Methodology for Multifractal Analysis of Heart Rate Variability: From LF/HF Ratio to Wavelet Leaders

P. Abry, H. Wendt, S. Jaffard, H. Helgason, P. Goncalvès, E. Pereira, Cl. Gharib, P. Gaucherand, M. Doret

***Abstract*— The present contribution aims at proposing a comprehensive and tutorial introduction to the practical use of wavelet Leader based multifractal analysis to study heart rate variability. First, the theoretical background is recalled. Second, practical issues and pitfalls related to the selection of the scaling range or statistical orders, minimal regularity, parabolic approximation of spectrum and parameter estimation, are discussed. Third, multifractal analysis is connected explicitly to other standard characterizations of heart rate variability: (mono)fractal analysis, Hurst exponent, spectral analysis and the HF/LF ratio. This review is illustrated on real per partum fetal ECG data, collected at an academic French public hospital, for both healthy fetuses and fetuses suffering from acidosis.**

I. INTRODUCTION

Heart Rate Variability. Heart Rate Variability (HRV) refers to the characterization of the *short time* (within a minute) fluctuations of Heart Rate (HR) time series. Often, in medical practice, a *large* HRV is considered a sign of good health [1]. For instance, in per partum fetal HRV analysis, the variability is commonly defined as the largest oscillation of the heart rate observed within a window of 3.75s, and observing it weaker than 5 beats-per-minute constitutes an indication for fetal suffering and acidosis. Hence, the precise characterization of HRV is of major practical and clinical importance and has been, and continues to be, the subject of numerous and advanced research efforts.

Heart Rate Variability Analysis. HRV has been analyzed using various statistical signal processing tools and approaches. Essentially, all techniques aim at measuring the degree of irregularity of the HR time series. Entropy based approaches, such as those proposed in e.g., [2], evaluate the variability of a time series via the complexity of its marginal distribution. They therefore measure a static property of the data that does not account for their intrinsic dynamic: Two different times series with same marginals but different correlations for instance still have the same entropy. To account for the dynamical properties of the data, one often resorts to spectral analysis, which, after the seminal contribution of Akselrod and collaborators (cf. e.g., [3]), has been considered as a reference tool to characterize HRV (cf., e.g., [4], [5] for reviews on adult HRV or [6], [1] for more specific reviews of fetal HRV analysis). Classically, HRV spectral analysis relies on the use of pre-defined frequency bands, referred to as the low frequency (LF) and high frequency (HF) bands. Notably, the so-called LF/HF ratio, that measures the ratio between the total energies in the LF and HF bands, is considered as a key indicator for HRV analysis. It has indeed been shown to measure the sympathic-parasympathic balance, hence reflecting the regulation mechanisms within the central autonomous nervous system. However, spectral analysis requires stationary data, a property that can legitimately be called into question for HR time series. To overcome this limitation, fractal analysis has been envisaged as an alternative approach to characterize HRV. Fractal analysis, in essence, consists in measuring a variability related quantity (increments, oscillations, wavelet coefficients) simultaneously at different analysis scales a. These measures are assumed to evolve as a power law with respect to the analysis scales, and the corresponding power law (or scaling) exponent to characterize HRV. Amongst such exponents, the most commonly used are the fractal dimension, the Hurst or long memory exponents. Multifractal analysis can be envisaged as an extension to fractal analysis insofar as it characterizes data variability with a collection of scaling exponents rather than with a single one. It has recently been used to investigate HRV, cf. e.g., [7], [8], [9].

Goals and Contributions. Goals of the present contribution are twofold. First, it aims at providing readers with a methodological introduction to the recently proposed wavelet Leader based multifractal analysis and to its practical use for studying HRV. Second, it aims at comparing the wavelet Leader based multifractal analysis with the wavelet coefficient based one, the Hurst exponent, and the LF/HF ratio.

