candidate decision. In addition to the completeness of information, it is usually assumed in classical approaches to multiple criteria decision making that knowledge is consistent.

In such a case, it is not possible to have conflicting evaluations of a criterion for a choice. Consequently, the whole power of our argumentation setting will not be used, in particular all arguments will be accepted.

4.3 Retrieving some classical multiple criteria aggregations

The aim of this subsection is to show the agreement of the argumentation-based approach with some classical approaches to multiple criteria decision making. It is worth mentioning that until recently, most multiple criteria approaches use only positive evaluations, i.e. unipolar scales ranging from "bad" to "good" values, rather than distinguishing genuinely good values from really bad ones that have to be rejected. The argumentation-based approach makes natural the reference to the distinction between what is favored and what is disfavored by the decision maker for giving birth to arguments for and arguments against candidate decisions. Here, only two types of arguments are needed: positive arguments pro of type $\texttt{PP}$, and negative arguments con of type $\texttt{NC}$ since rejections in this case are just the complement of goals. Indeed, the negative values of a criterion reflect the fact that we are below some threshold while the positive values express to what extent we are above. Thus, $\mathcal{A}_p = \mathcal{A}_{\texttt{PP}} \cup \mathcal{A}_{\texttt{NC}}$.

In what follows, the base $\mathcal{K}$ is supposed to be *consistent*, fully certain (i.e. $\mathcal{K} = \mathcal{K}_n$), and to contain complete information w.r.t. the evaluation of criteria. Thus, the set $\mathcal{A}$ of arguments is exactly $\mathcal{A}_e \cup \mathcal{A}_p$. Since $\mathcal{K}$ is consistent, then the two attack relations $\mathcal{R}_e$ and $\mathcal{R}_m$ are empty (i.e. $\mathcal{R}_e = \mathcal{R}_m = \emptyset$). Consequently, $\texttt{Def}_e = \texttt{Def}_m = \emptyset$, and the set of skeptically accepted arguments of the decision system $\texttt{AF} = (\mathcal{D}, \mathcal{A}, \texttt{Def} = \emptyset)$ is exactly $\texttt{Acc}(\texttt{AF}) = \mathcal{A}$.

The first category of classical approaches to multiple criteria decision making that we will study is the one that gives the same importance to the different criteria of the set $\mathcal{C}$. The idea is to prefer the alternative that satisfies positively more criteria. Let $c'_i(d) = 1$ if $c_i(d) > 0$ and $c'_i(d) = 0$ if $c_i(d) < 0$, where $c_i(d)$ is the evaluation of choice d by the i-th criterion. In order to capture this idea, a particular unipolar principle is used. Before introducing this principle, let us first define a function $\texttt{Results}$ that returns for a given set $\mathcal{B}$ of practical arguments, all the consequences of those arguments, i.e. all the goals and rejections to which arguments of $\mathcal{B}$ refer to.

Definition 22 *Let $d_1, d_2 \in \mathcal{D}$. $d_1 \succeq d_2$ iff* $\texttt{Results}(\mathcal{F}_p(d_2)) \subseteq \texttt{Results}(\mathcal{F}_p(d_1))$.

Note that in our case, $\mathcal{F}_p(d) \subseteq \mathcal{A}_{\texttt{PP}}$ and $\mathcal{F}_c(d) \subseteq \mathcal{A}_{\texttt{NC}}$ for a given $d \in \mathcal{D}$.

Property 6 *Let $\texttt{AF} = (\mathcal{D}, \mathcal{A}, \texttt{Def})$ be a decision system. Let $d_1, d_2 \in \mathcal{D}$. When $\mathcal{C} = \mathcal{C}_n$, $d_1 \succeq d_2$ (according to Definition 22) iff $\sum_i c'_i(d_1) \geq \sum_i c'_i(d_2)$.*

If we focus on arguments con, the idea is to prefer the option that violates less criteria. Let $c''_i(d) = 0$ if $c_i(d) > 0$ and $c''_i(d) = 1$ if $c_i(d) < 0$. This idea is captured by the following unipolar principle.

Definition 23 *Let* $d_1, d_2 \in \mathcal{D}$. $d_1 \succeq d_2$ *iff* $\texttt{Results}(\mathcal{F}_c(d_1)) \subseteq \texttt{Results}(\mathcal{F}_c(d_2))$.

Property 7 *Let* $(\mathcal{D}, \mathcal{A}, \texttt{Def})$ *be a decision system. Let* $d_1, d_2 \in \mathcal{D}$. *When* $\mathcal{C} = \mathcal{C}_n$, $d_1 \succeq d_2$ *(according to Definition 23) iff* $\sum_i c''_i(d_1) \leq \sum_i c''_i(d_2)$.

When the criteria do not have the same level of importance, the promotion focus principle given in Definition 7 amounts to use $max_i \ c'_i(d)$ with $c'_i(d) = c_i(d)$ if $c_i(d) > 0$ and $c'_i(d) = 0$ if $c_i(d) < 0$ as an evaluation function for comparing decisions. Recall that the promotion focus principle is based on a preference relation $\geq_p$ between arguments. In what follows, we will propose a definition of the force of an argument, as well as a definition of $\geq_p$. The two definitions are chosen in such a way that they will allow us to retrieve the above idea on the promotion focus principle.

