Drowsiness detection from polysomnographic data using multivariate selfsimilarity and eigen-wavelet analysis

Charles-Gérard Lucas[1], Patrice Abry[1], Herwig Wendt[2] and Gustavo Didier[3]

Abstract— **Because drowsiness is a major cause in vehicle accidents, its automated detection is critical. Scale-free temporal dynamics is known to be typical of physiological and body rhythms. The present work quantifies the benefits of applying a recent and original multivariate selfsimilarity analysis to several modalities of polysomnographic measurements (heart rate, blood pressure, electroencephalogram and respiration), from the MIT-BIH Polysomnographic Database, to better classify drowsiness-related sleep stages.**

Clinical relevance— **This study shows that probing jointly temporal dynamics amongst polysomnographic measurements, with a proposed original multivariate multiscale approach, yields a gain of above 5% in the Area-under-Curve quantifying drowsiness-related sleep stage classification performance compared to univariate analysis.**

I. Introduction

Context: Physiology, selfsimilarity and drowsiness. Physiological and body temporal rhythms are well-described by arhythmic or scale-free dynamics. This is notably the case of infraslow brain activity [1], [2], [3], heart rate variability [4], [5], [6], [7], [8], human gait variability [9], [10] or sleep-stage dynamics [11]. Such scale-free dynamics are often well-modeled by selfsimilarity and quantified by the corresponding so-called selfsimilarity or Hurst exponent [12]. However, physiological data very often consist of several time series recorded jointly to study one same biological mechanisms or a pathology. This is for instance the case for polysomnographic measurements, of interest here, aiming to characterize sleep stages and to quantify sleep quality, from several non-invasive modalities related to cardiovascular, respiratory or macroscopic brain activities. However, most often, selfsimilarity is assessed in individual time series, resulting in a collection of univariate analyses. This is mostly due to the fact that, up to a recent past, selfsimilarity was defined only in univariate settings and modeled by the univariate fractional Brownian motion. However, recently a proper definition of multivariate selfsimilarity was proposed [13] together with multivariate eigen-wavelet-based analysis [14], [15]. The present work thus aims to show the potential of multivariate selfsimilarity in physiological and body rhythm analyses through the example of drowsiness detection from polysomnographic data.

Work supported by PhD Grant DGA/AID (no 01D20019023)

[1]Univ Lyon, Ens de Lyon, Univ Claude Bernard, CNRS, Laboratoire de Physique, Lyon, France, `firstname.lastname@ens-lyon.fr`

[2]IRIT, Univ. Toulouse, CNRS, Toulouse, France, `herwig.wendt@irit.fr`

[3]Math. Dept., Tulane University, New Orleans, USA, `gdidier@tulane.edu`

Related work: Drowsiness detection. Drowsiness is usually defined as an intermediate state between wake and sleep [16]. Drowsiness is documented to play a major role in vehicle accidents. Therefore, drowsiness detection from portable non-invasive biomedical measurements constitutes a significant societal stake. It has often been performed from electromyogram data, ECG data or EEG data, making use of non-linear statistical signal processing tools such as sample entropy, selfsimilarity, multifractality, cf., e.g., [16], [17], [18], [19], yet leaving essentially unexplored the use of multivariate scale-free dynamics (see a contrario [11]).

Goals, contributions and outline. The present work aims to quantify the benefits of using the recently devised multivariate selfsimilarity-based eigen-wavelet analysis [14], [15] to detect drowsiness in polysomnographic data. To that end, the estimation of a set of M selfsimilarity exponents from M-variate data using a multivariate wavelet eigen-value based analysis is decribed in Section II. These tools are applied to $M=4$-variate polysomnographic data, obtained from the *MIT-BIH Polysomnographic Database*, well-documented in [20], [21] and described in Section III. Drowsiness detection performance are reported in Section IV.

II. Multivariate eigen-wavelet analysis

Multiscale or Wavelet spectrum. The M-variate time series to be analyzed are denoted $Y=\{Y_1(t),\ldots,Y_M(t)\}_{t\in\mathbb{R}}$. In the next sections, Y will correspond to the collection of polysomnographic signals used to detect drowsiness.

The discrete wavelet transform (DWT) coefficients of the component Y_m of Y, $D_{Y_m}(2^j,k)$ are defined as inner products between Y_m and dilated and translated templates $\psi_0(2^{-j}t-k)$ of a well-chosen reference pattern ψ_0 called the mother wavelet: $D_{Y_m}(2^j,k)=\langle 2^{-j}\psi_0(2^{-j}t-k)|Y_m(t)\rangle$, [22].