Per Partum Fetal Heart Rate. The theoretical and practical notions related with multifractal analysis are discussed below and illustrated on actual per partum fetal HR data (rather than on synthetic time series). Data were collected at the department of obstetrics of the academic *Hôpital Femme-Mère-Enfant* (Lyon, France), where F-ECG monitoring is routinely performed to monitor risk of fetal asphyxia. The data were recorded with a STAN system (Neoventa Medical, Moelndal, Sweden) and consist of the list of time occurrences $\{t_n, n = 1, \ldots, N\}$ (in ms) of the R-peaks. Interpolating the points $\{(t_n, (t_{n+1} - t_n)^{-1}), n = 1, \ldots, N\}$, resampling at some frequency F_s and changing units provides practitioners with regularly sampled HR time series, in beats-per-minute. The database contains both healthy fetuses and fetuses suffering from acidosis.

P. Abry, H. Helgason, P. Gonçalvès, E. Pereira are with ENS Lyon, CNRS, INRIA, Université de Lyon, France, `patrice.abry@ens-lyon.fr`

H. Wendt is with Math. Dept., Purdue Univ., Lafayette, USA

S. Jaffard is with Math. Dept., Paris Est Univ., Créteil, France

M. Doret, Cl. Gharib, P. Gaucherand are with Hospices Civils de Lyon, Hôpital, Femme, Mère, Enfant, Université Lyon 1, France

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II. Multifractal analysis: Theory

Local Regularity and Hölder Exponent. Let $X(t)$ denote the bounded function to be analyzed. Its regularity around time t_0 is measured locally by the so-called Hölder exponent $h(t_0)$, defined as the largest $\alpha > 0$, such that there exist a constant $C > 0$ and a polynomial P_{t_0} of degree less than α, such that $|X(t) - P_{t_0}(t)| \leq C|t - t_0|^\alpha$ in a neighborhood of t_0. To understand this, let us consider the case where $P_{t_0}(t) \equiv X(t_0)$ and $0 < h(t_0) < 1$. Qualitatively, $h(t_0)$ is hence the fractional order at which $X(t)$ remains differentiable around t_0. When $h(t_0)$ is close to 1, $X(t)$ is close to be differentiable, and hence is locally almost as regular as a line. Conversely when $h(t_0)$ is close to 0, X(t) is highly irregular at t_0. When the degree of P_{t_0} is $m \geq 1$, then $h(t_0) > m$ and the above interpretation translates to the m-th derivative of X.

Multifractal Spectrum. Although based on a local regularity measure, multifractal analysis intends to provide a global analysis of the variability of the data: It characterizes the geometrical structure of the subset E_h of points t_i on the real line where $h(t_i) = h$. Because such geometrical structures are inherited from the time evolution of the data, multifractal analysis hence measures globally the local dynamics (or variability) of X. This measure is based on the Haussdorf dimension of E_h, denoted by $D(h)$, and is referred to as the multifractal spectrum. The Haussdorf dimension is essentially a mathematical extension of the box counting dimension (cf. e.g., [10] for technical definitions). A theoretical multifractal spectrum is sketched in Fig. 1 while multifractal spectra estimated over 30-min long per partum fetal ECG data are shown in Fig. 2.

Wavelet Leader Multifractal Formalism. To practically measure $D(h)$ from a given time series, one cannot measure directly $h(t)$ for all t and then estimate the dimension of the E_h, essentially because actual data always have a finite resolution. Instead one has to resort to a multifractal formalism procedure. We present here the recently proposed formalism based on wavelet Leaders [10], [11]. Let ψ denote the mother wavelet, and $\psi_{j,k}(t) = 2^{-j}\psi(2^{-j}t - k)$ the dilated and translated wavelets. Let $d_X(j,k)$ denote the (L^1-normalized) discrete wavelet transform coefficients of X, where j refers to the analysis scale ($a = 2^j$) and k to time ($t = 2^j k$). Wavelet Leaders $L_X(j,k)$ are defined as the local supremum of wavelet coefficients taken within a spatial neighborhood over all finer scales [11]: $L_X(j,k) = \sup_{\lambda' \subset 3\lambda_{j,k}} |d_X(\lambda')|$, where $\lambda_{j,k} = [k2^j, (k+1)2^j)$ and $3\lambda_{j,k} = \bigcup_{m\{-1,0,1\}} \lambda_{j,k+m}$. The scaling exponents $\zeta_L(q)$ are defined for $a = 2^j \to 0$ as

$$S_L(2^j, q) = \frac{1}{n_j} \sum_k L_X(j,k)^q \simeq S_0(q) 2^{j\zeta_L(q)}. \quad (1)$$