In our application, the force of an argument depends on two components: the *certainty level* of the knowledge involved in the argument, and the *importance degree* of the goal (or rejection). Formally:

Definition 24 (Force of an argument) *Let* $\delta = \langle S, d, g\rangle \in \mathcal{A}_{\texttt{PP}}$ *(resp.* $\delta = \langle S, d, r\rangle \in \mathcal{A}_{\texttt{NC}}$*). The* strength *of* δ *is a pair* $(\texttt{Lev}(\delta), \texttt{Wei}(\delta))$ *s.t.*

- *The* certainty level *of the argument is* $\texttt{Lev}(\delta) = min\{i | 1 \leq i \leq n$ *such that* $S_i \neq \emptyset\}$, *where* S_i *denotes* $S \cap \mathcal{K}_i$. *If* $S = \emptyset$ *then* $\texttt{Lev}(\delta) = n$.
- *The* weight *of the argument is* $\texttt{Wei}(\delta) = j$ *if* $g \in \mathcal{G}_j$ *(resp.* $r \in \mathcal{R}_j$*).*

The levels of satisfaction of the criteria should be balanced with their relative importance. Indeed, for instance, a criterion c_i highly satisfied by d is not a strong argument in favor of d if c_i has little importance. Conversely, a poorly satisfied criterion for d is a strong argument against d only if the criterion is really important. Moreover, in case of uncertain criteria evaluation, one may have to discount arguments based on such evaluation. In other terms, the force of an argument represents to what extent the decision will satisfy the most important criteria. This suggests the use of a *conjunctive* combination of the certainty level, the satisfaction / dissatisfaction degree and the importance of the criterion. This requires the commensurateness of the scales.

Definition 25 (Conjunctive strength) *Let* $\delta, \delta' \in \mathcal{A}_{\texttt{PP}}$.
$\delta >_p \delta'$ *iff* $min(\texttt{Lev}(\delta), \texttt{Wei}(\delta)) > min(\texttt{Lev}(\delta'), \texttt{Wei}(\delta'))$.

Property 8 *Let* $\texttt{AF} = (\mathcal{D}, \mathcal{A}, \texttt{Def})$ *be a decision system. Let* $d_1, d_2 \in \mathcal{D}$. *When* $\mathcal{C} = \mathcal{C}_n$, $d_1 \succeq d_2$ *(according to Definition 7 and using Definition 25 for the relation* $\geq_p$*) iff* $max_i \ c'_i(d_1) \geq max_i \ c'_i(d_2)$.

The prevention focus principle (see Definition 10) amounts to use $min_i\ c''_i(d)$ with $c''_i(d) = 0$ if $c_i(d) > 0$ and $c''_i(d) = -c_i(d)$ if $c_i(d) < 0$.

Property 9 *Let* $\mathtt{AF} = (\mathcal{D}, \mathcal{A}, \mathtt{Def})$ *be a decision system. Let* $d_1, d_2 \in \mathcal{D}$. *When* $\mathcal{C} = \mathcal{C}_n$, $d_1 \succeq d_2$ *(according to Definition 10 and using Definition 25 for the relation* $\geq_p$*) iff* $min_i\ c''_i(d_1) \leq min_i\ c''_i(d_2)$.

When each criterion c_i is associated with a level of importance w_i ranging on the positive part of the criteria scale, the above $c'_i(d)$ is changed into $min(c'_i(d), w_i)$ in the promotion case.

Property 10 *Let* $\mathtt{AF} = (\mathcal{D}, \mathcal{A}, \mathtt{Def})$ *be a decision system. Let* $d_1, d_2 \in \mathcal{D}$. $d_1 \succeq d_2$ *(according to Definition 7 and using Definition 25 for the relation* $\geq_p$*) iff* $max_i\ min(c'_i(d_1), w_i) \geq max_i\ min(c'_i(d_2), w_i)$.

This expresses that d is all the more preferred as there is an important criterion that is positively evaluated. A similar proposition holds for the prevention focus principle. Thus, weighted disjunctions and conjunctions defined in [27] are retrieved. It would even be possible to provide the argumentative counter-part of a general qualitative weighted conjunction of the form $min_i max(c_i(d), neg(w_i))$, where neg is the reversing map of the discrete scale where w_i takes its value. However, this would be quite similar to the qualitative decision making under uncertainty problem which is now discussed in great detail, and where aggregations having the same structure are encountered.

5 Application to decision making under uncertainty

Decision making under uncertainty relies on the comparative evaluation of different alternatives on the basis of a decision principle, which can be usually justified by means of a set of rationality postulates. This is, for example, the Savage view of decision making under uncertainty based on expected utility [45]. Thus, standard approaches for making decisions under uncertainty consist in defining decision principles in terms of analytical expressions that summarize the whole decision process, and for which it is shown that they encode a preference relation obeying postulates that are supposedly meaningful. Apart from quantitative principles such as expected utility, another example of such an approach is provided by the qualitative *pessimistic* or *optimistic* decision principles, which have been more recently proposed and also axiomatically justified [28,35]. The qualitative nature of these decision evaluations make them more liable to be unpacked in terms of arguments in favor/against each choice, in order to better understand the underpinnings of the evaluation. We successively study the pessimistic and optimistic decision principles. Note, however, that these qualitative decision criteria do not make use of a bipolar univariate scale, so in the following we apply our general decision system with an empty set of rejections. Thus, we will consider a decision theory $\mathcal{T} = \langle \mathcal{D}, \mathcal{K}, \mathcal{G} \rangle$. Consequently, the set of practical arguments built from such a theory is $\mathcal{A}_p = \mathcal{A}_{\texttt{PP}} \cup \mathcal{A}_{\texttt{PC}}$. Recall that arguments of type PP are pro their conclusions whereas arguments of type PC are cons their conclusions. In classical decision systems, the knowledge base and the goals base are assumed to be consistent. Thus, in what follows, we will assume that they are consistent as well. Thus, the three defeat relations $\texttt{Def}_e$, $\texttt{Def}_p$ and $\texttt{Def}_m$ are empty. Consequently, the decision system that will be used is $(\mathcal{D}, \mathcal{A}_e \cup \mathcal{A}_p, \texttt{Def} = \emptyset)$. In such a system, the whole decision process is reduced to the last step, which consists of ordering pairs of options. This means that in order to show how pessimistic and optimistic principles are captured, it is sufficient to choose the most suitable decision principle among the ones proposed in Section 2.5.