To study scale-free dynamics, one usually forms the wavelet spectrum $S(2^j)$ defined, at a given scale 2^j, as a $M\times M$ matrix of wavelet coefficients intercorrelation functions [14], [15]:

$$S_{m,m'}(2^j)\triangleq\frac{1}{n_j}\sum_{k=1}^{n_j}D_{Y_{m'}}(2^j,k)D_{Y_{m'}}(2^j,k)^*, \qquad (1)$$

where n_j is the number of wavelet coefficients at scale 2^j.

In addition, for each pair of components $(Y_m,Y_{m'})$, the wavelet coherence function $C_{m,m'}(2^j)=S_{m,m'}(2^j)/\sqrt{S_{m,m}(2^j)S_{m',m'}(2^j)}$, measures a series of scale-dependent correlation coefficients [23], [24].

Univariate selfsimilarity parameter estimation. The classical univariate analysis of Y consists in estimating one selfsimilarity parameter independently for each time series. In other words, it amounts to using only the diagonal entry of the matrix wavelet spectrum $S(2^j)$. It has been well-documented that, for the increments of univariate selfsimilar signals such as fractional Gaussian noise, $S_{m,m}(2^j)$ should (asymptotically) behave as a power law with respect to the scales 2^j, with scaling exponents driven by the selfsimilarity parameters as $2H_m^{True}-1$ [25]. Thus, parameter estimation can be efficiently performed by means of linear regressions of $\log_2(S_{m,m}(2^j))$ against $\log_2(2^j)=j$ [26]:

$$\hat{H}_m^U = \left(\sum_{j=j_1}^{j_2} v_j \log_2(S_{m,m}(2^j))\right)/2-\frac{1}{2}, \quad m=1,\ldots,M, \quad (2)$$

with v_j such that $\sum_j v_j = 1$ and $\sum_j v_j = 0$.

Classical multivariate selfsimilarity parameter estimation. To account for cross-dependencies amongst component temporal dynamics, a natural approach is to apply the same procedure to each entry of the wavelet spectrum $S_{m,m'}(2^j)$, thus defining a set of cross-selfsimilarity exponents:

$$\hat{H}_{m,m'} = \left(\sum_{j=j_1}^{j_2} v_j \log_2(S_{m,m'}(2^j))\right)/2-\frac{1}{2}, \quad m,m'=1,\ldots,M. \quad (3)$$

Eigen wavelet selfsimilarity parameter estimation. However, it has been recently documented that multivariate selfsimilarity can be better conducted based on a change of paradigm: Instead of considering for each pair of components, m and m', the behavior of $S_{m,m'}(2^j)$ along scales and then comparing between pairs of components the estimated selfsimilarity parameter $\hat{H}_{m,m'}$, as above, one can first consider at a given scale 2^j all components together, by computing the eigenvalues $\lambda_m(2^j)$ of the spectrum matrix $S(2^j)$, and then study the behavior of these eigen-values along scales 2^j. It has indeed been shown that for multivariate selfsimilar processes the eigenvalues $\lambda_m(2^j)$ should (asymptotically) behave as a power law with respect to the scales 2^j, with scaling exponents related to the selfsimilarity parameters as $2H_m^{True}-1$ [14], [15].

However, the practical computation of the eigenvalues of the set of matrices $S(2^j)$ from finite sample size multivariate data suffers from a bias, referred to as the *repulsion bias* of significant impact at coarse scales, where the number of available wavelet coefficients becomes limited compared to the number of components M. This recently lead to the construction of a robust estimation based on the eigenvalues, from a collection of estimated wavelet spectra obtained from non-overlapping windows of width w, using an equal number n_{j_2} of wavelet coefficients $D_Y(2^j,k)$ at each scale 2^j (with n_{j_2} the number of wavelet coefficients at coarsest scale 2^{j_2}) [27]:

$$S_{m,m'}^{(w)}(2^j) \triangleq \frac{1}{n_{j_2}} \sum_{k=1+(w-1)n_{j_2}}^{wn_{j_2}} D_{Y_m}(2^j,k) D_{Y_{m'}}(2^j,k)^*. \quad (4)$$