The Legendre transform $D_L(h)$ of the function $\zeta_L(q)$ provides an upper bound of the multifractal spectrum, i.e., $D_L(h) \geq D(h)$. For further theoretical details on multifractal analysis and wavelet Leader formalism, the reader is referred to, e.g., [10] and [11], respectively.

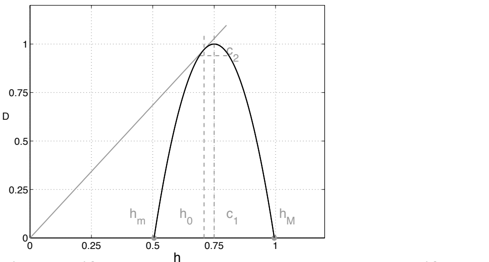

Fig. 1. **Schematic multifractal spectrum and related multifractal attributes.**

III. Multifractal analysis: Practice

Concave Spectrum and Parabolic Approximation. For practical purposes, $D_L(h)$ is assimilated to the true $D(h)$, which hence is always concave and practically resembles a bell-shaped curve, taking values between 0 and 1 (for a 1D signal) over a finite range of Hölder exponents: $h \in [h_m, h_M]$. Practically, $D(h)$ is efficiently characterized by the position of its maximum c_1, the width c_2 of the best parabolic approximation around its maximum (note that, by concavity, $c_2 \leq 0$), and its minimal and largest Hölder exponents h_m and h_M. Often, in practice, the spectrum can hence be efficiently approximated by a parabola: $D(h) \simeq 1 + (h - c_1)^2/(2c_2)$. Equivalently, this amounts to saying that, for q close to 0, $\zeta_L(q) \simeq c_1 q + c_2 q^2/2$.

Ranges of qs. In practice, a natural question that arises is related to the ranges of values of q that one should use. Here it is crucial to note that (i) by nature of the Legendre transform, both positive and negative values of q are needed to obtain the entire curve $D_L(h)$; and (ii) when the function X takes values of Hölder exponent into a bounded set $h \in [h_m, h_M]$ (a very natural assumption for actual data), then, the Legendre transform implies that both for $q \geq q_*^+$ and $q \leq q_*^-$, $\zeta_L(q)$ is a linear function of q, with $q_*^+ = dD(h_m)/dh$ and $q_*^- = dD(h_M)/dh$ (cf. [12]). The range of useful values of q is hence centered around zero: $q \in [q_*^-, q_*^-]$. For the parabolic approximation, $q_*^\pm = \pm\sqrt{2/|c_2|}$. Typically, on HRV data (cf. e.g., [9]), $|c_2|$ ranges from 0.02 to 0.2, hence $q_*^\pm$ from ± 10 to ± 3. The ranges of qs to be actually used is narrow and around 0.

Minimal Regularity and Fractional Integration. Multifractal analysis is theoretically defined for bounded functions only, i.e., essentially for functions that have a positive minimal regularity, defined as:

$$h_m = \liminf_{2^j \to 0} \frac{\ln \sup_k |d_X^{(m)}(j,k)|}{\ln 2^j}. \quad (2)$$

This is of major practical importance: Indeed, often, HR time series resemble fractional Gaussian noise, fGn, (increment process of the fractional Brownian motion, fBm) (cf. e.g. [4], [5]). However, for fGn with Hurst parameter $H (\in (0,1))$, $h_m = H - 1 < 0$, and hence multifractal analysis cannot be applied to it! To overcome this limitation, it has been proposed first to measure h_m, according to Eq. (2) ; second, whenever $h_m < 0$, to fractionally integrate the data with an integration order $\gamma > -h_m$ [13]. It has been shown that practically performing fractional integration can be avoided by applying the wavelet Leader formalism above to the pseudo-wavelet coefficients: $d_X^{\gamma}(j,k) = 2^{j\gamma} d_X(j,k)$ [13]. In practice, one then uses: $D(h) = D^{\gamma}(h-\gamma)$, $\zeta_L(q) = \zeta_L^{\gamma}(q) - \gamma q$, $c_1 = c_1^{\gamma} - \gamma$, $c_2 = c_2^{\gamma}$. Note that these relations are known not to hold in general [13], yet are regarded as valid approximations for most practical purposes.