5.1 Pessimistic Criterion

The pessimistic decision criterion is defined as follows: given a possibility distribution π_d restricting the plausible states that can be reached when a decision d takes place, and a qualitative utility function μ, the so-called pessimistic qualitative decision principle, which estimates a kind of qualitative expectation, is defined as [28]:

$$E_*(d) = \min_{\omega} \max\left(\mu(\omega), neg(\pi_d(\omega))\right) \tag{1}$$

where π_d is a mapping from a set of interpretations Ω to a linearly ordered scale $U = \{0, 1, ..., n\}$, and μ is a mapping from Ω to the same scale U, and neg is the involutive order-reversing map on $U = \{0, 1, \ldots, n\}$ such that $neg(0) = n$ and and $neg(n) = 0$, where 0 and n are the bottom and the top elements of U. Namely, $neg(n - k) = k$. Thus, $\pi_d(\omega)$ (resp. $\mu(\omega)$) is all the greater as ω is more plausible (resp. satisfactory), 0 standing for the minimal level, and n for the maximal level. Moreover, π_d and μ are assumed to be normalized, i.e. $\exists\ \omega \in \Omega$ such that $\pi_d(\omega) = 1$, and similarly $\exists\ \omega' \in \Omega$, $\mu(\omega') = 1$.

$E_*(d)$ is all the greater as *all* the states ω that have some plausibility according to π_d are among the most preferred states according to μ. $E_*(d)$ is in fact a degree of inclusion of the fuzzy set of plausible states (when d is applied) into the fuzzy set of preferred states. The pessimistic utility $E_*(d)$ is small as soon as there exists a possible consequence of d that is both highly plausible and has a low satisfaction level with respect to preferences. This is clearly a risk-averse and thus a pessimistic attitude.

In [26], it has been shown that a stratified knowledge base has a possibility distribution as semantic counterpart. See annex for a refresher on possibilistic logic. Let $\mathcal{K}_d$ be the knowledge base built from the base $\mathcal{K}$ to which the decision d is added to the stratum $\mathcal{K}_n$. Let π_d be the possibility distribution associated with $\mathcal{K}_d$, and μ be the possibility distribution associated with the base $\mathcal{G}$ of goals. The normalization of π_d and μ is equivalent to the non-emptiness of the highest strata $\mathcal{K}_n$ and $\mathcal{G}_n$. It has been shown in [24] that it is possible to compute $E_*(d)$, as expressed by formula 1, by only using a classical logic machinery on x-level cuts of the two bases $\mathcal{K}_d$ and $\mathcal{G}$.

Proposition 1 *[24] $E_*(d)$ is the maximal value of $x \in U$ s.t.*

$$(\mathcal{K}_d)_x \vdash (\mathcal{G})_{\underline{neg(x)}} \tag{2}$$

where $(\mathcal{B})_x$ (resp. $(\mathcal{B})_{\underline{x}}$) is the set of formulas of a base $\mathcal{B}$ that appear in the strata $x, \ldots, n$ (resp. in the strata $x+1, \ldots, n$). Mind that $(\mathcal{B})_x$ is a set of strata, while $\mathcal{B}_x$ is a stratum. By convention, $E_*(d) = 0$ if there is no such x.

$E_*(d)$ is equal to n ($x = n$) if the completely certain part of $\mathcal{K}_d$ entails the satisfaction of all the goals, even the ones with low priorities.

In the pessimistic view, as pointed out by Proposition 1, we are interested in finding a decision d (if it exists) such that $\mathcal{K}_x \cup \{d\} \vdash \mathcal{G}_y$ with x high and y low, i.e. such that the decision d together with the most certain part of $\mathcal{K}$ entails the satisfaction of the goals, even those with low priority (provided that those with higher priority are also satisfied).

Example 9 (Surgery example continued) *The example is about having or not a surgery, knowing that the patient has colonic polyps. The knowledge base is $\mathcal{K} = \mathcal{K}_n \cup \mathcal{K}_x$, with $\mathcal{K}_n = \{cp, sg \rightarrow se, \neg sg \rightarrow \neg se, sg \rightarrow \neg ll, ca \wedge \neg sg \rightarrow ll\}$, and $\mathcal{K}_x = \{cp \rightarrow ca\}$, $(0 < x < n)$ where se: having side-effect, ca: cancer, ll: loss of life, sg: having a surgery, cp: having colonic polyps. The integer $x < n$ refers to a lack of complete certainty.*

The goals base is $\mathcal{G} = \mathcal{G}_n \cup \mathcal{G}_y$ with $\mathcal{G}_n = \{\neg ll\}$, and $\mathcal{G}_y = \{\neg se\}$ (where $0 < y < n$). We do not like to have side effects after a surgery, but it is more important to not lose life.

The set of decisions is $\mathcal{D} = \{sg, \neg sg\}$.

Note that $(\mathcal{K}_{sg})_n \vdash (\mathcal{G})_n$. However, $(\mathcal{K}_{sg})_n \nvdash (\mathcal{G})_y$. Note also that $(\mathcal{K}_{sg})_1 \vdash (\mathcal{G})_n$ while $(\mathcal{K}_{sg})_1 \nvdash (\mathcal{G})_y$. It is clear that $(\mathcal{K}_{sg})_n \nvdash (\mathcal{G})_{\underline{neg(n)}} = (\mathcal{G})_1$. Thus, the only value that satisfies Proposition 1 is $neg(y)$. Indeed, $(\mathcal{K}_{sg})_{neg(y)} \vdash (\mathcal{G})_{\underline{y}}$. Consequently, $E_*(sg) = neg(y)$.

In its side, the option $\neg sg$ violates the most important goal ($\neg ll$). Thus, there is no value that satisfies Proposition 1. Consequently, $E_*(\neg sg) = 0$.

We are going to show that the result of Proposition 1 can be captured in terms of arguments pro, i.e., arguments of type PP are underlying the pessimistic criterion. We first relate the absence of PP arguments to some consequences on the value of E_*. We then conversely relate the value of E_* to the existence of some PP arguments. Lastly, we show that the comparison of decisions in terms of tcriterion E_* can also be handled directly in terms of PP arguments.

Let us first recall the definition of the strength of an argument already used in the previous section (Definition 24).