The log-eigenvalues $\lambda_1^{(w)}(2^j),\ldots,\lambda_M^{(w)}(2^j)$ of the matrices $S^{(w)}(2^j)$ are then averaged to yield $\bar{\lambda}_m(2^j) \triangleq 2^{j_2-j}\sum_{w=1}^{2^{j-j_2}} \log_2(\lambda_m^{(w)}(2^j))$. Eigenwavelet-based estimates $\hat{H}_m^M$ of the multivariate selfsimilarity parameters are finally obtained by linear regressions of the log-averaged eigenvalues $\bar{\lambda}_m(2^j)$ against $\log_2 2^j = j$:

$$\hat{H}_m^M = \left(\sum_{j=j_1}^{j_2} v_j \bar{\lambda}_m(2^j)\right)/2-\frac{1}{2}, \quad m=1,\ldots,M. \quad (5)$$

III. Databasis

MIT-BIH Polysomnographic database. Data used here are those made available in the MIT-BIH Polysomnographic Database[1], documented in [20], [21]. Data consist of a collection of 4 to 7 physiological measurements (cardiovascular, respiratory or macroscopic brain activities, stroke volume, oxygen saturation, eye movements or chin muscle responses) recorded for 16 male subjects in Boston's Beth Israel Hospital Sleep Laboratory in the context of chronic obstructive sleep apnea syndrome; the recordings of the first two subjects are split into two consecutive portions, resulting in a total of 18 multivariate time series (sampling 250Hz, data lengths from 77 to 390 min). Sleep stage expert annotations are available for each 30-second long non overlapping window.

Data for the present study. Because the focus of this work is on detecting drowsiness, defined as transitions between the *awake* and *stage1* stages, the goal here is to perform a classification of these two stages. Use is thus made only of the time windows corresponding to such annotations.

Further, to investigate the benefits of a joint analysis based on multivariate selfsimilarity, one needs to analyze the different modalities of polysomnographic data in the same frequency range. Because respiratory and heart rate rhythms contain relevant information in time scales ranging from 1 second to 1 minute, only the infraslow brain activity is considered and data are filtered and resampled at 4Hz.

Finally, to ensure a consistent number of features for detection and classification, only those 4 modalities of the polysomnographic data are used that are available for all subjects: heart rate (HR), blood pressure (BP), electroencephalogram (EEG) and respiration (RESP).

IV. Sleep stage classification

A. Analysis/classification set-up

1) Physiological time series analysis: Analysis is performed in 2-min long sliding windows, with 75% of overlap between successive windows, each hence containing $N_w = 480 = 4\times2\times60$ samples.

Classification will be restricted to sequences sharing the same annotations for at least 4 consecutive 30-second windows. In total, 1753 and 561 such windows are available for the awake and stage 1 states, respectively.

The DWT coefficients $D_{Y_m}(2^j,k)$ are computed using the least asymmetric Daubechies3 wavelet [22]. Linear regressions are performed across scales ranging from $2^{j_1}=2^1$ to $2^{j_2}=2^4$ corresponding to frequencies from 1/8 to 2 Hz, or equivalently to time scales from 1/2 to 8 seconds.

Figs. 1 and 2 plot, as an example, log-wavelet spectrum and log-eigenvalues, for one 2-min long window, indicating

[1] https://physionet.org/content/slpdb/1.0.0/

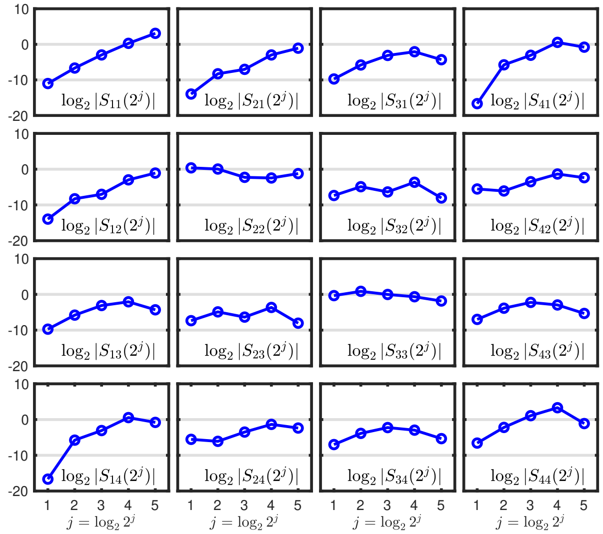

Fig. 1: **Log-wavelet spectrum** for one subject and one 2-min long window.