Linear Regression and Scaling Range. The definition of the $\zeta_L(q)$ above (cf. Eq. 1) suggests that they can be estimated by linear regression of the $\log_2 S_L(2^j, q)$ versus $\log_2 2^j = j$. Equivalently, c_1, c_2, h_m and the parametric form $D_L(q)$ (the Legendre transform of $h_L(q)$) can be estimated via linear regressions. These procedures are described in depth in [11] and hence not recalled here. A key practical issue yet lies in deciding over which range of scales $j \in [j_1, j_2]$ the linear regressions are to be performed. This can be envisaged in two ways. Either, one decides that range *a priori* and from a physiological understanding of the data (for example, one can decide that the range of scales should cover the classical HF and LF bands). Or, one inspects, visually, or by means of statistical procedures, the $\log_2 S(2^j, q)$ versus $\log_2 2^j = j$ plots to decide *a posteriori* in which range the data actually show a scaling behavior. In both cases, that range of scales must, by nature, be the same for all parameters. Note moreover that the interpolation and resampling procedure transforming the R-peak arrival into regularly sampled time series creates artificially regular high frequency behavior. Such high frequencies (or fine scales) must not be used for variability analysis and hence must not be included in the regression range. For example, for intrapartum fetal ECG, HR naturally varies around 120 bpm. This implies that all frequencies larger than $2Hz$ or, equivalently, all scales below $F_s/2 \le 2^j$ do not account for the actual data variability and shall not be used.

Practical Estimation Procedures. MATLAB routines implementing the procedures listed above are available upon request. They are complemented by a non-parametric time-scale bootstrap procedure enabling the estimation of not only the multifractal parameters but also of confidence intervals for these estimates as well as the implementation of hypothesis tests.

IV. Wavelet Leaders versus Wavelet Coefficients, Hurst Parameter and LF/HF Ratio

Wavelet Leaders versus Wavelet Coefficients. Prior to the wavelet Leader multifractal formalism, another formalism based on wavelet coefficient had been proposed, relying on $S_d(2^j, q) = \frac{1}{n_j}\sum_k d_X(j,k)^q \simeq S_0(q) 2^{j\zeta_d(q)}$. It has long been known that such $S_d(2^j, q)$ are numerically instable and hence turn useless, in practice, for negative qs. Therefore, practically, only the increasing portion of $D(h)$ can be measured with wavelet coefficients. Furthermore, it can be shown theoretically [10] that $D_L(h)$ and $D_d(h)$ (Legendre transform of $\zeta_d(q)$) coincide exactly only for $h < h_0$, where h_0 is defined as: $h_0 D'(h_0) = D(h_0)$ (as sketched in Fig. 1). This also implies that $\zeta_d(q)$ and $\zeta_L(q)$ coincide exactly only for $q \ge q_0 \ge 0$. Interestingly, for the parabolic approximation, it yields $h_0 = \sqrt{c_1^2 + 2c_2}$ and

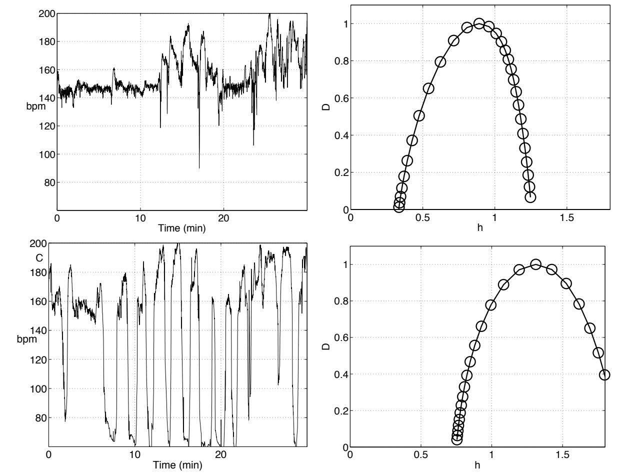

Fig. 2. **Wavelet Leader based estimated multifractal spectra computed from real per partum fetal ECG.** Top: healthy fetus ; Bottom: fetus suffering from acidosis.