Definition 26 (Strength of an Argument) *Let $\delta = \langle S, d, g\rangle \in \mathcal{A}_p$. The* strength *of δ is a pair (*$\texttt{Lev}(\delta)$, $\texttt{Wei}(\delta)$*) s.t.*

- *The* certainty level *of the argument is* $\texttt{Lev}(\delta) = min\{i | 1 \leq i \leq n$ *such that* $S_i \neq \emptyset\}$*, where S_i denotes $S \cap \mathcal{K}_i$. If $S = \emptyset$ then* $\texttt{Lev}(\delta) = n$.
- *The* weight *of the argument is* $\texttt{Wei}(\delta) = y$ *s.t* $g \in \mathcal{G}_y$.

The two following theorems state that the absence of strong PP argument can only weaken $E_*(d)$.

Theorem 4 *Let $d \in \mathcal{D}$. If $\exists g \in \mathcal{G}_k$ s.t. $\nexists\ \delta = \langle S, d, g\rangle \in \mathcal{A}_{\texttt{PP}}$ then $E_*(d) \leq neg(k)$.*

Proof *By reduction ab absurbo. Assume $E_*(d) \not\leq neg(k)$. Then $E_*(d) \geq neg(k) + 1$. By Proposition 1, $(\mathcal{K}_d)_{neg(k)+1} \vdash (\mathcal{G})_{\underline{neg(neg(k)+1)}}$. But $neg(neg(k) + 1) = k - 1$. Thus, $(\mathcal{K}_d)_{neg(k)+1} \vdash (\mathcal{G})_k$. This clearly contradicts the hypothesis that $\exists g \in \mathcal{G}_k$ s.t. $\nexists\ \delta = \langle S, d, g\rangle \in \mathcal{A}_{\texttt{PP}}$.* ■

Theorem 5 *Let $d \in \mathcal{D}$. If $\forall g \in \mathcal{G}$, $\nexists\ \delta = \langle S, d, g\rangle \in \mathcal{A}_{\texttt{PP}}$ with $\texttt{Lev}(\delta) > l$ and $l \geq 1$, then $E_*(d) \leq l$.*

Proof *By reduction ab absurbo also. Assume that $E_*(d) \not\leq l$. Then $E_*(d) \geq l + 1$. By Proposition 1, $(\mathcal{K}_d)_{(l+1)} \vdash (\mathcal{G})_{\underline{neg(l+1)}}$. Since $l \geq 1$, $neg(l + 1) \leq n - 2$. Thus, $(\mathcal{K}_d)_{(l+1)} \vdash (\mathcal{G})_{(n-1)}$. This contradicts the hypothesis that $\forall g$, $\nexists\ \delta = \langle S, d, g\rangle \in \mathcal{A}_{\texttt{PP}}$ with $\texttt{Lev}(\delta) > l$, which means that $\forall\ \langle S, d, g\rangle \in \mathcal{A}_{\texttt{PP}}$, $\texttt{Lev}(\delta) \leq l$, since $(\mathcal{G})_{(n-1)}$ cannot be empty.* ■

The third theorem states that the value of $E_*(d)$ is determined by the existence of sufficiently certain PP arguments in favor of decision d with respect to all important goals whose priority is above some value, and the absence of any more certain PP argument in favor of decision d with respect to some goal whose priority may be smaller.

Theorem 6 *Let $d \in \mathcal{D}$. If $E_*(d) = x$, then $\forall g \in (\mathcal{G})_{neg(x)+1}$, $\exists\ \delta = \langle S, d, g\rangle \in \mathcal{A}_{\texttt{PP}}$, and $\texttt{Lev}(\delta) \geq x$. Moreover, $\exists g \in \mathcal{G}$ s.t. $\nexists\ \delta = \langle S, d, g\rangle \in \mathcal{A}_{\texttt{PP}}$ with $\texttt{Lev}(\delta) \geq x+1$ and $\texttt{Wei}(\delta) \geq neg(x)$.*

Proof *Assume that $E_*(d) = x$. By Proposition 1, $(\mathcal{K}_d)_x \vdash (\mathcal{G})_{\underline{neg(x)}}$. Thus, $\forall g \in (\mathcal{G})_{neg(x)+1}$, $\exists\ \delta = \langle S, d, g\rangle \in \mathcal{A}_{\texttt{PP}}$ and $\texttt{Lev}(\delta) \geq x$. Besides, $(\mathcal{K}_d)_{x+1} \nvdash (\mathcal{G})_{\underline{neg(x+1)}}$, and then $\exists g \in \mathcal{G}$ s.t. $\nexists\ \delta = \langle S, d, g\rangle \in \mathcal{A}_{\texttt{PP}}$ with $\texttt{Lev}(\delta) > x$ and $\texttt{Wei}(\delta) \geq neg(x+1)+1 = neg(x)$.* ■

However, as illustrated by the following example, $E_*(d) = x$ does not necessarily mean that there does not exist a sufficiently good PP argument for some goal in $\mathcal{G}_{neg(x)}$.

Example 10 *Let $n = 4$. Let $\mathcal{G}_4 = \{g_1\}$, $\mathcal{G}_3 = \{g_2\}$, $\mathcal{G}_2 = \{g_3\}$. Assume $(\mathcal{K})_4 \vdash \{g_2, g_3\}$, and $(\mathcal{K})_2 \vdash \{g_1\}$, but $(\mathcal{K})_3 \nvdash \{g_1\}$. Then $(\mathcal{K})_2 \vdash (\mathcal{G})_3$, i.e. $(\mathcal{K})_2 \vdash (\mathcal{G})_{\underline{2}}$, and then $E_*(d) = 2$. Note that here $(\mathcal{K})_2 \vdash (\mathcal{G})_{\underline{1}}$, but $(\mathcal{K})_3 \nvdash (\mathcal{G})_{\underline{1}}$ (here $neg(3) = 1$), since the most important goal has only a rather weak proof when d takes place, namely $(\mathcal{K})_3 \nvdash \{g_1\}$, although stronger proofs exist for less important goals: $(\mathcal{K})_2 \vdash (\mathcal{G})_{\underline{1}}$, and thus $E_*(d) \neq 3$.*