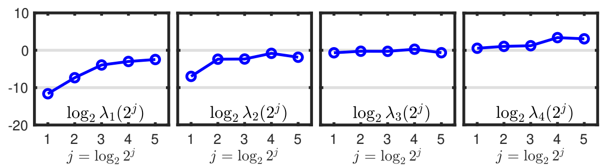

Fig. 2: **Log-wavelet eigenvalues** for one subject and one 2-min long window.

power law scaling thus selfsimilarity across the chosen analysis scales. Scaling exponents are estimated using Eqs. 3, 2 and 5 from such functions.

2) Multifeature classification: We make use of a standard Random Forest Classifier [28], a learning method consisting in training a large number of decision trees from data resampled with replacement. Each tree is trained from a subset of features randomly selected among the N_f available features, to reduce correlation between trees. The size of the subset list is chosen here as $\sqrt{N_f}$ as in [28], and decision is made by majority vote.

Random Forest Classifiers are performed with $N_{\text{trees}} \in \{10, 25, 50\}$ trees. A diagonal cost matrix is set with coefficients $w_1 = W_v N_a / N_W$ and $w_2 = N_s / N_W$, where N_a is the number of windows related to the state "awake" (class 0), N_s is the number of windows related to the state "stage 1" (class 1), $N_W = N_a + N_s$ is the total number of windows and $W_v \in [0.001, 6]$ to tune the classification false alarm rate. Performances are assessed by cross-validation over $N_{MC} = 100$ repetitions, with 80% of the available windows randomly and independently selected for the training sets.

Receiver Operational Characteristic (ROC) curves are computed by varying W_v during training. ROC areas-under-curve (AUC) are used as performance score.

B. Single-feature classification

As a baseline, classification is first performed for each of the four modalities independently, using as a single feature the univariate selfsimilarity parameter, $\hat{H}_m^U$, estimated using Eq. 2. Classification hence simply consists in comparing the feature against a threshold, and ROC curves are computed by varying the classification threshold as shown in Fig. 3. AUC, reported in Table I, show that Blood Pressure yields the best classification performance. However, single-feature performance remains quite low.

feature $\hat{H}_1^U$	feature $\hat{H}_2^U$	feature $\hat{H}_3^U$	feature $\hat{H}_4^U$
52.71	67.59	55.15	57.44

TABLE I: **AUC** for univariate classifications.

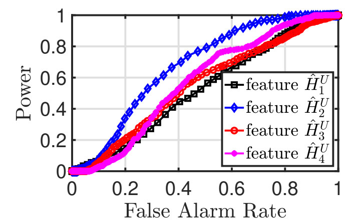

Fig. 3: **ROC curves** (average $\pm$ standard deviation) for single-feature classification.

C. Multi-feature classification

The goal is now to quantify the benefits of using several polysomnographic modalities jointly in classifying drowsiness related sleep stages.

1) Features: Three different strategies to combine information stemming from the different modalities are tested.

The simplest way consists in concatenating the four univariate $\hat{H}_m^U$ into a feature vector of dimension 4. This will be referred to as *features1*. Such a joint modality classification does not account for information related to cross-temporal dynamics between modalities as it only makes use of features computed independently on each modality. However, as illustrated in Fig. 1, there exist non-negligible cross-temporal dynamics at all scales between modalities, thus prompting for the use of such information to improve classification.

To account for cross-temporal dynamics across modalities, a second classification will be performed by adding to the 4 univariate features $\hat{H}_m^U$, the $6 = 4 \times 3/2$ cross-selfsimilarity parameters $\hat{H}_{m,m'}$ ($m' \neq m$) estimated from the nondiagonal entries of the wavelet spectrum (3), resulting in a feature vector of dimension 10 (referred to as *features2*).

In addition, an original contribution of this work is to promote the use of the eigenwavelet analysis as a new way to quantify cross-temporal dynamics. Therefore, a third classification will be performed by adding to the 4 univariate features $\hat{H}_m^U$, the 4 eigenvalue based multivariate selfsimilarity parameters $\hat{H}_m^M$ estimated as in (5), resulting in a feature vector of dimension 8. This will be referred to as *features3*.