$q_0 = (\sqrt{c_1^2 + 2c_2} - c_1)/c_2$. Assuming $|c_2|/c_1 \ll 1$, this yields, at first order, $h_0 \simeq c_1 + c_2$ and $q_0 \simeq 1$!

Fractal Analysis and Hurst Exponent. As mentioned above, fractal analysis amounts to assuming that HR is well modeled with fGn and that its Hurst parameter H measures the corresponding variability of the data. It is well known that this Hurst exponent can be well estimated using wavelet coefficients and $S_d(2^j, q = 2)$ [14]. Because for fGn, $h_m = H - 1$, and because $\zeta_d(2) = \zeta_L(2)$ (see previous paragraph), one obtains that the wavelet Leaders yield an estimate of H via $\hat{H} = (\zeta_L^{\gamma}(2) - 2\gamma)/2$, for any $\gamma > 1 - H$. Note that in practice, taking the limit case $\gamma = 1$ amounts to analyzing the cumulated sum of fGn, hence fBm. Fig. 3 (left) reports the Hurst exponents estimated using wavelet coefficients and Leaders for per partum fetal ECG. The estimations are close and their correlation coefficients for the entire database is above 0.9. Incidentally, Fig. 3 (left) also shows that fetuses actually suffering from acidosis have higher $\hat{H}$, indicating lesser variability and hence corroborating that a decrease in variability indicates a non-healthy situation.

Self-similarity (or Monofractal) versus Multifractal. Let Y denote the cumulated sum of the HR time series X. Practically, Y is often modeled as fBm, a self-similar model classically referred to as monofractal, since a single scaling exponent H controls its entire dynamic and notably induces that $h(t) \equiv H$, $\forall t$. In other words, the (local) Hölder exponents are all identical and equal to the (global) self-similarity or Hurst exponent H. For fBm, it has been shown that: $\zeta_L(q) = qH$, hence $c_1 = H$ and $c_2 \equiv 0$ [11]. Therefore, deciding whether HR time series are mono- or multi-fractal (more precisely, whether HR time series should be modeled with fGn or with the increments of a multifractal process) amounts to testing whether $c_2^{\gamma} = 0$ for $\gamma > -h_m$. This can actually be tested precisely using the wavelet Leader based multifractal formalism and the non-parametric bootstrap procedures described in details in [11]. Although theoretically possible with the wavelet coefficient formalism, the corresponding tests show significantly less power in rejecting the null hypothesis $c_2^\gamma = 0$ with wavelet coefficients than with wavelet Leaders [11]. This is another major practical benefit of using wavelet Leaders for HRV analysis. As can be seen in Fig. 2, per partum fetal ECG data are found with clear negative c_2 both for healthy fetuses and fetuses suffering from acidosis.