The above results show the links between PP arguments supporting candidate decisions and the pessimistic values E_* assigned to those decisions. In what follows, we will show that an instantiation of our decision system returns the same ordering on the set $\mathcal{D}$ as the one obtained by comparing the pessimistic values of elements of $\mathcal{D}$. As already said, this amounts to choose the most appropriate decision principle. In the case of pessimistic decision making, the most suitable principle is the one proposed in Definition 7. Let us recall that principle:

Definition 27 *Let $\texttt{AF} = (\mathcal{D}, \mathcal{A}, \texttt{Def})$ be a decision system and $\texttt{Acc}(\texttt{AF})$ its accepted arguments. Let d_1, $d_2 \in \mathcal{D}$.*

$$d_1 \succeq d_2 \text{ iff } \exists \delta \in \mathcal{F}_p(d_1) \cap \texttt{Acc}(\texttt{AF}) \text{ such that } \forall \delta' \in \mathcal{F}_p(d_2) \cap \texttt{Acc}(\texttt{AF}), \delta \geq_p \delta'.$$

Note that this principle is based on a preference relation $\geq_p$ between practical arguments. For our purpose, this relation prefers the argument that is based on a subset of $\mathcal{K}$ made of beliefs that are more certain and that together entail a goal having a higher priority. Formally, using the usual Pareto strict partial order between vectors:

Definition 28 (Comparing arguments of type PP) *Let $\delta, \delta' \in \texttt{PP}$. δ is* stronger *than δ', denoted $\delta >_p \delta'$, if and only if $(\texttt{Lev}(\delta), neg(\texttt{Wei}(\delta))) >_{Pareto} (\texttt{Lev}(\delta'), neg(\texttt{Wei}(\delta')))$.*

Let us now relate the result of our decision system to that of pessimistic decision making.

Theorem 7 *Let $\mathcal{T} = \langle \mathcal{D}, \mathcal{K}, \mathcal{G} \rangle$ be a decision theory, and* $\texttt{AF} = (\mathcal{D}, \mathcal{A}_e \cup \mathcal{A}_p, \texttt{Def} = \emptyset)$ *be the decision system. If $\forall g \in \mathcal{G}$ s. t. $\forall\ \delta' = \langle S, d_2, g \rangle \in \mathcal{A}_{\texttt{PP}}$, $\exists\ \delta = \langle S', d_1, g \rangle \in \mathcal{A}_{\texttt{PP}}$ and $\delta >_p$* [8] *δ', then $E_*(d_1) \geq E_*(d_2)$.*

Proof *Assume $0 < E_*(d_1) < E_*(d_2)$. Then $\exists x$ s. t. $E_*(d_1) = x$. Then $(\mathcal{K}_{d_1})_x \vdash (\mathcal{G})_{\underline{neg(x)}}$ and $(\mathcal{K}_{d_1})_{x+1} \nvdash (\mathcal{G})_{\underline{neg(x+1)}}$, while $(\mathcal{K}_{d_2})_{x+1} \vdash (\mathcal{G})_{\underline{neg(x+1)}}$. This contradicts the hypothesis. Indeed $\forall \delta \in \mathcal{A}_{\texttt{PP}}$ s.t. $\texttt{Conc}(\delta) = d_1$, $\texttt{Lev}(\delta) < x+1$, but $\exists \delta' \in \mathcal{A}_{\texttt{PP}}$ s.t. $\texttt{Conc}(\delta) = d_2$ and $\texttt{Lev}(\delta) = x+1$. Assume $0 = E_*(d_1) < E_*(d_2)$. Then $(\mathcal{K}_{d_1})_{\underline{0}} \nvdash (\mathcal{G})_n$, and $\nexists \delta \in \mathcal{A}_{\texttt{PP}}$ s.t. $\texttt{Conc}(\delta) = d_1$.* ■

The converse of the above theorem is false as shown by the example below, where $E_*(d_1) > E_*(d_2)$.

Example 11 *Let $\mathcal{G} = \{g_1, g_2\}$, $\mathcal{G}_n = \{g_1\}$, $\mathcal{G}_{n-1} = \{g_2\}$. Assume $(\mathcal{K}_{d_1})_n \vdash \{g_1\}$, $(\mathcal{K}_{d_1})_{n-2} \vdash \{g_2\}$ (but $(\mathcal{K}_{d_1})_n \nvdash \{g_2\}$ and $(\mathcal{K}_{d_1})_{n-1} \nvdash \{g_2\}$). Similarly, $(\mathcal{K}_{d_2})_n \vdash \{g_2\}$, $(\mathcal{K}_{d_2})_{n-1} \vdash \{g_1\}$, and $(\mathcal{K}_{d_2})_n \nvdash \{g_1\}$. We can take $n = 3$. Thus, $E_*(d_1) = 2$ since $(\mathcal{K}_{d_1})_2 \vdash (\mathcal{G})_{\underline{2}}$, while $E_*(d_2) = 1$ since $(\mathcal{K}_{d_1})_1 \vdash (\mathcal{G})_{\underline{1}}$. So, $E_*(d_1) > E_*(d_2)$. Let $\delta_2 = \langle (\mathcal{K})_3, d_2, g_2 \rangle \in \mathcal{A}_{\texttt{PP}}$, so $\texttt{Lev}(\delta_2) = 3$ and $neg(\texttt{Wei}(\delta_2)) = neg(2) = 1$. But, regarding d_1, $\nexists\ \delta_1 \in \mathcal{A}_{\texttt{PP}}$ s.t. $\texttt{Conc}(\delta_1) = d_1$ and $\texttt{Lev}(\delta_1) = 3$ and $neg(\texttt{Wei}(\delta_1)) = 2 = neg(1)$, i.e. s.t. $\delta_1 >_p \delta_2$ (according to Definition 28). Indeed, the best arguments for d_1 are $\delta_1 = \langle (\mathcal{K})_3, d_1, g_1 \rangle$ with $\texttt{Lev}(\delta_1) = 3$ and $neg(\texttt{Wei}(\delta_1)) = 0$, and $\delta'_1 = \langle (\mathcal{K})_1, d_1, g_2 \rangle$ with $\texttt{Lev}(\delta'_1) = 2$ and $neg(\texttt{Wei}(\delta'_1)) = 1$.*

Going back to our running example, we have,

Example 12 (Surgery example continued) *In the above example, there is an argument of type* PP *in favor of sg: $\delta = \langle \{sg \rightarrow \neg ll\}, sg, \neg ll \rangle$, and there is an argument of type* PP *in favor of $\neg sg$: $\delta' = \langle \{\neg sg \rightarrow \neg se\}, \neg sg, \neg se \rangle$.*

The strength of δ is $<n, n>$, whereas the strength of δ' is $<n, \sigma>$. Thus, δ is preferred to δ' (according to Definition 28). Consequently, the decision sg is preferred to the decision $\neg sg$.

