BRUEGEL'S DRAWINGS UNDER THE MULTIFRACTAL MICROSCOPE

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SUMMARY Recently, a growing interest in *image processing* tools for *art analysis* has emerged. Here, we investigate the use of the *wavelet leader based multifractal formalism* for this purpose, a mathematical tool for characterizing the *regularity properties* of homogeneous textures. We apply this tool to a set of digitized version of authentic *drawings by Bruegel* and imitations. Multifractal attributes estimated on the paintings enable us to *discriminate the authentic drawings from imitations*, give interesting insights into the regularity properties of their textures and thus show that multifractal analysis is a promising tool for *stylometry*.

MULTIFRACTAL ANALYSIS OF IMAGES

MULTIFRACTAL SPECTRUM

- LOCAL REGULARITY:

  locally bounded function $X(\mathbf{x})$, $\mathbf{x} = (x_1, x_2)$

  $\longrightarrow$ *local power law behavior*

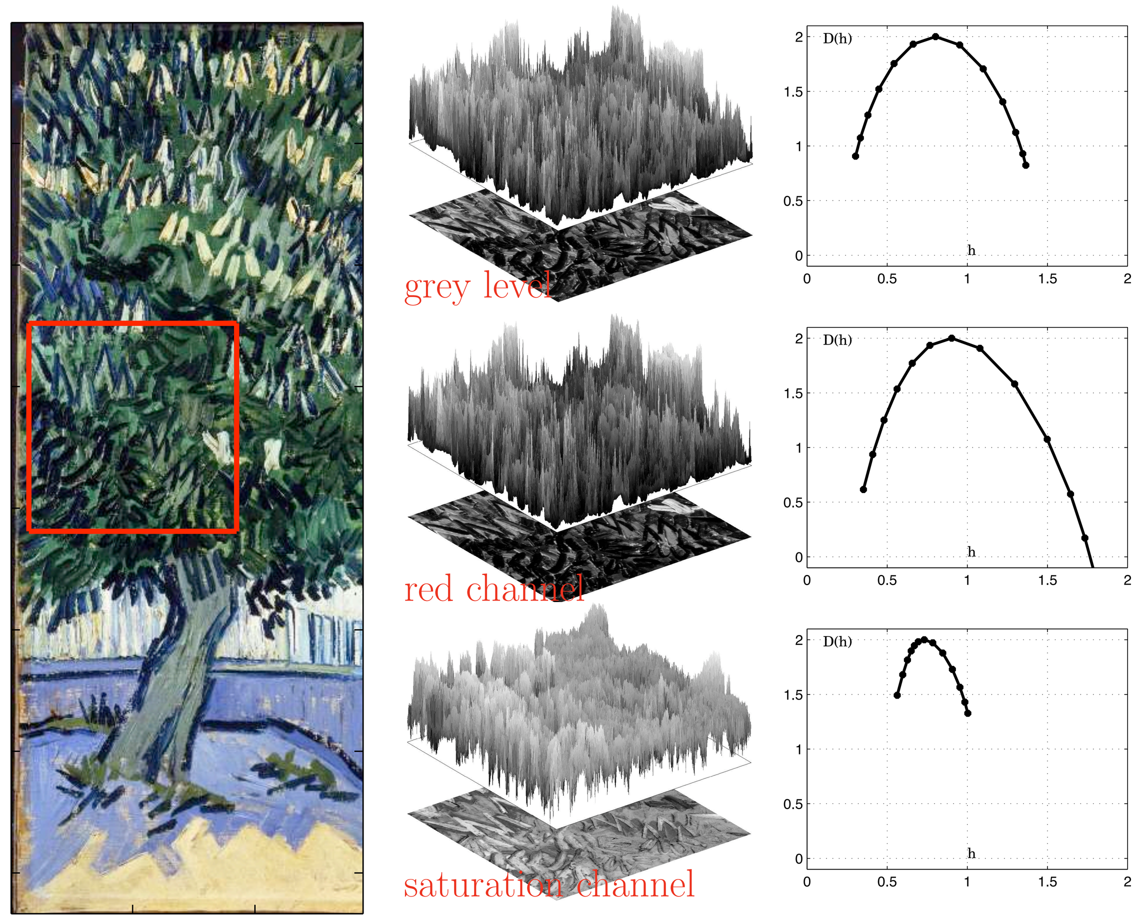
  $|X(\mathbf{x}) - X(\mathbf{x}_0)| \le C|\mathbf{x} - \mathbf{x}_0|^\alpha \quad C > 0, \quad \alpha > 0$

  largest such α: Hölder exponent $h(\mathbf{x}_0)$

- MULTIFRACTAL SPECTRUM:

  $\longrightarrow$ *geometric structure of subsets* E_h : $h(\mathbf{x}_i) = h$

$$\mathcal{D}(h) = \dim_{Hausdorff}\{\mathbf{x} : h(\mathbf{x}) = h\} \quad (1)$$



[Van Gogh F752 — within the Image Processing for Art Investigation (IP4AI) research program (www.digitalpaintinganalysis.org)]

MULTIFRACTAL FORMALISM

- WAVELET LEADERS:

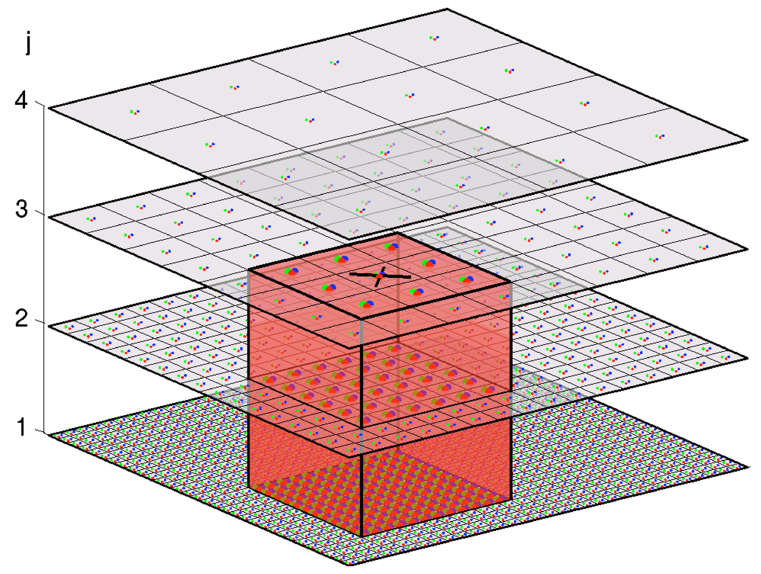
$$L_X(j, k_1, k_2) = \sup_{m, \lambda' \subset 3\lambda_{j,k_1,k_2}} |d_X^{(m)}(\lambda')| \quad (3)$$

$d_X^{(m)}(j, \mathbf{k})$ – DWT coefficients of locally bounded function (2D orthonormal wavelet basis, L^1 normalized)

λ_{j,k_1,k_2} – dyadic cube $[k_1 2^j, (k_1+1)2^j) \times [k_2 2^j, (k_2+1)2^j)$

$3\lambda_{j,k_1,k_2}$ – union with eight closest neighbors

$\longrightarrow$ *local supremum of wavelet coefficients*



- MULTIFRACTAL FORMALISM:

  Scaling function $\quad (S(2^j, q) = \frac{1}{n_j} \sum_k L_X(j, k_1, k_2)^q)$

$$\zeta(q) = \liminf_{2^j \to 0} \log_2 S(2^j, q) / \log_2 2^j \quad (4)$$

  Legendre transform:

$$\mathcal{L}(h) = \min_q (2 + qh - \zeta(q)) \ \ge \ \mathcal{D}(h) \quad (5)$$

  $\longrightarrow$ *upper bound for multifractal spectrum*

MINIMUM REGULARITY

$\mathcal{D}(h)$, L_X: locally bounded functions only!

- MINIMUM REGULARITY

$$h_m = \liminf_{2^j \to 0} \frac{\ln \sup_k |d_X(j, k_1, k_2)|}{\ln 2^j} \quad (2)$$

  $\longrightarrow$ *X locally bounded: $h_m > 0$*

- FRACTIONAL INTEGRATION

  - if $h_m < 0$:

    $\longrightarrow$ fractional integral of order $\gamma = \max(0, -h_m)$

    $\longrightarrow$ $FI_\gamma(X)$ locally bounded

  - equivalently: apply multifractal formalism (3-5) to

$$d_X^{(m),\gamma}(j, \mathbf{k}) = 2^{\gamma j} d_X^{(m)}(j, \mathbf{k})$$

CUMULANT EXPANSION

Polynomial expansion around $q = 0$:

- $\zeta(q) = \sum_{p \ge 1} c_p \frac{q^p}{p!}$

- $\mathcal{L}(h) \simeq 2 - (h - c_1)^2 / (2|c_2|) + \cdots$

  c_1 – *position of maximum*

  c_2 – *typical width*

  c_3 – *asymmetry*

- $C_p(2^j)$ – p-th cumulant of $\ln L_X(j, \mathbf{k})$

$$C_p(2^j) = c_p^0 + c_p \ln 2^j \quad (6)$$

ESTIMATION

Eqs. (2), (4), (6) $\longrightarrow$ *linear regressions* (cf. e.g. [1,2])