LF/HF **Ratio.** As mentioned in the introduction, spectral analysis has been often used to characterize HRV. Let $\Gamma_X(\nu)$ denote the frequency spectrum of X (not to be confused with the multifractal spectrum $D(h)$) and $\hat\Gamma(\nu)$ any standard spectral estimator. Let $[\nu_m, \nu_I]$ and $[\nu_I, \nu_M]$ denote respectively the LF and HF frequency bands, the LH/HF ratio is defined as $\rho = \int_{\nu_m}^{\nu_I} \hat\gamma(\nu)d\nu / \int_{\nu_I}^{\nu_M} \hat\gamma(\nu)d\nu$. Let us assume that HR is well modeled by fGn with parameter H. Then its spectrum reads approximately $\Gamma_X(\nu) = C|\nu|^{-(2H-1)}$. In that case, one can show that: $\rho = (\nu_I^{2-2H} - \nu_m^{2-2H})/(\nu_M^{2-2H} - \nu_I^{2-2H})$, and hence that there is a clear and obvious relation between the Hurst parameter H and the LF/HF ratio: the larger H, the larger the energy at low frequency, and the larger ρ. Even when data are only grossly modeled by fGn, the estimated Hurst parameter $\hat H$ remains related with the estimated LF/HF ratio $\hat\rho$, as long as the linear regression yielding $\hat H$ is performed over a range of scales $j \in [j_1, j_2]$ that closely matches the union of the LF and HF frequency bands, i.e., $\nu \in [\nu_m, \nu_M]$. Notably, they will vary in a related manner, the larger $\hat H$, the larger $\hat\rho$. This is clearly evidenced on per partum fetal ECG data (correlation above 70%, cf. right plot on Fig. 3) and can be explained as follows. Let Ψ denote the Fourier transform of ψ. It has then be shown that [14]: $\mathbb{E}d_X(j,k)^2 = \int_{\mathbb{R}} \gamma_X(\nu)|\Psi(2^j\nu)|^2 d\nu$, which indicates that the variance of the wavelet coefficients at scale 2^j are actually related to the content of the frequency spectrum of the data around frequency $\nu_0/2^j$ (with ν_0 depending only on ψ). $S_d(2^j, q=2)$ can hence be read as a wavelet spectrum, i.e., an estimator of $\Gamma(\nu_0/2^j)$. As a consequence, the linear regression yielding $\hat H$ essentially results from a balance between the energies in the LF and HF bands and is hence naturally closely related to $\hat\rho$ (cf. Fig. 4 for illustration).

V. Conclusion

The wavelet Leader based multifractal analysis extends and enriches the spectral and fractal based analyses of HRV. In a recent study [9], it has been shown to discriminate healthy fetuses, for which the standard clinical practice had incorrectly lead to an acidosis diagnosis (and hence to an unnecessary operative delivery), from fetuses actually suffering from acidosis.

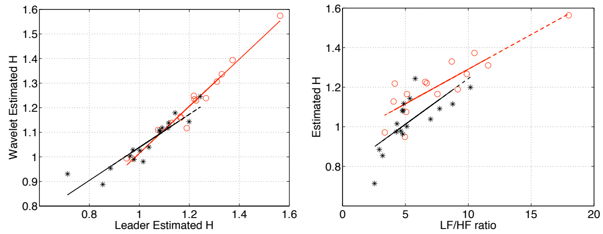

Fig. 3. **Hurst parameter and** LF/HF **ratio.** Left: wavelet Leaders versus wavelet coefficients based Hurst parameter estimation. Estimates are highly correlated. Right: estimated Hurst parameter versus estimated LF/HF ratio. Both estimates are highly correlated. Black (stars): healthy fetuses. Red (circles): Fetuses suffering from acidosis.

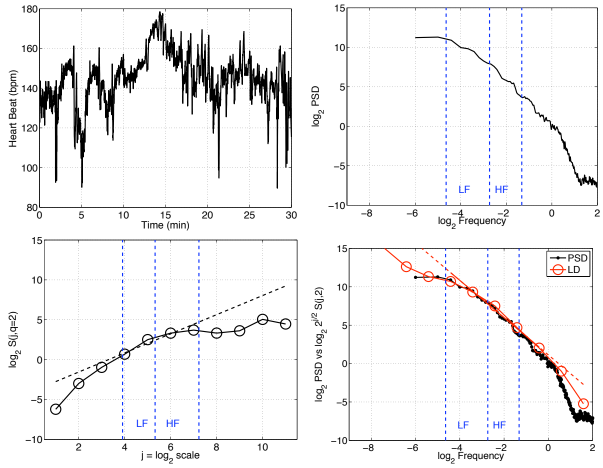

Fig. 4. **Wavelet vs. Frequency Spectrum.** Top Left, real FHR; Top Right, frequency spectrum; bottom left, wavelet spectrum ; bottom right, superimposition: $\hat H$ measures the balance between LF and HF energies.

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