The agreement between the pessimistic qualitative decision criterion and the argument-based view is due to a decomposability property of arguments of type PP w.r.t the conjunction of goals. Namely, $\mathcal{K}_x \cup \{d\} \vdash g$ and $\mathcal{K}_x \cup \{d\} \vdash g'$ is equivalent to $\mathcal{K}_x \cup \{d\} \vdash g \wedge g'$. Indeed, the pessimistic evaluation sanctions the fact that all the most important goals are satisfied for sure up to a level where this is no longer true. However, things are not as simple with consistency since one may have $\mathcal{K}_x \cup \{d\}$ consistent with both g and g' separately without

[8] According to Definition 28.

having it consistent with $g \wedge g'$. This means that the absence of arguments of type PC is only a necessary condition for consistency w.r.t the whole set of goals. Thus, the optimistic criterion can only be approximated in terms of the evaluation of elementary practical arguments. Indeed, as it will be recalled in the next section the optimistic evaluation refers to the fact that the most certain part of $\mathcal{K}$, and the most important goals are consistent together in presence of a candidate decision.

5.2 Optimistic Criterion

The optimistic qualitative criterion [28] is given by

$$E^*(d) = \max_{\omega} \min \left(\mu(\omega), \pi_d(\omega)\right). \tag{3}$$

It is a consistency evaluation since it amounts to estimate to what extent the intersection of the fuzzy set of good states (in the sense of μ) with the fuzzy set of plausible states (when d is applied) is not empty. The criterion $E^*(d)$ corresponds to an optimistic attitude since it is high as soon as there exists a possible consequence of d that is both highly plausible and highly prized. $E^*(d)$ is equal to n (is maximal) as soon as one fully acceptable choice ω (i.e., such that $\mu(\omega) = n$) is also completely plausible. As for the pessimistic case, the optimistic utility can be expressed in logical terms.

Proposition 2 *[24] $E^*(d)$ is equal to the greatest $x \in U$ such that $(\mathcal{K}_d)_{\underline{neg(x)}}$ and $(\mathcal{G})_{\underline{neg(x)}}$ are logically consistent together.*

The above proposition means that in the optimistic point of view, we are interested in finding a decision d (if it exists) which is consistent with the knowledge base and the goals (i.e. $\mathcal{K} \wedge \{d\} \wedge \mathcal{G} \neq \bot$). This is optimistic in the sense that it assumes that goals may be attained as soon as their negation cannot be proved.

Example 13 (Surgery example continued) *In this example, $E^*(sg) = neg(y)$ and $E^*(\neg sg) = neg(x)$. Thus the best decision in the optimistic case depends on the values x and y.*

In order to capture the result of Proposition 2, arguments of type PC are needed. The strength of such arguments is given using Definition 24.

Theorem 8 *Let $d \in \mathcal{D}$. If $\exists\ \delta \in \mathcal{A}_{\mathtt{PC}}$ s.t. $\mathtt{Conc}(\delta) = d$, then $E^*(d) \leq max(neg(\mathtt{Lev}(\delta)), neg(\mathtt{Wei}(\delta)))$.*

Proof $E^*(d) = max_{\omega} min(\pi_{\mathcal{K}_d}(\omega), \mu_{\mathcal{G}}(\omega)) = max[max_{\omega:\omega\models(\mathcal{K}_d)_\beta}\ min(\pi_{\mathcal{K}_d}(\omega),$

$\mu_{\mathcal{G}}(\omega))$, $max_{\omega:\omega\not\models(\mathcal{K}_d)_\beta}$ $min(\pi_{\mathcal{K}_d}(\omega), \mu_{\mathcal{G}}(\omega))]$. Let $\delta = \langle S, d, g\rangle$ be a `PC` *argument with* $\texttt{Lev}(\delta) = x$ *and* $\texttt{Wei}(\delta) = y$. *Then, $\mathcal{K}_x \cup \{d\} \vdash \neg g$. Thus, $\forall\omega$, $\omega \models (\mathcal{K}_d)_x$, then $\pi_{\mathcal{K}_d}(\omega) \leq neg(x)$. Thus, $E^*(d) \leq max[min(n, neg(y)), min(neg(x), n)] = max(neg(y), neg(x))$.* ■

Conversely, we have the following theorem

Theorem 9 *Let $d \in \mathcal{D}$. If $E^*(d) = x$, then there is a* `PC` *argument $\delta = \langle S, d, g\rangle$ for d such that* $\texttt{Wei}(\delta) \leq neg(x)$.

Proof *If $E^*(d) = x$, then x is the maximal value such as $(\mathcal{K}_d)_{\underline{neg(x)}} \cup (\mathcal{G})_{\underline{neg(x)}}$ is consistent, from Proposition 2. This entails that $(\mathcal{K}_d)_{neg(x')} \vdash \neg\mathcal{G}_{\underline{neg(x')}}$ for $neg(x) \geq neg(x')$.* ■

Example 14 (Surgery example continued) *In the above example, there is one strong argument against the decision 'sg': $<\{sg \rightarrow se\}, sg, \neg se>$. There is also a unique strong argument against the decision $\neg sg$: $<\{cp, cp \rightarrow ca, ca \wedge \neg sg \rightarrow ll\}, \neg sg, \neg ll>$.*

The level of the argument $<\{sg \rightarrow se\}, sg, \neg se>$ is n whereas its weight is y. Concerning the argument $<\{cp, cp \rightarrow ca, ca \wedge \neg sg \rightarrow ll\}, \neg sg, \neg ll>$, its level is x, and its weight is n.