TRUE BRUEGEL VS. FORGERIES

FRACTAL & SCALING PROPERTIES

- ANALYSIS: grey level intensity images

  3 patches 1024×1024 pixel per drawing

  $N_\psi = 2$, $\gamma = 0.75$

  $\longrightarrow$ estimates consistent for different patches of single drawing

- POWER LAW BEHAVIORS:

  $\longrightarrow$ scales 16×16 to 128×128 pixel (3 octaves)

  $\longrightarrow$ *fine scales*

  $\longrightarrow$ *hand style of artist*

MULTIFRACTAL PROJECTIONS

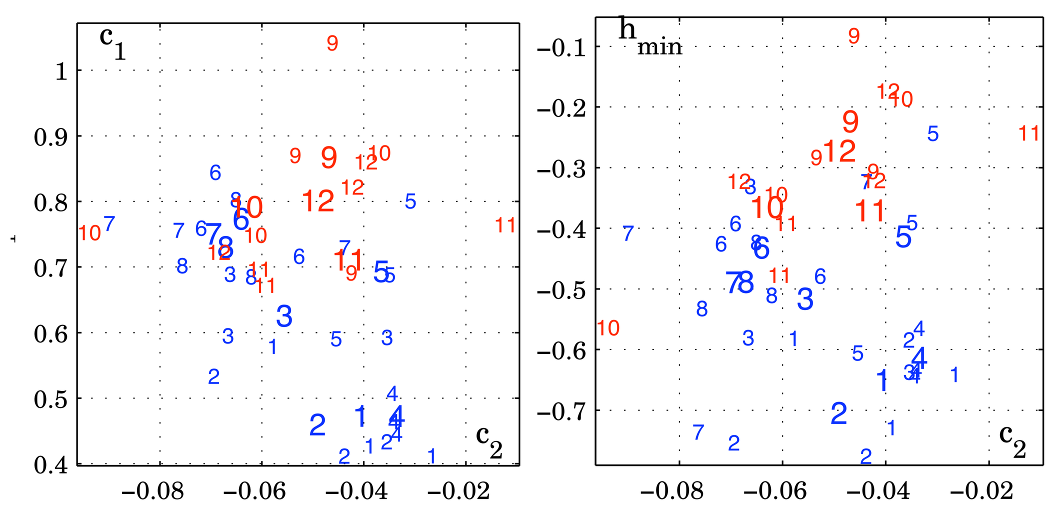
projections on sub-spaces of multifractal attributes