2) Performance: To compare performance between these three different multimodal classification strategies, Fig. 4(right plots) displays ROC curves and Table II further reports the AUC. These results yield the following conclusions. Multi-feature classifications significantly outperform single-feature classification. For multi-feature classifications, the performance is robust to the choice of the number of trees in the random forest procedure. Amongst the multi-feature classifications, the one based on the classical measures of cross-temporal dynamics (features2) does not improve classification performance compared to the simplest concatenation of the univariate features (features1). To the contrary, the promoted eigen-wavelet approach to probing cross-temporal dynamics, combined with the univariate features (features3), improves significantly drowsiness related sleep-stage classification performance.

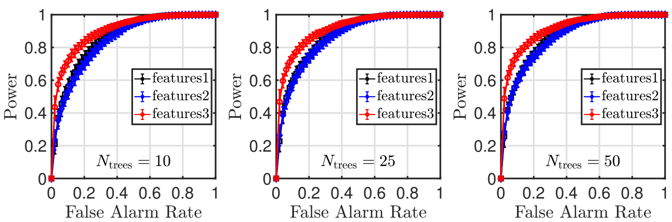

Fig. 4: **ROC curves** (average ± standard deviation), for the multivariate classifications with three different numbers of trees.

AUC	features1	features2	features3
$N_{\text{trees}} = 10$	85.34 ± 0.70	83.71 ± 0.71	89.31 ± 0.54
$N_{\text{trees}} = 25$	86.25 ± 0.64	85.29 ± 0.85	90.11 ± 0.54
$N_{\text{trees}} = 50$	86.68 ± 0.66	85.68 ± 0.68	90.46 ± 0.54

TABLE II: **AUC** (average ± standard deviation) for the multivariate classification.

V. Conclusions

The present work has first quantified the benefits of sleep multimodal monitoring in phase sleep classification. Second, it has shown that compared to simply combining by concatenation univariate features, the proposed multivariate eigenwavelet approach strengthens sleep stage classification. It clearly quantifies that cross-temporal dynamics amongst modalities do carry information relevant to the sleep state evaluation. This is because the smallest eigenvalues are able to sense to low-intensity yet meaningful $M-$wise dependencies amongst components that classical pairwise cross-correlations may miss.

References

[1] B. J. He, "Scale-free properties of the functional magnetic resonance imaging signal during rest and task," *J. Neurosci.*, vol. 31, no. 39, pp. 13786–13795, 2011.

[2] P. Ciuciu, P. Abry, and B. J. He, "Interplay between functional connectivity and scale-free dynamics in intrinsic fMRI networks," *Neuroimage*, vol. 95, pp. 248–263, 2014.

[3] D. La Rocca, N. Zilber, P. Abry, V. van Wassenhove, and P. Ciuciu, "Self-similarity and multifractality in human brain activity: A wavelet-based analysis of scale-free brain dynamics," *J. Neurosci. Methods*, vol. 309, pp. 175–187, 2018.

[4] Y. Yamamoto and R. L. Hughson, "Coarse-graining spectral analysis: new method for studying heart rate variability," *J. Appl. Physiol.*, vol. 71, no. 3, pp. 1143–1150, 1991.

[5] P. C. Ivanov, L. A. N. Amaral, A. L. Goldberger, S. Havlin, M.G. Rosenblum, Zbigniew R. Struzik, and H. E. Stanley, "Multifractality in human heartbeat dynamics," *Nature*, vol. 399, no. 6735, pp. 461–465, 1999.

[6] L. A. N. Amaral, A. L. Goldberger, P. C. Ivanov, and H. E. Stanley, "Modeling heart rate variability by stochastic feedback," *Comput. Phys. Commun.*, vol. 121, pp. 126–128, 1999.

[7] M. Doret, J. Spilka, V. Chudáček, P. Gonçalves, and P. Abry, "Fractal analysis and hurst parameter for intrapartum fetal heart rate variability analysis: a versatile alternative to frequency bands and lf/hf ratio," *PLoS One*, vol. 10, no. 8, pp. e0136661, 2015.

[8] T. Nakamura, K. Kiyono, H. Wendt, P. Abry, and Y. Yamamoto, "Multiscale analysis of intensive longitudinal biomedical signals and its clinical applications," *Proc. IEEE*, vol. 104, no. 2, pp. 242–261, 2016.