In this example, the comparison of the two arguments amounts to compare x with y. Namely, if y (the priority of the goal "no side effect") is small then the best decision will be to have a surgery. If the certainty degree x of having cancer in presence of colonic polyps for the particular patient is small enough then the best optimistic decision will not be to have a surgery.

In order to retrieve the exact value of $E^*(d)$ as weight of an argument, we would have to use a non-elementary notion of arguments, described in [7], that considers as a whole the goals base $\mathcal{G}$.

6 Related Works

Different works have combined the ideas of argumentation and decision in artificial intelligence systems. In particular, Fox and Parsons [32] have developed an inference-based decision support machinery, which have been implemented in medical applications (see e.g. Fox and Das in [31]). In this approach the knowledge base is made of recommendation rules that conclude on candidate decisions. However, in [32,33], no explicit distinction is made between knowledge and goals. However, in their examples, values (belonging to a linearly ordered scale) are assigned to formulas which represent goals. These values provide an empirical basis for comparing arguments using a symbolic combination of strengths of beliefs and goals values. This symbolic combination is performed through dictionaries corresponding to different kinds of scales that may be used. Only one type of arguments in favor of or against is used. Another recent example of argument-based decision system that is purely based on an inference system is proposed by Chesnevar et al. in [21] for advising about language usage assessment on the basis of corpus available on the web.

We now survey works that handle classical multiple decision or decision making under uncertainty problems in an argumentative manner. This means that recommended decisions have to be found or explained from user's preferences and information about the current state of the world. Moreover, no "precompiled" rules that explicitly recommend decisions in a given situation are supposed to be available in these works.

In [15], Bonet and Geffner have also proposed an original approach to qualitative decision, inspired from Tan and Pearl [47], based on "action rules" that link a situation and an action with the satisfaction of a *positive* or a *negative* goal. However in contrast with the previous work and the work presented in this paper, this approach does not refer to any model in argumentative inference. In their framework, there are four parts:

(1) a set $\mathcal{D}$ of *actions* or *decisions*.
(2) a set $\mathcal{I}$ of input propositions defining the possible input situation. A degree of plausibility is associated with each input. Thus, $\mathcal{I} = \{(k_i, \alpha_i)\}$ with $\alpha_i \in \{likely, plausible, unlikely\}$.
(3) a set $\mathcal{G}$ of *prioritized goals* such that $\mathcal{G} = \mathcal{G}^+ \cup \mathcal{G}^-$. $\mathcal{G}^+$ gathers the *positive* goals that one wants to achieve and $\mathcal{G}^-$ gathers the *negative* goals that one wants to avoid. Thus, $\mathcal{G} = \{(g_i, \beta_i)\}$ with $\beta_i \in [0, 1, ...N]$.
Note that in our framework what they call here negative goals are considered in our goal base as negative literals.
(4) a set of *action rules* $\mathcal{AR} = \{(A_i \wedge C_i \Rightarrow x_i, \lambda_i), \lambda_i \geq 0\}$, where A_i is an action, C_i is a conjunction of input literals, and x_i is a goal. Each action rule has two measures: a *priority* degree which is exactly the priority degree of the goal x_i, and a *plausibility* degree. This plausibility is defined as follows: A rule $A \wedge C \Rightarrow x$ is likely if any conjunct of C is likely. A rule $A \wedge C \Rightarrow x$ is unlikely if some conjunct of C is unlikely. A rule $A \wedge C \Rightarrow x$ is plausible if it is neither likely nor unlikely.

In this approach only input propositions are weighted in terms of plausibility. Action rules inherit these weights through the three above rules in a rather empirical manner which depends on the chosen plausibility scale. The action rules themselves are not weighted since they are potentially understood as defeasible rules, although no non-monotonic reasoning system is associated with them.

In contrast, our approach makes use of an abstract scale. Moreover, weighted possibilistic clauses have been shown to be able to properly handle non-monotonic inference in the sense of Kraus, Lehmann and Magidor [37]' preferential system augmented with rational monotony. So a part of our weighted knowledge may be viewed as the encoding of a set of default rules. From the above four bases, reasons are constructed for (against) actions in [15]. Indeed, goals provide reasons for (or against) actions. Positive goals provide reasons *for* actions, whereas negative goals provide reasons *against* actions. The basic idea behind this distinction is that negative goals should be discarded, and consequently any action which may lead to the satisfaction of such goals should be avoided. However, the approach makes no distinction between what we call pessimism and optimism. The definition of a 'reason' in [15] is quite different from our definition of an argument. Firstly, a reason considers only one goal and secondly, the definition is poor since it only involves facts. Finally, in Bonet and Geffner's framework, decisions which satisfy the most important goals are privileged. This is also true in our approach, but the comparison between decisions can be further refined, in case of several decisions yielding to the satisfaction of the most important goals, by taking into account the other goals which are not violated by these decisions.

Amgoud and Prade in [7] have already proposed an argumentation-based reading of possibilistic decision criteria. However, their approach has some drawbacks from a pure argumentation point of view. In their approach, there was only one type of arguments pros and one type of arguments con. Moreover, these arguments were taking into account the goal base as a whole, and a consequence for a given decision there was at most a unique argument pros and a unique argument cons. This does not really fit with the way human are discussing decisions, for which there are usually several arguments pro and cons, rather than a summarized one. On the contrary in this paper, we have discussed all the possible types of arguments pro and cons in a systematic way, and each argument pertains to only one goal.