- RESULTS: imitations have

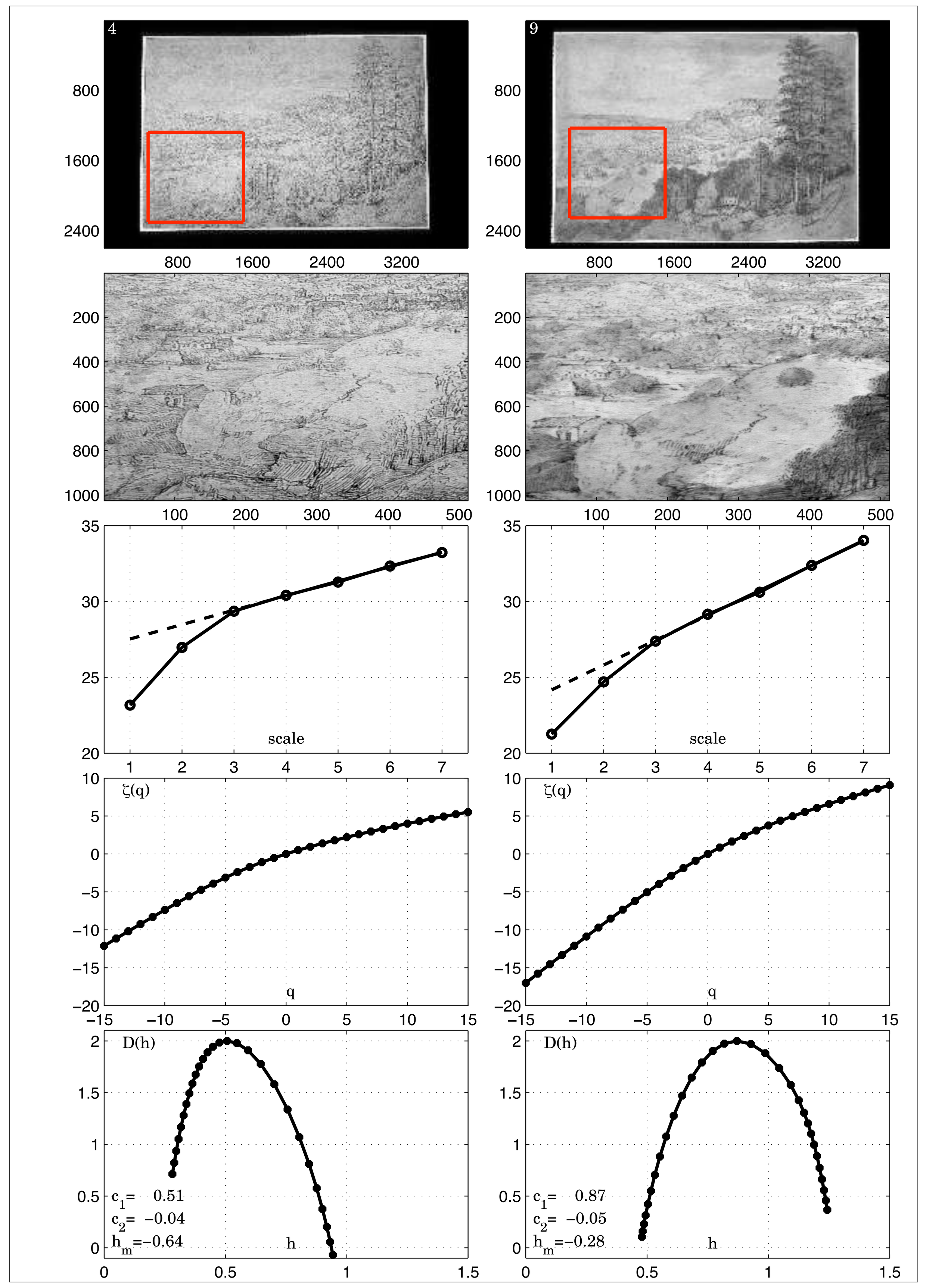
  $\longrightarrow$ *globally more regularity* (c_1 and h_{min} larger)

  $\longrightarrow$ *less regularity fluctuations along space* ($|c_2|$ smaller)

  $\longrightarrow$ *stylometry*



$\longrightarrow$ consistent with results on *Princeton experiment: paintings – original/copy by same artist*



CLASSIFICATION

Quadratic Discriminant Analysis: 3-tuple $\{c_1, c_2, h_{min}\}$

$\longrightarrow$ joint Gaussian – different means / covariance per class

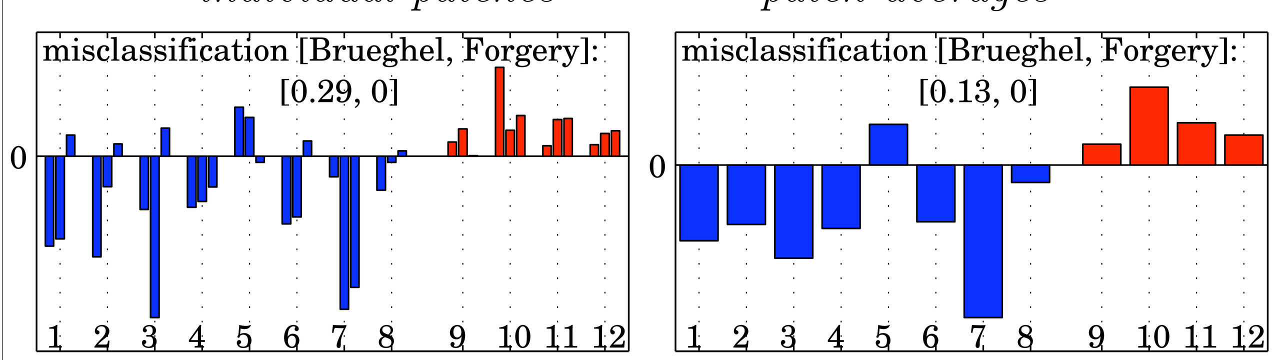
$\longrightarrow$ classification: log-likelihood ratio

- RESULTS:

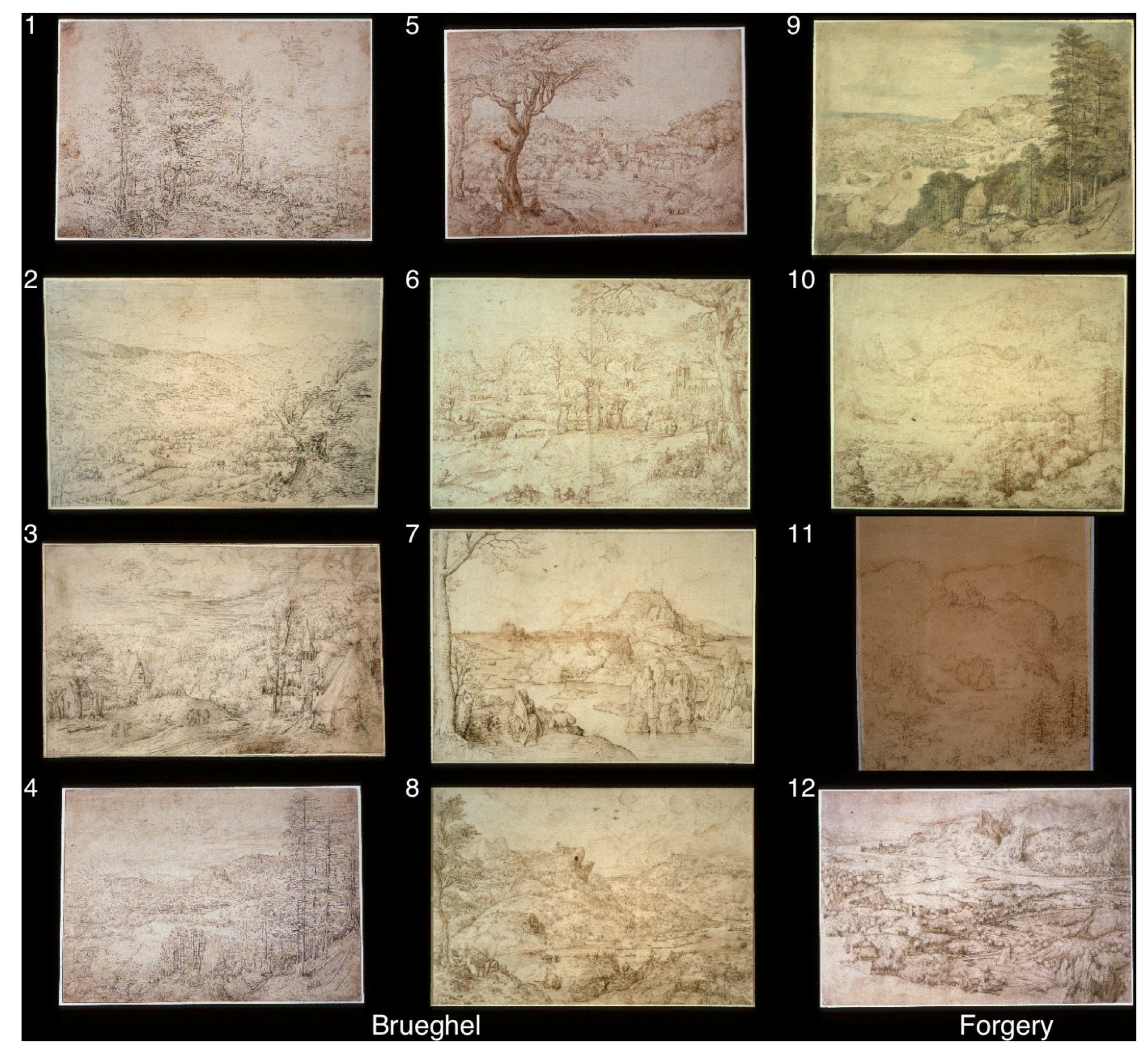
  $\longrightarrow$ *perfect detection of forgeries*

  $\longrightarrow$ misclassification of 7 out $8 \cdot 3 = 24$ authentic patches

  $\longrightarrow$ one single false detection for patch averages



$\longrightarrow$ *use any pair* $\{c_1, c_2\}$, $\{c_1, h_{min}\}$, $\{c_2, h_{min}\}$ *instead: decreased performance*



Drawings courtesy of NY Metropolitan Museum of Art.

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REFERENCES

[1] P. Abry, S. Jaffard, H. Wendt, "When Van Gogh meets Mandelbrot: Multifractal classification of painting textures," *Signal Proces.*, 2012, *to appear.*

[2] S. Jaffard, P. Abry, H. Wendt, "Irregularities and Scaling in Signal and Image Processing: Multifractal Analysis," in *Benoit Mandelbrot: A Life in Many Dimensions, M. Frame, Ed., World scientific*, Fall 2012, *to appear.*