[9] C. BenAbdelkader, R. G. Cutler, and L. S. Davis, "Gait recognition using image self-similarity," *EURASIP J. Adv. Signal Process.*, vol. 2004, no. 4, pp. 1–14, 2004.

[10] L. M. Decker, F. Cignetti, and N. Stergiou, "Complexity and human gait," *Rev. Andal. Med. Deport.*, vol. 3, no. 1, pp. 2–12, 2010.

[11] L. Leon, H. Wendt, J.-Y. Tourneret, and P. Abry, "A bayesian framework for bivariate multifractal analysis," *submitted*.

[12] V. Pipiras and M. S. Taqqu, *Long-Range Dependence and Self-Similarity*, vol. 45, Cambridge University Press, 2017.

[13] G. Didier and V. Pipiras, "Integral representations and properties of operator fractional Brownian motions," *Bernoulli*, vol. 17, no. 1, pp. 1–33, 2011.

[14] P. Abry and G. Didier, "Wavelet estimation for operator fractional Brownian motion," *Bernoulli*, vol. 24, no. 2, pp. 895–928, 2018.

[15] P. Abry and G. Didier, "Wavelet eigenvalue regression for *n*-variate operator fractional Brownian motion," *J. Multivar. Anal.*, vol. 168, pp. 75–104, November 2018.

[16] J. Yu, S. Park, S. Lee, and M. Jeon, "Driver drowsiness detection using condition-adaptive representation learning framework," *IEEE trans. Intell. Transp. Syst.*, vol. 20, no. 11, pp. 4206–4218, 2018.

[17] F. Wang, H. Wang, and R. Fu, "Real-time ecg-based detection of fatigue driving using sample entropy," *Entropy*, vol. 20, no. 3, pp. 196, 2018.

[18] B.N. Cahyadi, W. Khairunizam, I. Zunaidi, L. H. Ling, A.B. Shahriman, M.R. Zuradzman, W.A. Mustafa, and N.Z. Noriman, "Muscle fatigue detections during arm movement using emg signal," in *IOP Conf. Ser.: Mater. Sci. Eng.* IOP Publishing, 2019, vol. 557, p. 012004.

[19] S. Ahn, T. Nguyen, H. Jang, J. G. Kim, and S. C. Jun, "Exploring neuro-physiological correlates of drivers' mental fatigue caused by sleep deprivation using simultaneous eeg, ecg, and fnirs data," *Front. Hum. Neurosci.*, vol. 10, pp. 219, 2016.

[20] Y. Ichimaru and G.B. Moody, "Development of the polysomnographic database on cd-rom," *Psychiatry Clin. Neurosci.*, vol. 53, no. 2, pp. 175–177, 1999.

[21] A. Goldberger, L.A. Amaral, L. Glass, J.M. Hausdorff, P.C. Ivanov, R.G. Mark, J.E. Mietus, G.B. Moody, C.-K. Peng, and H.E. Stanley, "Physiobank, physiotoolkit, and physionet: components of a new research resource for complex physiologic signals," *Circulation*, vol. 101, no. 23, pp. e215–e220, 2000.

[22] S. Mallat, *A Wavelet Tour of Signal Processing*, Academic Press, San Diego, CA, 1998.

[23] B. Whitcher, P. Guttorp, and D. B. Percival, "Wavelet analysis of covariance with application to atmospheric time series," *J. Geophys. Res. Atmos.*, vol. 105, no. D11, pp. 14941–14962, 2000.

[24] H. Wendt, G. Didier, S. Combrexelle, and P. Abry, "Multivariate Hadamard self-similarity: testing fractal connectivity," *Physica D*, vol. 356-357, pp. 1–36, 2017.

[25] P. Flandrin, "Wavelet analysis and synthesis of fractional Brownian motion," *IEEE Trans. Info. Theory*, vol. 38, no. 2, pp. 910–917, 1992.

[26] D. Veitch and P. Abry, "A wavelet-based joint estimator of the parameters of long-range dependence," *IEEE Trans. Info. Theory*, vol. 45, no. 3, pp. 878–897, 1999.

[27] C.-G. Lucas, P. Abry, H. Wendt, and G. Didier, "Bootstrap for testing the equality of selfsimilarity exponents across multivariate time series," in *Proc. Eur. Signal Process. Conf (EUSIPCO)*, Dublin, Ireland, August 2021.

[28] L. Breiman, "Random forests," *Mach. Learn.*, vol. 45, no. 1, pp. 5–32, 2001.