Dubois and Fargier [25] have studied a framework where a candidate decision d is associated with two distinct sets of positive arguments (pros) and negative arguments (cons). It is assumed that positiveness and negativeness are not a matter of degrees. If one considers that the arguments refers to criteria, this means that an implicit scale $\{-, 0, +\}$ would be used for evaluating a candidate decision according to each criterion. Moreover, there is no uncertainty. However, a function π assesses the level of importance of each argument for the decision maker. Importance ranges on a totally ordered scale from "no important at all" to a maximal level of importance. If $\pi(x) > \pi(y)$, "the strength of x is considered at least one order of magnitude higher than the one of y, so that y is negligible in front of x". The authors provide an axiomatic characterization of different natural rules in this setting, with a possibility theory interpretation of their meaning. In particular, a "bipolar lexicographic" preference relation (which is among the decision principles discussed in Section 6) is characterized. It amounts to compare two candidate decisions d and d' by comparing the difference of the cardinalities of the sets of positive and negative arguments they have (thus allowing for cancellation between positive and negative arguments), starting with the highest level of importance; in case of equality at a given level, the level immediately below is considered to solve the ties and so on. In direct relation to this work, an empirical study of the different decision rules considered (Bonnefon and Fargier [16]) has shown that the bipolar lexicographic rule is largely favored by humans in practice.

Another trend of works relating argumentation and decision is mainly interested in the use of arguments for explaining and justifying multiple criteria decisions once they have been made using some definite aggregation function. A systematic study for different aggregation functions can be found in [40,39]. The implemented system developed by Carenini and Moore [19] is an example of such a use for an aggregation process based on weighted sums associated to value trees.

7 Conclusion

The paper has proposed an abstract argumentation-based framework for decision making. The main idea behind this work is how to define a complete preorder on a set of candidate decisions on the basis of arguments. The framework distinguishes between two types of arguments: epistemic arguments that support beliefs and practical arguments that justify candidate decisions. Each practical argument concerns only one candidate decision, and may be either in favor of that decision or against it. The framework follows two main steps:

(1) An inference step in which arguments are evaluated using acceptability semantics. This step amounts to return among the practical arguments, those which are warranted in the current state of information, i.e. the "accepted" arguments.
(2) A pure decision step in which candidate decisions are compared on the basis of accepted practical arguments.

For the second step of the process, we have proposed three families of principles for comparing pairs of choices. An axiomatic study and a cognitive validation of these principles are worth developing, in particular in connection with [16,25].

The abstract framework is then instantiated in order to handle decision under uncertainty and multiple criteria decision making. For that purposes, the framework emphasizes clearly the bipolar nature of the consequences of choices by distinguishing goals to be pursued from rejections to be avoided. These bipolar preferences are encoded by two sets of stratified formulas stating goals and rejections with their level of importance. In addition, the knowledge about the current state of the world in encoded in another stratified base which may be inconsistent. The bipolar nature of the setting has led us to identify two types of arguments pro a choice (resp. against a choice).

The proposed approach is very general and includes as particular cases already studied argumentation-based decision systems. Moreover it is suitable for multiple criteria decision making as well as decision making under uncertainty. In particular, the approach has been shown to fully agree with qualitative decision making under uncertainty, and to distinguish between a pessimistic and an optimistic attitude of the decision maker.

Although our model is quite general, it may be still worth extending along different lines. First, the use of default knowledge could be developed. Second, our approach does not take into account rules that recommend or disqualify decisions in given contexts. Such rules should incorporate modalities for distinguishing between strong and weak recommendations. Moreover, they are fired by classical argumentative inference. This contrasts with our approach where the only arguments pertaining to decisions have an abductive structure. Recommendation rules may also turn to be inconsistent with other pieces of knowledge in practical arguments pro or cons w.r.t. a decision. Lastly, agents may base their decision on two types of information, namely generic knowledge and a repertory of concrete reported cases. Then, past observations recorded in the repertory may be the basis of a new form of arguments by exemplification of cases where a decision has succeeded or failed. This would amount to relate argumentation and case-based decision.

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Appendix: Brief refresher on possibility logic

A possibilistic logic base $\mathcal{K}$ can be viewed as a stratified set of classical logical formulas, such that $\mathcal{K} = \mathcal{K}_1 \cup \ldots \cup \mathcal{K}_i \cup \ldots \cup \mathcal{K}_n$, with $\forall i, j$, $\mathcal{K}_i \cap \mathcal{K}_j = \emptyset$. It is assumed that formulas in $\mathcal{K}_i$ are associated with a higher level of certainty or priority than formulas in $\mathcal{K}_{i-1}$. Thus, $\mathcal{K}_n$ contains the formulas with the highest level, and $\mathcal{K}_1$ the ones with the smallest non-zero level.

Let ρ be a function that returns the rank of stratum to which a formula belongs, i.e. $\rho(k_j) = i$ such that $k_j \in \mathcal{K}_i$. In the following, $\rho(k_j)$ will be denoted for short ρ_j. Thus, $\mathcal{K}$ can be rewritten as $\mathcal{K} = \{(k_j, \rho_j); j = 1, l\}$, as often done in possibilistic logic [26]. $\mathcal{K}$ may represent the available knowledge about the world, or goals having different levels of priority. The pair (k_j, ρ_j) is understood as $N(k_j) \geq \rho_j$, where N is a necessity measure obeying the characteristic decomposability axiom $N(p \wedge q) = min(N(p), N(q))$. Namely (k_j, ρ_j) encodes that the piece of knowledge "k_j is true" is certain or prioritized at least at level ρ_j, where ρ_j belongs to a linearly ordered valuation scale whose top and bottom elements are resp. n and 1. At the semantic level, a possibilistic base $\mathcal{K}$ is associated with a possibility distribution defined by

$$\pi_{\mathcal{K}}(\omega) = min_{j=1,l}\ max(v_\omega(k_j), neg(\rho_j)),$$

where neg is the order reversing map of the scale $(0, 1, \ldots, n)$, and where $v_\omega(k_j)$ $= n$ if ω is a model of k_j and $v_\omega(k_j) = 0$ if ω falsifies k_j. An interpretation ω is thus all the less plausible or satisfactory, as it falsified a proposition k_j associated with a high level ρ_j. It rank-orders the more or less plausible states of the world.

The equivalence between a possibilistic logic base and its possibility distribution-based semantic counter-part has been established in terms of correction and completeness of the inference mechanism that is associated with these representations [26]. This inference mechanism is governed at the syntactic level by the resolution rule $(\neg p \vee q, \rho)$, $(p \vee r, \lambda) \vdash (q \vee r,\ \min(\rho, \lambda))$.

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